Enrollment No. 04801172022

MID-TERM EXAMINATION

B. Tech. CSE-AI / ECE-AI/Reappear Semester: IV

(March, 2024) OFF LINE mode

Code: BAI 204	**Optimization Techniques and Decision Making**
Time : 1 ½Hours	**Maximum Marks : 30**
Note: Q. 1 is compulsory.	

Q1		**(2.5*4)**
	(a) Explain Optimization, and its applications in Engineering	
	(b) Depending on whether a particular point belongs to the acceptable or unacceptable region, it can be identified as one of the four types. Define and explain these types?	
	(c) Explain the merits and limitations of the graphical method?	
	(d) Explain decision variables, objective function, and constraints. Write an LPP to illustrate these terms?	
Q2	**(Attempt any Two Parts) UNIT-1**	**(5,5)**
	(a) A soft drink manufacturing company has 300 ml and 150 ml canned cola as its products with profit margin of Rs. 4 and Rs. 2 per unit respectively. Both the products have to undergo process in three types of machine. The following data indicates the time required on each machine and the available machine-hours per week. Formulate the optimization problem as an LPP to maximize the total profit considering the limited resources.	

Requirement	Cola 300 ml	Cola 150 ml	Available machine hours per week
Machine 1	3	2	300
Machine 2	2	4	480
Machine 3	5	7	560

	(b) Discuss briefly about multiple and unbounded optimization Linear Programming Problems. Use appropriate example to justify your answer.	
	(C) Explain in detail the steps involved in formulating problems as mathematical programming problems? Explain the process, including the translation of design objectives and constraints into mathematical formulations	
Q3	**(Attempt any Two Parts) UNIT-2**	**(5,5)**
	(a) An airplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine using a graphical method how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?	
	(b) Use Simplex method to solve the following LP problem Maximize Z = 50x + 60y subject to: 2x + y <=300 ; 3x + 4y <=509; 4x + 7y <=812; x, y >=0.	
	(C) Write the algorithm to solve LPP using the simplex method OR explain the Integer Programming Problems in Optimization with an example.	

[END-TERM EXAMINATION

B. TECH. CSE-AI / ECE-AI SEMESTER: IV

(May, 2024) OFFLINE MODE

Code: BAI 204	Optimization Techniques & Decision Making
Time : 3 Hours	Maximum Marks :60

Note: Q.1 is compulsory. Attempt one question each from the Units I, II, III & IV.

Q1		(5*4 =20)
	(a) Discuss two applications of optimization in engineering. Explain the steps and general structure of optimization algorithms.	
	(b) A logistics company must decide which routes to use for transporting goods from warehouses to retail stores. There are five possible routes, each with different costs and capacities. Develop an integer programming model to minimize transportation costs while ensuring that all demand is met and each route is used at most once.	
	(c) What is crossover in the context of genetic algorithms, and how does it combine genetic information from two parent solutions to generate offspring? Explain the applications of Ant Colony Optimization problems?	
	(d) Explain the steps in Decision Analysis and the applications of decision trees in optimization methods in engineering.	
	UNIT-I	
Q2	Explain classification of optimization problems based on the nature of the equations involved, and give an example for each type of optimization problem in engineering.	(10)
Q3	Define and explain role of constraints in defining feasible region. Illustrate how constraints are incorporated into the formulation of optimization problems using an example.	(10)
	UNIT-II	
Q4	a) A manufacturer produces two products A and B. Both products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of A requires 3 hours on both machines and each unit of B requires 2 hours on first machine and 1 hour on second machine. Each unit of A is sold at Rs 7 profit and that of B at a profit of Rs 4 per unit. Compute maximum profit using Graphical Method. b) Discuss the limitations of the Graphical Method compared to more advanced solution techniques like the Simplex Method.	(10)
Q5	a) Discuss conditions under which Simplex method terminates b) Discuss the possibility of an unbounded solution in linear programming and how Simplex method detects it?	(10)

UNIT-III

Q6 a) Explain how Genetic Algorithms are applied to solve Knapsack Problem, highlighting the encoding, fitness function, selection, crossover, and mutation (10)

b) Compare and contrast different selection mechanisms in Genetic Algorithms

Q7 a) Design Ant Colony Optimization algorithm for Traveling Salesman Problem, outlining the pheromone update rule, ant movement strategy, and construction of solutions. (10)

b) Implement a simple PSO algorithm to optimize a basic mathematical function, specifying the initialization, update rules, termination condition, and parameter settings

UNIT-IV

Q8 A glass factory that specializes in crystal is developing a substantial backlog and for this the firm's management is considering three courses of action: To arrange for subcontracting (S1), to begin overtime production (S2), and to construct new facilities (S3). The correct choice depends largely upon the future demand, which may be low, medium, or high. By consensus, management ranks the respective probabilities as 0.10, 0.50 and 0.40. A cost analysis reveals the effect upon the profits. This is shown in the table below: (10)

Demand	*Probability*	*Course of Action*		
		S_1 (*Subcontracting*)	S_2 (*Begin Overtime*)	S_3 (*Construct Facilities*)
Low (L)	0.10	10	− 20	− 150
Medium (M)	0.50	50	60	20
High (H)	0.40	50	100	200

Formulate this situation in the form of a decision tree and indicate the most preferred decision and its corresponding expected value.

Q9 The following matrix gives the payoff (in Rs) of different strategies (alternatives) S_1, S_2 and S_3 against the four states of nature (events) N_1, N_2, N_3 and N_4: (10)

Strategy	*State of Nature*			
	N_1	N_2	N_3	N_4
S_1	4,000	− 100	6,000	18,000
S_2	20,000	5,000	400	0
S_3	20,000	15,000	− 2,000	1,000

Indicate the decision taken under the following approaches: (i) Maximin criterion (ii) Maximax criterion (iii) Equal probability (iv) Regret criterion (v) Hurwicz criterion where the degree of optimism is 0.7

(Please write your Enrollment Number) **Enrollment No.__________________**

End-Term Examination
(CBCS)(SUBJECTIVE TYPE)(OffLine)
Course Name: B.TECH, Semester:5th
(December, 2024)

Subject Code: BAM 301	**Subject:** Optimization Techniques and Decision Making
Time :3 Hours	**Maximum Marks :60**

Note:Q1 is compulsory. Attempt one question each from the Units I, II, III & IV.

Q1		**(2.5*8 =20)**	**CO Mapping**
	a) State associated Issues with Iterative Methods.	**2.5**	**1**
	b) What are the complementary slackness conditions?	**2.5**	**1**
	c) Explain Curvature Analysis.	**2.5**	**2**
	d) State disadvantages of Newton's Method.	**2.5**	**2**
	e) What is Bias-Variance tradeoff?	**2.5**	**3**
	f) Differentiate overfitting and underfitting.	**2.5**	**3**
	g) What are the "positive ideal solution" and "negative ideal solution" in the TOPSIS method?	**2.5**	**4**
	h) What is the role of the kernel in Support Vector Machines?	**2.5**	**4**
UNIT I			**CO Mapping**
Q2	The manager of an oil refinery must decide on the optimal mix of two possible blending processors of which the input and output production runs as follows-	**(10)**	**1**

Process	Input	Input	Output	Output
	Crude A	Crude B	Gasoline X	Gasoline Y
1.	6	4	6	9
2.	5	6	5	5

The maximum amounts available of Crude A and B are 250 units and 200 units respectively. Market demand shows that atleast 150 units of Gasoline X & 130 units of Gasoline Y must be produced. The profits per production run from Process 1 & Process 2 are Rs.4 and Rs.5 respectively. Formulate the problem for maximizing the profit.

Q3	Minimize the quadratic function using Steepest Descent Method. $f(x)=x^2+4x+4$	**(10)**	**1**
UNIT II			**CO Mapping**
Q4	Explain the concept of lagrange variables. Find minimum value of the objective function $f(x,y)=x^2+y^2$ subject to: $x+y=1$, using Lagrange variables method.	**(10)**	**2**
Q5	What is the advantage of the Quasi Newton Method over Newton Method? Minimize the function using the Quasi Newton Method. $f(X)=x^2+2x+1$ starting from an initial guess of $x_0=0$ and Take $\alpha=1$.	**(10)**	**2**

<table>
<tr><td colspan="2">UNIT III</td><td></td><td>CO Mapping</td></tr>
<tr><td>Q6</td><td>Explain K Nearest Neighbors Algorithm with an example.</td><td>(10)</td><td>3</td></tr>
<tr><td>Q7</td><td>Define Prescriptive Analytics? What are its types and differentiate between them?</td><td>(10)</td><td>3</td></tr>
<tr><td colspan="2">UNIT IV</td><td></td><td>CO Mapping</td></tr>
<tr><td>Q8</td><td>Explain k-means algorithm. For the given data, compute two clusters using K-means algorithm for clustering where initial cluster centers are (1.0, 1.0) and (5.0, 7.0). Execute for two iterations.
<table>
<tr><td>Record No</td><td>R1</td><td>R2</td><td>R3</td><td>R4</td><td>R5</td><td>R6</td><td>R7</td></tr>
<tr><td>A</td><td>1</td><td>1.5</td><td>3</td><td>5</td><td>3.5</td><td>4.5</td><td>3.5</td></tr>
<tr><td>B</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>5</td><td>4.5</td></tr>
</table></td><td>(10)</td><td>4</td></tr>
<tr><td>Q9</td><td>(a) What is a Markov Decision Process (MDP), and why is it important in reinforcement learning?
(b) What is reinforcement learning? Write its applications.</td><td>(10)</td><td>4</td></tr>
</table>

Non-Linear Programming (NLP)

Optimization problems where either the objective function or the constraints (or both) are non-linear fall into the category of Non-Linear Programming.

1. First-Order and Second-Order Conditions in NLP

Consider a general unconstrained optimization problem:

$$\min f(x), \quad x \in \mathbb{R}^n$$

where $f(x)$ is continuously differentiable.

1.1 First-Order Necessary Condition (FONC)

- At a **local minimum** x^*:

  $$\nabla f(x^*) = 0$$

  That is, the **gradient (vector of first derivatives)** vanishes.
- Geometric interpretation:
  At the minimum, the tangent plane is flat → no slope in any direction.
- Note: This is a **necessary** condition, not sufficient — because it also holds at local maxima and saddle points.

1.2 Second-Order Conditions (SOC)

- Let $H(x) = \nabla^2 f(x)$ denote the **Hessian matrix** (matrix of second derivatives).

At a candidate point x^* where $\nabla f(x^*) = 0$:

- If $H(x^*)$ is **positive definite**, then x^* is a **strict local minimum**.
- If $H(x^*)$ is **positive semi-definite**, then x^* may be a local minimum (not strict).
- If $H(x^*)$ is **negative definite**, then x^* is a **strict local maximum**.
- If $H(x^*)$ is **indefinite** (some eigenvalues positive, some negative), then x^* is a **saddle point**.

1.3 Constrained NLP

- With constraints:

$$\min f(x), \quad g_i(x) \leq 0, \quad h_j(x) = 0$$

Karush-Kuhn-Tucker (KKT) conditions are the generalization of first-order conditions.

- KKT introduces **Lagrange multipliers** to handle constraints.
  (This belongs to later lectures, but good to keep in mind.)

2. Iterative Methods in NLP

- Analytical solutions (closed-form) are rare in NLP. Instead, we use **iterative methods**:

$$x_{k+1} = x_k + \alpha_k d_k$$

where:

- x_k: current iterate,
- d_k: search direction,
- α_k: step size (learning rate).

2.1 Gradient Descent (Steepest Descent)

- $d_k = -\nabla f(x_k)$.
- Moves in the direction of steepest decrease.
- Convergence can be **very slow** in ill-conditioned problems.

2.2 Newton's Method

- Uses second-order (Hessian) information:

$$d_k = -H(x_k)^{-1}\nabla f(x_k)$$

- Quadratic convergence near the optimum (very fast).
- Issues: computing Hessian is costly, may not be positive definite.

2.3 Quasi-Newton Methods

- Approximate Hessian instead of computing exactly.
- Example: **BFGS algorithm** (widely used in optimization libraries).

2.4 Issues with Iterative Methods

1. **Convergence to Local vs. Global Minimum**
   - Non-convex functions may trap algorithms in local minima.
2. **Choice of Initial Guess x_0**
   - Strongly affects performance and final result.
3. **Step Size (Learning Rate) Selection**
   - Too large → divergence.
   - Too small → very slow convergence.
4. **Ill-conditioning**
   - If level curves are elongated (like a narrow valley), gradient descent zig-zags and converges slowly.
   - Preconditioning or using Newton-type methods helps.

3. Line Search Methods

Line search methods focus on choosing **optimal step size** α_k in each iteration.

General iteration:

$$x_{k+1} = x_k + \alpha_k d_k$$

3.1 Stationarity of Limit Points in Steepest Descent

- If:
  1. The objective function $f(x)$ is continuously differentiable.
  2. Step sizes α_k are chosen by exact or inexact line search (satisfying descent conditions).

Then:

- Any **accumulation point** of the sequence $\{x_k\}$ generated by steepest descent is a **stationary point** (i.e., satisfies $\nabla f(x^*) = 0$).
- In practice: steepest descent may take many iterations to reach acceptable accuracy.

3.2 Successive Step-Size Reduction Algorithms

- Instead of fixing step size, start large and **reduce until progress is adequate**.
- Common rules:

(a) Backtracking Line Search

1. Choose initial $\alpha = 1$.
2. While:

$$f(x_k + \alpha d_k) > f(x_k) + c\alpha \nabla f(x_k)^T d_k$$

   (Armijo condition not satisfied), reduce α (e.g., $\alpha \leftarrow \beta\alpha$, with $\beta \in (0, 1)$).
3. Accept the reduced α.

- Guarantees sufficient decrease in each step.

(b) Wolfe Conditions

- Stronger conditions to balance **sufficient decrease** and **curvature condition**.
- Ensures both progress and stability.

Remarks on Line Search

- Backtracking is simple and widely used.
- Exact line search (finding optimal α\alphaα analytically) is rare in practice because it may require solving another optimization problem.
- In machine learning and data analytics, **fixed learning rate with occasional reduction** is common (a practical variant of step-size reduction).

First-order condition: Gradient must vanish at local optima.

Second-order condition: Hessian determines nature of stationary point (min/max/saddle).

Iterative methods: Gradient descent, Newton's, Quasi-Newton are main techniques.

Issues: Step-size selection, local minima, ill-conditioning, sensitivity to starting point.

Line search: Essential for efficient convergence; backtracking and Wolfe conditions widely used.

Steepest descent: Limit points are stationary, but convergence may be slow.

First and Second Order Conditions

Concept Recap

- At local optimum, gradient must vanish: $\nabla f(x^*) = 0$.
- Nature of point determined by Hessian matrix.

Problem 1:

Find the stationary points of

$$f(x) = x^2 - 4x + 5$$

and determine their nature.

Solution:

- Gradient:
  $\frac{df}{dx} = 2x - 4$.
- Set = 0: $2x - 4 = 0 \Rightarrow x = 2$.
- Hessian: $f''(x) = 2 > 0$.
- **Conclusion:** Minimum at $x = 2$, value = $f(2) = 1$.

Problem 2:

For the function

$$f(x, y) = x^2 + y^2 - 2x - 4y + 5$$

find the minimum point.

Solution:

- Gradient:
  $\nabla f(x, y) = (2x - 2,\ 2y - 4)$.
- Stationary point: $2x - 2 = 0 \Rightarrow x = 1$, $2y - 4 = 0 \Rightarrow y = 2$.
- Hessian:
  $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (positive definite).
- **Conclusion:** Minimum at $(1, 2)$, value = $f(1, 2) = 0$.

Problem 3:

Check the nature of stationary point for:

$$f(x, y) = x^2 - y^2$$

Solution:

- Gradient: $(2x, -2y)$.
- Stationary point: $(0, 0)$.
- Hessian:
  $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$.
- Eigenvalues: $+2, -2$ → indefinite.
- **Conclusion:** $(0, 0)$ is a **saddle point**.

Remarks:

First-order → stationarity.

Second-order → min/max/saddle classification.

Iterative Methods in NLP

Concept

- Use iterative updates:

$$x_{k+1} = x_k + \alpha_k d_k$$

- Gradient descent: $d_k = -\nabla f(x_k)$.
- Newton's method: $d_k = -H^{-1}(x_k)\nabla f(x_k)$.

Problem 1: Gradient Descent in 1D

Minimize

$$f(x) = (x - 3)^2$$

using gradient descent, starting from $x_0 = 0$, with $\alpha = 0.1$.

Solution:

- Gradient: $\nabla f(x) = 2(x - 3)$.
- Iteration: $x_{k+1} = x_k - 0.1(2(x_k - 3))$.
- Step 1: $x_1 = 0 - 0.1(-6) = 0.6$.
- Step 2: $x_2 = 0.6 - 0.1(-4.8) = 1.08$.
- Step 3: $x_3 = 1.08 - 0.1(-3.84) = 1.464$.
- Converges towards $x = 3$.

Problem 2: Newton's Method in 1D

Minimize

$$f(x) = x^2 + 4x + 4$$

(start from $x_0 = 2$).

Solution:

- Gradient: $f'(x) = 2x + 4$.
- Hessian: $f''(x) = 2$.
- Newton update: $x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$.
- Step 1: $x_1 = 2 - (8/2) = -2$.
- Gradient at -2 is 0 → convergence.
- **Minimum at $x = -2$.**

Problem 3: Gradient Descent in 2D

Minimize

$$f(x, y) = (x - 1)^2 + (y - 2)^2$$

starting from $(0, 0)$, $\alpha = 0.1$.

Solution:

- Gradient: $\nabla f(x, y) = (2(x - 1), 2(y - 2))$.
- Start: $(0, 0)$. Gradient = $(-2, -4)$.
- Update: $(x, y) = (0, 0) - 0.1(-2, -4) = (0.2, 0.4)$.
- Next step: Gradient = $(-1.6, -3.2)$.
- Update: $(0.36, 0.72)$.
- Converges towards $(1, 2)$.

Remarks:

- Gradient descent = slow but general.
- Newton = fast but needs Hessian.

OPTIMIZATION TECHNIQUES for DECISION MAKING

UNIT-1

Dr Ravi Prakash Shahi

Prof & SME in Analytics, AI, Machine Learning, Data Science, IoT, Software Engineering, Security, Computer Vision, Big Data

ravishahi71@gmail.com

Need of Optimization Techniques

- The ever-increasing demand on engineers to lower production costs to withstand global competition has prompted engineers to look for rigorous methods of decision making, such as optimization methods, to design and produce products and systems both economically and efficiently.
- Optimization techniques, having reached a degree of maturity in recent years, are being used in a wide spectrum of industries, including aerospace, automotive, chemical, electrical, construction, and manufacturing industries.
- With rapidly advancing computer technology, computers are becoming more powerful, and correspondingly, the size and the complexity of the problems that can be solved using optimization techniques are also increasing.

INTRODUCTION

- **Optimization is the act of obtaining the best result under given circumstances**. In design, construction, and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages.
- **The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit**. Since the effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables, ***optimization*** can be defined as the process of finding the conditions that give the maximum or minimum value of a function.
- There is **no single method available for solving all optimization problems** efficiently. Hence a number of optimization methods have been developed for solving different types of optimization problems. The optimum seeking methods are also known as ***mathematical programming techniques*** and are generally studied as a part of **Operations Research**.

SCOPE OF OT / OR

O.R. has a wide scope in everyday life as it provides better solutions to various decision-making problems with great speed and competence. It finds applications in a wide range of areas including defence operations, planning, agriculture, industry (finance, marketing, personal management, production management), research and development. We now describe the applications briefly.

Areas where Optimization is applied are:

1. **Science**
2. **Engineering**
3. **Management**
4. **Finance**
5. **Business**

In Planning for Economic Development

Careful planning is necessary for economic development of any country. Operations Research is used to frame future economic and social policies.

In Agriculture

Agricultural output needs to be increased due to increasing needs for adequate quantity and quality of food for our increasing population. But there are a number of restrictions under which agricultural production is studied. Problems of agricultural production under various restrictions such as optimum allocation of land to various crops in accordance with the climatic conditions, optimum distribution of water from various resources for irrigation purposes can easily be solved by application of Operations Research techniques.

In Industry

Now-a-days, due to complexities of operations and huge sizes of industries, important decisions regarding various sections of the organisation, e.g., planning, procurement, marketing, finance, etc. have to be taken division wise. For example, the production department needs to minimise the cost of production, but maximise output; the finance department needs to optimise capital investment; the personnel department needs to appoint competent work force at minimum cost. Each department has to plan its own objectives which may be in conflict with the objectives of other departments and may not conform to the overall objectives of the organisation. For example, the sales department of an organisation may want to keep sufficient stocks in the inventory, whereas the finance department may want to have minimum investment. In that case, both departments would be in conflict with each other. The applications of O.R. techniques to such situations help in overcoming this difficulty by evolving an optimal strategy and serving efficiently the interest of the organisation as a whole.

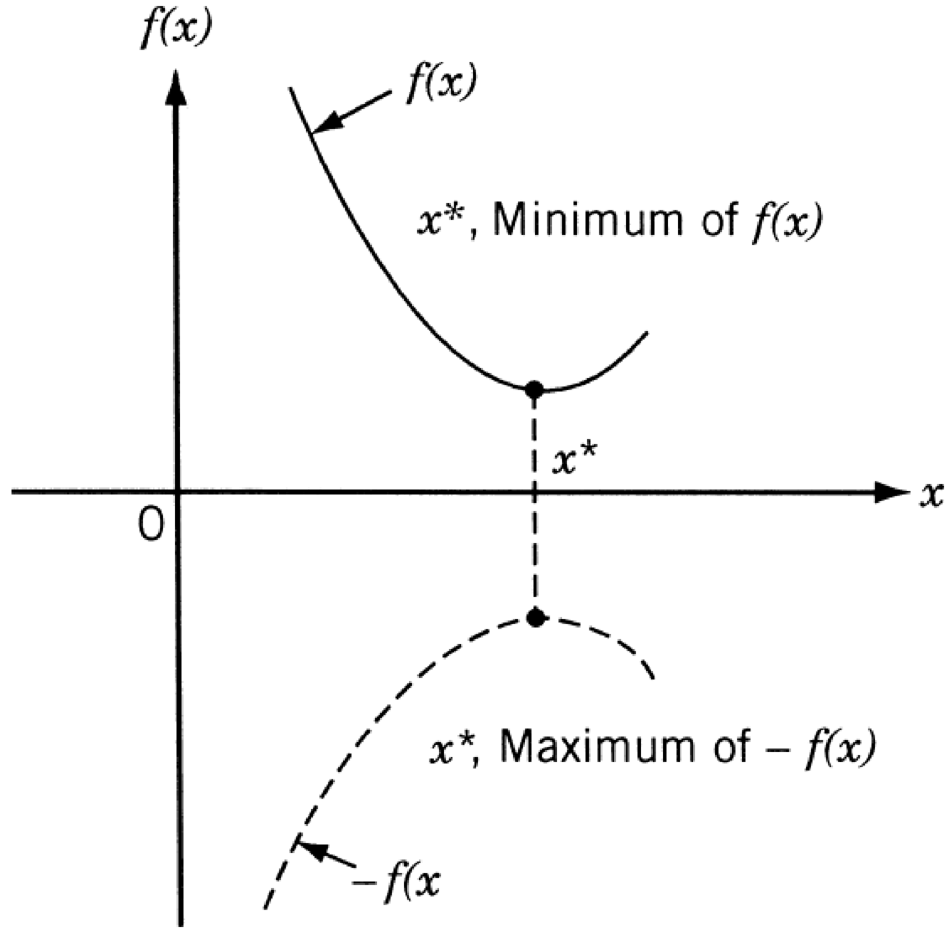

Figure 1.1 Minimum of $f(x)$ is same as maximum of $-f(x)$.

ENGINEERING APPLICATIONS OF OPTIMIZATION

Some typical applications from different engineering disciplines include :

1. Design of aircraft and aerospace structures for minimum weight

2. Finding the optimal trajectories of space vehicles

3. Minimum-weight design of structures for earthquake, wind, other types of random loading

4. Selection of machining conditions in metal-cutting for minimum production cost

5. Shortest route taken by a salesperson visiting various cities during one tour

6. Optimal production planning, controlling, and scheduling

7. Design of optimum pipeline networks for process industries

8. Selection of a site for an industry

9. Planning of maintenance and replacement of equipment to reduce operating costs

10. Inventory control management

11. Allocation of resources among several activities to maximize the profit.

12. Controlling the waiting and idle times in production lines to reduce the costs

13. Optimum design of control systems in design of electronic appliances.

GENERAL OPTIMIZATION PROBLEM

Minimize (Maximize) $f(X)$ → objective function

where $f: R^n \rightarrow R$, $X = (x_1, x_2, x_3 \ldots x_n)$ → decision values parameters

s.t. $X \in S \subseteq R^n$ where S is defined by

$g_k(X) \geq 0$, $k=1,2, \ldots m$ → inequality constraints

$h_j(X) = 0$, $j=1,2, \ldots l$ → equality constraints

$a_i \leq x_i \leq b_i$ → lower & upper bounds

COMPONENTS OF AN OPTIMIZATION MODEL

Decision variables

Objective function

Constraints

CLASSIFICATION

Linear Programming Problems (LPP)

Nonlinear Programming Problems (NLPP)

Unconstrained Optimization Problems

Constrained Optimization Problems

LINEAR PROGRAMMING PROBLEM

A **Linear Programming Problem** is an optimization problem where the **objective function** and all the **constraints** are linear in nature.

Objective function → A linear function of decision variables to be **maximized or minimized** (e.g., profit, cost, revenue).

Constraints → Linear equations or inequalities representing resource limitations.

Decision variables → Unknowns to be determined (e.g., number of products, allocation of resources).

The General form of LPP is as follows:

Maximize or Minimize:

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

Subject to constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$

...

$$x_i \geq 0 \quad (i = 1, 2, ..., n)$$

Where:

- Z = Objective function
- $x_1, x_2, ..., x_n$ = Decision variables
- c_i = Coefficients of objective function
- a_{ij} = Coefficients of constraints
- b_j = Resource availability

Problem:

A company produces two products A and B.

Profit from A = ₹3 per unit, profit from B = ₹2 per unit.

Constraints:

- Each unit of A requires 1 hour on machine 1 and 2 hours on machine 2.
- Each unit of B requires 1 hour on machine 1 and 1 hour on machine 2.
- Machine 1 is available for 8 hours, Machine 2 is available for 10 hours.

Formulation:

$$\text{Maximize } Z = 3x_1 + 2x_2$$

Subject to:

$$x_1 + x_2 \leq 8 \quad (Machine1)$$

$$2x_1 + x_2 \leq 10 \quad (Machine2)$$

$$x_1, x_2 \geq 0$$

Solution (Graphical):

- Plot the inequalities, find feasible region.
- Corner points = (0,0), (5,0), (2,6), (0,8).
- Evaluate Z:
  - (0,0) → 0
  - (5,0) → 15
  - (2,6) → 18
  - (0,8) → 16

Optimal Solution: Produce 2 units of A and 6 units of B → **Maximum Profit = ₹18**

NON-LINEAR PROGRAMMING

Non-Linear Programming involves optimization of a **non-linear objective function** and/or **non-linear constraints**.

General Formulation

$$\text{Maximize/Minimize } Z = f(x_1, x_2, \ldots, x_n)$$

Subject to:

$$g_j(x_1, x_2, \ldots, x_n) \leq b_j \quad \text{for } j = 1, 2, \ldots, m$$

$$x_i \geq 0$$

Where:

- $f(x)$ = non-linear objective function
- $g_j(x)$ = non-linear constraint functions

Methods of Solving NLPP

1. **Lagrange Multiplier Method** (for equality constraints)
2. **Kuhn-Tucker (KKT) Conditions** (for inequality constraints)
3. **Gradient Descent / Newton's Method** (iterative numerical methods)

CLASSIFICATION BASED ON TYPE OF DECISION VARIABLES

Dynamic Programming

Geometric Programming

Integer Programming

Quadratic Programming

Separable Programming

STATEMENT OF AN OPTIMIZATION PROBLEM

An optimization or a mathematical programming problem can be stated as follows.

$$\text{Find } \mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \text{ which minimizes } f(\mathbf{X})$$

subject to the constraints

$$\begin{aligned} g_j(\mathbf{X}) &\leq 0, \qquad j = 1, 2, \ldots, m \\ l_j(\mathbf{X}) &= 0, \qquad j = 1, 2, \ldots, p \end{aligned} \tag{1.1}$$

where $\mathbf{X}$ is an n-dimensional vector called the *design vector*, $f(\mathbf{X})$ is termed the *objective function*, and $g_j(\mathbf{X})$ and $l_j(\mathbf{X})$ are known as *inequality* and *equality* constraints, respectively. The number of variables n and the number of constraints m and/or p need not be related in any way. The problem stated in Eq. (1.1) is called a *constrained optimization problem*.† Some optimization problems do not involve any constraints and can be stated as

OPTIMIZATION PROBLEM(2)

$$\text{Find } \mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \text{ which minimizes } f(\mathbf{X})$$

Such problems are called *unconstrained optimization problems*.

CONSTRAINTS IN OPTIMIZATION

- In many practical problems, the design variables cannot be chosen arbitrarily; rather, they have to satisfy certain specified functional and other requirements.
- The restrictions that must be satisfied to produce an acceptable design are collectively called *design constraints*.
- Constraints that represent limitations on the behavior or performance of the system are termed *behavior* or *functional constraints*.
- Constraints that represent physical limitations on design variables, such as availability, fabricability, and transportability, are known as *geometric* or *side constraints*.

CONSTRAINT SURFACE

For illustration, consider an optimization problem with only inequality constraints $g_j(\mathbf{X}) \leq 0$. The set of values of $\mathbf{X}$ that satisfy the equation $g_j(\mathbf{X}) = 0$ forms a hypersurface in the design space and is called a *constraint surface*. Note that this is an $(n-1)$-dimensional subspace, where n is the number of design variables. The constraint surface divides the design space into two regions: one in which $g_j(\mathbf{X}) < 0$ and the other in which $g_j(\mathbf{X}) > 0$. Thus the points lying on the hypersurface will satisfy the constraint $g_j(\mathbf{X})$ critically, whereas the points lying in the region where $g_j(\mathbf{X}) > 0$ are infeasible or unacceptable, and the points lying in the region where $g_j(\mathbf{X}) < 0$ are feasible or acceptable. The collection of all the constraint surfaces $g_j(\mathbf{X}) = 0,\ j = 1, 2, \ldots, m$, which separates the acceptable region is called the *composite constraint surface*.

CONSTRAINT SURFACE(2)

- Figure 1.4 shows a hypothetical two-dimensional design space where the infeasible region is indicated by hatched lines.
- A design point that lies on one or more than one constraint surface is called a *bound point* , and the associated constraint is called an *active constraint* .
- Design points that do not lie on any constraint surface are known as *free points*.
- Depending on whether a particular design point belongs to the acceptable or unacceptable region, it can be identified as one of the following four types:

1. Free and acceptable point

2. Free and unacceptable point

3. Bound and acceptable point

4. Bound and unacceptable point

All four types of points are shown in Fig. 1.4.

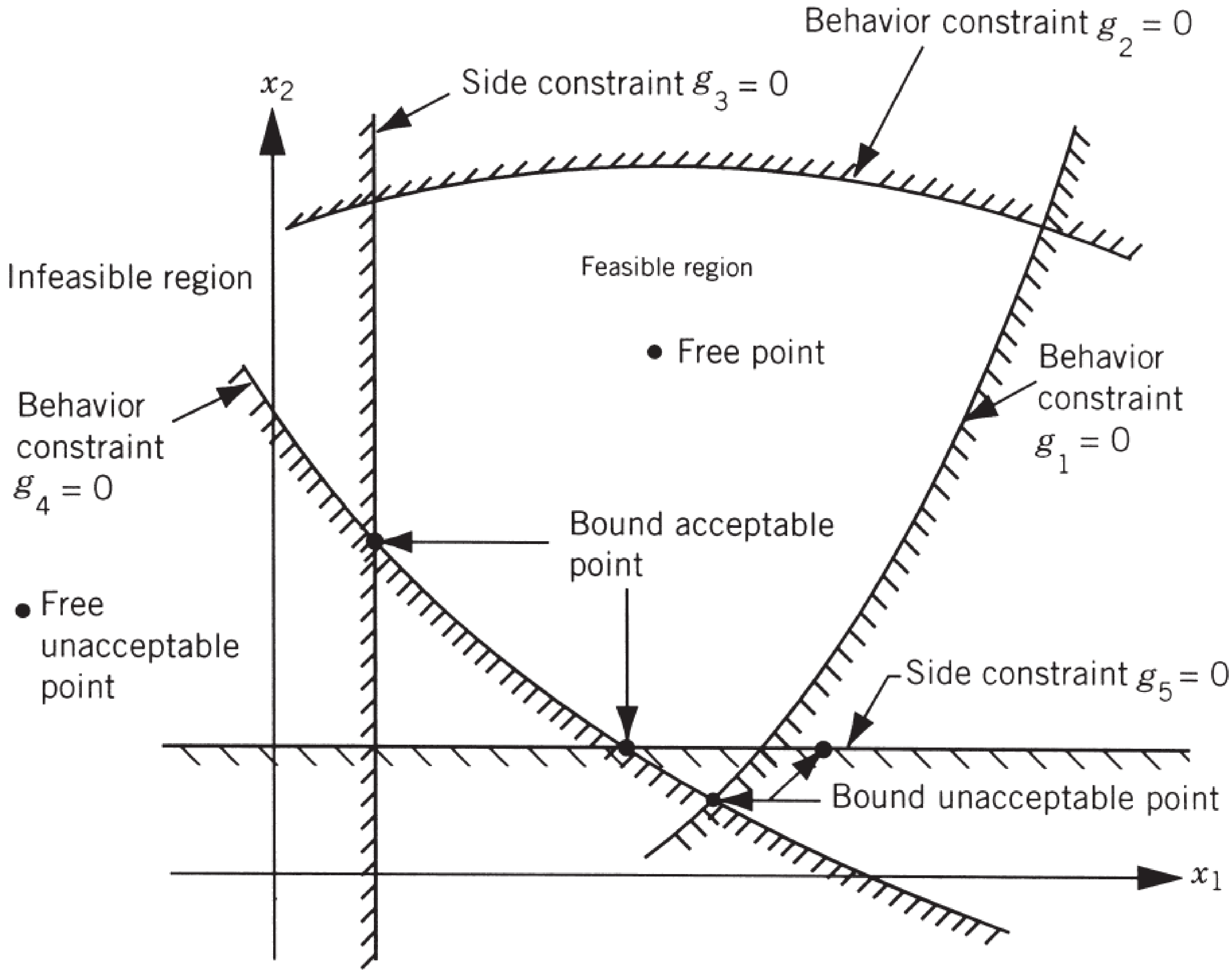

Figure 1.4 Constraint surfaces in a hypothetical two-dimensional design space.

CONSTRAINT SURFACE(2)

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4. Bound and unacceptable point

All four types of points are shown in Fig. 1.4.

OBJECTIVE FUNCTION

- In general, there will be more than one acceptable design, and the purpose of optimization is to choose the best one of the many acceptable designs available.
- Thus a criterion has to be chosen for comparing the different alternative acceptable designs and for selecting the best one. The criterion with respect to which the design is optimized, when expressed as a function of the design variables, is known as the *criterion* or *merit* or *objective function*.
- The choice of objective function is governed by the nature of problem.
- The objective function for minimization is generally taken as weight in aircraft and aerospace structural design problems. In civil engineering structural designs, the objective is usually taken as the minimization of cost.
- In some situations, there may be more than one criterion to be satisfied simultaneously.
- For example, a gear pair may have to be designed for minimum weight and maximum efficiency while transmitting a specified horsepower.
- An optimization problem involving multiple objective functions is known as a *multiobjective programming problem*.

PROBLEM 1

1. Reddy Mikks produces both interior and exterior paints from two raw materials M1 and M2. Following table provides the basic data of problem:
2. A market survey indicates that the daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton.
3. Also, the maximum daily demand for interior paint is 2 tons.
4. Reddy Mikks wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit?
5. Find the optimal solution to this problem

Tons of raw material per ton			
	Exterior paint	Interior paint	Max daily availability
Raw material, M1	6	4	24
Raw material, M2	1	2	6
profit per ton ($1000)	5	4	

SOLUTION

- The LP model, has three basic components:

1. Decision variables that we seek to determine.

2. Objective that we need to optimize (maximize or minimize).

3. Constraints that solution must satisfy.

The **decision variables** of model are defined as

$x1$= tons produced daily of exterior paint

$x2$= tons produced daily of interior paint

Objective function : The company wants to maximize (i.e., increase as much as possible) the total daily profit of both paints.

Given that the profits per ton of exterior and interior paints are 5 and 4 (thousand) dollars, respectively, it follows that

Total profit from exterior paint = $5x_1$

Total profit from interior paint = $4x_2$

Let Z denote the total daily profit, then objective of the company is

Maximize $\mathbf{Z = 5x_1 + 4x_2}$

CONSTRAINTS : To construct the constraints that restrict raw material usage and product demand.

The raw material restrictions are expressed verbally as

(usage of a raw material by both paints) ≤ (max. raw material availability)

Daily usage of raw material M1 is 6 tons of exterior paint and 4 tons of interior paint.

So, Usage of raw material M1 by exterior paint = $6x_1$ tons/day ,

Usage of raw material M1 by interior paint = $4x_2$ tons/day

Therefore, Usage of M1 by both paints = $6x_1 + 4x_2$ tons/day

By similar logic,

Usage of M2 by both paints = $1x_1 + 2x_2$ tons/day

Because the daily availabilities of raw materials M1 and M2 are limited to 24 and 6 tons respectively, the associated restrictions are given as

$$6x_1 + 4x_2 \le 24 \quad \textbf{(Raw material M1)}$$

$$x_1 + 2x_2 \le 6 \quad \textbf{(Raw material M2)}$$

Demand restriction stipulates that the excess of daily production of interior over exterior paint, x_2-x_1, should not exceed 1 ton, which translates to $x_2 - x_1 \le 1$ **(market limit)**

Second demand restriction- the max. daily demand of interior paint is limited to 2 tons, which translates to $x_2 \le 2$ **(demand limit)**.

An implicit (or "understood- to –be") restriction is that variables x_1 and x_2 cannot assume negative values. So $x_1 \ge 0$, $x_2 \ge 0$ **(nonnegativity restrictions)**

The complete Reddy Mikks Model is

$$\textbf{Maximize} \quad \mathbf{Z = 5x_1 + 4x_2}$$

Subject to:

$$6x_1 + 4x_2 \leq 24 \qquad (1)$$

$$x_1 + 2x_2 \leq 6 \qquad (2)$$

$$x_2 - x_1 \leq 1 \qquad (3)$$

$$x_2 \leq 2 \qquad (4)$$

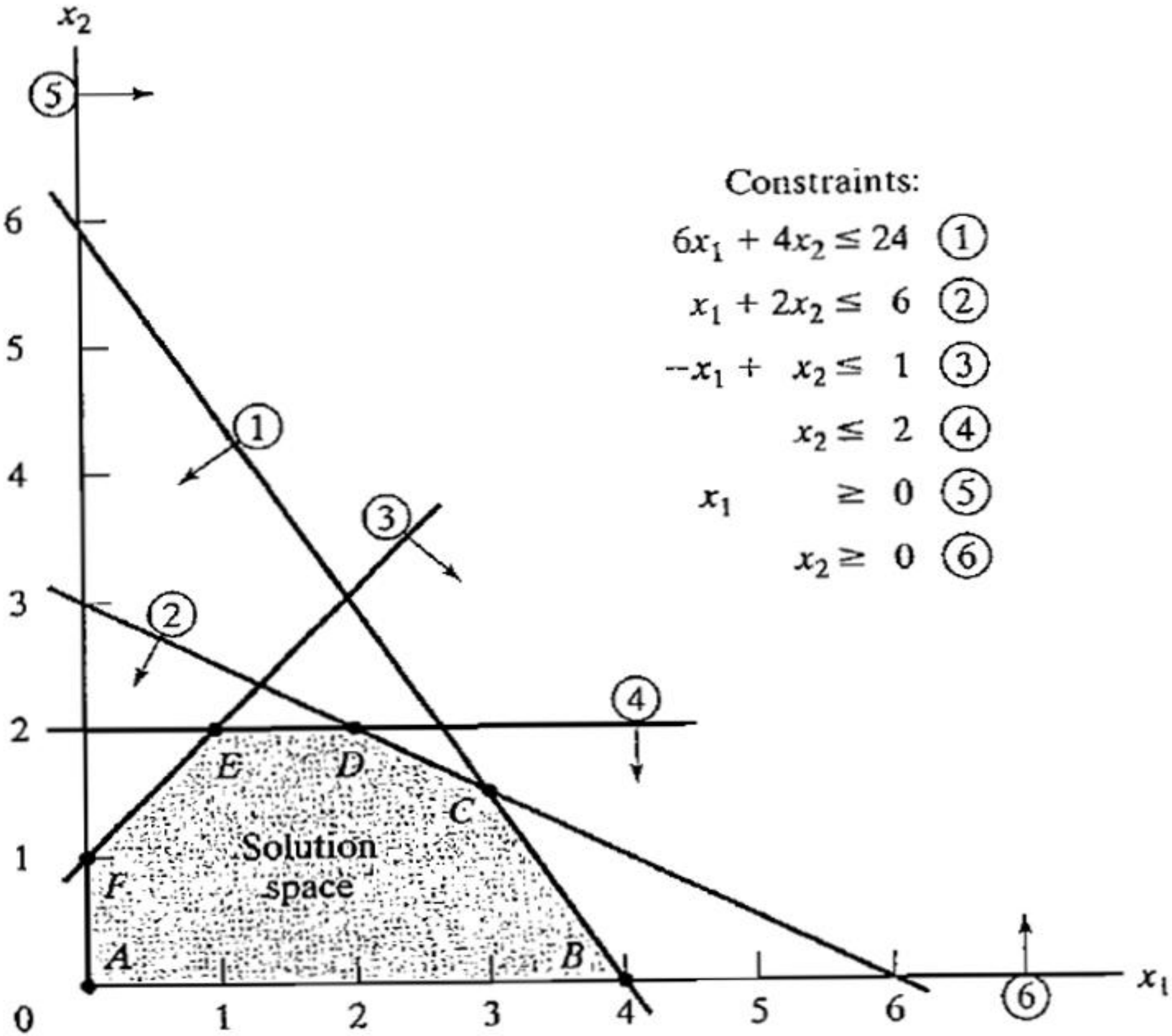
$$x_1,\ x_2 \geq 0 \qquad (5)$$

Any values of x_1 and x_2 that satisfy all five constraints constitute a feasible solution. Otherwise, the solution is infeasible.

For example, the solution, $x_1 = 3$ tons per day and $x_2 = 1$ ton per day, is feasible because it does not violate any of the constraints.

The goal of the problem is to find the best feasible solution, or the optimum, that maximizes the total profit. Before we can do that, we need to know how many feasible solutions the Reddy Mikks problem has. The answer, as we will see from the the graphical solution is an infinite number, which makes it impossible the solve the problem by enumeration.

GRAPHICAL SOLUTION OF LPP



The graphical procedure includes 2 steps:

1. Determination of the feasible solution space.

2. Determination of the optimum solution from among all the feasible points in the solution space

Determination of Optimum Solution

- The **feasible space** in figure is delineated by the line segments joining the points A, B, C, D, E, and F. **Any point within or on the boundary of the space ABCDEF is feasible.** Because the feasible space ABCDEF consists of an infinite number of points, we need a systematic procedure to identify the optimum solution.
- **An important characteristic of the optimum LP solution is that it is always associated with a corner point of the solution space** (where two lines intersect). This is true even if the objective function happens to be parallel to a constraint. For example, if the objective function is $z = 6x_1 + 4x_2$, which is parallel to constraint 1, we can always say that the optimum occurs at either corner point B or comer point C.
- The observation that the LP optimum is always associated with a corner point means that **the optimum solution can be found simply by enumerating all the corner points** as the following table shows:

Determination of Optimum Solution(2)

- As the number of constraints and variables increases, the number of corner points also increases.
- Nevertheless, the idea shows that, from the standpoint of determining the LP optimum, the solution space ABCDEF with its infinite number of solutions can, in fact, be replaced with a finite number of promising solution points-namely, the corner points, A, B, C, D, E, and F.
- The optimum solution is $x_1 = 3$ and $x_2 = 1.5$ with $Z = (5 * 3) + (4 * 1.5) = 21$. **This calls for a daily product mix of 3 tons of exterior paint and 1.5 tons of interior paint. The associated daily profit is $21,000.**

Corner point	(x_1, x_2)	z
A	(0, 0)	0
B	(4, 0)	20
C	**(3, 1.5)**	**21 (OPTIMUM)**
D	(2, 2)	18
E	(1, 2)	13
F	(0, 1)	4

PRACTICE PROBLEM 1

A company produces two products, A and B. The sales volume for A is at least 80% of the total sales of both A and B.

However, the company cannot sell more than 100 units of A per day. Both products use one raw material, of which the maximum daily availability is 240 lb. The usage rates of the raw material are 2 lb per unit of A, and 4 lb per unit of B.

The profit units for A and B are \$20 and \$50, respectively.

Formulate the LPP for the same.

SOLUTION TO PRACTICE PROBLEM

If we let A = units of product A and B =units of product B, then we'll

$$\text{maximize } z = 20A + 50B$$

subject to

$$\begin{aligned} 2A + 4B &\leq 240 && \text{(raw material availability)} \\ A &\leq 100 && \text{(sales limit of A)} \\ -0.2A + 0.8B &\leq 0 && \text{(sales of A at least 80\%)} \\ A, B &\geq 0 && \text{(sign restrictions)} \end{aligned}$$

The sales volume for A is at least 80% of the total sales of both A and B. So, $A \geq 0.8\,(A + B)$ which gives us $0 \geq -0.2\,A + 0.8\,B$

PRACTICE PROBLEM 2

A company produces two types of items P and Q that require gold and silver.

Each unit of type P requires 4g silver and 1g gold while that of type Q requires 1g silver and 3g gold. The company can produce 8g silver and 9g gold.

If each unit of type P brings a profit of `44 and that of type Q `55, determine the number of units of each type that the company should produce to maximise the profit. What is the maximum profit?

Degeneracy in Linear Programming

- Degeneracy in LPP occurs when a basic feasible solution has one or more basic variables equal to zero, which can lead to the Simplex method cycling or failing to reach an optimal solution in a finite number of steps.
- It manifests when there is a tie in the minimum ratio test while selecting an outgoing variable in the Simplex table (**Ties in Replacement Ratios)**. Also degeneracy can arise if a**t least one of the constraints has a zero value on the right-hand side (Zero-Valued Basic Variables).** Degeneracy can also arise from **redundant constraints** that overly restrict the solution space.
- In the graphical method for LPPs, degeneracy occurs when **a basic feasible solution has at least one basic variable equal to zero,** , or when **multiple constraints intersect at the same corner point**, resulting in a degenerate corner point. Graphically, it means that more than one constraint line passes through a single corner of the feasible region.
- **Degeneracy can lead to cycling or difficulty in identifying unique optimal solutions.**

Consequences of Degeneracy

1. The primary consequences of degeneracy in LPP are the potential for the simplex algorithm to experience **cycling** (repeatedly visiting the same set of basic feasible solutions without improving the objective function) or **stalling** (failing to make progress toward the optimal solution).

2. **Increased iterations**: Even if it doesn't lead to cycling, degeneracy can significantly increase the number of iterations required for the simplex algorithm to converge to the optimal solution.

3. Degeneracy typically occurs when a pivot operation results in **no improvement to the objective function value**, often due to a tie in determining the outgoing variable in a simplex tableau, making it difficult (potentially requiring more iterations) to reach the optimal solution.

- Degeneracy does not affect the **existence of an optimal solution**.
- **Feasibility:** Degeneracy does not make a basic feasible solution infeasible.

Duality in LPP

- In LPPs, **duality is the concept that every LPP, called the Primal, has an associated LPP called the Dual**, derived from the same data and sharing the same solution.

Relationship between Primal and Dual :

- **Variables and Constraints:** The variables in the primal problem become the constraints in the dual problem, and vice-versa.
- **Objective Functions:** The objective function coefficients of primal become the RHS constants of the dual's constraints, and the RHS constants of the primal constraints become the objective function coefficients of the dual.
- **Optimization Direction:** If the primal is a maximization problem, its dual will be a minimization problem, and vice versa.

Duality Theorem: This theorem states that if the primal problem has an optimal solution, the dual problem also has an optimal solution, and the optimal values of their objective functions are equal.

Observations in Duality

- The number of constraints in the primal problem is equal to the number of variables in the dual problem.
- Similarly, the number of variables in the primal problem corresponds to the number of constraints in the dual problem.
- When primal is in maximization form, the dual is in minimization form.
- The coefficients in the objective function of the primal problem become the right-hand side(RHS) of the constraints in the dual problem.
- The right-hand side of the primal problem becomes the coefficients in the objective function of the dual problem.
- The coefficients of the variables in the constraints of the primal problem are transposed to form the coefficients of the variables in the constraints of the dual problem.

Primal-Dual Relationship

PRIMAL	CONVERSION	DUAL
Maximization Problem	↔	Minimization Problem
Minimization Problem	↔	Maximization Problem
Objective Coefficients	↔	Right Hand Side (RHS) values
Right Hand Side (RHS) values	↔	Objective Coefficients
Number of Variables	↔	Number of Constraints
Number of Constraints	↔	Number of Variables
Variables are in terms of X_n	↔	Variables are in terms of Y_n

Primal Problem (LPP)

$$
\begin{aligned}
\text{Maximize} \quad & Z = 3x_1 + 4x_2 \\
\text{subject to} \quad & \frac{1}{2}x_1 + 2x_2 \leq 30 \\
& 3x_1 + x_2 \leq 25 \\
& x_1, x_2 \geq 0.
\end{aligned}
$$

Dual LPP

$$
\begin{aligned}
\text{Minimize} \quad & Z = 30y_1 + 25y_2 \\
\text{subject to} \quad & \frac{1}{2}y_1 + 3y_2 \geq 3 \\
& 2y_1 + y_2 \geq 4 \\
& y_1, y_2 \geq 0
\end{aligned}
$$

Primal-Dual Relationship

Normal Primal Problem	Normal Dual Problem
Maximize $Z = \mathbf{c}^\top \mathbf{x}$	Minimize $W = \mathbf{b}^\top \mathbf{y}$
subject to $\mathbf{A}\mathbf{x} \leq \mathbf{b}$	subject to $\mathbf{A}^\top \mathbf{y} \geq \mathbf{c}$
$\mathbf{x} \geq \mathbf{0}$	$\mathbf{y} \geq \mathbf{0}$

$$
\begin{aligned}
\text{Minimize} \quad & Z = 2x_1 + 3x_2 + 4x_3 \\
\text{subject to} \quad & x_1 + 2x_2 + x_3 \geq 5 \\
& 3x_1 + x_2 + 2x_3 \geq 8 \\
& -3x_1 - x_2 - 2x_3 \geq -8 \\
& -x_1 - 4x_2 - 3x_3 \geq -10 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

$$
\begin{aligned}
\text{Minimize} \quad & Z = 2x_1 + 3x_2 + 4x_3 \\
\text{subject to} \quad & x_1y_1 + 2x_2y_1 + x_3y_1 \geq 5y_1 \\
& 3x_1y_2 + x_2y_2 + 2x_3y_2 \geq 8y_2 \\
& -3x_1y_3 - x_2y_3 - 2x_3y_3 \geq -8y_3 \\
& -x_1y_4 - 4x_2y_4 - 3x_3y_4 \geq -10y_4 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
\quad \rightarrow \quad
\begin{aligned}
\text{Maximize} \quad & Z = 5y_1 + 8y_2 - 8y_3 - 10y_4 \\
\text{subject to} \quad & y_1 + 3y_2 - 3y_3 - y_4 \leq 2 \\
& 2y_1 + y_2 - y_3 - 4y_4 \leq 3 \\
& y_1 + 2y_2 - 2y_3 - 3y_4 \leq 4 \\
& y_1, y_2, y_3, y_4 \geq 0
\end{aligned}
$$

Primal Problem (or Dual Problem)	**Dual Problem (or Primal Problem)**
Maximize Z (or W)	Minimize W (or Z)
Constraint i:	Variable y_i (or x_i):
$\leq$ form $\longleftrightarrow$	$y_i \geq 0$
$=$ form $\longleftrightarrow$	Unconstrained
$\geq$ form $\longleftrightarrow$	$y_i' \leq 0$
Variable x_j (or y_j):	Constraint j:
$x_j \geq 0$ $\longleftrightarrow$	$\geq$ form
Unconstrained $\longleftrightarrow$	$=$ form
$x_j' \leq 0$ $\longleftrightarrow$	$\leq$ form

Benefits of Duality in LPP

- **Alternative Formulations:** Provides another way to view and solve the same problem.
- **Solution Bounds:** Helps in establishing upper or lower bounds for the optimal solution of the primal problem.
- **Sensitivity Analysis:** Facilitates the calculation of shadow prices, which indicate the value of additional units of a resource.
- **Feasibility and Optimality:** Helps in evaluating whether a solution is feasible or optimal.

Fundamental Theorem of Linear Programming

If a linear programming problem (LPP) has an optimal solution, then at least one optimal solution occurs at a **corner point (vertex)** of the feasible region.

•Implications:

- Search for optimal solutions can be restricted to corner points of the feasible region.
- There may be:
  - **Unique solution** at one vertex.
  - **Multiple optimal solutions** if the objective function is parallel to a constraint.
  - **Unbounded solution** if feasible region is open in the direction of optimization.
  - **Infeasible problem** if feasible region is empty.

Degenerate Solutions in LPP

A solution is **degenerate** if one or more basic variables take the value zero at a basic feasible solution (BFS).

- **Causes:**
  - Redundant constraints.
  - Intersection of more than ‘m’ constraints at a BFS (where *m = number of constraints*).
- **Implications:**
  - May lead to **stalling** in the simplex method.
  - Could cause **cycling** (repetition of same BFS).

Simplex-Based Methods

- **Purpose:** Solve LPPs by moving from one BFS to another, improving the objective function until optimality.
- **Key Components:**
  - **Initial Basic Feasible Solution (IBFS):** Obtained using slack/surplus/artificial variables.
  - **Pivot Operations:** Exchange of basic and non-basic variables.
  - **Optimality Test:** When all reduced costs are ≥ 0 (for maximization).
  - **Unboundedness Check:** If entering variable has no positive ratio for leaving variable test.

Cycling in Simplex

- **Problem:** Simplex method may revisit the same set of BFS repeatedly due to degeneracy.
- **Result:** Infinite loop, no progress toward optimality.
- **Prevention Techniques:**
  - **Bland's Rule:** Always choose entering and leaving variables with smallest index.
  - **Perturbation Technique:** Slightly adjust constraints to remove degeneracy.

Note: Q. 1 is compulsory.

Q1		(2.5*4)	CO
	(a) Explain Optimization, and its applications in Engineering		CO1
	(b) Depending on whether a particular point belongs to the acceptable or unacceptable region, it can be identified as one of the four types. Define and explain these types?		CO1
	(c) Explain the merits and limitations of the graphical method?		CO2
	(d) Explain decision variables, objective function, and constraints. Write an LPP to illustrate these terms?		CO2

Q2 **(Attempt any Two Parts) UNIT-1 (CO1)** **(5,5)**

(a) A soft drink manufacturing company has 300 ml and 150 ml canned cola as its products with profit margin of Rs. 4 and Rs. 2 per unit respectively. Both the products have to undergo process in three types of machine. The following data indicates the time required on each machine and the available machine-hours per week. Formulate the optimization problem as an LPP to maximize the total profit considering the limited resources.

Requirement	**Cola 300 ml**	**Cola 150 ml**	**Available machine hours per week**
Machine 1	3	2	300
Machine 2	2	4	480
Machine 3	5	7	560

(b) Discuss briefly about multiple and unbounded optimization Linear Programming Problems. Use appropriate example to justify your answer.

(C) Explain in detail the steps involved in formulating problems as mathematical programming problems? Explain the process, including the translation of design objectives and constraints into mathematical formulations

Q3 **(Attempt any Two Parts) UNIT-2 (CO2)** **(5,5)**

(a) An airplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine using a graphical method how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

(b) Use Simplex method to solve the following LP problem Maximize Z = 50x + 60y subject to: 2x + y <=300 ; 3x + 4y <=509; 4x + 7y <=812; x, y >=0.

(C) Write the algorithm to solve LPP using the simplex method OR explain the Integer Programming Problems in Optimization with an example.

OTDM SOLUTION

SOLUTION OF 2a)

Let x_1 be the number of units of 300 ml cola and x_2 be the number of units of 150 ml cola to be produced respectively. Formulating the given problem, we get

$$Max.Z = 4x_1 + 2x_2$$

Subject to:

$$3x + 2x \leq 300$$
$$2x_1 + 4x_2 \leq 480$$
$$5x_1 + 7x_2 \leq 560$$
$$x_1, x_2 \geq 0$$

SOLUTION OF 3a)

Let x, y denote the number of executive class tickets and economy class tickets sold resp.

Since, aeroplane can carry maximum 200 passengers.

$\therefore x + y \leq 200$ ...(1)

Since, at least 20 tickets is reserved for executive class.

$\therefore x \geq 20$ ...(2)

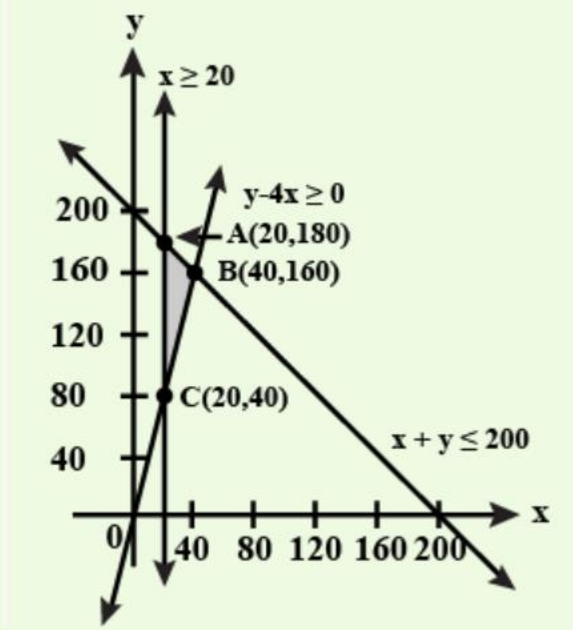
Since the number of tickets for economy class should be at least 4 times the executive class.

$\therefore y \geq 4x \Rightarrow y - 4x \geq 0$ ...(3)

Also, the number of tickets can't be negative. So, $x, y \geq 0$ ...(4)

Profit on an executive class ticket is 1000 Rs and profit on an economy class ticket is 600 Rs

So, Objective function is Maximize Z=1000x + 600y



Corner points	Value of Z = 1000X + 600Y
A (20, 180)	128000
B (40, 160)	136000 (Maximum)
C (20,80)	68000

We have to maximize the total profit. After plotting all the constraints given by equation (1), (2), (3) and (4), we get the feasible region as shown in the image above.

Hence, **Maximum profit will be 136000 Rs**, when number of executive class and economy class tickets sold will be 40 and 160 respectively.

Q3 b)
Use Simplex method to solve the following LP problem
Maximize Z = 50x + 60y subject to: 2x + y <=300 ; 3x + 4y <=509; 4x + 7y <=812; x, y >=0.

To solve this LPP using Simplex, let's first rewrite the problem in standard form:

Maximize Z = 50x + 60y Subject to:

1. 2x + y + s1 = 300
2. 3x + 4y + s2 = 509
3. 4x + 7y + s3 = 812
4. x, y, s1, s2, s3 >= 0

Basic Variables	x	y	s1	s2	s3	RHS
s1	2	1	1	0	0	300
s2	3	4	0	1	0	509
s3	4	7	0	0	1	812
Z-row	-50	-60	0	0	0	0

After the first iteration:

Basic Variables	x	y	s1	s2	s3	RHS
y	1	2	1/2	0	0	150
s2	0	1	-3/2	1	0	259
s3	0	1	-2	0	1	412
Z-row	0	10	0	0	0	3000

Now, since all coefficients in the Z-row are non-negative, we have reached the optimal solution.

Solution: x = 150 y = 259 , Z = 3000

Simplex method

The Simplex method is an approach to solving LPP involving 2 or more decision variables by hand using slack variables, tableaus, and pivot variables as a means to finding the optimal solution. Following steps are necessary:

- Standard form
- Introducing slack variables
- Creating the tableau
- Pivot variables
- Creating a new tableau
- Checking for optimality
- Identify optimal values

Basic variables:

Are the variables which coefficients One in the equations and Zero in the other equations.

Non-Basic variables:

Are the variables which coefficients are taking any of the values, whether positive or negative or zero.

Slack, surplus & artificial variables:

a) If the inequality be (less than or equal, then we add a slack variable + S to change to =.

b) If the inequality be (greater than or equal, then we subtract a surplus variable - S to change to =.

c) If we have = we use artificial variables.

ALGORITHM OF SIMPLEX METHOD:

Step 1:
Determine a starting basic feasible solution.

Step 2:
Select an entering variable using the optimality condition. Stop if there is no entering variable.

Step 3:
Select a leaving variable using the feasibility condition.

Optimality condition:

The entering variable in a maximization (minimization) problem is the non-basic variable having the most negative (positive) coefficient in the Z-row.
The optimum solution is reached at the iteration where all the Z-row coefficient of the non-basic variables are non-negative (non-positive).

Feasibility condition:

For both maximization and minimization problems the leaving variable is the basic associated with the smallest non-negative ratio (with strictly positive denominator).

Pivot row:

a) Replace the leaving variable in the basic column with the entering variable.
b) New pivot row equal to current pivot row divided by pivot element.
c) All other rows:
   New row=current row - (pivot column coefficient) *new pivot row

LIMITATION OF GRAPHICAL METHOD

The main limitation of graphical method for solving LPPs are that it is only applicable to problems with two decision variables and becomes awkward and impractical as the number of variables or constraints increases.
Additionally, it is not suitable for problems with three or more variables, yields approximate rather than precise results, and can be challenging to interpret if the feasible region is not convex or the objective function isn't linear.

1. **Limited to two variables:**

The graphical method relies on plotting constraints on a 2D plane (x-y axes), making it impossible to visualize problems with more than two decision variables.

2. Impractical for larger problems:

Even for two-variable problems, the method becomes tedious and difficult to manage as the number of constraints increases.

3. Results can be approximate:

The solution obtained from a graph is often an estimate, and precision depends on the accuracy of the drawing.

4. Not applicable to problems with three or more variables:

For LPPs with three or more decision variables, more advanced methods like the Simplex method are required.

5. Difficulty with non-convex or non-linear scenarios:

While the graphical method is for linear programming, it's difficult to find the optimal solution if the feasible region is not convex or the objective function is not strictly linear.

6. May not have a feasible solution:

In some cases, the constraints can be contradictory, resulting in no overlapping area on the graph and thus no feasible solution for the problem.

7. Multiple optimal solutions:

If the objective function line is parallel to one of the boundaries of the feasible region, there can be multiple optimal solutions, which the graphical method can identify but may be difficult to work with in complex cases.

Graphical solution of an LPP

Solving a maximization LPP

Solving a minimization LPP

Steps to solving an LPP graphically

- Step 1: Modify each constraint by replacing the inequality sign ($\leq \geq$) with the equal to sign (=)
- Step 2: Solve each equation to obtain two points such that the equation can be plotted on the graph paper.
  - This can be done by finding the value of 'y' when $x = 0$ and then the value of 'x' when $y = 0$.
  - A minimum of 2 points are needed to draw a line.
- Step 3: Taking an appropriate scale for the 'x' & 'y' axis, draw the constraint equations on the graph paper.

STEPS TO SOLVING AN LPP GRAPHICALLY

- Step 4: Identify the solution region for each constraint equation.
  - If the original constraint was $\leq$ type, the solution to the constraint lies between its line and the origin. This means the solution includes the origin (0,0).
  - If the original constraint was $\geq$ type, the solution to the constraint lies between its line and infinity. This means the solution does not include the origin (0,0).
  - If the original constraint was = type, all points on the line are a part of its solution.
- Step 5: Identify the *common solution region* that satisfies all given constraints.

Steps to solving an LPP graphically

- Step 6: Name the *corner points of the feasible region* and read their values from the graph.
- Step 7: Now find the *value of the objective function* at each of these points.
- Step 8: The *optimum solution* is the one that gives largest (for Max Z) and least (for Min Z) value of the objective function.
- Step 9: The values of the identified corner point is the *optimum value of the decision variable* of the LPP.

Maximization case

An example of graphical solution of LPP

THE COMPLETED LPP *(FROM LECTURE 1)*

Objective function

$$\text{Max } Z = 40\,x + 35\,y$$

Subject to:

raw material $\quad 2\,x + 3\,y \leq 60$

labor hours $\quad 4\,x + 3\,y \leq 96$

NNC $\quad x \geq 0,\ y \geq 0$

Solve to obtain 2 points per constraint

Material constraint $2x + 3y = 60$

When $x=0$, $y = 60/3 = 20$ hence the point is $(0, 20)$

When $y=0$, $x=60/2 = 30$ hence the point is $(30, 0)$

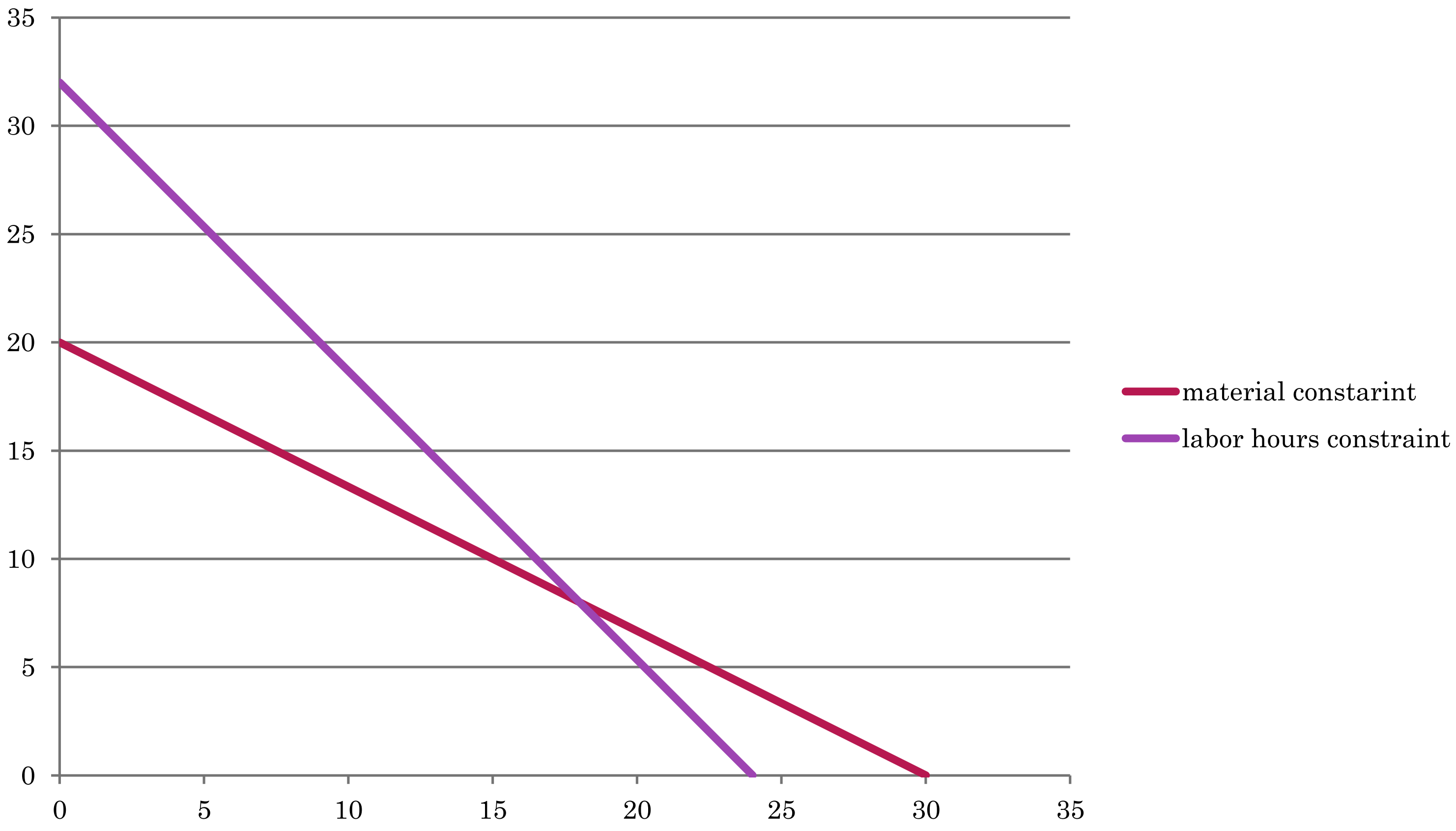
labor hours constraint $4x + 3y = 96$

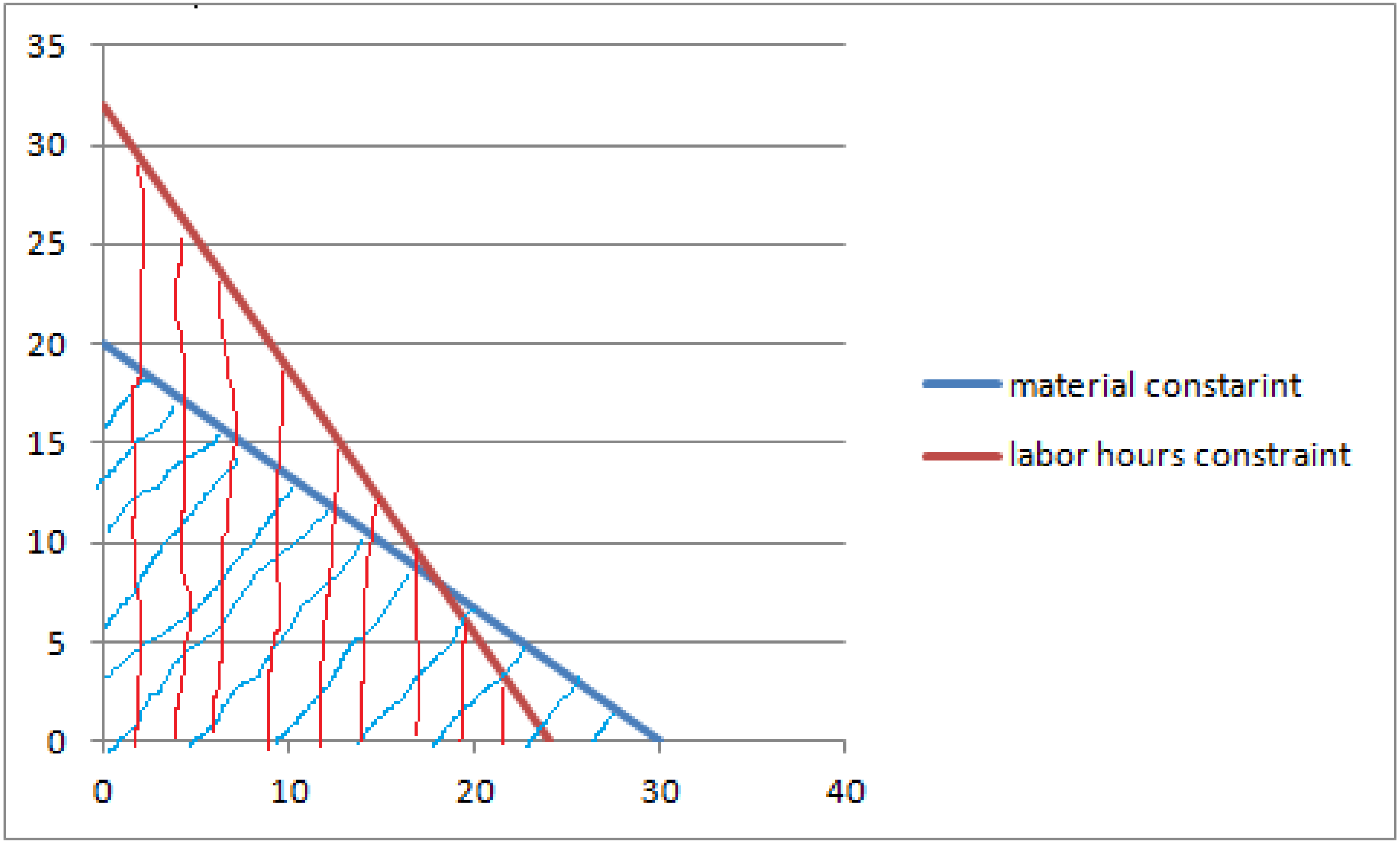
When $x=0$, $y = 96/3 = 32$ hence the point is $(0, 32)$

When $y=0$, $x= 96/4 = 24$ hence the point is $(24, 0)$

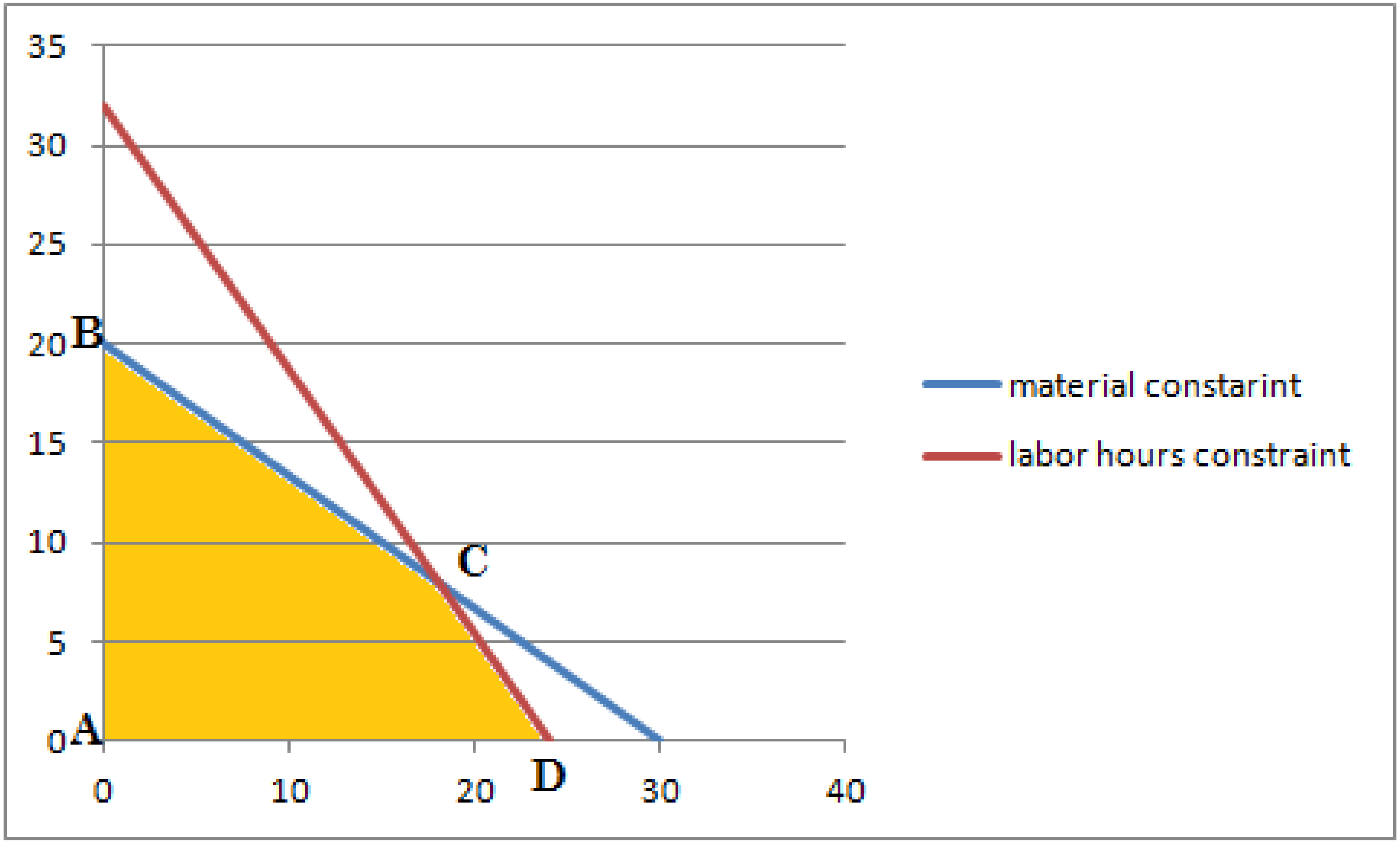
Drawing these on the graph paper will give us the following graph:

The graph of LPP





The feasible region of each equation



The feasible solution region- ABCD

The corner points of the solution region – A-B-C-D

- A (0, 0)
- B (0, 20)
- C (18, 8)
- D (24, 0)

Now calculate the value of the objective function at each of these points:

The objective function is

Max $Z = 40x + 35y$

- A (0, 0) = 40 (0) + 35 (0) = 0
- B (0, 20) = 40 (0) + 35 (20) = 700
- C (18, 8) = 40 (18) + 35 (8) = 1000
- D (24, 0) = 40 (24) + 35 (0) = 960

as can be observed, the largest value of objective function is obtained at point C (18, 8).

SOLUTION:

The optimum solution of the LPP:

The firm should manufacture18 units of product A and 8 units of product B at a profit of Rs 1000.

Minimization case

An example of formulation of LPP

COMPLETE LPP *(FROM LECTURE 1)*

Objective function

$$\text{Min } Z = 40\,x + 24\,y$$

Subject to:

$20\,x + 50\,y \geq 4800$ (Ph requirement)

$80\,x + 50\,y \geq 7200$ (N requirement)

$x \geq 0,\ y \geq 0$

SOLVE TO OBTAIN 2 POINTS PER CONSTRAINT

(Ph constraint) $20x + 50y = 4800$

When $x=0$, $y = 4800/50 = 96$ hence the point is (0, 96)

When $y=0$, $x = 4800/20 = 240$ hence the point is (240,0)

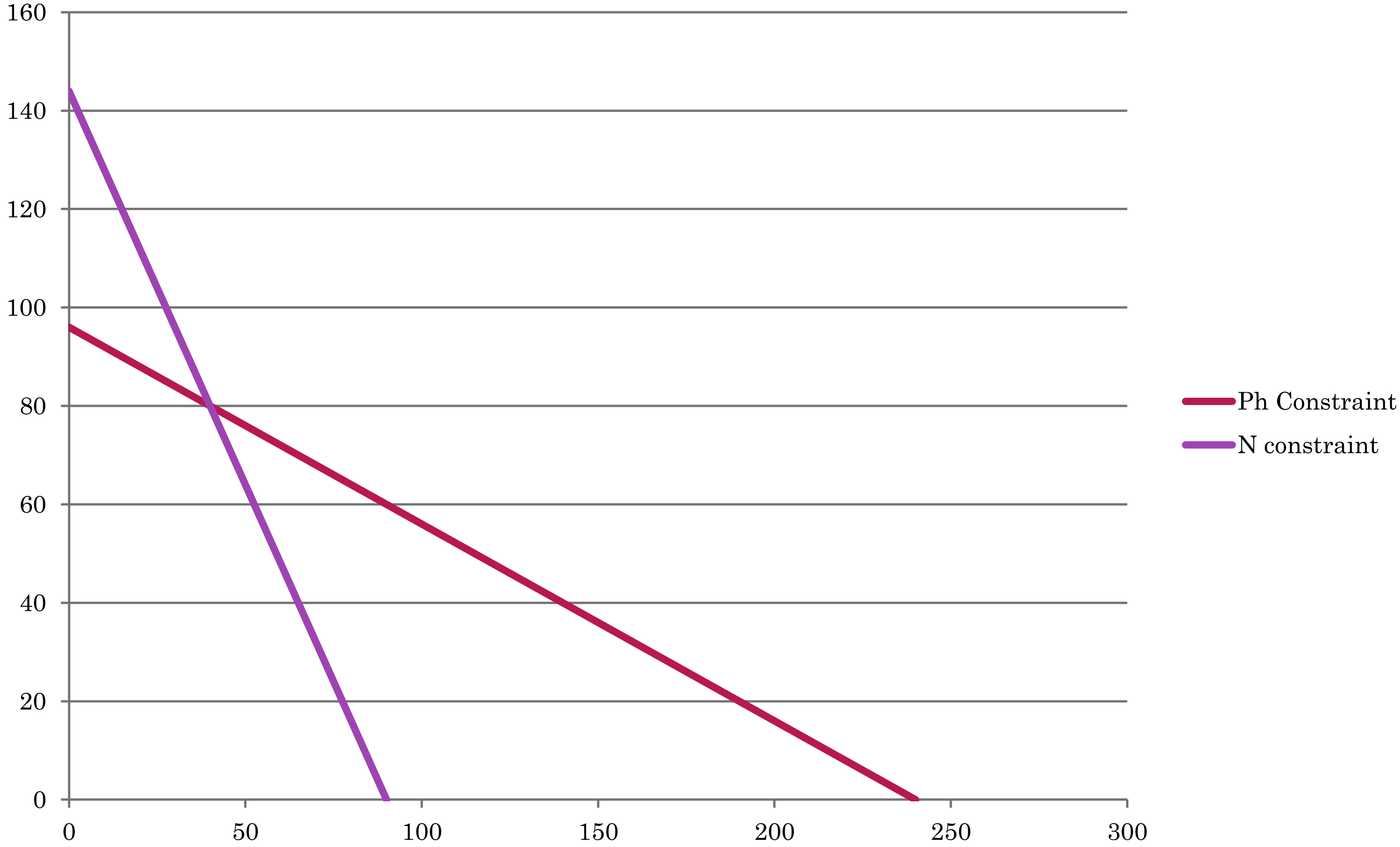
(N constraint) $80x + 50y = 7200$

When $x=0$, $y = 7200/50 = 144$ hence the point is (0,144)

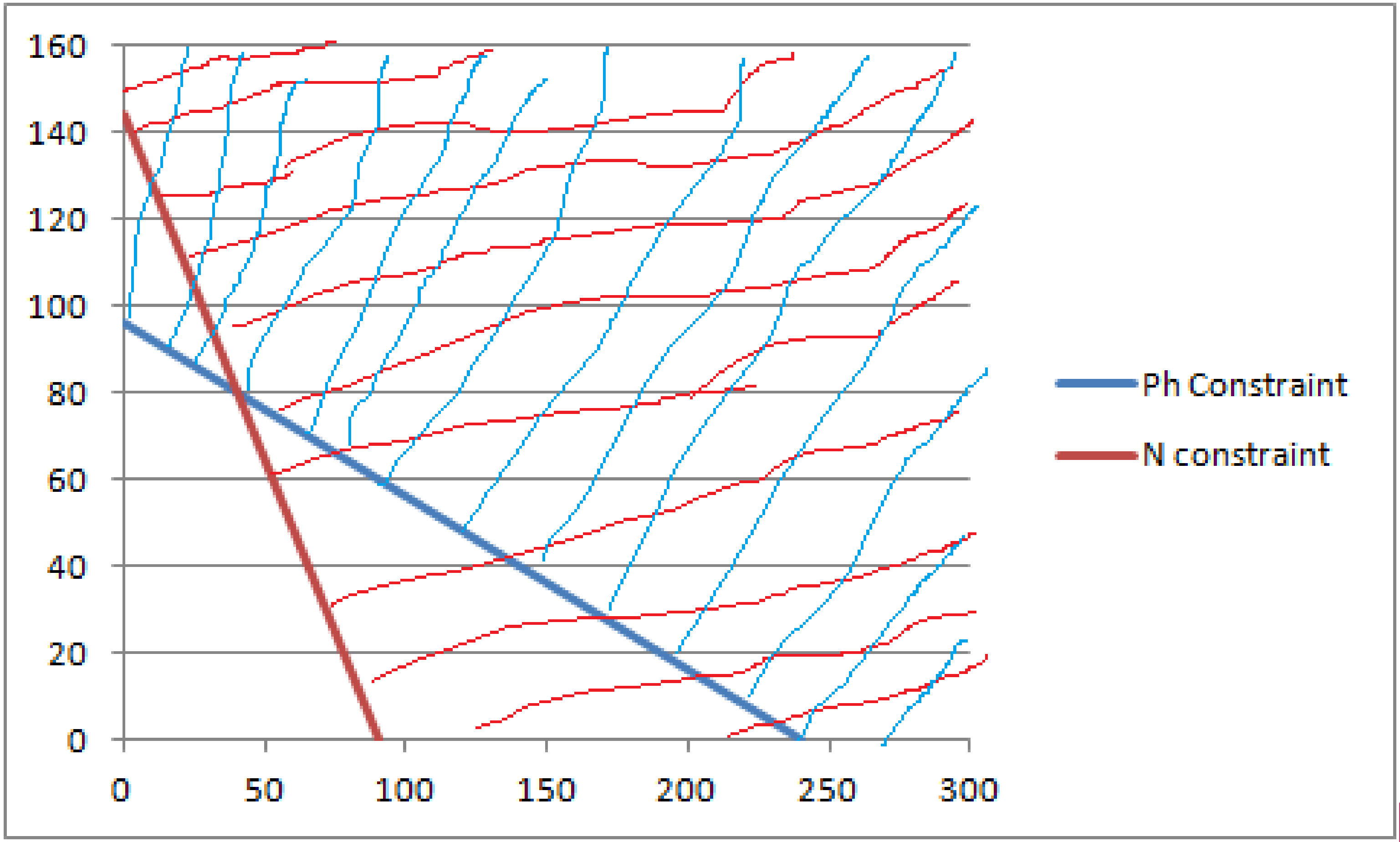
When $y=0$, $x = 7200/80 = 90$ hence the point is (90, 0)

Drawing these on the graph paper will give us the following graph:

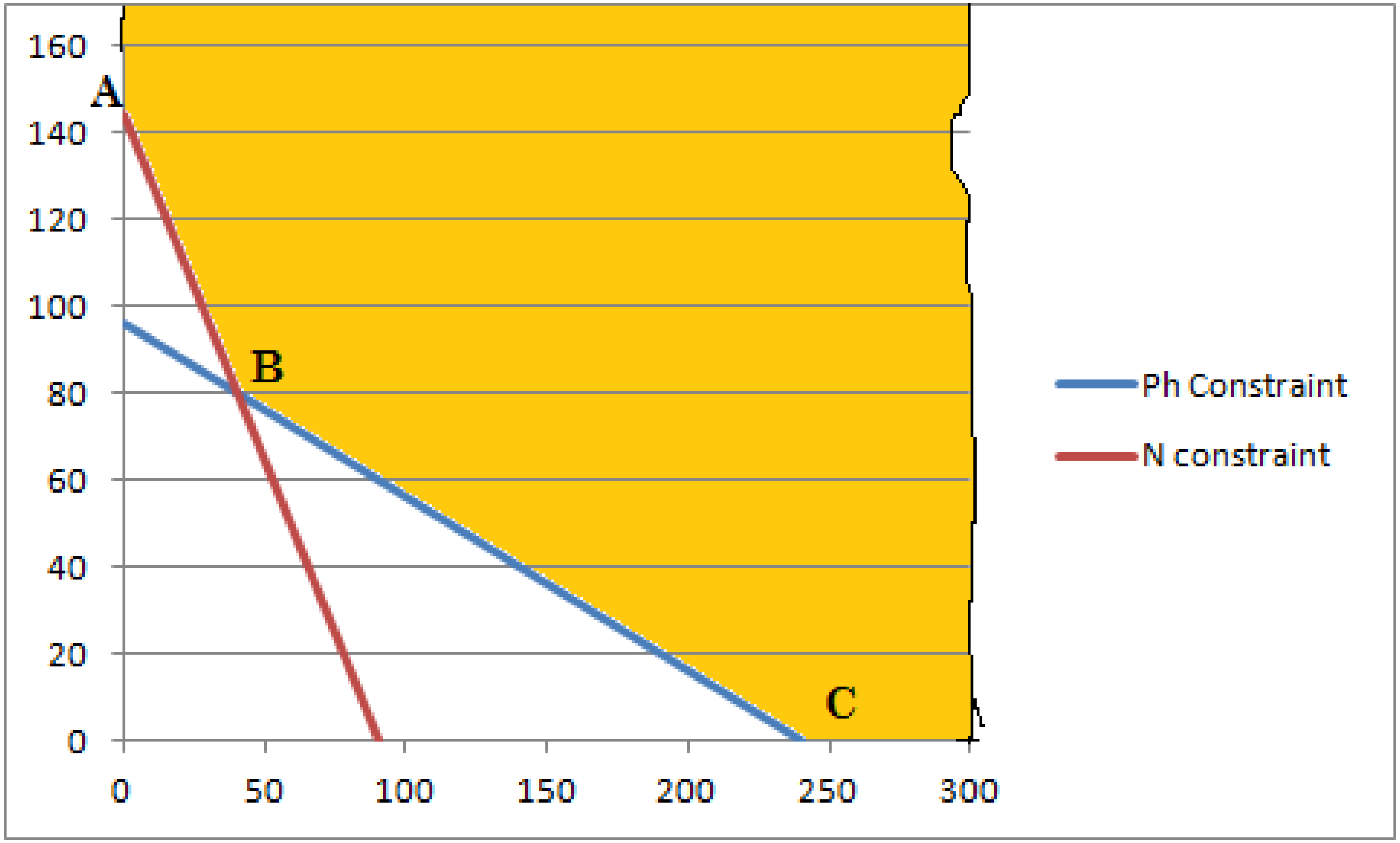
The graph of LPP



Identify the feasible region of each constraint



The feasible area and its corner points – A-B-C



The corner points of the solution region – A-B-C

- A (0,144)
- B (40,80)
- C (240,0)

Now calculate the value of the objective function at each of these points:

Objective function

Min $Z = 40x + 24y$

- A (0,144) = 40 (0) + 24 (144) = 3456
- B (40,80) = 40 (40) + 24 (80) = 3520
- C (240,0) = 40 (240) + 24 (0) = 9600

as can be observed, the smallest value of objective function is obtained at point A (0, 144).

SOLUTION:

The optimum solution of the LPP:

The farmer should purchase 0 bags of mixture A and 144 bags of mixture B at a cost of Rs 3456.