Optimization Techniques and Decision Making Unit I

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Optimization:-

It is the mathematical discipline concerned with finding the maxima and minima of functions possibly subject to constraints.

Optimization is an important tool in making decisions and analyzing physical systems.

In mathematical terms, an optimization problem is the problem of finding best sol^n from set of all possible sol^ns.

In various fields, optimization is employed to improve performance, efficiency and resource utilization.

Whether in engineering, finance, operation research, machine learning or other areas, OT play an important role in decision making and problem solving.

Eg:- Going home from office

	Time	Money	Savings
Taxi	less	more	less
Auto	more	little more	less
Train	more	less	more
Bus	more	less	even more

Terminologies for constructing a model:-

First of all, in the optimization process we construct an appropriate model. Modeling is the process of identifying and expressing in mathematical form

(1) Objective function:-

It is a quantitative measure of the performance of the system that we want to minimize or maximize.

In manufacturing or creating, we may want to maximize the profit or minimize the cost of production, whereas, in fitting experimental data to a model, we may want to minimize the total deviation of observed data from the predicted data.

(II) Decision variables:-

Variables are called unknowns. These are the components of the system for which we want to find values.

In manufacturing or creating, variables may be the amount of each resource consumed or the time spent on each activity, whereas, in fitting data, the variables would be the parameters of the model.

(III) Constraints of the problem:-

Constraints are the functions that describe relationship among the variables and the allowable values for the variables.

In manufacturing or creating, the amount of resource con--sumed cannot exceed the available amount. In other words, we put some constraints on the decision variable of the problem in order to control the range of each variable.

(IV) Feasible Region:-

Set of all possible values for decision variables that satisfy the constraints is known as feasible region. The goal is to find the best solution within its region.

(V) Optimal Solution:-

It is the set of values for decision variable that minimize or maximize the objective function while satisfying all constraints. There can be multiple solutions and the best one depends on the specific problem and context.

(VI) Local and global optimum:-

A local optimum is a solution that is best within specific neigh--bourhood but may not be overall best solution. A global optimum is the best solution across the entire feasible region.

(VII) Optimization algorithms:-

Various algorithms are used to find optimal solution such as linear programming, non-linear programming, genetic programming, gradient descent among others. The choice of algorithm depends on the nature of problem and its specific characteristics.

(VIII) Trade-off:-

Optimization involves trade off between conflicting objectives, eg: minimizing cost might conflict with maximizing quality and finding right balance is essential.

Engineering applications of OT:-

(I) Algorithmic efficiency:-

Optimization algorithms are used to enhance efficiency of algorithm, reducing time complexity and improving overall performance of computation.

(II) Database query optimization:-

Database systems use optimization techniques to improve the efficiency of every query processing, indexing and data retrieval resulting in faster response time.

(III) Optimization for compilers:-

Compilers employ optimization techniques to generate more efficient machine code improving the execution speed and reducing the memory footprint of compiled programs.

(IV) Network routing:-

Optimization algorithms are applied in routing protocols to find the most efficient path for data transmission, minimizing latency and optimizing network utilization

(V) Code optimization:-

Developers use optimization

techniques to improve performance of source code such as loop unrolling, inline expansion and other compiler optimization.

(vi) Memory management:-

Optimization is applied in memory allocation and de-allocation strategies to reduce the memory fragmentation and improve the overall memory utilization.

(vii) Cyber Security:-

optimization is used in cryptographic algorithms to secure the communication while minimizing the computational overhead and response time.

(viii) Data compression:-

Optimization techniques are applied in data compression algorithms to reduce the size of file or data stream improving storage and transmission efficiency.

formulation of design problem as mathematical programming problem

formulating design problems as mathematical programming problems involves expressing the design objectives, constraints and decision variables in mathematical framework that can be solved using optimization techniques.

Mathematical programming or optimization is a branch of mathematics that deals with finding best solution from a set of feasible solution.

There is general outline of how design problem can be formulated:-

(1) Define decision variable:-

Identify the decision parameters that can be adjusted or controlled in the design process. Assign symbols to represent these variables.

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(II) Specify objective function:-

Define an objective function that ~~requires~~ quantifies the goal of the design that would be minimizing or maximizing a certain parameter such as cost, efficiency and performance. Express the objective function as a mathematical equation involving the decision variable.

(III) Set constraints:-

Identify the constraints that must be specified during the design process. Constraints could be related to physical limitation, resource availability, legal requirements, etc. Express these constraints as mathematical inequalities involving the decision variables.

(IV) Formulate the mathematical programming problem:-

(a) Combine the decision variable, objective function and the constraints of problem.

(b) General form of optimization problem is to either minimize or maximize the objective function subject to constraints given.

eg: Generic mathematical programming problem can be written as a function $f(x_1, x_2, \ldots, x_n)$ subject to constraints

$$g_i(x_1, x_2, \ldots, x_n) \leq 0$$
$$h_j(x_1, x_2, \ldots, x_n) = 0$$
where $i = 1, 2, \ldots, m$
$j = 1, 2, \ldots, p$

here $x_1, x_2, \ldots, x_n$ are the decision variables, f is objective function, g is inequality and h is equality function.

(V) Select optimization algorithm:

Choose an appropriate optimization technique to solve mathematical programming problem. The selection depends on the nature of problem such as linear programming, non-linear programming, integer programming, etc.

By formulating design problem in this way, designers can leverage mathematical optimization techniques to find optimal solⁿ ensuring that design meets specific objective while satisfying the relevant conditions.

General structure of optimizⁿ algorithms:-

The structure of OA can be outlined through several common components and steps while specific OA may have unique characteristics, the following provides a high level overview of general structure:-

(I) Initialisation:-

(I) Input parameter:- Define the optimization problem by specifying the objective function to be minimized or maximized, decision variables or any other constraints.

(II) Initial solution:- Set an initial solution by assigning values to the decision variables. this serves as starting point for optimization process.

(II) Optimization function and constraints:-

(I) Objective function:- Clearly define the mathematical expression representing the objective to be optimized. This could involve minimizing or maximizing a certain parameter

(II) Constraints:- Identify and formulate any constraint associated with the optimizⁿ problem. Constraint could be equality or inequality relationship involving decision variables

(III) Iteration loop / rule:-

(I) Evaluate objective function:- Compute the value of objective function based on current set of values of decision variables.

(II) Evaluate constraints:- Check if current solⁿ satisfies all constraints.

(III) Convergence check:- Determine whether the optimization process has converged, meaning the algorithm has found a satisfactory solution.

(IV) Update decision variables:-

(I) Adjustment strategies:- Employ a strategy of updating the decision variable values based on optimization algorithm principle. This may involve moving towards a better solution accor--ding to the objective function.

(II) Search direction:- Determine the direction or combination of direction in which algorithm could explore for improving the solution.

(V) Feasibility check:-

Ensure that updated solution remains feasible, meaning it satisfies all the constraints, if not then take correct action to maintain feasibility.

(VI) Convergence criteria:-

Specify criteria for convergen. which could include reaching a certain objective value, a specified level of improvement or pre-determined number of iterations.

(VII) Termination condition:-

Define conditions for terminati the optimization process. This could be based on reaching convergence, exceeding a maximum number of iteratio or other criterias.

(VIII) Output:-

Including the values of decisio variables that optimize the objective function while satisfying the constraints are the final solution.

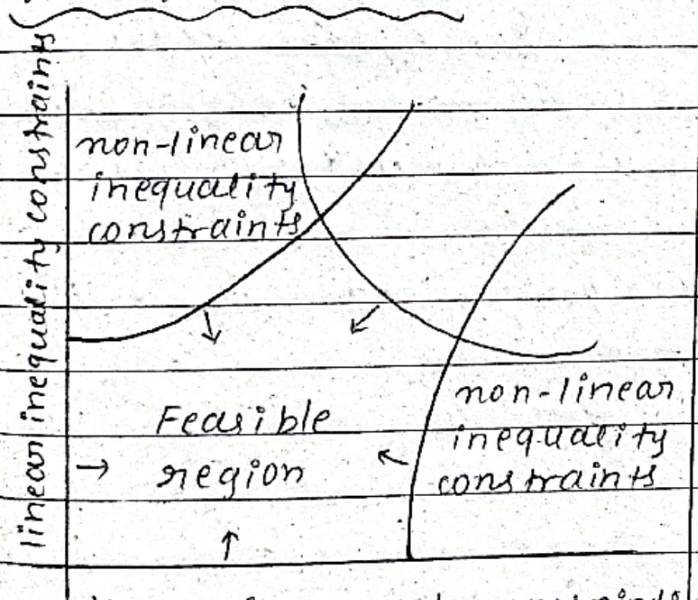
(IX) Post processing:-

Perform any necessary post processing steps such as analyzin result, visualizing solutions or decision making based on optimized solution.

(X) Adjust parameters:-

If applicable, find turning algorithms parameters based on optimization performance may involve for sensitivity analysis or adaptive adjustment.

Types of constraints:-



(I) Budget constraints:-
Each apple costs Rs 1, orange costs Rs 2 and total budget available is Rs 10

$x + 2y = 10$

(II) Nutrition constraints:-
Each apple gives 3u, orange gives 4u and requirement is 15u vitamin C.

$3x + 4y \geq 15$

(III) Equality constraints:-
A person wants 2 apples, i.e $x = 2$, it represents equality constraint, indicating that you must buy exactly 2 apples.

(IV) Inequality constraints:-
⊙ Budget ensures total cost of apples and oranges doesn't exceed Rs 10.
⊙ Nutrition ensures that total vitamin C from apples and oranges meets daily requirement of 15 units.

(V) Feasible region:-
set of valid combinⁿ of apples and oranges that satisfy all constraints, it is area where budget and nutrition conditions are met.

(VI) Optimization goal is to minimize the fxⁿ, i.e cost within budget.

Optimization Techniques and ...

Unit 2

Optimization in mathematical programming covers a wide range of branches, each focusing on specific type of optimization problem.

Branches of mathematical programming:-

1. Linear programming
2. Non-linear programming
3. Quadratic programming
4. Integer programming
5. Semi-definite programming
6. mixed-Integer programming
7. Convex optimization
8. Dynamic programming
9. Stochastic programming
10. Combination optimization
11. meta-Heuristic optimization
12. Game theory
13. Goal programming
14. Global optimization
15. Robust optimization
16. Network optimization
17. Multi-objective optimization
18. Cutting-stock problem
19. Knapsack problem
20. Portfolio optimization

Classification of optimization problems:-

1. Based on existence of constraints:-
Constraint / Unconstraint

2. Based on nature of design variable:- Design variable is a function of one or more parameter, known as dynamic optimization problem.

3. Based on physical structure of problem:- Optimal and non-optimal control problems.

4. Based on nature of equation involved:- linear, non-linear, geometric and quadratic programming problems.

5. Based on permissible values of design variable:- Integer and real-valued programming problems.

6. Based on deterministic nature of variables:- Deterministic and stochastic

programming problems.

7. Based on separability of objective fxn and constraint separable and non-separable programming problems.

8. Based on number of objective functions:- Single and multi objective programming problem.

Optimization using Calculus:-

1. Single variable:-

It involves finding maximum or minimum of a fxn wrt single independent variable.

(I) Find the objective function.

(II) Find the derivative of objective fxn wrt single independent variable.

(III) Find the critical points by setting the derivative equal to zero.

(IV) Find whether each critical point corresponds to minimum/maximum or neither using first derivative test.

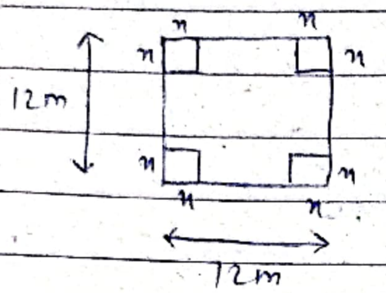
4.1: If derivative changes from -ve to +ve at a critical point, the function has a minimum at that point.

4.2: If derivative changes from +ve to -ve at a critical point, the function has a maximum at that point.

4.3: If derivative does not change sign at critical point, the test is inconclusive regarding whether there is a local extremum at that point.

(V) Interpret the result once the critical points and boundary values are determined. Compare the function values to the absolute maximum or minimum values and points.

An open box is made by cutting small squares from corners of 12X12 metred sheet. How large should these squares cut from the corner be, to make the box hold as much as possible?



maximize volume

$$V(x) = (12-x)^2(x)$$
$$= 144x - 48x^2 + 4x^3$$

$$V'(x) = 0$$
$$144 - 96x + 12x^2 = 0$$
$$(2-x)(6-x) = 0$$
$$x = 2 \text{ or } 6$$

$V' = +ve$ $V' = -ve$ $V' = +ve$

2   6

maximum at 2

Boundary values $0 \le x \le 6$

max point $x = 2m$

max volume $V = 128\,m^3$

2. multi-variable:-

(I) Define the objective function, i.e you want to optimize. The fxn typically represents a quantity that you want to minimize or maximize.

(II) Identify the constraints, if there are constraints on variable then only.

(III) Find partial derivatives: Compute all the derivatives required for Hessian matrix

$$H_f = \begin{bmatrix} \frac{d^2f}{dx^2} & \frac{d^2f}{dx \cdot dy} & \frac{d^2f}{dx \cdot dz} \\ \frac{d^2f}{dy \cdot dx} & \frac{d^2f}{dy^2} & \frac{d^2f}{dy \cdot dz} \\ \frac{d^2f}{dz \cdot dx} & \frac{d^2f}{dz \cdot dy} & \frac{d^2f}{dz^2} \end{bmatrix}$$

(IV) Set the partial derivative to 0, if evaluation points are not given.

(V) Find Hessian matrix and put points in matrix.

(VI) Compute determinant of H

$|H| = +ve$ potential extremum

$|H| = -ve$ saddle point

$|H| = 0$ need more analysis

(VII) Compute eigen values of H

- if all eigen values are positive → minimum
- if all eigen values are negative → maximum
- if eigen values are both +ve or -ve → saddle point
- if eigen value is zero or inconclusive → need more analysis

ques Find relative minimum or maximum of function

$f(x) = x^3+y^3+z^3 - 9xy - 9xz + 27x$

$\frac{df}{dx} = 0$

$3x^2 - 9y - 9z + 27 = 0$ — (I)

$\frac{df}{dy} = 0$

$3y^2 - 9x = 0$

$y^2 = 3x$ — (II)

$\frac{df}{dz} = 0$

$3z^2 - 9x = 0$

$z^2 = 3x$ — (III)

(II) and (III)

$y^2 = z^2 = 3x$

$y = \pm z$

putting $y = \pm z$ in (I)

$y = z$

$3x^2 - 18y + 27 = 0$

$y = -z$

$3x^2 - 9y + 9y + 27 = 0$

$3x^2 + 27 = 0$

$x^2 + 9 = 0$ → not possible

$\Rightarrow y = z$

solve $3x^2 - 18y + 27 = 0$

$x^2 - 6y + 9 = 0$ put (II)

$\frac{y^4}{3} - 18y + 27 = 0$

$y^4 - 54y + 81 = 0$

$y = 3,\ z = 3,\ x = 3$

evaluation point = (3, 3, 3)

$$H = \begin{bmatrix} 6x & -9 & -9 \\ -9 & 6y & 0 \\ -9 & 0 & 6z \end{bmatrix}$$

$$= \begin{bmatrix} 18 & -9 & -9 \\ -9 & 18 & 0 \\ -9 & 0 & 18 \end{bmatrix}$$

$|H| = 18[(18)^2 - 0] - (-9)[(-9)(18) - 0] + (-9)[0 - (-9)(18)]$

$= (18)^3 - (9\times9\times18) - (9\times9\times18)$
$= 18\times18\times9$
$= 2916$
$= +ve$

i.e potential extremum

$|H - \lambda I| = 0$

$$\begin{vmatrix} 18-\lambda & -9 & -9 \\ -9 & 18-\lambda & 0 \\ -9 & 0 & 18-\lambda \end{vmatrix} = 0$$

$(18-\lambda)[(18-\lambda)^2] + 9[-9(18-\lambda)]$
$-9[0 - (-9)(18-\lambda)] = 0$

$(18-\lambda)^3 - 81(18-\lambda) - 81(18-\lambda) = 0$

$-\lambda^3 + 54\lambda^2 - 810\lambda + 2916 = 0$

$\lambda = 9(2+\sqrt{2}),\ 18,\ 9(2-\sqrt{2})$
$=$ all +ve

hence, point of minimum

Linear programming:-

Optimization using
(i) Graphs
(ii) simplex method
(iii) Big-M method
(iv) 2-Phase method

Quadratic Programming

It is a type of optimization problem that involves the minimizing or maximizing quadratic objective function subject to linear to quadratic constraints.

1. objective function:-
It consists of quadratic terms in terms of decision variable. It can be either maximization or minimization problems.

2. Decision variable:-
Quadratic programming involves finding values for a set of decision variables that optimize the objective function while satisfying the constraints.

3. Constraints :- Quadratic programming can be linear or quadratic equations. These constraints define the feasible region within which the optimal solutions must lie.

4. Matrix formulation:- Quadratic programming problems are often formulated in matrix form where objective function and constraints are represented using matrices.

General mathematical formulation of quadratic programming problems :-

min / max $f(x)$

$= x^T Q x + c^T x$

subject to constraints

$Ax \le B$, $x \ge 0$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} \qquad c = \begin{bmatrix} \text{coeff of } x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1}$$

$$Q = \begin{bmatrix} \text{coefficient of } x_1^2 & \frac{1}{2}\left(\text{coefficient of } x_1x_2\right) \\ \frac{1}{2}\left(\text{coefficient of } x_1x_2\right) & \text{coefficient of } x_2^2 \end{bmatrix}$$

where,

x = decision variables

Q = coefficient matrix of quadratic terms

c = coefficient matrix of linear terms in $f(x)$

A = coefficient of constraints (LHS of constraints)

B = RHS of constraints

que: max $f(x)$

$= 3x_1^2 + 4x_2^2 + 2x_1x_2 - 2x_1 - 3x_2$

subject to $3x_1 + 2x_2 \le 6$

$x_1 + x_2 \le 2$

$x_1, x_2 \ge 0$.

$f(x) = x^T Q x + c^T x$

$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$Ax \le B$

$$\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

Integer programming:-

It is a type of optimization problem where decision variables are required to take integer values. It is an optimization problem where solution must be an integer.

Optimisation with discrete decision:- In this type of problem decision variables are required to take integer values rather than continuous values.

Objective fxn and constraints:- like linear programming, integer programming also involves optimizing an objective fxn subject to a set of constraints. Objective fxn and constraints can be linear or non-linear.

Integer programming problems can be categorized into two main parts:- Pure and mixed integer programming

In pure integer programming problem, all decision variables must takes integer values while in mixed integer programming problem, some variables can be continuous.

Solving integer programming problem can be more challenging than solving linear programming problems due to discrete nature of decision variables.

Integer feasibility:-
In some cases, finding feasible integer solution can be challenging specially when the problem involves complex constraints or non-convex objective fxn.

Semi-definite programming:-

It is a subfield of optimization that deals with optimizing a linear objective fxn subject to linear equality constraints and linear matrix inequality constraints where variables are symmetric matrices.

Semi-definite programming problem involves finding a symmetric matrix that satisfies certain linear matrix inequalities while optimizing a linear objective fxn.

$$\min \; Tr(C^T x)$$

subject to $Tr(A_i^T x) = B_i$

$i = 1, 2, \ldots, m$

Formulation of linear programming problem:-

Drawbacks of LPP :- we can only use two variables and the two are then known as decision variables.

$x \geq 0$, $y \geq 0$ always exists

que

A company produces P and Q items that requires gold and silver. Each unit of type P requires 4gm silver and 1gm gold. While that of Q requires 1gm silver and 3gm gold. The company can produce 8gm silver and 9gm gold. Suppose each unit of type P brings a profit of 44 and Q brings of 55. Formulate a LPP to maximize the profit.

Let x and y be the number of units of P and Q.

max $P = 44x + 55y$

$4x + y \leq 8$

$x + 3y \leq 9$

$x \geq 0, y \geq 0$

Graphical Method:-

(i) Draw the given constraints

(ii) Find all points of intersection

(iii) Put in Z and find min/max

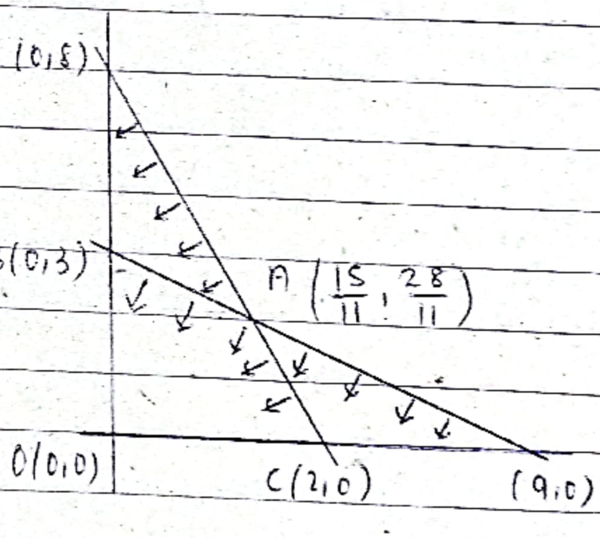
que. max $Z = 44x + 55y$

subject to constraints

$4x + y \leq 8$

$x + 3y \leq 9$

$x \geq 0, y \geq 0$



Points of intersection: A, B, O, C

$Z(A) = 200$

$Z(B) = 165$

$Z(O) = 0$

$Z(C) = 88$

hence, max $Z = 200$ at $\left(\frac{15}{11}, \frac{28}{11}\right)$

Slack Variables:- $\max z = x_1 + 2x_2$

$x_1 + x_2 \le 2$

$3x_1 + x_2 \le 4$

$x_1, x_2 \ge 0$

$\max z = x_1 + 2x_2 + 0x_3 + 0x_4$

$x_1 + x_2 + x_3 + 0x_4 = 2$

$3x_1 + x_2 + 0x_3 + x_4 = 4$

$x_1, x_2, \underbrace{x_3, x_4}_{\text{slack variables}} \ge 0$

Surplus Variables:- $\min z = x_1 + 2x_2$

$x_1 + x_2 \ge 4$

$3x_1 + x_2 \ge 2$

$x_1, x_2 \ge 0$

$\min z = x_1 + 2x_2 + 0x_3 + 0x_4$

$x_1 + x_2 - x_3 + 0x_4 = 4$

$3x_1 + x_2 + 0x_3 - x_4 = 2$

$x_1, x_2, \underbrace{x_3, x_4}_{\text{surplus variables}} \ge 0$

LP in standard form:-

Step1: all constraints should be converted to equations

Step2: RHS of constraints should be non-negative

$3x_1 + x_2 \le -4$

$-3x_1 - x_2 \ge 4$

$-3x_1 - x_2 - x_3 = 4$

Step3: All variables must have non-negative values

If x is unrestricted in sign, replace x with $x' - x''$ where x' and x'' are non-negative.

Step4: objective function should be of maximizing form.

$\min z = 3x_1 + x_2$

$\max z = -\min z$

$= -3x_1 - x_2$

que $\min z = x_1 - 2x_2 + x_3$

$2x_1 + 3x_2 + 4x_3 \ge -4$

$3x_1 + 5x_2 + 2x_3 \ge 7$

$x_1, x_2 \ge 0$ and

x_3 is unrestricted in sign.

$\max z = -x_1 + 2x_2 - (x_3' - x_3'') + 0x_4 + 0x_5$

$-2x_1 - 3x_2 - 4(x_3' - x_3'') + x_4 + 0x_5 = 4$

$3x_1 + 5x_2 + 2(x_3' - x_3'') + 0x_4 - x_5 = 7$

$x_1, x_2, x_3', x_3'', x_4, x_5 \ge 0$

most -ve in $z_j - c_j$

least +ve in min ratio, i.e -ve and 0 is not allowed.

Simplex : max = cost of extras is 0

Big-M method : max = cost of extras is 0
cost of artificials is $-M$

Two-phase method : Phase 1, max = cost of artificials is -1
cost of rest all is 0

Phase 2, max = cost of extras = 0
rest will have their original costs

Simplex method :-

ques max $z = 5x_1 + 3x_2$

$3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$

$x_1, x_2 \ge 0$

max $z = 5x_1 + 3x_2 + 0x_3 + 0x_4$

$3x_1 + 5x_2 + x_3 = 15$

$5x_1 + 2x_2 + x_4 = 10$

$x_1, x_2 \ge 0$

		c_j	5	3	0	0	
C_B	X_B	b	x_1	x_2	x_3	x_4	ratio
0	x_3	15	3	5	1	0	5
0	x_4	10	5 key	2	0	1	← 2
	$z_j - c_j$		-5 ↑	-3	0	0	
0	x_3	9	0	19/5 key	1	-3/5	← 45/19
5	x_1	2	1	2/5	0	1/5	5
	$z_j - c_j$		0	-1 ↑	0	1	
3	x_2	45/19	0	1	5/19	-3/19	
5	x_1	20/19	1	0	-2/19	5/19	
	$z_j - c_j$		0	0	5/19	16/19	

all $z_j - c_j \ge 0$, stop

$x_1 = 20/19$, $x_2 = 45/19$

max $z = 5x_1 + 3x_2$
$= 235/19$

Big-M Method:-

que: $\min z = x_1 + x_2$

$2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$

$\max z = -x_1 - x_2 + 0x_3 + 0x_4 - Mx_5 - Mx_6$

$2x_1 + x_2 - x_3 + 0x_4 + x_5 + 0x_6 = 4$

$x_1 + 7x_2 + 0x_3 - x_4 + 0x_5 + x_6 = 7$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

		c_j	-1	-1	0	0	-M	-M	
C_b	X_b	b	x_1	x_2	x_3	x_4	x_5	x_6	ratio
-M	x_5	4	2	1	-1	0	1	0	4
-M	x_6	7	1	7 key	0	-1	0	1	← 1
	$z_j - c_j$		1-3M	1-8M ↑	M	M	0	0	
-M	x_5	3	$13/7$ key	0	-1	$1/7$	1		← $21/13$
-1	x_2	1	$1/7$	1	0	$-1/7$	0		7
	$z_j - c_j$		$\frac{6-13M}{7}$ ↑	0	M	$\frac{1-M}{7}$	0		
-1	x_1	$21/13$	1	0	$-7/13$	$1/13$			
-1	x_2	$10/13$	0	1	$1/13$	$-2/13$			
	$z_j - c_j$		0	0	$6/13$	$3/13$			
			all $z_j - c_j \geq 0$, stop						

$x_1 = 21/13$ $\quad x_2 = 10/13$ $\quad \min z = x_1 + x_2$

$= 31/13$

Two-phase method:-

que: $\min z = x_1 + x_2$

$2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$

$\max z = -x_1 - x_2 + 0x_3 + 0x_4 - x_5 - x_6$

$2x_1 + x_2 - x_3 + 0x_4 + x_5 + 0x_6 = 4$

$x_1 + 7x_2 + 0x_3 - x_4 + 0x_5 + x_6 = 7$

Phase 1: $\max z = 0x_1 + 0x_2 + 0x_3 + 0x_4 - x_5 - x_6$

	c_j		0	0	0	0	-1	-1	
C_b	X_B	b	x_1	x_2	x_3	x_4	x_5	x_6	ratio
-1	x_5	4	2	1	-1	0	1	0	4
-1	x_6	7	1	7 key	0	-1	0	1	← 1
	$z_j - c_j$		-3	-8	1	1	0	0	
-1	x_5	3	13/7 key	0	-1	1/7	1		← 21/13
0	x_2	1	1/7	1	0	-1/7	0		7
	$z_j - c_j$		-13/7	0	1	-1/7	0		
0	x_1	21/13	1	0	-7/13	1/13			
0	x_2	10/13	0	1	1/13	-2/13			
	$z_j - c_j$		0	0	0	0			

all $z_j - c_j \geq 0$

Phase 2: $\max z = -x_1 - x_2 + 0x_3 + 0x_4$

	c_j		-1	-1	0	0
C_b	X_b	b	x_1	x_2	x_3	x_4
-1	x_1	21/13	1	0	-7/13	1/13
-1	x_2	10/13	0	1	1/13	-2/13
	$z_j - c_j$		0	0	6/13	1/13

all $z_j - c_j \geq 0$, stop

$x_1 = 21/13$ $\quad x_2 = 10/13$ $\quad \min z = x_1 + x_2$

$= 31/13$

Exceptional cases:-

1. multiple optimal solution:-

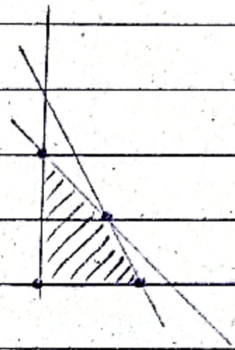
If slope of z is equal to slope of any constraints then multiple optimal solution exists.

$\max z = 10x_1 + 6x_2$

$5x_1 + 3x_2 \le 30$

$x_1 + 2x_2 \le 18$

$x_1, x_2 \ge 0$



$z_1 = 60$

$z_2 = 0$ same

$z_3 = 54$

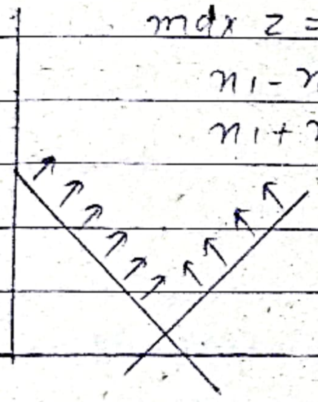
$z_4 = 60$

2. Unbounded solution

$\max z = 3x_1 + 2x_2$

$x_1 - x_2 \ge 1$

$x_1 + x_2 \ge 3$

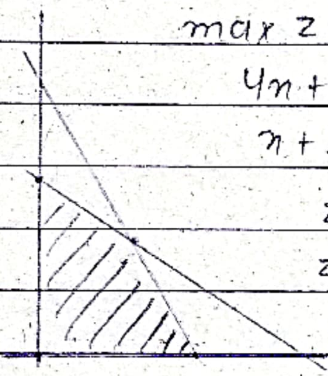


3. Feasible solution

$\max z = 44x + 55y$

$4x + y \le 8$

$x + 3y \le 9$



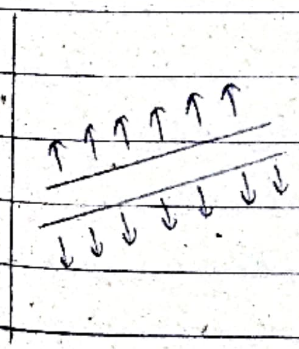
$z_1 = 200$ ✓ ans

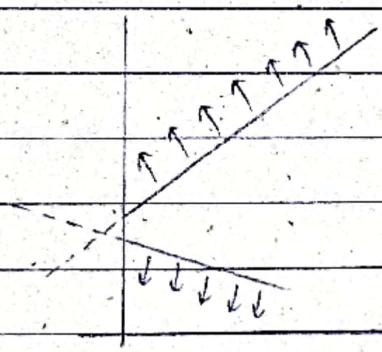
$z_2 = 165$

$z_3 = 88$

$z_4 = 0$

4. Infeasible solution





$\max z = 6x_1 - 4x_2$

$2x_1 + 4x_2 \le 4$

$4x_1 + 8x_2 \ge 16$

$x_1, x_2 \ge 0$

$\max z = x_1 + x_2$

$x_1 + x_2 \le 1$

$-3x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$

Optimization Techniques & Decision making

Unit 3

Genetic Algorithm:-

- Developed by John Holland.
- Abstraction of real biological evolution
- Solve complex problems such as NP hard, Travelling Salesman, 0/1 knapsack
- Focus on optimization
- Population of possible solutions for a given problem
- From a group of individuals, the best will survive

Phenotype (data in raw or real world form) $\xrightarrow{\text{Encode}}$ Genotype (data in computational form)

Genotype $\xrightarrow{\text{Decode}}$ Phenotype

Initial population

↓

Calculate fitness

↓

Selection

↓

Cross-over

↓

mutation

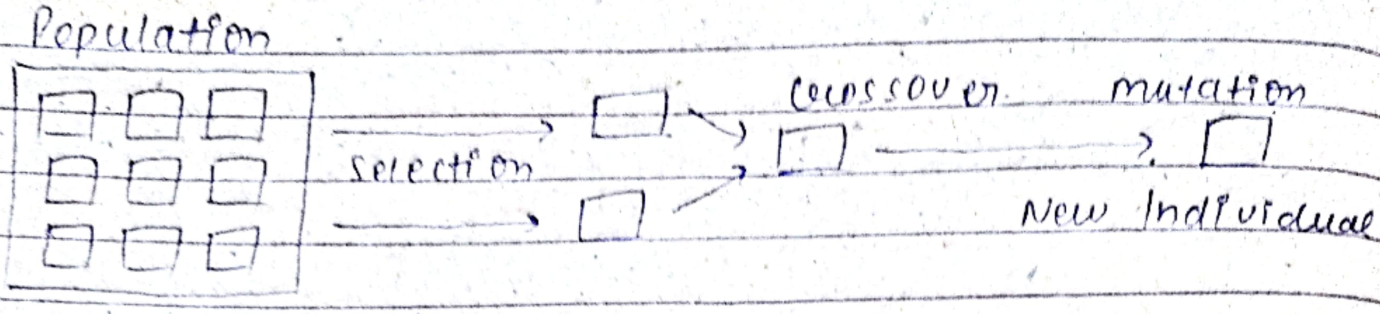
↓

Stopping Criteria — No → (back to Calculate fitness)

↓ Yes

Optimal Solution

Selection, Cross-over, mutation } Genetic operations



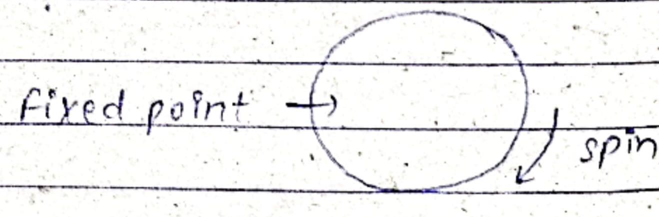
1. Selection :-

Parent selection is a process of choosing a set of chromosomes as a parent for the next generation of the population, based on its fitness value, threshold etc.

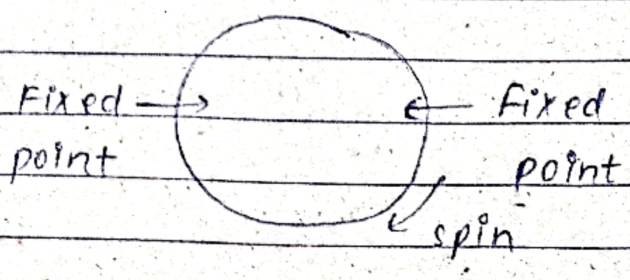
It can be done using
- Roulette wheel selection
- Stochastic universal selection
- Ranking selection
- Tournament selection
- Truncation selection

Roulette wheel selection :-

Chromonsome	Fitness
A	8.2
B	3.2
C	1.4
D	1.2
E	4.2



Stochastic Universal Sampling (SUS) Selection :-



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Ranking Selection:- Every individual is ranked according to their fitness. The higher ranked individuals are preferred more.

Tournament Selection:- Select k-individuals at random and select the best with highest fitness value to become a parent.

Truncation Selection:- Individuals are sorted according to their fitness and best are selected!

2. Crossover :-

A genetic operator used to combine the genetic information of two parents to generate new offspring.

One-point crossover:- A random crossover point is selected and tails of its two parents are swapped to get new offsprings.

012|3456 = 0124763
589|4763   5893456

multi-point crossover:- It is a generalization of one-point crossover where alternating segments are swapped to get new offsprings.

0123|456|789 = 0123235789
5894|235|758   5894456758

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Uniform crossover:- altering bits are swapped

0 1 2 3 4 5 = 0 8 2 4 4 3
5 8 9 4 2 3   5 1 9 3 2 5

3. mutation:-
Mutation is the part of genetic algorithm which is related to the exploration of the search space. It has been observed that mutation is essential to the covergence of genetic algorithm while crossover is not.

Binary mutation:- mutation operator changes 1 to 0 or vice versa with a mutation probability (generally kept low for steady convergence)

Bit Flip 1101000 before mutation
1100000 after mutation

Swap mutation:- 1 2 3 4 5 6 7
1 6 3 4 5 2 7

Scramble mutation:- 0 1 | 2 3 4 | 5   0 1 4 3 2 5 random

Inversion mutation:- 0 1 | 2 3 4 | 5   0 1 4 3 2 5 Inverse

Que: Consider the function of maximizing the function $f(x) = x^2$, where x is permitted to vary [0,31]

Step1:- Select encoding technique
value ∈ [0,31] = [00000, 11111]

Step2:- Select initial population randomly
size of population = 4

S. No.	Initial Population	X Value	Fitness $f(x) = x^2$	Prob	% Prob	Expected Count	Actual Count
1	01100	12	144	0.1247	12.47	0.4987	1
2	11001	25	625	0.5411	54.11	2.1645	2
3	00101	5	25	0.0216	2.16	0.0866	0
4	10011	19	181	0.3126	31.26	1.2502	1
Sum			1155	1.0	100	4	4
Avg			288.75	0.25	25	1	1
max			625	0.5411	54.11	2.1645	2

$$\text{Prob} = \frac{f(x)}{\sum f(x)}, \quad \% \text{ Prob} = 100 \times \text{Prob}$$

$$\text{Expected count} = \frac{f(x_i)}{avg(\sum f(x))} \quad \text{i.e} \quad \frac{144}{288.75} = 0.4987$$

Actual count = Round off of expected count to the nearest integer value.

						actual
For next step :-	01100	will	be selected		once	1
	11001	"	"	"	twice	2
	00101	"	not be	"	at all	0
	10011	"	be	"	once	1

Step 3:- Cross over

Select crossover points randomly, if not given.

S. No.	Mating Pool	Crossover Point	Offspring after Crossover	X value	Fitness $f(x) = x^2$
1	0110|0	4	01101	13	169
2	1100|1		11000	24	576
3	11|001	2	11011	27	729
4	10|011		10001	17	289
Sum					1763
Avg					440.75
Max					729

For original value, max fitness was 625.

Step 4:- Mutation

mutation chromosome for flipping = Choose randomly

S. No.	Offspring after Crossover	mutation chromosome	Offspring after mutation	X Value	Fitness $f(x) = x^2$
1	01101	10000	11101	29	841
2	11000	00000	11000	24	576
3	11011	00000	11011	27	729
4	10001	00101	10100	20	400
Sum					2546
Avg					636.5
Max					841

offspring after mutation = offspring after crossover (EXOR) mutation chromosome

Que maximize the value of function :- $f(x) = -x^2 + 2x$ over the range of real number from 0 to 2 with initial population 11010, 00111, 10110, 00101, with random number 0.4, 0.15, 0.7, 0.9

Step1 :- Encoding is already given.

Step2 :- Initial population is also given
size = 4

Decode individual into a real number

$$\text{real of binary} = \text{min range} + \left[\frac{\text{max range} - \text{min range}}{2^{\text{bits}} - 1}\right] \times \text{decimal of given binary}$$

$$11010 = 0 + \left(\frac{2-0}{2^5-1}\right) \times (26) = \frac{52}{31} = 1.677$$

$$00111 = 0 + \left(\frac{2-0}{2^5-1}\right) \times (7) = \frac{14}{31} = 0.451$$

$$10110 = 0 + \left(\frac{2-0}{2^5-1}\right) \times (22) = \frac{44}{31} = 1.419$$

$$00101 = 0 + \left(\frac{2-0}{2^5-1}\right) \times (5) = \frac{10}{31} = 0.322$$

S. No.	Initial Population	X Value	Fitness $f(x) = -x^2 + 2x$	Prob	Cumulative	Intervals of Random no.
1	11010	1.677	0.541	0.21	0.21	0 to 0.21
2	00111	0.451	0.699	0.27	0.48	0.22 to 0.48
3	10110	1.419	0.824	0.32	0.8	0.49 to 0.8
4	00101	0.322	0.541	0.2	1	0.81 to 1

Random Number	Region	Chosen String
0.4	0.22 to 0.48	00111
0.15	0 to 0.21	11010
0.7	0.49 to 0.8	10110
0.9	0.81 to 1	00101

Step 3 :- Crossover points → first and fifth digits

00111	0¦011¦1	01011	0.709
11010	1¦101¦0	10110	1.419
10110	1¦011¦0	10100	1.29
00101	0¦010¦1	00111	0.451

S. No.	New Population	X Value	Fitness $f(x) = -x^2 + 2x$	Prob.	Cumulative	Interval of RN
1	01011	0.709	0.915			
2	10110	1.419	0.824			
3	10100	1.29	0.915			
4	00111	0.451	0.699			

Previous max was 0.824
after first generation, max becomes 0.915

Applications :-
(i) Engineering design optimization (aerodynamic surfaces)
(ii) Robotics
(iii) Financial modelling
(iv) Game playing
(v) Bioinformatics

Advantages :- (I) Versatile
(II) Parallelism
(III) Population diversity
(IV) No gradients needed.
(V) Global search

Disadvantages :- (I) Computationally intensive
(II) Premature convergence
(III) Parameter sensitivity
(IV) Lack of adaptation
(V) Difficulty with high dimensions.

Limitations :- (I) Not suitable for continuous domains
(II) Limited scalability
(III) Knowledge requirement
(IV) Limited handling of constraints
(V) Inefficient for fine-tuning

Particle Swarm Optimization (PSO):-

Particle: Birds or fishes or either a solution are the particle

Swarm: Population of moving particles.

- PSO is inspired by the social behaviour of birds or fishes
- PSO is computational method that optimize a problem.
- PSO is applied in artificial neural network training and fuzzy control system.

Search Space:-

The range in which the algorithm computes ~~original~~ optimal control variable. If the optimal control value of the particle exceed the search space, the values will be re-initialized.

Strategy:-

(I) Suppose a group of birds is randomly flying and searching for food in an area and there is only one piece of food. Birds don't know where the food is, but they know the distance of food in each iteration.

(II) Best search strategy is to follow the birds nearest to food.

(III) Starting with the randomly initialized population and moving in randomly chosen direction, each particle goes through a search space and remembers its best previous position of itself and its neighbourhood.

Algorithm :-

(I) Initialize each particle randomly
(II) (a) Calculate fitness of each
(b) If fitness is better than P_{best}, set current value as P_{best}
(III) Choose particle with best fitness value of all as g_{best}
(IV) (a) Calculate velocity of each particle as

$$V_{i+1} = [w \times V_i] + [rand \times c_1 \times (P_{best} - x_i)] + [rand \times c_2 \times (g_{best} - x_i)]$$

$$x_{i+1} = x_i + V_{i+1}$$

(V) Update till max iteration or min error criteria obtained.

Que: $\max f(x) = x_1^2 + x_2^2 - x_1 x_2 + 2x_1 + 4x_2 + 3$
where $-5 \le x_1, x_2 \le 5$, population size = 5,
$c_1 = c_2 = 1.5$, max iteration = 2, weight = 0.9

Velocity $\in [0,1]$		Position $[-5,5]$		$f(x)$	P_{best}		
0.21	0.24	3.14	-4.02	31.96	3.14	-4.02	
0.19	0.22	4.05	-2.21	32.61	4.05	-2.21	
0.38	0.32	-3.7	0.46	13.20	-3.7	0.46	
0.39	0.35	4.13	4.57	48.68	4.13	4.57	= g_{best}
0.09	0.37	1.32	4.64	41.45	1.32	4.64	

0.34	0.32	3.47	-2.5	27.86	3.14	-4.02	
0.18	1.07	4.32	-1.14	31.03	4.05	-2.21	
1.35	0.81	-2.37	+1.28	13.75	-2.37	1.28	
0.35	0.31	4.49	4.89	53.70	4.49	4.89	= g_{best}
0.44	0.33	1.76	4.97	45.56	1.76	4.97	

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Ques max $f(x) = -x^2 + 2x + 11$ in range $-2 \le x \le 2$ using PSO, use 4-particles with initial position $x_1 = -1.5$, $x_2 = 0$, $x_3 = 0.5$, $x_4 = 1.25$, show detailed computation till 3^{rd} iteration, weight = 0.8, $c_1 = c_2 = 2.05$.

Velocity	Position	f(x)	Pbest	
0	-1.5	5.75	-1.5	
0	0	11	0	
0	0.5	11.75	0.5	
0	1.25	11.93	1.25	= gbest
3.38	1.88	11.22	1.88	
1.53	1.53	11.71	1.53	
0.92	1.42	11.82	1.42	
0	1.25	11.93	1.25	gbest
1.92	3.8 ≈ 2	11	1.88	
0.87	2.4 ≈ 2	11	1.53	
0.52	1.94	11.11	1.42	
0	1.25	11.93	1.25	gbest

Applications:-

(i) Engineering design (antenna design)
(ii) Control systems
(iii) Image processing
(iv) Data clustering
(v) Wireless Sensor Networks.

Advantages:- (i) Simple implementation
(ii) Fewer parameters
(iii) Fast convergence
(iv) Global search
(v) No gradients needed

Disadvantages:- (i) Premature convergence
(ii) Lack of diversity
(iii) limited handling of constraints
(iv) Sensitivity to parameters
(v) Stagnation

Limitations:- (i) limited memory
(ii) Poor handling of discrete variables
(iii) Difficulty with dynamic optimization
(iv) limited scalability
(v) Lack of robustness

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Ant Colony Optimization (ACO):-

- Ants can easily communicate with each other using pheromones.
- Pheromones are chemical signals released by ants in danger to alert other ants for help.
- Ants detect pheromones through their mobile antenn
- They leave pheromones on the soil that can be easily followed by other ants.
- ACO algorithm is a meta-heuristic algorithm which i inspired by social behaviour of ants, i.e pheromones based ant communication.
- It is developed by marco Dorigo in 1922.
- It is used to find optimal paths.
- most popular problem is Travelling salesman Problem.
- It can be applied to continuous optimization problem.

ACO algorithm is basically inspired by the foraging behaviour of ants searching for suitable paths between their colonies and food source.

How do ants find out the shortest path between colony and food?

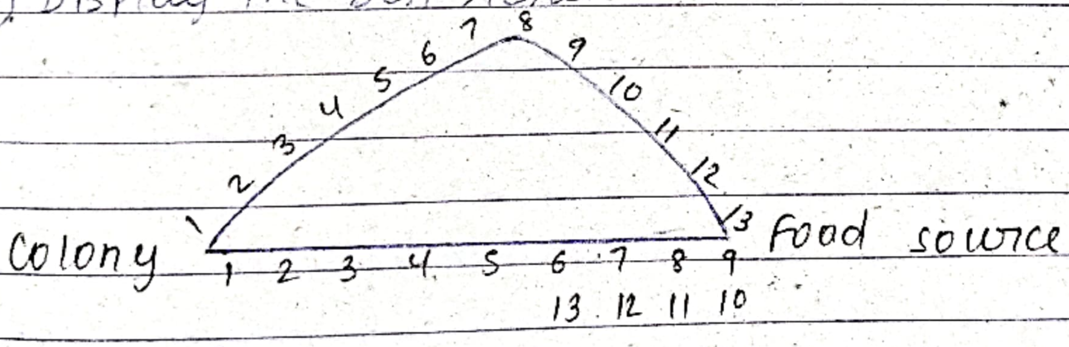
(i) with the help of pheromones, ants can easily find shortest path between colony and food.

(ii) Ants foraging behaviour
→ explore area randomly
→ chemical release
→ when ants find food source they choose the way having strong pheromones.

(iii) A group of ants foraging in the environment.
(iv) A forger finds food.
(v) Forger will mark trails on the way using pheromones while going back to the colony.
(vi) When food source is finished, no new marks on the way are marked by returning ants.

Algorithm:-

(i) Initialize ACO parameters.
(ii) Ant solution construction.
(iii) Position each ant in the stating node.
(iv) Each ant will select next node by applying state transition rule.
(v) Repeat until ant build the best solution, then compute the fitness value.
(vi) Update best solution.
(vii) Apply offline pheromone update.
(viii) Display the best result.



→ high pheromone signals
→ ants will follow this.

1. All ants are in the colony. There is no pheromone marks on the ground.

2. Foraging starts :-
    50% ants will take shortest path
    50% ants will take longest path
3. Ants used shortest path to arrive earlier to the food source.
4. Pheromone marks on the shorter paths have strong pheromone signals. Probability of this path selected by other ants increases.

Applications :-
(I) Travelling Salesman Problem
(II) Network Routing Algorithm
(III) Job Scheduling Algorithm
(IV) Resource allocation
(V) Logistics optimizations.

Advantages:- (I) Robustness

(II) Global search

(III) Distributed nature

(IV) Adaptability

(V) Suitable for combinatorial problems

Disadvantages:- (I) Slow convergence

(II) Complexity

(III) Sensitivity to parameters
(IV) Memory consumption
(V) Exploration - Exploitation tradeoff

Limitations:- (I) Lack of guaranteed convergence.
(II) Limited appilcability
(III) Sensitivity to problem representation
(IV) Difficulty with dynamic environments
(V) Performance dependency on problem characteristics.

Similarities in the three:-

(I) Nature inspired:- Evolution, birds and ants.
(II) Population based:- Maintains a population of individual solutions.
(III) Global search:- Explores search space efficientey.
(IV) Stochastic optimization:- Randomness & Probabilitic components.
(V) Metaheuristic methods:- General-purpose algorithms applicable to a wide range of optimization algorithms.

Real life problems and their mathematical formulation as standard programming problems:-

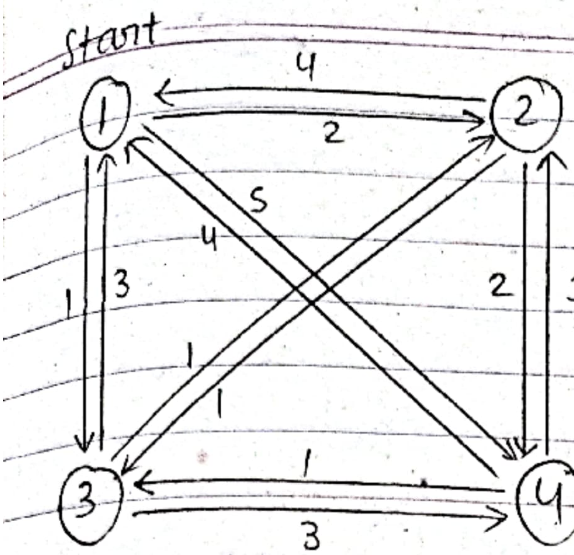
1. Travelling Salesman Problem:-

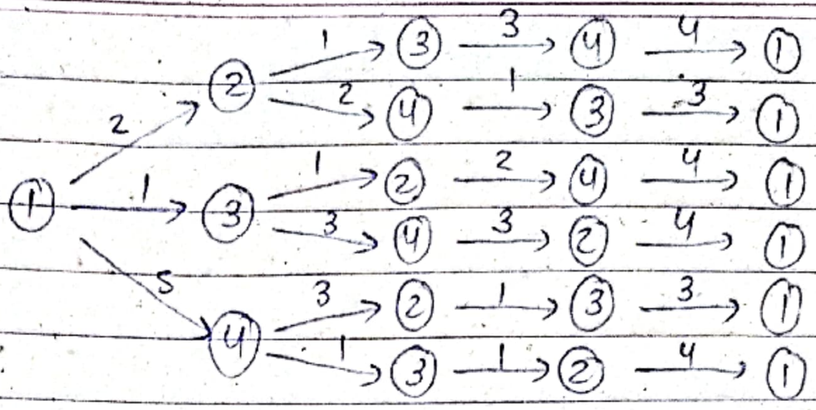
Salesman want to visit each city exactly once for minimum travelling distance.

TASK:- (i) find shortest route

(ii) Visit each city exactly once

(iii) Return to the current city.





$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & & & & \\
2 & & & & \\
3 & & & & \\
4 & & & &
\end{array}
\quad
\begin{array}{c}
 \\ 1 \\ 2 \\ 3 \\ 4
\end{array}
\begin{array}{c}
\quad 1 \quad\; 2 \quad\; 3 \quad\; 4 \\
\begin{bmatrix} 0 & 2 & 1 & 5 \\ 4 & 0 & 1 & 2 \\ 3 & 1 & 0 & 3 \\ 4 & 3 & 1 & 0 \end{bmatrix}
\end{array}
$$

$$T(1,\{2,3,4\}) = \min \begin{cases} (1,2) + T(2,\{3,4\}) = 5 \\ (1,3) + T(3,\{2,4\}) = 4 \checkmark \\ (1,4) + T(4,\{2,3\}) = 7 \end{cases}$$

$$T(2,\{3,4\}) = \min \begin{cases} (2,3) + T(3,4) = 4 \\ (2,4) + T(4,3) = 3 \checkmark \end{cases}$$

$$T(3,\{2,4\}) = \min \begin{cases} (3,2) + T(2,4) = 3 \checkmark \\ (3,4) + T(4,2) = 6 \end{cases}$$

$$T(4,\{2,3\}) = \min \begin{cases} (4,2) + T(2,3) = 4 \\ (4,3) + T(3,2) = 2 \checkmark \end{cases}$$

2. Assignment Problem :-

Assigning task to the workers to minimize the total completion time.

$\min \sum C_{ij} \times x_{ij}$ x = binary variable
0 = not assigned
1 = assigned
c = time taken to solve a task

3. Knapsack problem :-

You are given a set of items, each with a weight and a value, and a knapsack with a maximum weight capacity.

The goal is to determine the combination of items to include in the knapsack that maximizes the total value while not exceeding the weight capacity.

It can be categorized into two parts :-

(i) Fractional Knapsack, you can take fractional part of item. It can be solved using greedy approach.

(ii) 0/1 Knapsack, where you can either take an item or leave it. It can be solved using branch and bound technique.

4. Shortest Path algorithm :-
It involves finding the shortest path between two nodes in a weighted graph.

There are various algorithms to solve this problem, with Dijkstra's and Bellman Ford algorithm.

Dijkstra algorithm:- It is a dynamic programming based algorithm and is most popular for non-negative weighted edges.

Bellman-Ford algorithm:- It is a dynamic programming based algorithm and can handle graphs with negative edge weights, as well as detect negative weight cycles.

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Decision making:-

Decision making is the process of choosing actions that are directed towards the resolution.

It involves thinking and deciding before doing and so is inherent in every activity.

Decisions maybe (I) routine : repititive nature
(II) strategic : requires systematic and scientific analysis.

Significance of Decision making:-

Decision making is the cornerstone of progress, achievement and adaptation. It empowers individuals and shapes the destinies of organizations.

(I) Guides us towards goals
(II) Problem solving
(III) Resource allocation and management
(IV) Empowers us
(V) learning and growth
(VI) Strategic direction
(VII) Innovation
(VIII) Risk mitigation

Types of decision making:-

	Programmed decisions	Non-programmed decisions
Nature of Problem	Structured / Routine / Well-defined	Unstructured / Novel / Ill-defined
Recurrence of problem	Repititive	Non-Repititive
Method of solving	Policies / Rules	Managerial Initiativ
Judgement	Objective	Subjective.
Probability of outcome	Some degree of certainty is involved	Uncertain
Level of management	middle / low level	Top-level
Types	Organisational / Operational / Research / Opportunity	Personal / Crisis Intuitive / Problem solving

Models of decision making:-

(i) Contingency:- Based on specific situation

(ii) Economic man:- Focuses on maximizing benefits and acting rationally.

(iii) Administrative:- Acknowledges limitations in rationality and prioritizes finding an acceptable solution.

(iv) Social:- Considers social norms and group influences on decisions.

Techniques of decision making:-

1. Traditional techniques:-
(for programmed and non-programmed)
(i) Habits
(ii) Operating procedures
(iii) Organization structure

2. Modern techniques:-

for programmed decisions:-
(i) Break even techniques
(ii) Inventory models
(iii) linear programming
(iv) Simulation
(v) Probability theory
(vi) Decision tree
(vii) Queing theory
(viii) Gaming theory
(ix) Network theory

for non-programmed decisions:-
(i) Creative techniques
(ii) Brainstorming
(iii) Participative technique
(iv) Heuristic techniques
(v) Decision support system
(vi) Expert systems

Basic steps in decision analysis:-

1. Identify the decision problem:-
Identify the nature of decision we want to make.

eg:- Which mode of transportation to take to work?

2. Gather relevant information:-
Objective, constraint, uncertainty.
Collect some information before you make decision, what info is needed, best source of information and how to get it.

eg:- Consider factors like cost, time, convenience.

3. Identify the alternatives:-
You will probably identify several possible path of action. Here, you will list all possible and desirable alternatives

eg:- Various transportation options such as driving, taking the bus, biking or walking.

4. Weigh the evidence:-
Draw your information - what would it be if you carried out each of the alternatives to the end. Evaluate whether the need identified in step 1 could be met or resolved through the use of each alternative.

eg:- Driving: Expensive, flexible time, convenient
Bus: Cost-effective, bus timetable, less-convenient
Biking: Cost-effective, flexible time, convenient

5. Choose among alternatives:-
Once you have evaluated all the evidence, you are ready to select the alternative that seems to be the best one. You may even choose combination of alternatives.

eg:- Biking is best option for you.

6. Take action:-
Now you are ready to take some action to implement the alternative you choose.

eg:- You start biking to work as it's cost-effective, flexible and convenient.

7. Review your decision and its consequences:-
In the final step, consider the result of your decision and evaluate whether or not, it has resolved the need you identified in step 1. If the decision has not meet the identified need you may want to repeat certain steps.

eg:- Biking has been a positive experience overall.

Decision-making environments:- conditions in which decisions are made.

1. Decision making under certainity:-
"In a certainity environment, decision maker knows the complete certainity of the outcome associated with each alternative.

Simplest decision making environment but relatively rare in real world situation.

In another words, in decision making environment of certainity, all relevant information is known and outcomes of alternatives are perfectly predictable.

Eg:- Store manager knows that spending on marketing will lead to higher sales.

2. Decision making under uncertainity:-

In uncertainity environment, decision maker does not have complete information about conferences or probabilities associated with each alternatives.

It is characterized by lack of information about future events.

Inother words, in an uncertain decision making environment, there is incomplete or ambiguous information and outcomes of alternatives are not known.

States.

$$\begin{array}{ll} & \begin{array}{cccc} S1 & S2 & S3 & S4 \end{array} \\ \text{Decisions} \begin{array}{c} D1 \\ D2 \\ D3 \end{array} & \begin{bmatrix} 3 & 5 & 8 & -1 \\ 6 & 5 & 2 & 0 \\ 0 & 5 & 6 & 4 \end{bmatrix} \end{array}$$

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(i) minimax
(max of states - states)

	S1	S2	S3	S4	max
D1	3	0	0	5	5
D2	0	0	6	4	6
D3	6	0	2	0	6

minimax = 5
D1

(ii) maximin

	S1	S2	S3	S4	min
D1	3	5	8	-1	-1
D2	6	5	2	0	0
D3	0	5	6	4	0

maximin = 0
D2, D3

(iii) maximax

	S1	S2	S3	S4	max
D1	3	5	8	-1	8
D2	6	5	2	0	6
D3	0	5	6	4	6

maximax = 8
D1

(iv) Laplace

	S1	S2	S3	S4	avg
D1	3	5	8	-1	3.75
D2	6	5	2	0	3.25
D3	0	5	6	4	3.75

3.75
D1, D3

(v) Hurwicz α = given
for each decision, find $(\alpha \times max) + [(1-\alpha) \times (min)]$

	S1	S2	S3	S4	max	min	0.4 max + 0.6 min
D1	3	5	8	-1	8	-1	$(0.4)(8) + (0.6)(-1) = 2.6$
D2	6	5	2	0	6	0	$(0.4)(6) + (0.6)(0) = 2.4$
D3	0	5	6	4	6	0	$(0.4)(6) + (0.6)(0) = 2.4$

$\alpha = 0.4$

2.6
D1

Eg:- weather forecasting
Stock market investments

3. Decision making under risk:-

Decision maker has partial information about outcome and their associated probabilities.

While decision maker may not know the exact outcome, they have some information that allows them to termi--nate the probabilities associated with different outcomes.

In other words, in decision making environment of risk, there is uncertainity about the outcomes but the probabilities of different outcomes are known or can be estimated.

	S1	S2	S3	S4
D1	3	5	8	-1
D2	6	5	2	0
D3	0	5	6	4
Prob	0.6	0.1	0.2	0.1

(i) Expected monetary Value:-

$3(0.6) + 5(0.1) + 0.2(8) + 0.1(-1) = 3.8$

$6(0.6) + 5(0.1) + 0.2(2) + 0.1(0) = 4.5$ ✓

$0(0.6) + 5(0.1) + 0.2(6) + 0.1(4) = 2.1$

D2

(ii) Expected Opportunity Loss (EOL):-

(max of states - states)

	S1	S2	S3	S4
D1	3	0	0	5
D2	0	0	6	4
D3	6	0	2	0
Prob	0.6	0.1	0.2	0.1

$3(0.6) + 0 + 0(0.2) + 5(0.1) = 2.3$

$0(0.6) + 0 + 6(0.2) + 4(0.1) = 1.6$

$6(0.6) + 0 + 2(0.2) + 0(0.1) = 4$ ✓

D3

(iii) Expected Value of Perfect Information (EVPI):-

$$\begin{array}{c|cccc} & S1 & S2 & S3 & S4 \\ \hline D1 & 3 & 5 & 8 & -1 \\ D2 & 6 & 5 & 2 & 0 \\ D3 & 0 & 5 & 6 & 4 \\ & 0.6 & 0.1 & 0.2 & 0.1 \\ max & 6 & 5 & 8 & 4 \end{array}$$

$6(0.6)+5(0.1)+8(0.2)+4(0.1)$

$-\text{EMV}$

$= 6.1 - 4.5$

$= 1.6$

eg:- (i) Project management
(ii) Business managing investments

Utility theory:-

It is a concept used in decision theory to model and quantify the preferences of decision makers.

According to utility theory, individual makes decision by maximizing their expected utility (EU), where utility represent the subjective value or satifaction derived from different functions.

Utility functions ($U(x)$) are used to represent individual preferences and attitudes towards risk.

By accessing the utility and different outcomes, decision maker can choose the alternatives that maximize their expected utility.

$$EU(A) = \sum_{i=1}^{n} U(x_i) \times P(x_i)$$

eg:- $U(x) = \sqrt{x}$

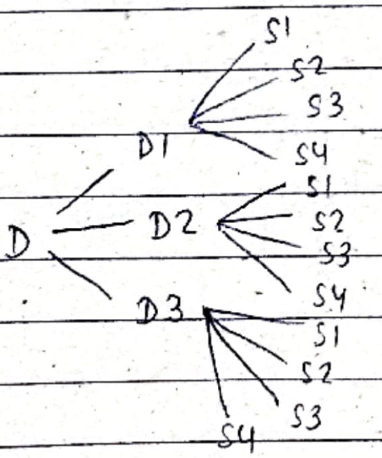
A : outcome 1   10,000 $P_1 = 0.6$
outcome 2   5000 $P_2 = 0.4$

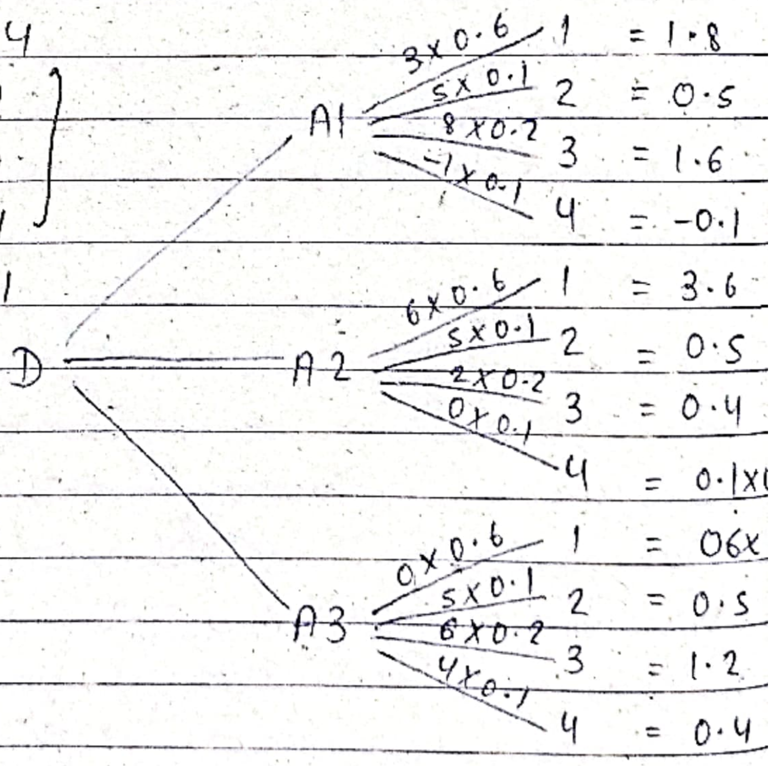
$$EU(A) = (10000)^{1/2} \cdot (0.6) + (5000)^{1/2} \cdot (0.4)$$
$$= 88.28$$

Decision Tree :-

It is a graphical representation of a decision problem that involves sequential decisions and uncertain outcomes. It consists of
- nodes represent decision points
- branches represent possible outcomes

$$\begin{array}{c|cccc} & S1 & S2 & S3 & S4 \\ \hline D1 & 3 & 5 & 8 & -1 \\ D2 & 6 & 5 & 2 & 0 \\ D3 & 0 & 5 & 6 & 4 \\ \text{Prob} & 0.6 & 0.1 & 0.2 & 0.1 \end{array}$$





Sum =>

A1 = 3.8   A2 = 4.5   A3 = 2.1

Group Decision making (GDM):-

It is defined as decision situation in which there are more than one individual involved. These group members have their own attitudes and motivation, energize the existence of a common problem and attempt to reach a collective decision. Moving from single decision making to multiple decision making introduces a great deal of complexity into analysis.

Group decision methods:-

(I) Brainstorming
(II) Ranking group technique
(III) Surveys and questionnaires
(IV) Multi-voting
(V) Analytic hierarchy process
(VI) Consensus decision making
(VII) Group decision support system

Types of GDM:-

1. Content oriented approach:-
- focuses on content of problem.
- Makers use their expertize to evaluate content of decision to identify most suitable solution.
- Course of action includes collecting all the relevant data then working on it and judging different outcomes.
- Eg:- Businesses (market, needs, loss)
  Healthcare (past records, guidelines)

2. Process oriented approach:-

- Focuses on "how decisions are made"
- Methods, procedures and steps involved in decision making or problem solving
- Makers specially take care of decision making roles, communication channels, collaboration mechanism and decision making techniques.
- eg:- (I) Project management
  (II) Organisational governance

Multi-criteria Decision Making (MCDM):-

- When there are multiple factors to consider, each with its own weight or importance
- Helps in decision making within alternative options based on different criterias.

Steps of MCDM:- (I) Identify criteria
(II) Define alternatives
(III) Weighing criteria
(IV) Evaluate alternative
(V) Aggregation of criteria
(VI) Sensitivity analysis
(VII) Decision making

Methods:- (I) Weighted sum method
(II) Analytic Hierarchy Process (AHP)
(III) Preference Ranking Method

Applications:- Project selection, supplier selection, Product design, Investment decision, Healthcare