Jr. Inter physics important problems

1. **The error in measurement of radius of a sphere is 1%. What is the error in the measurement of volume?**

   Sol: Error in the radius of sphere, $\frac{\Delta r}{r}$ x 100 = 1%

   Volume of the sphere, V = $\frac{4}{3}\pi r^3$;

   Error in the measurement of volume $\frac{\Delta V}{V}$x100 = 3$\left(\frac{\Delta r}{r} \text{ x } 100\right)$ = 3x1% =**3%**

2. **The percentage error in the mass and speed are 2% and 3% respectively. What is the maximum error in kinetic energy calculated using these quantities?**

   Sol: $\frac{\Delta m}{m}$x100 = 2% $\frac{\Delta V}{V}$x100 = 3%

   K.E = $\frac{1}{2}mv^2$; Max .error in K.E $\frac{\Delta E}{E}$x100 = $\frac{\Delta m}{m}$x100 + 2$\left(\frac{\Delta v}{v} \text{ x } 100\right)$

   $\frac{\Delta E}{E}$x100 = 2% + 2(3%) = **8%**

3. **A man walks on a straight road from his home to a market 2.5kmph. finding the market closed, he instantly turns and walks back home with a speed of 7.5kmph. what is the (a)magnitude of average velocity and (b) average speed of the man over the time interval 0 to 50 min?**

   Sol: **a)** Average velocity = $\frac{\text{total displacement}}{\text{total time}} = \frac{\text{Zero}}{t}$ = **0 (Zero)**

   b) Average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{2.5+2.5}{t_1+t_2} = \frac{5}{\frac{2.5}{5}+\frac{2.5}{5}}$ = **6kmph.**

4. **A car travels the first third of a distance with a speed of 10kmph, the second third at 20kmph, and the last third at 60kmph. What is its mean speed over the entire distance?**

   Sol: Average speed = $\frac{\text{total distance}}{\text{total time taken}}$

   Let the total distance = X km

   Time taken to travel first third distance $t_1 = \frac{\text{distance}}{\text{time}} = \frac{x}{3x10}$

   Time taken to travel second third distance $t_2 = \frac{x}{3x20}$

   Time taken to travel last third distance $t_3 = \frac{x}{3x60}$

   Total time taken T = $t_1+t_2+t_3 = \frac{x}{3x10} + \frac{x}{3x20} + \frac{x}{3x60} = \frac{x}{3}\left(\frac{1}{10}+\frac{2}{20}+\frac{1}{60}\right) = \frac{x}{3}\left(\frac{6+3+2}{60}\right) = \frac{x}{3}\left(\frac{10}{60}\right)$

   T = $\frac{x}{18}$ s

   ∴ Average or mean speed = $\frac{\text{total distance}}{\text{total time taken}} = \frac{X}{x/18}$ = **18kmph**

   (OR)

   Average speed V = $\frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1} = \frac{3(10x20x60)}{10x20+20x60+60x10} = \frac{36000}{200+1200+600} = \frac{36000}{2000}$ = **18kmph**

5. **A bullet moving with a speed of 150 m/s strikes a tree and penetrates 3.5cm before stopping. What is the magnitude of its retardation in the tree and the time taken for it to stop after striking the tree?**

   Sol: From $V^2 - U^2 = 2aS$

   U = 150m/s; V = 0 S = 3.5cm = $3.5x10^{-2}$m

   Retardation a = $\frac{U^2}{2s} = \frac{(150)^2}{2x3.5x10^5}$ = **$3.214x10^5 m/s^2$**

   From v=u+at

   Time taken to comes to rest t = $\frac{u}{a} = \frac{150}{3.21x10^5}$ = **$4.67 \text{ x } 10^{-4}$ s**

6. **Two balls are projected from the same point in directions 30^0 and 60^0 with respect to the horizontal. What is the ratio of their initial velocities if they (a) attain the same height? (b) have the same range?**

   Sol: **(a)** Let u_1 and u_2 be their initial velocities.

   Given, maximum height of 1st body = maximum height of 2nd body

$\Longrightarrow \frac{u_1^2 \sin^2 30^0}{2g} = \frac{u_2^2 \sin^2 60^0}{2g} \Longrightarrow u_1^2 x \frac{1}{4} = u_2^2 x \frac{3}{4} \Longrightarrow u_1^2 x1 = u_2^2 x3 \Longrightarrow \frac{u_1^2}{u_2^2} = \frac{3}{1} \Longrightarrow \frac{u_1}{u_2} = \frac{\sqrt{3}}{1}$

(b) Range of 1st body = Range of 2nd body

$\Longrightarrow \quad \frac{u_1^2 \sin(2x30^0)}{g} = \frac{u_1^2 \sin(2x60^0)}{g} \Longrightarrow u_1^2 \sin 60^0 = u_2^2 \sin 120^0 \Longrightarrow u_1^2 x \frac{3}{4} = u_2^2 x \frac{3}{4} \Longrightarrow u_1^2 = u_2^2 \Longrightarrow u_1 = u_2$

$\Longrightarrow \mathbf{u_1 : u_2 = 1:1}$

7. A car moving along a straight high way with speed of 126km/h is brought to a stop within a distance of 200m. what is the retardation of the the car (assumed uniform) and how long dose it take for the car to stop?

<u>**Sol**</u>: given u = 126km/h = 126 x $\frac{5}{18}$m/s = 35m/s , v = 0, s = 200m, a = ?

Form v^2-u^2=2as $\Longrightarrow 0^2-(35)^2=2a(200) \Longrightarrow a = \frac{35x35}{2(200)} = \frac{-49}{16} m/s^2$

And from v=u+at $\Longrightarrow 0 = 35 - \frac{49}{16} t \Longrightarrow \frac{49}{16} t = 35 \Longrightarrow t = \frac{35(16)}{49} = \mathbf{\frac{80}{7}}$**s**

8. A particle moves in a straight line with uniform acceleration. Its velocity at time t = 0 is v_1 and at time t=t is v_2. The average velocity of the particle in this time interval is $\frac{v_1+v_2}{2}$. Is this correct? Substantiate your answer?

<u>**Sol**</u>: Yes, it is correct.

We know that $s = v_1t+\frac{1}{2}at^2 \Longrightarrow s = v_1t+\frac{1}{2}\left(\frac{v_1+v_2}{t}\right)t^2 \Longrightarrow s = v_1t + \frac{1}{2}(v_2-v_1)\, t$

$\Longrightarrow s = v_1t + \frac{v_2t}{2} - \frac{v_1t}{2} \Longrightarrow s = \left(v_1t - \frac{v_1t}{2}\right) + \frac{v_2t}{2} \Longrightarrow s = \frac{v_1t}{2} + \frac{v_2t}{2} \Longrightarrow \mathbf{s = \left(\frac{v_1+v_2}{2}\right)t}$

(OR)

From $s = ut + \frac{1}{2}at^2 \Longrightarrow s = v_1t + \frac{1}{2}\left(\frac{v_2-v_1}{t}\right)t^2 = \frac{v_2+v_1}{2}\, t$

Average velocity $V = \frac{\text{total displacement(s)}}{\text{total time(t)}} = \frac{v_2+v_1}{2}$.

9. If $\vec{A} = \vec{i} + \vec{j}$ what is the angle between vector $\vec{A}$ with x-asis?

Sol: comparing the vector $\vec{A}$ with x $\vec{i}$ + y $\vec{j}$, we get x=1 and y=1

If $\vec{A}$ = x $\vec{i}$ + y $\vec{j}$ makes an angle θ with the x-axis then tan θ = $\frac{y}{x} = \frac{1}{1}$ =1

∴ θ =45^0

10. The vertical component of a vector is equal to its horizontal component. What is the angle made by the vector with x-axis?

Sol: Let the vector is $\vec{A}$

Vertical component of $\vec{A}$ = A sinθ

Horizontal component of A = A cosθ

When θ is the angle with x-axis

Given A sinθ = A cosθ $\Longrightarrow$ tanθ = 1 $\Longrightarrow$ **θ = 45^0**

11. Two forces of magnitudes 3 units and 5 units act at 60^0 with each other, what is the magnitude of their resultant?

Sol: Given $\vec{A}$ = 3 units $\quad \vec{B}$ = 5 units $\quad$ θ = 60^0

Magnitude of resultant R = $\sqrt{\vec{A}^2 + \vec{B}^2 + 2\ \vec{A}\,\vec{B} cos\theta}$

$R = \sqrt{3^2 + 5^2 + 2.3.5\ cos60^0}$ = **7units**

12. When two right angled vectors of magnitude 7 units and 24 units combine, what is the magnitude of their resultant?

Sol: Let $\vec{A}$ = 7units $\quad \vec{B}$ = 24 units $\quad$ θ = 90^0

Magnitude of resultant R = $\sqrt{\vec{A}^2 + \vec{B}^2 + 2\ \vec{A}\,\vec{B} cos\theta}$

Magnitude of resultant R = $\sqrt{7^2 + 24^2 + 2.7.24\ cos90} = \sqrt{625}$ = **25 units**

13. If $\vec{P} = 2\,\vec{i} + 4\,\vec{j} + 14\,\vec{k}$ and $\vec{Q} = 4\,\vec{i} + 4\,\vec{j} + 10\,\vec{k}$ find the magnitude of $\vec{P} + \vec{Q}$?

Sol: $\vec{P}+\vec{Q} = (2+4)\,\vec{\boldsymbol{i}} + (4+4)\,\vec{\boldsymbol{j}} + (10+14)\,\vec{\boldsymbol{k}} = 6\vec{\boldsymbol{i}} + 8\vec{\boldsymbol{j}} + 24\vec{\boldsymbol{k}}$

Magnitude of $\vec{P}+\vec{Q} = \sqrt{36+64+576}$ = **26 units**

14. A force $2\,\vec{i} + \vec{j} - \vec{k}$ newton act on a body which is initially at rest. At the end of 20 seconds, the velocity of body is $4\,\vec{i} + 2\,\vec{j} - 2\,\vec{k}$ m/s. What is the mass of the body?

Sol: Given force $\vec{F} = 2\,\vec{i} + \vec{j} - \vec{k}$ newton $\quad \vec{V} = 4\,\vec{i} + 2\,\vec{j} - 2\,\vec{k}$ m/s

$\left|\vec{F}\right| = \sqrt{2^2+1^2+1^2} = \sqrt{6}$

$\left|\vec{V}\right| = \sqrt{4^2+2^2+2^2} = \sqrt{24}$

T = 20s, mass of the body $m = \frac{F}{a} = \frac{F}{v}t = \frac{\sqrt{6}}{\sqrt{24}} \times 20$ = **10kg**

15. Calculate the time needed for a net force of 5N to change the velocity of a 10kg mass by 2m/s.

Sol: net force $F = \frac{m(\Delta v)}{t} \Rightarrow t = \frac{m(\Delta v)}{F} = \frac{10x2}{5}$ = **4s**

(OR)

Force F = 5N, mass m = 10kg, change the velocity v-u = 2m/s, time t =?

From $F = ma \Rightarrow a = \frac{F}{m} = \frac{5}{10} = \frac{1}{2}$ m/s

But $a = \frac{v-u}{t} \Rightarrow \frac{1}{2} = \frac{2}{t} \Rightarrow$ **t = 4s**

16. A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of the ball is 12m/s, determine the mass of the ball is 0.15kg, determine the impulse imparted to the ball. (Assume linear motion of the ball)?

Sol: mass of the ball m = 0.15kg, initial velocity of the ball u = 12m/s

Final velocity of the ball v = -12m/s (opp. Direction)

Impulse = change in momentum

Change in momentum = 0.15 x 12 – (-0.15 x 12) = **3.6Ns**

Impulse = 3.6Ns in the direction from the batsman to the bowler.

17. A body freely falling from a certain height 'h', after striking a smooth floor rebounds and 'h' rises to a height h/2. What is the coefficient of restitution between the floor and the body?

Sol: given that $h_1 = h$ and $h_2 = h/2$

We know that coefficient of restitution, $e = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{\left(\frac{h}{2}\right)}{h}} = \frac{1}{\sqrt{2}}$

$\therefore \text{coefficient of restitution} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$

18. In a ballistics demonstration a police officer fires a bullet of mass 50g with speed 200m/s on soft plywood of thickness 2cm. The bullet emerges with only 10%of its initial kinetic energy. What is the emergent speed of the bullet?

Sol: the initial kinetic energy of the bullet is $\frac{1}{2}mv^2 = \frac{1}{2}(50x10^{-3})(200)^2$ = 1000J.

Final kinetic energy = 0.1x1000 = 100J.

If V_f is the emergent speed of the bullet. $\quad \frac{1}{2}mV_f^2 = 100J$

$V_f = \sqrt{\frac{2x100}{0.05}}$ = **63.2m/s**

19. A machine gun fires 360 bullets per minute and each bullet travels with a velocity of 600m/s. if the mass of each bullet is 5gm, fined the power of the machine gun?

Sol: Number of bullets n =360, time t = 1min = 60s velocity of the bullet v = 600m/s

Mass of each bullet m = 5gm = $5x10^{-3}$kg

Power of the machine gun $P = \frac{w}{t} = \frac{n(\frac{1}{2}mv^2)}{t} = \frac{360x\frac{1}{2}x5x10^{-3}x600^2}{60}$ = 5400W = **5.4kW**

20. A pump is required to lift 600kb of water per minute from a well 25m deep and to eject it with a speed of 50m/s calculate the power required to perform the above task?

Sol: Mass of water m = 600kg, time t = 1min = 60s, depth of well h = 25m

Velocity of ejected water V = 50m/s power p =?

Power p = $\frac{mgh+\frac{1}{2}mv^2}{t}$ = $\frac{600x9.8x25+\frac{1}{2}x600x50^2}{60}$ = 14950W = **14.95kW**

21. Fined the total energy of a body of 5kg mass, which is at a height of 10m from the each and falling downwards straightly with a velocity of 20m/s ($g=10ms^{-2}$)

Sol: mass of the body m = 5kg, height of the body from earth h = 10m

Velocity of the body V = 20m/s, Acceleration due to gravity g =$10ms^{-2}$

Potential energy of the body P.E = mgh = 5x10x10 = **500J**

Kinetic energy of the body K.E = $\frac{1}{2}mv^2$ = ½ x 5 x 20^2 = **1000J**

∴total energy of the body E = P.E + K.E = 500+1000 = **1500J**

22. Consider a drop of mass 1kg falling from a height of 1km. what is the work done by the gravitational force? ($g = 10m/s^2$)

Sol: mass of the drop m = 1g = $1x10^{-3}$ kg, g = $10m/s^2$, d = 1km = $1x10^3$m

Work done by the gravitational force w = mgd = $(1x10^{-3})(10)(1x10^3)$ = **10J**

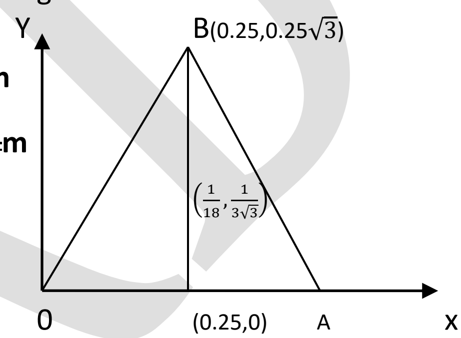
23. Fined the centre of mass of three particles at the vertices of an equilateral triangle. The masses of the particles are 100g, 150g, and 200g respectively. Each side of the equilateral triangle is 0.5m long.

Sol: O,A and B forming the equilateral triangle are respectively (0,0),(0.5,0),(0,25, $0.25\sqrt{3}$). Let the masses 100g, 150g and 200g Be located at O,A and B be respectively. Then,

$$X = \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3} = \frac{100(0)+(150(0.5)+200(0.25)}{100+150+200} = \frac{75+50}{450} = \frac{5}{18}m$$

$$Y = \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3} = \frac{100(0)+(150(0)+200(0.25\sqrt{3})}{100+150+200} = \frac{50\sqrt{3}}{450} = \frac{1}{3\sqrt{3}}m$$

The centre of mass = $\left(\frac{1}{18}, \frac{1}{3\sqrt{3}}\right)$



23. Fined the scalar and vector product of two vectors $\vec{A} = 3\,\vec{i} + 4\vec{j} + 5\vec{k}$ and $\vec{B} = -2\,\vec{i} + \vec{j} - 3\vec{k}$.

Sol: $\vec{A}+\vec{B}$ = ($3\,\vec{i} + 4\vec{j} + 5\vec{k}$) + ($-2\,\vec{i} + \vec{j} - 3\vec{k}$)= -6-4-15 = -25

$$\vec{A}X\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -3 & 5 \\ -2 & 1 & -5 \end{vmatrix} = \mathbf{-2\,\vec{i} + \vec{j} - 5\vec{k}}$$

24. Find the torque of force $7\,\vec{i} + 3\vec{j} - 5\vec{k}$ about the origin. The force act on a particle whose position vector is $\vec{i} - \vec{j} + \vec{k}$.

Sol: Here $\vec{r} = \vec{i} - \vec{j} + \vec{k}$ and $\vec{F} = 7\,\vec{i} - 3\,\vec{j} + 5\vec{k}$

We use the determinant rule to find the torque $\vec{\tau} = \vec{r} \times \vec{k}$

$$\vec{\tau} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 7 & 3 & 5 \end{vmatrix} = (5-3)\,\vec{i} - (-5-7)\,\vec{j} + (3-(-7))\,\vec{k} \Longrightarrow \boldsymbol{\vec{\tau} = 2\vec{i} + 12\,\vec{j} + 10\vec{k}}$$

25. On an average a human heart is found to beat 75times in a minute. Calculate its frequency and time period?

Sol: the beat frequency of heart = 75/1min = 75/60s = $1.25s^{-1}$ = **1.25Hz**

The time period T = 1/1.25 = **0.8s**

26. What is the length of a simple pendulum, which ticks seconds?

Sol: the time period of a simple pendulum T = $2\pi\sqrt{\frac{l}{g}} \Longrightarrow \boldsymbol{L = \frac{gT^2}{4\pi^2}} = \frac{9.8 \times 4}{4\pi^2}$ = **1m**

27. A particle executes SHM such that, the maximum velocity during the oscillation in numerically equal to half the maximum acceleration. What is the time period?

Sol: given that $V_{max}=\frac{1}{2}\,a_{max} \Longrightarrow A\omega = \frac{1}{2}\omega^2 \Longrightarrow \omega = 2\ \text{rads}^{-1}$

But $\omega = \frac{2\pi}{T} = 2 \Longrightarrow$ **$T = \pi s$**

28. What happens to the time period of a simple pendulum if length is increased upto four times?

Sol: time period of a simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

From the above expression we have $T \propto \sqrt{L}$

If the length of the pendulum is increased by 4 times then the time period increases by 2 times

29. What should be the radius of a capillary tube if water has to rise to a height of 6cm in it? (surface tension of water = $7.2 \times 10^{-2}Nm^{-1}$)

Sol: height of the water in the capillary tube h = 6cm = $6x10^{-2}$m

Surface tension of water T = $7.2x10^{-2}Nm^{-1}$

Acceleration due to gravity g = 10 m/s^2,

For water and glass angle of contact $\theta = 0^0$

Radius of the capillary tube $r = \frac{2T}{h\rho g} = \frac{2x7.2x10^{-2}}{6x10^{-2}x1000x10} = 2.4x10^{-4}$ m = **0.24mm**

30. If the diameter of a soap bubble is 10mm, and its surface tension is 0.04N/m find the excess pressure inside the bubble?

Sol: diameter of the soap bubble 2r = 10mm = $10x10^{-3}$m

Radius of the soap bubble r = $5x10^{-3}$m , surface tension T = 0.04N/m

Excess pressure inside the bubble, $P = \frac{4T}{r} = \frac{4x0.04}{5x10^{-3}}$ = **$32Nm^{-2}$**

31. What is the temperature for which the readings on Kelvin, Fahrenheit scales are same?

Sol: on the Kelvin and Fahrenheit scales $\frac{K-273.15}{100} = \frac{F-32}{180}$

Hear K = F $\Longrightarrow \frac{F-273.15}{100} = \frac{F-32}{180} \Longrightarrow F-273.15= \frac{5}{9}F - \frac{160}{9} \Longrightarrow \frac{4}{9}F - 273.15-17.77$

F $= \frac{9}{4}(255.38) = 574.6$

∴574.6^0K=574.6°F

32. Find the increase in temperature of aluminium rod if its length is to be increased by 1% (α for aluminium = $25x10^{-6}/^0C$)

Sol: coefficient of linear expansion of aluminium $\alpha_l = 25x10^{-6}\ /^0C$

We know that percentage increase in length = $\frac{\text{Increase in length}}{\text{Original length}}$ x 100

$\frac{l_2-l_1}{l_1}$ **x** 100 = $\alpha(t_2-t_1)$ x 100

∴increase in temperature = $\frac{\text{percentage increase in length}}{\alpha x100} = \frac{1}{25x10^{-6}x100}$ = **400**°C

33. If the maximum intensity of radiation for a black is found at 2.65μm what is the temperature of the radiating body? (Wien's constant = $2.9x10^{-3}$mK)

Sol: Wavelength corresponding to maximum intensity, λ_m = 2.65 μm = $2.65x10^{-6}$m

Wien's constant σ = $2.9x10^{-3}$mK

We know that, **λ_mT = Wien's constant (b)**

∴Temperature of the radiating body, $T = \frac{b}{\lambda_m} = \frac{2.90x10^{-3}}{2.65x10^{-6}}$ = **1094K**

34. A refrigerator is to maintain eatables kept inside at 9^0C. If room temperature is 36^0C, calculate the coefficient of performance.

Sol: T_H = 36+273 = 309K; T_L= 9+273 = 282K

∴Coefficient of performance (COP) of the refrigerator is

COP $= \frac{T_L}{T_H-T_L} = \frac{282}{309-282} = \frac{282}{27}$ = **10.44**

35. The absolute temperature of a gas is increase 3 times. What will be the increase in rms velocity of the gas molecule?

Sol: The relation between r.m.s velocity and absolute temperature of a gas is $V_{rms} \propto \sqrt{T}$

Therefore, the r.m.s velocity becomes $\sqrt{3}\, V_{rms}$

Hence increase in r.m.s velocity = $\sqrt{3}\, V_{rms} - V_{rms} = 0.732 V_{rms}$ = 73.2%

(OR)

The r.m.s velocity of gas molecule is given by $V_{rms} = \sqrt{\frac{3KT}{M}} \Longrightarrow V \propto \sqrt{T}$

$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{T}{3T}} \Longrightarrow V_2 = \sqrt{3}\, V_1$

∴ the r.m.s velocity of gas molecule is increase by $\sqrt{3}$ times of its original volume.

Increases in r.ms velocity = $V_2 - V_1 = \sqrt{3}V_2 - V_1 = \mathbf{0.732V_1}$

36. What is the ratio of r.m.s speed of Oxygen and Hydrogen molecules at the same temperature?

Sol: the r.m.s speed of gas molecules is given by $V_{rms} = \sqrt{\frac{3KT}{M}}$

That is r.m.s speed of gas molecule is inversely proportional to molecular weight of given mass of gas

That is $\frac{V_O}{V_H} = \sqrt{\frac{M_H}{M_O}} = \sqrt{\frac{2}{32}} = \sqrt{\frac{1}{16}} = \mathbf{\frac{1}{4}}$

∴r.m.s speed of hydrogen molecules is four times r.m.s speed of oxygen molecules.

37. Four molecules of a gas have speed 1,2,3,and 4km/s. Find the r.m.s speed of the gas molecule?

Sol: the r.m.s speed gas molecules interms of its speed is given by

$V_{rms} = \sqrt{\frac{V^2}{N}} = \sqrt{\frac{V_1^2+V_2^2+V_3^2+V_4^2}{4}} = \sqrt{\frac{1^2+2^2+3^2+4^2}{4}} = \sqrt{\frac{1+4+9+16}{4}} = \sqrt{\frac{30}{4}} = \sqrt{7.5}$ = **2.7km/s**

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