block reader.

OTDM Important Topics.

- Regression Analysis (linear, sum of square, logistic, multiple linear regressions, etc.
- KNN algorithm (K = 3)
- K-Means ~~algorithm~~ clustering
- Gradient descendent
- Newton Method
- Advanced newton method
- Revised method of newton method
- Stationary points, Hessian matrix
- LPP formulation / Graphical Method
- Decision making model (DA ya DM)
- Analytic Heirarchial process)
- TOPSIS method
- Markov Decision Process
- Reinforcement learning
- Decision Trees
- Regression (Unit-3 compare 2 models)
- Bias-variance diacotomy model
- Type of Analytics

- Classification
- LPP model Assumption
- Graphical method limitations
- Dual, Primal
- Use of duality
- Degeneracy
- Sentiment Analysis

General LPP not just graphical

(Please write your Enrollment Number) Enrollment No.________________

MID-TERM EXAMINATION

(Course Name : B. Tech. IT(AIML)) (Semester : 5)

(October 2024) OFF LINE mode

Subject Code: BAM 301	Subject: Optimization Techniques and Decision Making
Time : 1 ½Hours	Maximum Marks :30

Note: Q1 is compulsory. Attempt any two parts of Q2 and Q3. Calculators are permitted

Q1		(2.5*4=10)	CO Mapping
	a) State the Fundamental Theorem of Linear Programming. Discuss the conditions which guarantees the existence of an optimal solution.		CO1
	b) What is Cycling in Simplex Method, and how is it prevented?		CO1
	c) Find the Dual for following linear programming problem: Maximize $Z = 3x_1 + 2x_2$ Subject to: $2x_1 + x_2 = 4, x_1 + 3x_2 = 6,\ x_1, x_2 \geq 0$		CO1
	d) Define a regular point in the constrained optimization problems.		CO2
	UNIT I		CO Mapping
Q2	Attempt any two parts	(5*2=10)	
	a) Solve the following linear programming problem using the Simplex Method: Maximize $z = 2x_1 + 3x_2$ Subject to: $2x_1 + x_2 \leq 12,\ x_1 + 3x_2 \leq 15, x_1, x_2 \geq 0$		CO1
	b) Define a convex optimization problem with its key properties. Explain how to verify the convexity of a function using the Hessian matrix. Differentiate between convex and concave functions.		CO2
	c) Solve using Quasi-Newton method to find the minimum of the function: $f(x,y) = x^2 + y^2 + 2x + 2y$ starting from an initial guess of $(x_0, y_0) = (0,0)$. Take $\alpha = 0.5$		CO2
	UNIT II		CO Mapping
Q3	Attempt any two parts	(5*2=10)	
	a) Solve the following optimization problem using KKT conditions: Minimize $f(x,y) = (x-2)^2 + (y-3)^2$, Subject to: $g(x,y) = x + 2y - 5 \leq 0$. Also, indicate if constraint is active.		CO2
	b) What is Sensitivity Analysis in optimization problems? Explain its relevance in Linear Programming along with examples.		CO2
	c) Explain the concept of Complementary Slackness Conditions. Determine whether the slackness conditions hold for these primal variables: $x_1 + x_2 \leq 4, 2x_1 + x_2 \leq 6$ at (2,2).		CO1

Features

1. Special case of NLP
2. Obj fxn in QP is quadratic, i.e, a second order polynomial of decision variables
3. Global min. exist if the quadratic form is (+ve) definite (OR the fxn is ~~strictly~~ strictly convex).
4. There may be constraints which may or may not be binding.

General form of QP is:

Minimize : $f(x) = \frac{1}{2} x^T H x + c^T x$

s. t. $A_{eq} x = b_{eq}$

$A_{ineq} x \leq b_{ineq}$

$l \leq x \leq u$

where, x is vector of decision var.

H is symm. ~~vector~~ matrix representing the quad. terms

c is a vector representing linear terms.

A_{eq}, b_{eq}, A_{ineq}, b_{ineq}, l, u are matrices & vectors defining linear constraints & variable bounds.

~~Eg~~) Portfolio optimization

* NLP

⇒ Quadratic Programming (QP)

↳ QP is a type of NLP in which the variables (decision) are quadratic in nature.

- is an optimization method very popularly used in computing as the max or min. value of an objective fxn, (which is quadratic in nature), s.t. the constraints which could be linear equality or inequality.

- QP is a specific type of NLP where you have special constraints & a non-linear objective fxn. along with multiple local optima (local maxima or minima), i.e., unlike LPPs which guarantees a single optimum sol^n. (if it exists). So, NLPs methods like QP have multiple local optima, thus, making it difficult & challenging in finding the global optima.

The steps of the simplex method are

Step 1. Determine a starting basic feasible solution.

Step 2. Select an *entering variable* using the optimality condition. Stop if there is no entering variable; the last solution is optimal. Else, go to step 3.

Step 3. Select a *leaving variable* using the feasibility condition.

Step 4. Determine the new basic solution by using the appropriate Gauss-Jordan computations. Go to step 2.

3.3.3 Summary of the Simplex Method

So far we have dealt with the maximization case. In minimization problems, the *optimality condition* calls for selecting the entering variable as the nonbasic variable with the most *positive* objective coefficient in the objective equation, the exact opposite rule of the maximization case. This follows because max z is equivalent to min $(-z)$. As for the *feasibility condition* for selecting the leaving variable, the rule remains unchanged.

Optimality condition. The entering variable in a maximization (minimization) problem is the *nonbasic* variable having the most negative (positive) coefficient in the z-row. Ties are broken arbitrarily. The optimum is reached at the iteration where all the z-row coefficients of the nonbasic variables are nonnegative (nonpositive).

Feasibility condition. For both the maximization and the minimization problems, the leaving variable is the *basic* variable associated with the smallest nonnegative ratio (with *strictly positive* denominator). Ties are broken arbitrarily.

Gauss-Jordan row operations.

1. Pivot row

   a. Replace the leaving variable in the *Basic* column with the entering variable.
   b. New pivot row = Current pivot row ÷ Pivot element

2. All other rows, including z
   New row = (Current row) − (pivot column coefficient) × (New pivot row)

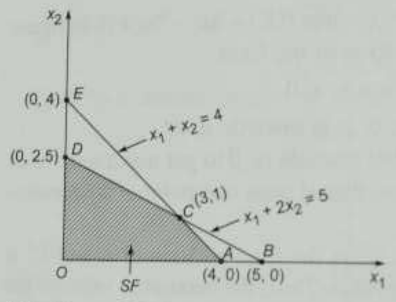
1.7.1 Alternative Optimal Solution

When the objective function assumes the same optimum value at more than one vertex of S_F, then we say that the LPP has an alternative optimal solution.

For example:

Maximize $Z = 2x_1 + 4x_2$

Subject to
$$x_1 + 2x_2 \le 5$$
$$x_1 + x_2 \le 4$$
$$x_1, x_2 \ge 0$$



Point	(x_1, x_2)	$Z = 2x_1 + 4x_2$
0	(0, 0)	0
A	(4, 0)	8
C	(3, 1)	10 ← Max.
D	(0, 2.5)	10 ← Max.

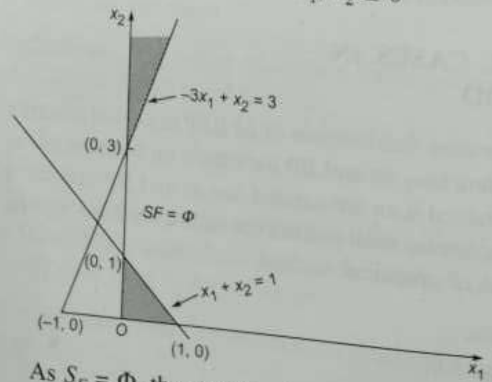
Solution: Maximum occurs at C (3, 1) and D (0, 2.5) and the value of $Z = 10$

1.7.2 Infeasible Solution

When the constraints are not satisfied simultaneously, the LPP has no feasible solution. This implies if $S_F = \Phi$. This situation can never occur if all the constraints are of the $\le$ type.

For example:

Maximize
$$Z = x_1 + x_2$$
Subject to
$$x_1 + x_2 \le 1$$
$$-3x_1 + x_2 \ge 3$$
$$x_1, x_2 \ge 0$$



As $S_F = \Phi$, the problem has no feasible solution.

1.7.3 Unbounded Solution

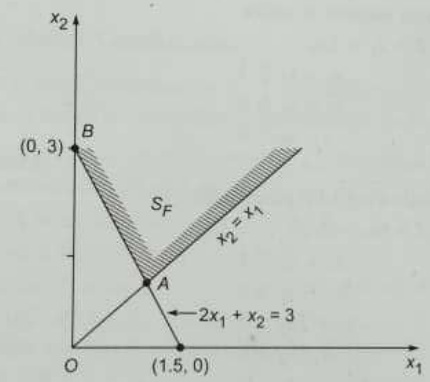
When the value of the decision variables may be increased indefinitely without violating any of the constraints the solution space S_F is unbounded. The value of objective function, in such cases, may increase (for maximization) or decrease (for minimization) indefinitely. Thus, both the solution space and the objective function value are unbounded.

For example:

Maximize $\quad Z = 6x_1 + x_2$

Subject to

$$2x_1 + x_2 \geq 3$$
$$x_2 - x_1 \geq 0$$
$$x_1, x_2 \geq 0$$



The graphical solution of the given LPP is depicted in the above figure. The two vertices of the feasible region are A and B. We observe, that the feasible region S_F is unbounded. The value of the objective function at the vertex A (1, 1) and B (0, 3) are 7 and 3, respectively.

But there exist number of points in feasible region for which the value of the objective function is more than 7. For example, the point (3, 6) lies in the feasible region and the objective function value at this point is 24 which is more than 7. Thus, both the variables x_1 and x_2 can be made arbitrarily large and the value of Z also increases. Hence, the problem has an unbounded solution.

Remark: An unbounded solution means that there exist an infinite number of solutions to the problem.

EXERCISE 1.3

1. Use graphical method to solve

   Maximize $\quad Z = 4x_1 + 3x_2$

   Subject to $\quad 2x_1 + x_2 \leq 1000$

Consequences of Degeneracy

1. The primary consequences of degeneracy in LPP are the potential for the simplex algorithm to experience **cycling** (repeatedly visiting the same set of basic feasible solutions without improving the objective function) or **stalling** (failing to make progress toward the optimal solution).
2. **Increased iterations**: Even if it doesn't lead to cycling, degeneracy can significantly increase the number of iterations required for the simplex algorithm to converge to the optimal solution.
3. Degeneracy typically occurs when a pivot operation results in **no improvement to the objective function value**, often due to a tie in determining the outgoing variable in a simplex tableau, making it difficult (potentially requiring more iterations) to reach the optimal solution.

- Degeneracy does not affect the **existence of an optimal solution**.
- **Feasibility:** Degeneracy does not make a basic feasible solution infeasible.

Q. Explain the condition in optimization when a Linear Programming Problem has an unbounded solution? use an example to justify your answe

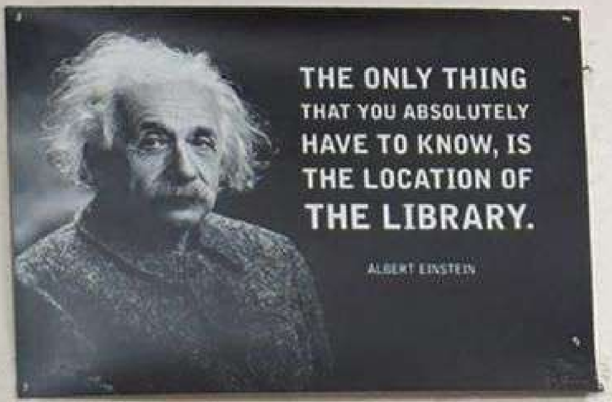
This occurs when:

An LPP is unbounded when the feasible region is open in the direction of optimization, and the objective function can increase (or decrease) indefinitely without violating any constraints. Meaning:

- Constraints do not restrict the feasible region in the direction of optimization.
- As a result, the value of the objective function can grow infinitely.

In Simplex Method, if all the coefficients in the entering column are ≤ 0, i.e., All entries are zero or negative, and therefore, no positive denominator exists for the ratio test then no leaving variable can be selected. This means the entering variable can increase indefinitely without violating any constraint.

So, this is the case of an unbounded solution of a LPP



If $f'(x_2) = \left| \dfrac{f_2^{+} - f_2^{-}}{2 \cdot \Delta x} \right| < \underset{(=0.01)}{\varepsilon}$ then the method converges

$$x_2 = 0.377882$$

$$
\begin{aligned}
f_2^{+} &= f(x_2 + \Delta x) \\
&= f(0.377882 + 0.01) \\
&= f(0.387882) = -\underline{0.304662}
\end{aligned}
$$

$$
\begin{aligned}
f_2^{-} &= f(x_2 - \Delta x) \\
&= f(0.377882 - 0.01) \\
&= f(0.367882) = -\underline{0.301916}
\end{aligned}
$$

$$= \frac{-0.304662 - (-0.301916)}{2 \times 0.01}$$

$$= 0.1373 > \varepsilon\ (= 0.01)$$

So, the method does not converge and we have to perform another Iteration (Iteration 2)

Newton method

Question: Minimize $f(x_1,x_2)=x_1-x_2+2x_1^2+2x_1x_2+x_2^2$ by taking the starting Point as $X_1=\begin{Bmatrix}0\\0\end{Bmatrix}$

Sol. To find X_2.

$$[J_1]=\begin{bmatrix}\frac{d^2f}{dx_1^2} & \frac{d^2f}{dx_1dx_2}\\ \frac{d^2f}{dx_2dx_1} & \frac{d^2f}{dx_2^2}\end{bmatrix}$$

$\frac{df}{dx_1}=1+4x_1+2x_2$ $\boxed{\frac{d^2f}{dx_1dx_2}=2}$

$\boxed{\frac{d^2f}{dx_1^2}=4}$

$\frac{df}{dx_2}=-1+2x_1+2x_2$ $\boxed{\frac{d^2f}{dx_2dx_1}=2}$

$\boxed{\frac{d^2f}{dx_2^2}=2}$

$$[J_1]=\begin{bmatrix}4&2\\2&2\end{bmatrix}$$

$$[J_1]^{-1}=\frac{1}{4\times2-2\times2}\begin{bmatrix}2&-2\\-2&4\end{bmatrix}$$

$$[J_1]^{-1}=\frac{1}{4}\begin{bmatrix}2&-2\\-2&4\end{bmatrix}=\begin{bmatrix}1/2&-1/2\\-1/2&1\end{bmatrix}$$

$$g_1=\begin{bmatrix}df/dx_1\\df/dx_2\end{bmatrix}_{X_1}=\begin{Bmatrix}1+4x_1+2x_2\\-1+2x_1+2x_2\end{Bmatrix}_{\begin{Bmatrix}0\to x_1\\0\to x_2\end{Bmatrix}}=\begin{Bmatrix}1\\-1\end{Bmatrix}$$

$$\therefore X_2=X_1-[J_1]^{-1}g_1=\begin{Bmatrix}0\\0\end{Bmatrix}-\begin{bmatrix}1/2&-1/2\\-1/2&1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}-\begin{bmatrix}1/2\times1+(-1/2)\times(-1)\\-1/2\times1+1\times(-1)\end{bmatrix}$$

$$X_2=\begin{Bmatrix}-1\\3/2\end{Bmatrix}$$

$$g_2=\begin{Bmatrix}df/dx_1\\df/dx_2\end{Bmatrix}_{X_2}=\begin{Bmatrix}1+4x_1+2x_2\\-1+2x_1+2x_2\end{Bmatrix}_{\begin{Bmatrix}-1\to x_1\\3/2\to x_2\end{Bmatrix}}\Rightarrow g_2=\begin{Bmatrix}0\\0\end{Bmatrix}$$

$$X_3=X_2-[J_1]^{-1}g_2$$

$$[\ \ ]\begin{bmatrix}0\\0\end{bmatrix}$$

$$\boxed{X_3=X_2}\begin{bmatrix}0\\0\end{bmatrix}_{g_2}$$

Newton vs. Gradient Descent

Feature	Gradient Descent	Newton's Method
Uses	Gradient only	Gradient + Hessian
Step Direction	Negative gradient	Inverse Hessian × gradient
Convergence Rate	Linear	Quadratic (faster near optimum)
Cost per Iteration	Low	High (Hessian inversion)
Suitable for	Large-scale, simple problems	Smaller, well-behaved quadratic problems