Note: Q. 1 is compulsory.

Q1		(2.5*4)	CO
	(a) Explain Optimization, and its applications in Engineering		CO1
	(b) Depending on whether a particular point belongs to the acceptable or unacceptable region, it can be identified as one of the four types. Define and explain these types?		CO1
	(c) Explain the merits and limitations of the graphical method?		CO2
	(d) Explain decision variables, objective function, and constraints. Write an LPP to illustrate these terms?		CO2

Q2	(Attempt any Two Parts) UNIT-1 (CO1)	(5,5)
	(a) A soft drink manufacturing company has 300 ml and 150 ml canned cola as its products with profit margin of Rs. 4 and Rs. 2 per unit respectively. Both the products have to undergo process in three types of machine. The following data indicates the time required on each machine and the available machine-hours per week. Formulate the optimization problem as an LPP to maximize the total profit considering the limited resources.	

Requirement	**Cola 300 ml**	**Cola 150 ml**	**Available machine hours per week**
Machine 1	3	2	300
Machine 2	2	4	480
Machine 3	5	7	560

	(b) Discuss briefly about multiple and unbounded optimization Linear Programming Problems. Use appropriate example to justify your answer.	
	(C) Explain in detail the steps involved in formulating problems as mathematical programming problems? Explain the process, including the translation of design objectives and constraints into mathematical formulations	

Q3	(Attempt any Two Parts) UNIT-2 (CO2)	(5,5)
	(a) An airplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine using a graphical method how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?	
	(b) Use Simplex method to solve the following LP problem Maximize Z = 50x + 60y subject to: 2x + y <=300 ; 3x + 4y <=509; 4x + 7y <=812; x, y >=0.	
	(C) Write the algorithm to solve LPP using the simplex method OR explain the Integer Programming Problems in Optimization with an example.	

OTDM SOLUTION

SOLUTION OF 2a)

Let x_1 be the number of units of 300 ml cola and x_2 be the number of units of 150 ml cola to be produced respectively. Formulating the given problem, we get

$$Max.Z = 4x_1 + 2x_2$$

Subject to:

$$3x + 2x \leq 300$$
$$2x_1 + 4x_2 \leq 480$$
$$5x_1 + 7x_2 \leq 560$$
$$x_1, x_2 \geq 0$$

SOLUTION OF 3a)

Let x, y denote the number of executive class tickets and economy class tickets sold resp.

Since, aeroplane can carry maximum 200 passengers.

$\therefore x + y \leq 200$ ...(1)

Since, at least 20 tickets is reserved for executive class.

$\therefore x \geq 20$ ...(2)

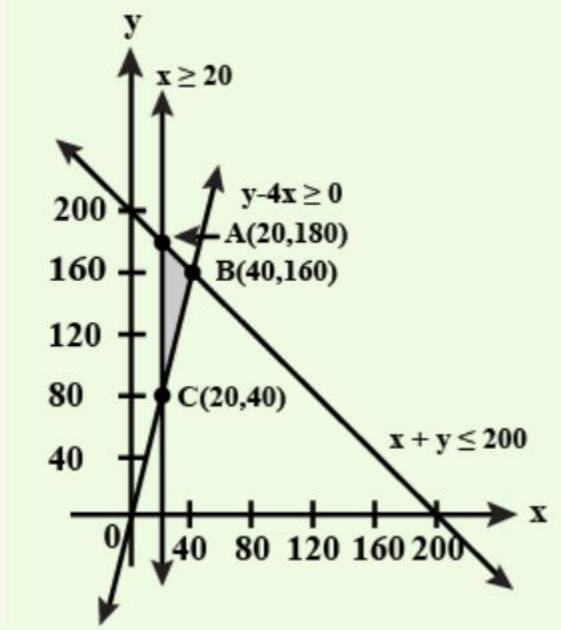
Since the number of tickets for economy class should be at least 4 times the executive class.

$\therefore y \geq 4x \Rightarrow y - 4x \geq 0$ ...(3)

Also, the number of tickets can't be negative. So, $x, y \geq 0$ ...(4)

Profit on an executive class ticket is 1000 Rs and profit on an economy class ticket is 600 Rs

So, Objective function is Maximize Z=1000x + 600y



Corner points	Value of Z = 1000X + 600Y
A (20, 180)	128000
B (40, 160)	136000 (Maximum)
C (20,80)	68000

We have to maximize the total profit. After plotting all the constraints given by equation (1), (2), (3) and (4), we get the feasible region as shown in the image above.

Hence, **Maximum profit will be 136000 Rs**, when number of executive class and economy class tickets sold will be 40 and 160 respectively.

Q3 b)

Use Simplex method to solve the following LP problem

Maximize Z = 50x + 60y subject to: 2x + y <=300 ; 3x + 4y <=509; 4x + 7y <=812; x, y >=0.

To solve this LPP using Simplex, let's first rewrite the problem in standard form:

Maximize Z = 50x + 60y Subject to:

1. 2x + y + s1 = 300
2. 3x + 4y + s2 = 509
3. 4x + 7y + s3 = 812
4. x, y, s1, s2, s3 >= 0

Basic Variables	x	y	s1	s2	s3	RHS
s1	2	1	1	0	0	300
s2	3	4	0	1	0	509
s3	4	7	0	0	1	812
Z-row	-50	-60	0	0	0	0

After the first iteration:

Basic Variables	x	y	s1	s2	s3	RHS
y	1	2	1/2	0	0	150
s2	0	1	-3/2	1	0	259
s3	0	1	-2	0	1	412
Z-row	0	10	0	0	0	3000

Now, since all coefficients in the Z-row are non-negative, we have reached the optimal solution.

Solution: x = 150 y = 259 , Z = 3000

Simplex method

The Simplex method is an approach to solving LPP involving 2 or more decision variables by hand using slack variables, tableaus, and pivot variables as a means to finding the optimal solution. Following steps are necessary:

- Standard form
- Introducing slack variables
- Creating the tableau
- Pivot variables
- Creating a new tableau
- Checking for optimality
- Identify optimal values

Basic variables:

Are the variables which coefficients <u>One</u> in the equations and <u>Zero</u> in the other equations.

Non-Basic variables:

Are the variables which coefficients are taking any of the values, whether positive or negative or zero.

<u>Slack, surplus & artificial variables:</u>

a) If the inequality be (less than or equal, then we add a slack variable + S to change to =.

b) If the inequality be (greater than or equal, then we subtract a surplus variable - S to change to =.

c) If we have = we use artificial variables.

ALGORITHM OF SIMPLEX METHOD:

<u>Step 1:</u>
Determine a starting basic feasible solution.

<u>Step 2:</u>
Select an entering variable using the optimality condition. Stop if there is no entering variable.

<u>Step 3:</u>
Select a leaving variable using the feasibility condition.

Optimality condition:

The entering variable in a maximization (minimization) problem is the non-basic variable having the most negative (positive) coefficient in the Z-row.
The optimum solution is reached at the iteration where all the Z-row coefficient of the non-basic variables are non-negative (non-positive).

<u>Feasibility condition:</u>

For both maximization and minimization problems the leaving variable is the basic associated with the smallest non-negative ratio (with strictly positive denominator).

<u>Pivot row:</u>

a) Replace the leaving variable in the basic column with the entering variable.
b) New pivot row equal to current pivot row divided by pivot element.
c) All other rows:
   New row=current row - (pivot column coefficient) *new pivot row

LIMITATION OF GRAPHICAL METHOD

The main limitation of graphical method for solving LPPs are that it is only applicable to problems with two decision variables and becomes awkward and impractical as the number of variables or constraints increases.
Additionally, it is not suitable for problems with three or more variables, yields approximate rather than precise results, and can be challenging to interpret if the feasible region is not convex or the objective function isn't linear.

1. **Limited to two variables:**

The graphical method relies on plotting constraints on a 2D plane (x-y axes), making it impossible to visualize problems with more than two decision variables.

2. Impractical for larger problems:

Even for two-variable problems, the method becomes tedious and difficult to manage as the number of constraints increases.

3. Results can be approximate:

The solution obtained from a graph is often an estimate, and precision depends on the accuracy of the drawing.

4. Not applicable to problems with three or more variables:

For LPPs with three or more decision variables, more advanced methods like the Simplex method are required.

5. Difficulty with non-convex or non-linear scenarios:

While the graphical method is for linear programming, it's difficult to find the optimal solution if the feasible region is not convex or the objective function is not strictly linear.

6. May not have a feasible solution:

In some cases, the constraints can be contradictory, resulting in no overlapping area on the graph and thus no feasible solution for the problem.

7. Multiple optimal solutions:

If the objective function line is parallel to one of the boundaries of the feasible region, there can be multiple optimal solutions, which the graphical method can identify but may be difficult to work with in complex cases.