GRAPHICAL SOLUTION OF AN LPP

Solving a maximization LPP

Solving a minimization LPP

Steps to solving an LPP graphically

- Step 1: Modify each constraint by replacing the inequality sign (≤ ≥) with the equal to sign (=)
- Step 2: Solve each equation to obtain two points such that the equation can be plotted on the graph paper.
  - This can be done by finding the value of 'y' when $x = 0$ and then the value of 'x' when $y = 0$.
  - A minimum of 2 points are needed to draw a line.
- Step 3: Taking an appropriate scale for the 'x' & 'y' axis, draw the constraint equations on the graph paper.

STEPS TO SOLVING AN LPP GRAPHICALLY

- Step 4: Identify the solution region for each constraint equation.
  - If the original constraint was $\leq$ type, the solution to the constraint lies between its line and the origin. This means the solution includes the origin (0,0).
  - If the original constraint was $\geq$ type, the solution to the constraint lies between its line and infinity. This means the solution does not include the origin (0,0).
  - If the original constraint was = type, all points on the line are a part of its solution.
- Step 5: Identify the *common solution region* that satisfies all given constraints.

Steps to solving an LPP graphically

- Step 6: Name the *corner points of the feasible region* and read their values from the graph.
- Step 7: Now find the *value of the objective function* at each of these points.
- Step 8: The *optimum solution* is the one that gives largest (for Max Z) and least (for Min Z) value of the objective function.
- Step 9: The values of the identified corner point is the *optimum value of the decision variable* of the LPP.

Maximization case

An example of graphical solution of LPP

The completed LPP *(from lecture 1)*

Objective function

$$\text{Max } Z = 40x + 35y$$

Subject to:

raw material $2x + 3y \leq 60$

labor hours $4x + 3y \leq 96$

NNC $x \geq 0,\ y \geq 0$

Solve to obtain 2 points per constraint

Material constraint $2x + 3y = 60$

When $x=0$, $y = 60/3 = 20$ hence the point is (0, 20)

When $y=0$, $x=60/2 = 30$ hence the point is (30, 0)

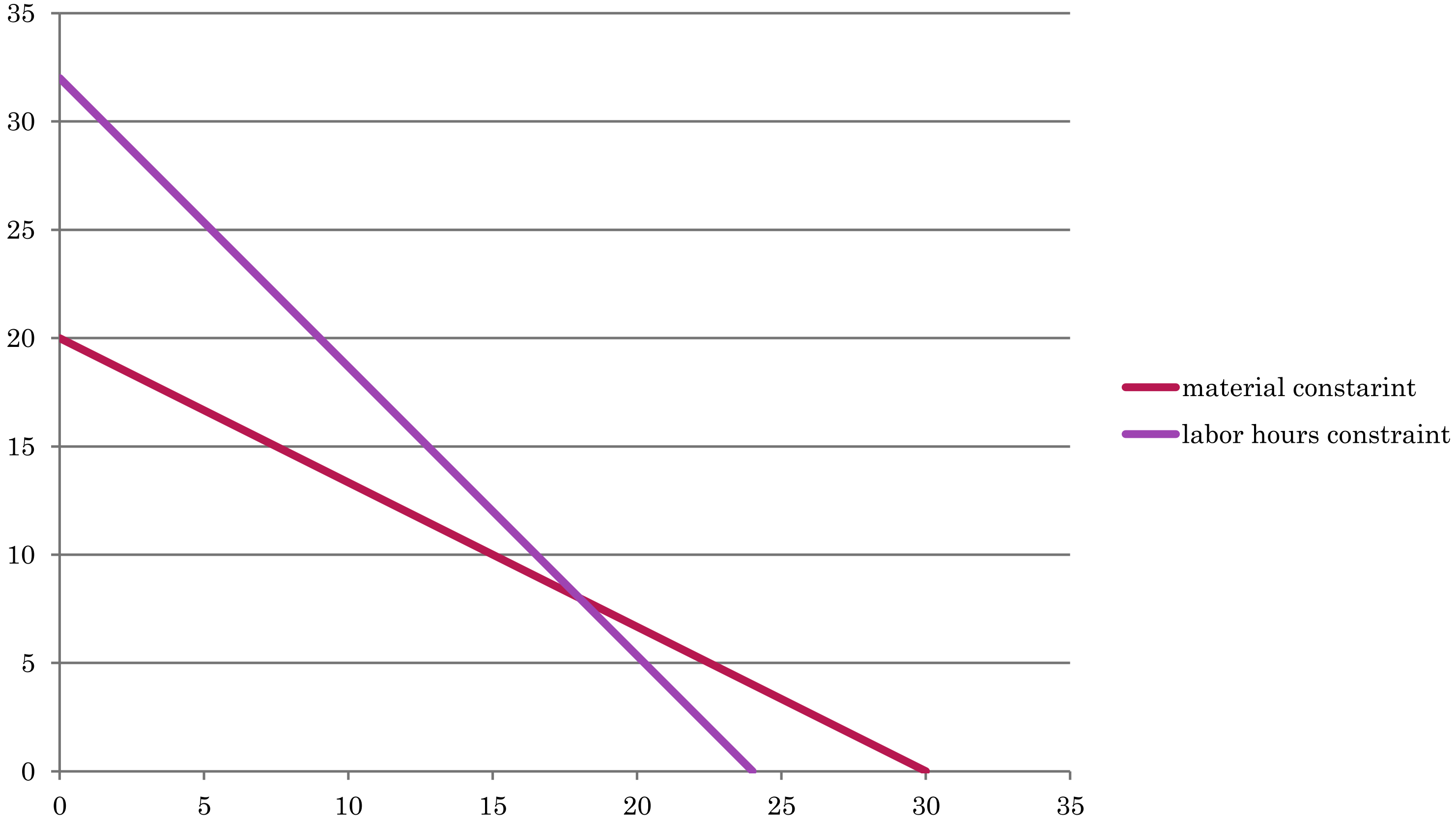
labor hours constraint $4x + 3y = 96$

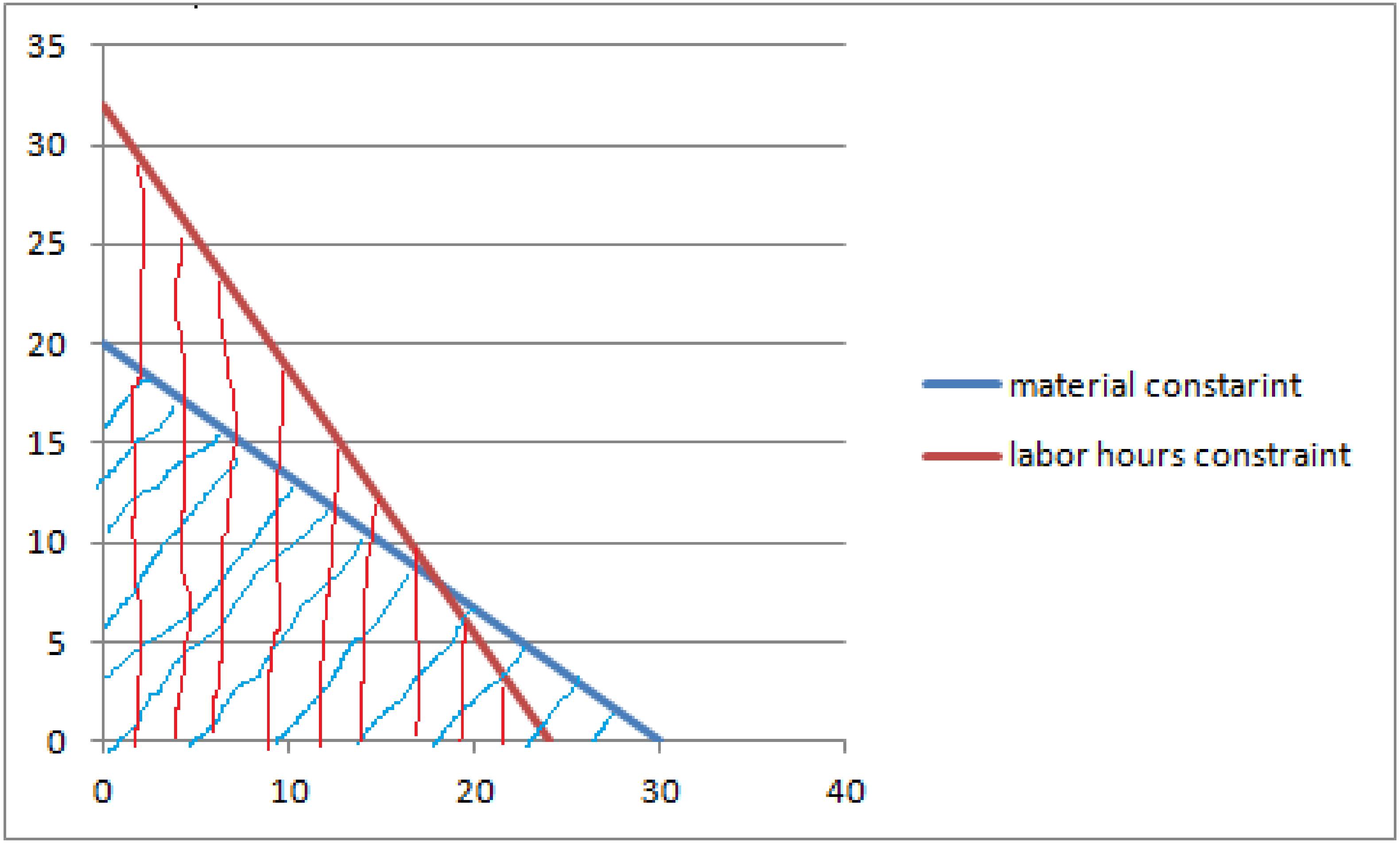
When $x=0$, $y = 96/3 = 32$ hence the point is (0, 32)

When $y=0$, $x= 96/4 = 24$ hence the point is (24, 0)

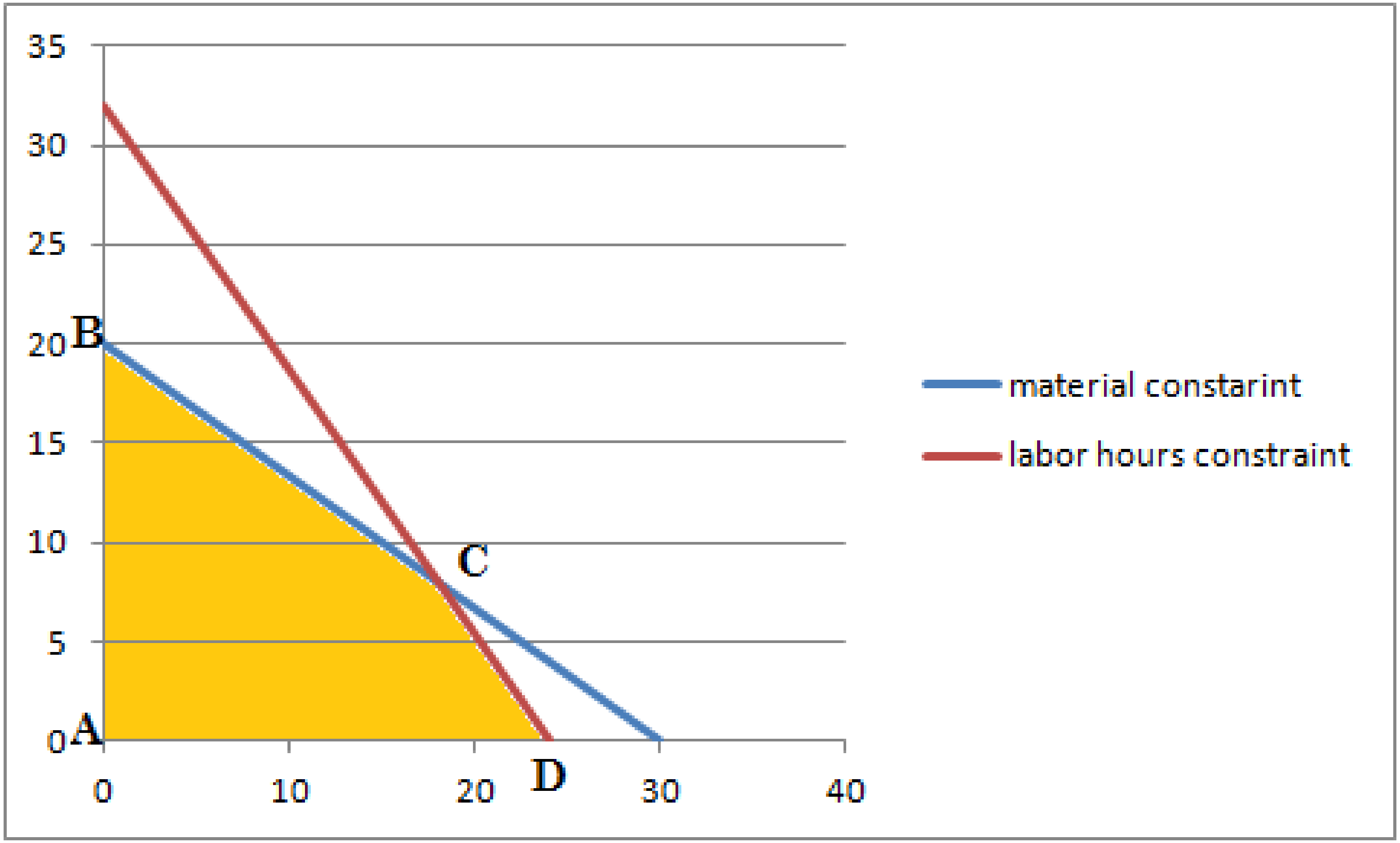
Drawing these on the graph paper will give us the following graph:

The graph of LPP





The feasible region of each equation



The feasible solution region- ABCD

The corner points of the solution region – A-B-C-D

- A (0, 0)
- B (0, 20)
- C (18, 8)
- D (24, 0)

Now calculate the value of the objective function at each of these points:

THE OBJECTIVE FUNCTION IS $\text{MAX } Z = 40x + 35y$

- A (0, 0) = 40 (0) + 35 (0) = 0
- B (0, 20) = 40 (0) + 35 (20) = 700
- C (18, 8) = 40 (18) + 35 (8) = 1000
- D (24, 0) = 40 (24) + 35 (0) = 960

as can be observed, the largest value of objective function is obtained at point C (18, 8).

SOLUTION:

The optimum solution of the LPP:

The firm should manufacture18 units of product A and 8 units of product B at a profit of Rs 1000.

Minimization case

An example of formulation of LPP

Complete LPP (*from lecture 1*)

Objective function

$$\text{Min } Z = 40\,x + 24\,y$$

Subject to:

$$20\,x + 50\,y \geq 4800 \quad \text{(Ph requirement)}$$

$$80\,x + 50\,y \geq 7200 \quad \text{(N requirement)}$$

$$x \geq 0,\ y \geq 0$$

Solve to obtain 2 points per constraint

(Ph constraint) $20x + 50y = 4800$

When $x=0$, $y = 4800/50 = 96$ hence the point is (0, 96)

When $y=0$, $x=4800/20 = 240$ hence the point is (240,0)

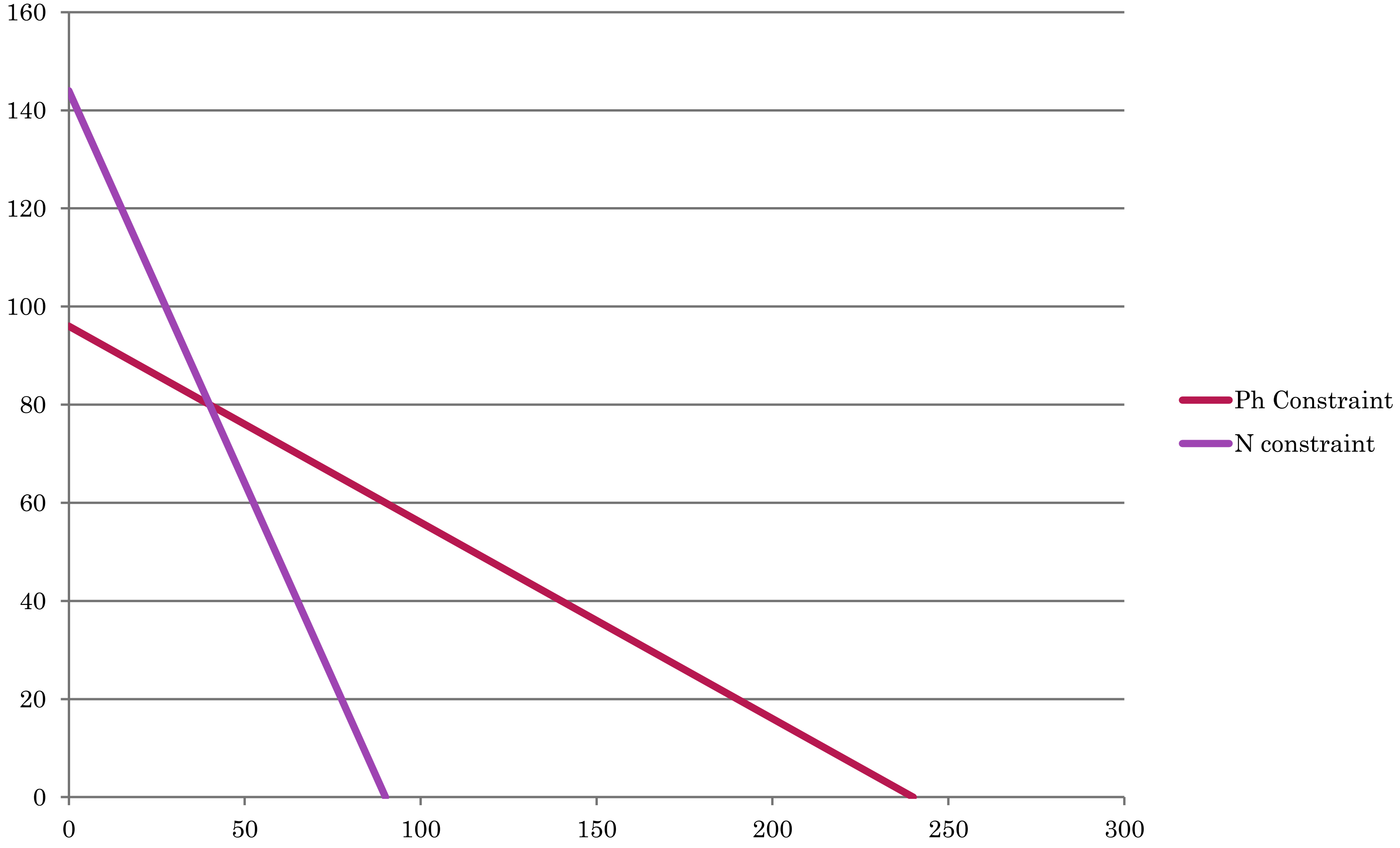
(N constraint) $80x + 50y = 7200$

When $x=0$, $y=7200/50 = 144$ hence the point is (0,144)

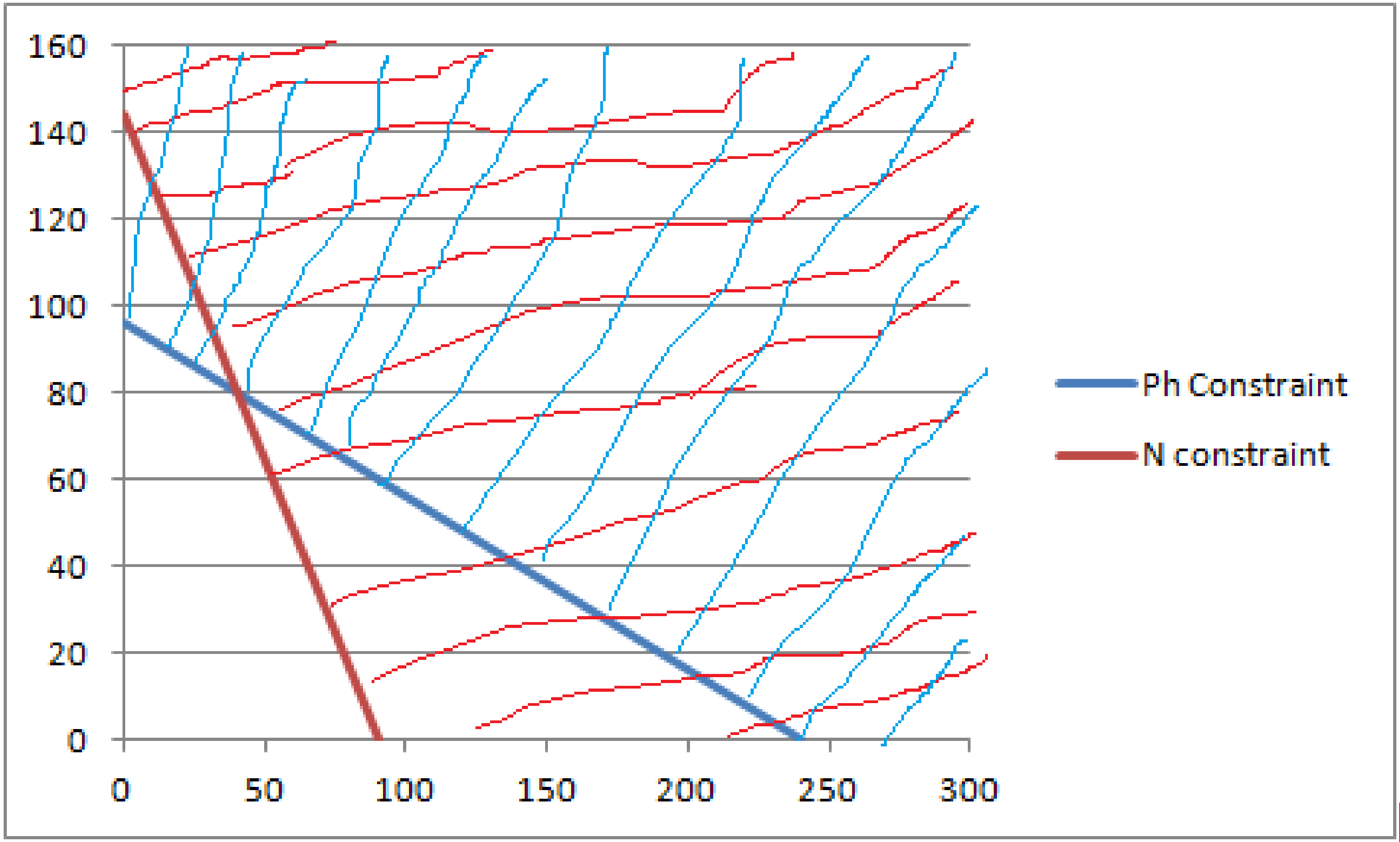
When $y=0$, $x= 7200/80 = 90$ hence the point is (90, 0)

Drawing these on the graph paper will give us the following graph:

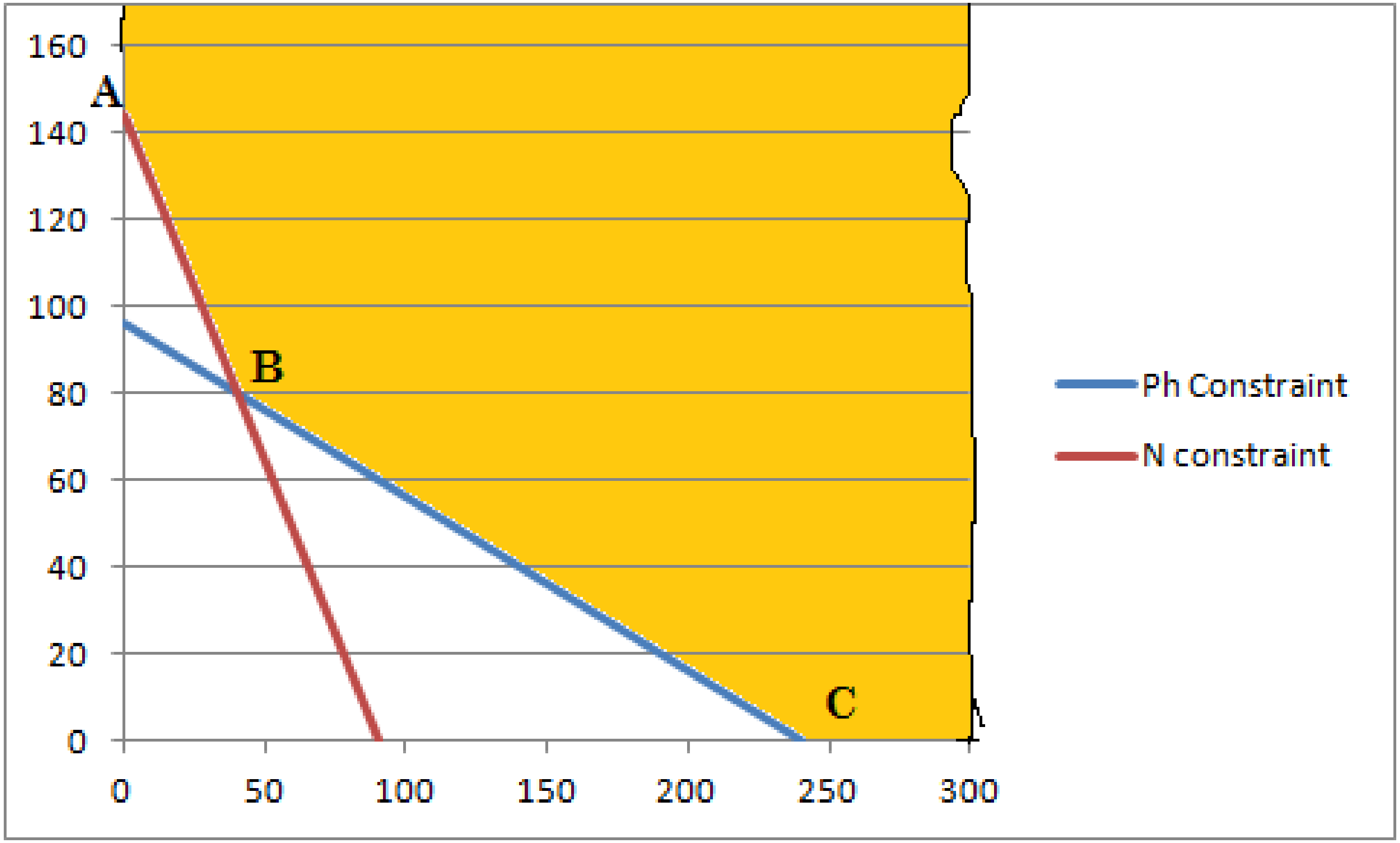
The graph of LPP



Identify the feasible region of each constraint



THE FEASIBLE AREA AND ITS CORNER POINTS – A-B-C



The corner points of the solution region – A-B-C

- A (0,144)
- B (40,80)
- C (240,0)

Now calculate the value of the objective function at each of these points:

Objective function

Min $Z = 40x + 24y$

- A (0,144) = 40 (0) + 24 (144) = 3456
- B (40,80) = 40 (40) + 24 (80) = 3520
- C (240,0) = 40 (240) + 24 (0) = 9600

as can be observed, the smallest value of objective function is obtained at point A (0, 144).

SOLUTION:

The optimum solution of the LPP:

The farmer should purchase 0 bags of mixture A and 144 bags of mixture B at a cost of Rs 3456.