

Study the behaviour of atoms and molecules

$$\partial_t f + \omega \cdot \nabla_x f = 0$$

Theorem

Let $p \in [1, +\infty]$.

The matrix A has no eigenvalues on the unit circle if and only if the problem

$$x(n+1) = Ax(n) + f(n), \quad n \in \mathbb{Z} \tag{1}$$

admits a unique solution $x \in \ell_p$ for all $f \in \ell_p$.

Theorem

Solving the problem (1) defines a linear continuous operator

$$\mathcal{R} : \ell_p \rightarrow \ell_p, \quad f \mapsto x.$$

The norm of this operator is bounded by

$$\|\mathcal{R}\| \leq \Delta_A = \text{something to copypaste},$$

*where matrices H_+ and H_- are defined somewhere else. **to copypaste***

Theorem

Suppose the matrix A has no eigenvalues on the unit circle. If the matrix B satisfies the condition

$$\|B\| \leq \Delta_A,$$

then the matrix $A + B$ has no eigenvalues on the unit circle, either. Moreover, matrices A and $A + B$ have the same number of eigenvalues (taken with multiplicity) inside the unit circle.

Theorem

Suppose the matrix A has no eigenvalues on the unit circle. If the sequence of matrices $B(n)$, $n \in \mathbb{Z}$ satisfies the condition

$$\sup_{n \in \mathbb{Z}} \|B(n)\| \leq \Delta_A,$$

then for all $f \in \ell_p$ the problem

$$y(n+1) = (A + B(n))y(n) + f(n), \quad n \in \mathbb{Z} \tag{2}$$

admits a unique solution $y \in \ell_p$.

TO DO

- prove projector approximation
- ideas for proofs for already known theorems
- motivation for work
- similar results for differential equation
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