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DFT-D: a cognitive-dynamical model of dynamic decision making

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Abstract The study of decision making has traditionally been dominated by axiomatic utility theories. More recently, an alternative approach, which focuses on the micro-mechanisms of the underlying deliberation process, has been shown to account for several "paradoxes" in human choice behavior for which simple utility-based approaches cannot. Decision field theory (DFT) is a cognitive-dynamical model of decision making and preferential choice, built on the fundamental principle that decisions are based on the accumulation of subjective evaluations of choice alternatives until a threshold criterion is met. This article extends the basic DFT framework to the domain of dynamic decision making. DFT-Dynamic is proposed as a new alternative to normative backward induction. Through its attention to the processes underlying planning and deliberation DFT-D provides simple, emergent explanations for violations of choice principles traditionally taken as evidence of irrationality. A recent multistage decision making study is used to showcase the model's efficacy for developing cognitive models of individual strategies.

Keywords Cognitive models · Dynamic decision making · Dynamic consistency · Planning · Decision trees · Sequential sampling · Preferential choice · DFT

The ability to choose among options is a key feature of human intelligence. Its importance in everyday life has led many cognitive scientists to develop models of how people make decisions. In his classic book on computational vision, Marr (1982) proposed three levels of theories about cognitive systems. At the highest level, theories aim to understand the abstract goals a system is trying to achieve; at an intermediate

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level, theories are designed to explain the dynamic processes used to achieve the top level goals; and at the bottom level, theories attempt to describe the neurophysiologic substrate of the second level.

Judgment and decision making researchers have traditionally developed theories at the higher and more abstract levels, opting to understand and predict behavior using simple, intuitive concepts. Their approach is typically agnostic to the underlying mechanisms that give rise to behavioral tendencies. This article presents an alternative view from the perspective of *decision field theory* (DFT), one of the most successful and widely tested computational models of decision making. By attending to lower level mental processes DFT elegantly explains many seemingly paradoxical findings in judgment and decision making. Moreover, DFT provides the basis for a new model aimed at understanding decision making in more complex environments. After reviewing the basic DFT framework, this article focuses on a new extension of the theory to dynamic decision making. We present several variants of the model to showcase its utility for identifying differences in information processing and decision strategy across individuals.

1 Utility-based models of decision making

There have been many theories of decision making developed within the utility framework. These models are used to explain why people apparently fail to maximize long-term expected value when making economic decisions. They assume that each choice alternative is evaluated according to its subjective utility and that individuals seek to maximize long-term expected utility. The strength of this approach is that it provides simple principles and functions meant to describe how physical values (e.g. dollars) and probabilities are interpreted on subjective, psychological scales.

Utility-based models have been quite successful in helping researchers understand why individuals engage in various risky behaviors like buying insurance or lottery tickets. They have also provided an explanation for some paradoxical findings in the judgment and decision making literature, like the Allais paradox (Allais 1953) and reflection effects (Kahneman and Tversky 1979). However, there are a several key features of people's choice behavior that have thus far resisted explanation by utility theories. In response, a new breed of cognitive models has been proposed. Rather than focusing on static maximization formulas, these describe the step by step computational processes that produce choices. They provide a richer view of the internal activities of the decision maker, and this attention to mental processes lends them superior explanatory power. The remainder of this article focuses on the process model that has been applied to greatest number of phenomena in judgment and decision making, *decision field theory* (Busemeyer and Townsend 1993; Hotaling et al. 2010; Roe et al. 2001).

2 A sequential sampling approach to decision making

While utility-based models provide high-level, abstract descriptions of individuals' preferences, DFT represents a different approach, focused on how preferences are

Table 1 Hypothetical investment decision

Action	Event g: Recovery	Event b: Recession
S: Savings Account	c_{11}: Missed opportunity	c_{12}: No loss; maintain wealth
R: Stocks	c_{21}: Large return on investment	c_{22}: Large loss

constructed. It employs many simple units, connected together to form a dynamical system from which preferences emerge. From a theoretical perspective, the model provides a psychologically plausible account of choice behavior built upon basic features of the cognitive and neurological systems. From a practical perspective, it can simultaneously account for a much broader range of the choice phenomena traditionally seen as paradoxes or anomalies.

The core of the model is a deliberation process that is assumed to produce overt decisions. It belongs to the general class of sequential sampling models, which aim to capture both choice and response time phenomena. They form the basis of models used in a variety of domains including sensory detection (Smith 1995), perceptual discrimination (Laming 1968; Link and Heath 1975; Usher and McClelland 2001; Vickers 1979), recognition memory (Ratcliff 1978), categorization (Ashby 2000; Nosofsky and Palmeri 1997), probabilistic inference (Wallsten and Barton 1982), preferential choice (Aschenbrenner et al. 1984; Busemeyer 1985) and decision neuroscience (Fehr and Rangel 2011).

To introduce DFT, consider the following situation. You receive a tax refund and are deciding what to do with the money. You consider two options: a safe option, S, deposit the money in a low yield savings account; or a risky option, R, invest in the stock market. Looking ahead, the economy will either recover (labeled event G for good) or slide into recession (labeled event B for bad). Table 1 displays the four possible outcomes.

According to DFT, the decision maker deliberates by thinking about the possible outcomes of each action. From moment to moment, different consequences come to mind and cause preferences to change. At one moment, you may remember the jobs report you read in the newspaper showing unemployment on the decline. This suggests the economy is headed toward recovery, and brings outcomes c_{11} and c_{21} to mind. The next moment, you remember hearing that real estate prices fell this month. This suggests that a recession is coming, and brings to mind outcomes c_{12} and c_{22}. At each moment, affective evaluations of outcomes are considered and compared, providing fluctuating input to the model. The probability of attending to a particular event at any moment reflects an individual's belief in the likelihood of that event, e.g. the subjective probability of economic recovery. The momentary evaluations of each alternative are compared to produce what is called the valence at time $t : V(t) = V_S(t) - V_R(t)$. These valences accumulate to form a preference state. Preference states evolve as new valences are integrated, until the accumulated preference for one alternative reaches a threshold.

Figure 1 illustrates an example sequential sampling process where the decision maker considers the two alternatives. The horizontal axis represents deliberation

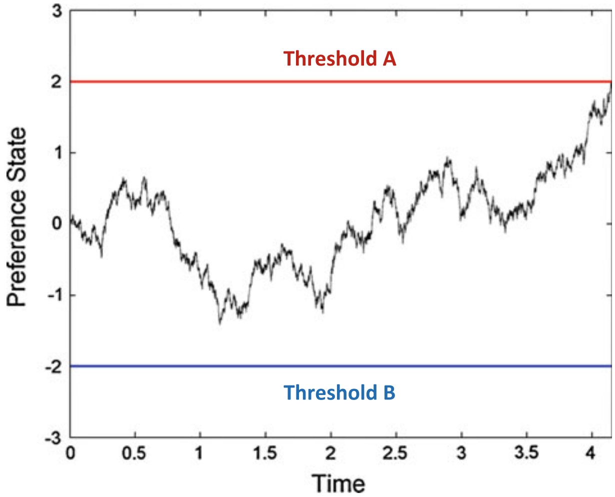

Fig. 1 Preference accumulation in DFT

time and the vertical axis represents the preference state. As attention switches stochastically between recovery and recession, the strengths and weaknesses of each alternative cause the preference state to evolve in a noisy manner. Eventually, sufficient evidence accumulates in favor of option A that the corresponding decision threshold is reached. The horizontal thresholds represent the amount of preference required for an individual to select each choice alternative. By focusing on deliberation, DFT provides a fuller picture of the decision process and accounts for both choices and response times. In addition to this internally controlled stopping version of the model, DFT can also explain externally controlled stopping by using a vertical threshold to represent the moment when some external force, e.g. an experimenter, ends deliberation and compels the decision maker to choose the option with the highest preference state.

Placement of the threshold is an important factor controlling speed and accuracy tradeoffs. High thresholds produce decisions based on many samples of evidence and yield slow responses and high accuracy. Low thresholds produces decisions based on relatively few samples, leading to fast, error-prone response. Factors, such as impulsivity and external time pressure may cause a person to lower their decision threshold. Sequential sampling in DFT provides an intuitive account of these factors.

3 Decision field theory: an overview

To explain in more detail how preference states are computed, some mathematics is required. Fortunately, the deliberation process posited by DFT—where attention switching guides sequential sampling of information about alternatives—has well defined formulas for calculating choice probabilities (see Diederich and Busemeyer 2003 for a derivation). Here we present a simple version of DFT. The preference state at time $t + 1$ is defined as the sum of the previous preference state and valence difference:

$$P(t+1) = P(t) + V(t). \tag{1}$$

$P(0)$ is the initial preference state, and can be used to explain initial bias and carry-over effects from past experience or previous decisions. The theoretical mean input to the sampling process is the expectation of the valence difference:

$$\mu = E\left[V(t)\right]. \tag{2}$$

This value can be calculated by taking the difference of mean inputs from each alternative, $\mu_S - \mu_R$, with each mean defined as:

$$\mu_i = W_G \cdot c_{i1} + W_B \cdot c_{i2}. \tag{3}$$

W_G and W_B are the average attention weights given to Event G and Event B, and $W_G = 1 - W_B$. Note the similarity between Eq. 3 and traditional weighted utility models. The main difference is that in DFT the momentary attention to outcomes randomly fluctuates across time and thus $V_i(t)$ also fluctuates across time. The traditional weighted utility emerges as the mean or time average of these stochastic evaluations. In this way, DFT provides a process account of these weighted evaluations. The variance of the valence difference is therefore a theoretically important concept, defined as:

$$\sigma_V^2 = E\left[V(t) - \mu\right]^2. \tag{4}$$

4 Applying DFT to decision making paradoxes

One of the major empirical result that challenges static utility models of choice deals with the time course of decision making. Growing evidence suggests that people's choices can be affected by the amount of time they are allowed to deliberate. For example, Svenson and Edland (1987) asked people to choose among several apartments that varied along multiple dimensions. Participants made their choice under two conditions: short time deadlines, and long time deadlines. Under the short time deadline, people tended to choose low rent apartments, but under long time deadlines more expensive apartments with other attractive features were selected. Diederich (2003) made a similar discovery in a gambling task where participant tried to maximize monetary rewards while avoiding a loud noise punishment. Several individuals reversed their preference for reward seeking versus punishment avoidance under time pressure. One explanation for these preference reversals has been that decision makers use qualitatively different strategies under different amounts of time pressure (see Payne et al. 1993). However, DFT successfully accounts for preference reversals under time pressure without assuming multiple strategies. This is a natural consequence of the sequential sampling approach. Note the preference trajectory in Fig. 1. DFT's stochastic attention switching mechanism causes preference for alternatives to accumulate in a noisy fashion. In this example, a preference value of at least ±2.00 is required to make a choice, and the threshold for selecting A is reached first. However, if a lower decision threshold had been set, e.g. ±1.00, alternative B would have been chosen. By manipulating DFT's initial starting point and the decision bound, preference reversals naturally emerge.

DFT has several other features that set it apart from traditional models of decision making. Some of these stem from its focus on the psychological processes that give rise to decisions. For instance, by allowing attention to fluctuate from one attribute to another across time, variability is incorporated into the model's predictions. This makes choice probabilistic, rather than deterministic. Due to limited space, there are many applications of DFT that cannot be properly dealt with here. To give just one example, Busemeyer and Goldstein (1992) and Johnson and Busemeyer (2005) developed a generalization of DFT that allowed it to model how individuals choose the prices at which they will agree to buy or sell an item. The model was successful in accounting for preference reversals between choice and pricing, as well as and differences in buying and selling prices, sometimes referred to as the *endowment effect*. Many other applications further expand the range of behavioral phenomenon that DFT can help us understand.

5 A new application for DFT: planning and multistage decision making

Much of the research into human decision making has focused on isolated choices, made independently from one another. Many real world choices are not so separable. For example, imagine Sam is contemplating asking his boss for a pay raise. This is not a decision that Sam should take lightly, without first considering the full consequences of his actions. Figure 2 uses a decision tree with three kinds of nodes to represent this scenario. Decision nodes (DNs) represent points where Sam chooses

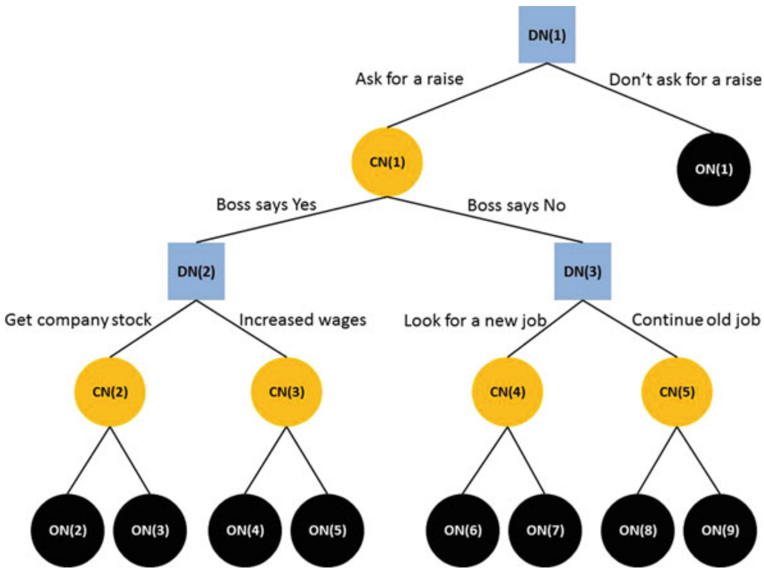

Fig. 2 A dynamic decision scenario, represented as a decision tree

an action. Chance nodes (CNs) denote random events that occur in the world, which Sam cannot control. Each branch of the tree terminates with an outcome nodes (ONs) representing a final outcome.

5.1 Backward induction

A normative strategy for solving such problems involves thinking backward from the end of the tree, and is known as *backward induction* (DeGroot 1970). The basic idea is to begin at the end of the tree and prune unwanted branches while working backward to the beginning. The process begins by assigning a utility value to each ON, representing its worth to the decision maker. For the example in Fig. 2, ON(1) – ON(9) are assigned $\mu_1 - \mu_9$, respectively. Next, an *expected utility* is computed for each CN as the weighted averaged utility of the node. For example, $\mu(4)$, the expected utility for CN(4), is calculated by weighting μ_6 and μ_7 by their probabilities, p_6 and p_7, respectively and summing the results. Third, each DN is assigned an expected utility by taking the maximum value of nodes branching out of it. For multistage decision trees, the process is repeated until utilities are propagated to the first DN. Following this strategy maximizes long-term utility.

Continuing the earlier example, assuming Sam already knows the utility of each outcome, he must next calculate expected utilities for CN(2) – CN(5), and use these to plan choices for DN(2) and DN(3). Imagine that choosing company stock, CN(2), offers Sam a greater expected value than choosing a wage increase, CN(3). According to the normative theory, Sam should prune the branch extending down to CN(3) and carry $\mu(2)$ up the tree to DN(2). This pruning reflects the assumption that, if Sam asks for a raise and his boss agrees, Sam will choose the action that is, on average, best for him. The same optimization is done at DN(3). Sam should then compute a weighting average of the DNs to get the expected utility of CN(1). All that remains is a choice between ON(1) and CN(1). Sam is again assumed to take the option with greatest utility and to continue down the tree as planned.

5.2 Tests of the normative model

Several studies have questioned backward induction as a model of how people make dynamic decisions. Busemeyer et al. (2000) tested a key assumption of the model, *dynamic consistency*, which stipulates that decision makers follow through on the plans that they make. This was tested by having participants complete a series of simple decision trees. On some trials, at the beginning of each tree they were asked to plan what they would choose if they found themselves at a specified future node. The computer would automatically carry out the plan if the node was reached. On other trials with the same tree, they simply made a final choice upon reaching the end of the tree. Busemeyer et al. (2000) found that these plans tended to be riskier than the final decision people made when they reached a decision node. They concluded that people are dynamically inconsistent, and do not necessarily follow their plans for future choices. Johnson and Busemeyer (2001) found that dynamic inconsistency increased with the length of the decision tree. That is, the further ahead a person was asked to plan, the more likely they were to deviate from that plan. A critical requirement of backward induction is that one follows through on one's planned decisions, so these findings challenge the idea of pure backward induction as a model of people's behavior.

Carbone and Hey (2001) conducted a study using large 3-stage decision trees comprised on interleaved binary DNs and CNs, terminating in sixty-four ONs. Participants were given unlimited time to explore and plan. They were free to make notes and calculations before working their way through the tree. Every action made by the participant during exploration, planning, and execution was recorded. Carbone and Hey replayed these actions to determine how people solved the problems. They found that many individuals did use some version of backward induction, but did not prune the tree according to their planned choices. They looked ahead to ONs, but neglected the *principle of optimality*, in that they seemed unaware that they would be making optimal choices in the future. Hey and Knoll (2011) further investigated individual differences with another 3-stage decision tree task. Again using the record of each decision maker's entire sequence of actions, they classified each person's strategy. Although, no specific classification scheme was reported, Hey and Knoll identified several distinct groups in terms of broad qualitative characteristics. For example, those who ignored information to minimize effort were distinguished from those who approximated an optimal strategy.

5.3 DFT-D: a cognitive model of multistage decision making

Taken together, previous research suggests two important conclusions: (1) there are significant differences in the strategies that individuals use to solve dynamic decision making problems, and (2) pure backward induction cannot account for most people's behavior. In light of these, we propose a new approach that uses cognitive models to understand the processes underlying dynamic decision making. We draw on the basic framework of DFT, extending the theory to multistage decisions. Below, we provide a brief introduction to the DFT-Dynamic (DFT-D) model, and develop several versions to understand individual differences in a multistage decision task.

To provide an intuition for DFT-D, recall the example decision tree in Fig. 2. DFT-D posits that, while deliberating at DN(1), Sam imagines what might happen if he asks for a raise. He simulates a sequence of events leading to an outcome, say ON(3), and compares this to ON(1) to produce a momentary valence. Different paths are simulated moment to moment, according to their subjective likelihoods. The probability of sampling a given ON comes from Sam's intuitive understanding of how likely each outcome is to occur. For ON(3), this involves his belief about how likely his boss is to agree to the raise and how likely the event at CN(2) is to lead to ON(3), as well as how likely he is to choose company stock at DN(2). Sam does not explicitly commit to one plan, but rather considers many, over time, based on his imperfect ability to predict his future choices. The model uses DFT to compute planned choice probabilities at DN(2) and DN(3). Each simulation results in a valence providing subjective evidence for one choice or the other, and preference accumulates until a threshold is reached. Thus, DFT-D imagines a forward-looking process where repeated simulations drive noisy sequential sampling.

Conveniently, this process is formally equivalent to a modified version of backward induction. The remainder of this section focuses on this formal representation of DFT-D because it allows for simple comparisons between the cognitive and normative models. As with backward induction, DFT-D can be conceptualized as beginning at the bottom of the tree. Utilities are assigned to each outcome and expected utilities are calculated for each CN. At the next level of the tree, DFT-D does not plan an optimal choice and prune the forgone branches. Instead, it computes choice probabilities, and therefore expected utilities, for DN(2) and DN(3). The normative model states that if $\mu_{\mathrm{cn}(2)} > \mu_{\mathrm{cn}(3)}$, Sam should entirely ignore CN(3) and carry $\mu_{\mathrm{cn}(2)}$ up the tree to DN(2). DFT-D posits that Sam is not sure what he will do at DN(2) and may simulate different choices from moment to moment. The result is an expected utility for DN(2) that weights $\mu_{\mathrm{cn}(2)}$ and $\mu_{\mathrm{cn}(3)}$ by the subjective probabilities that he will choose each. Once this is done for DN(2) and DN(3), an expected utility is computed for CN(1). Finally, Sam has a choice between this expectation and ON(1). According to the Wiener formulation of the diffusion model, and assuming that P(0) = 0 (i.e. no initial bias), it is possible to mathematically derive the probability of choosing the riskier of two alternatives (see Busemeyer and Diederich 2010, Ch. 4):

$$P\,(\text{choose risky option}) = \frac{1}{1 + e^{-\frac{\theta\mu}{\sigma}}} \tag{5}$$

where μ is the mean valence in favor of the risky option, σ is the standard deviation in valences, and θ is the decision threshold. This predicts that individuals will tend to choose the higher utility option, but allows for uncertainty about one's future decisions. Sensitivity to utility differences will decrease as outcomes become more different (i.e. as σ increases; see Eq. 4); a natural consequence of the noisy sampling process. Lower decision thresholds also serve to decrease sensitivity. Note that Eq. 5 is a formally equivalent representation of the sampling decision process described earlier. It conveniently allows choice proportions to be calculated analytically and for predictions of DFT-D to be compared to those of normative backward induction.

Thus, the model allows for probabilistic choice, dynamic inconsistency, uncertainty about future decisions, and sensitivity to outcome variance. We use this as the starting point for developing models of dynamic decision making strategies. The basic framework can be augmented by various components to represent how information is sampled, weighted, integrated, etc. One possibility is that individuals follow the above procedure entirely and implement a near-optimal strategy for maximizing payoffs. An alternative would be to simplify planning at DN(1) by initially ignoring much of the information at the bottom of the tree. Such behavior may appear myopic, with early decision based on a fuzzy view of future payoffs, but later decisions appearing more rational. Yet another strategy would be to minimize effort by not planning future choices at all. Individuals using this strategy may treat future DNs as if they were random events, out of their control. The models presented below share the basic DFT-D framework, but use different components and procedures to capture each of these unique strategies. They differ from one another in how μ is defined and whether or not it changes with the decision maker's perspective. Since this exercise was somewhat exploratory, emphasis was placed on analytical formulations of μ. Rather than test

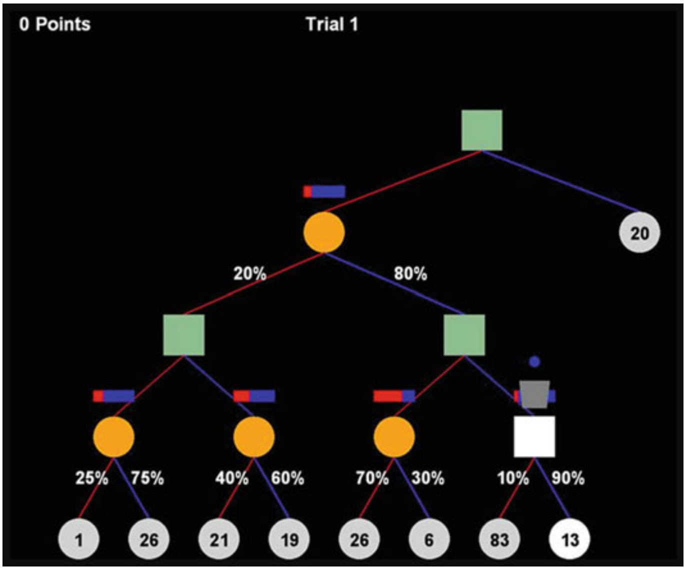

Fig. 3 One of the decision trees used in the experiment. The decision maker is about to move from CN(5) to ON(9), after a *blue* marble was drawn. (Color figure online)

myriad heuristic-based attentional processes, we focused on a relatively small number of functional forms meant to capture the degree to which individuals attend to particular types of information. The hope was that fitting these models to data would yield interpretable parameters that could be used when later developing more process-rich models of how attention guides sampling.

To test these predictions, a study was conducted in which twenty two individuals each completed a series of dynamic decision problems. On each trial, participants were shown a decision tree like the one in Fig. 3. All trees had the same structure, but CN probabilities and ON values varied, yielding twenty two unique trees representing many kinds of multistage decision problems. The goal was to earn as many points as possible, and participants were paid at the end of each session based on their winnings, with most earning between \$4 and \$13. A white square indicated the participant's current position in the tree. They began at DN(1) and made their first choice by clicking the mouse on either CN(1) or ON(1). If they chose the sure payoff, the position marker moved to ON(1). If they chose to gamble, the marker moved to CN(1) where an urn appeared and a marble was drawn. Participants were told that each urn contained a mixture of red and blue marbles and that the proportion was indicated by the colored bar and percentages above and below each CN. Drawing a red marble would send the marker down the path to DN(2), while a blue marble would send the marker to DN(3). Participants chose their next move by clicking on one of the CNs below their current position. The marker moved to the chosen node and another bucket and marble animation determined the final movement. Once an ON was reached, its points were added to the participant's total and a new tree was presented. Trees were presented in a random order, with each individual completing twelve repetitions of each type, for a total of 264 trials.

The above exposition is intended to familiarize the reader with our dynamic decision making task. We save a more thorough treatment for future publication. The remainder of this article focuses on applying DFT-D to the data collected. The diversity of decision trees provides an ideal dataset for testing model performance across many situations. Several variants of DFT-D were fit separately to each individual's data. The parameters of each model were simultaneously fit to every response that an individual made by maximizing the likelihood of the observed data. Each model shared the basic framework of DFT-D, as described above, but differed in ways meant to test hypotheses such as: whether or not gains were treated differently than losses; whether future decisions were deliberated over to the same degree as immediate ones; and whether some outcome or probability information was neglected in planning or execution of choices. Over one hundred such models, differing in the ways alluded to above, as well as in complexity (i.e. number of free parameters estimated), were fit separately to each individual's responses. A Bayesian information criterion (BIC) was computed for each fit in order to assess how well each DFT-D variant accounted for each individual's behavior. BIC measures goodness of fit, while penalizing models for the number of free parameters. The end result was a matrix of BICs corresponding to each combination of model and participant. Unfortunately, automated clustering methods, such as k-means and multidimensional scaling, did not yield unambiguously optimal solutions. Instead, clusters were constructed 'by hand', with the goal of selecting the models that provided the best fit (according to BIC) to the most individuals.

There is insufficient space to expound the full set of models tested, so we focus only on the most successful. From our analysis, three distinct groups immerged, which we refer to as *near-optimal planners*, *myopic planners*, and *non-planners*. These are intended as convenient labels for the qualitative differences in models. The primary purpose of the present article is to showcase DFT-D's ability to identify individual decision strategies, rather than to make strong claims about how these strategies should be classified. Although, we omit certain technical details, some attention must be paid to the parameters of each model. The decision threshold, θ, representing the amount of preference required to make a choice, was common to all and is therefore ignored in this discussion. Each model's unique features, however, shed light on different ways in which individuals approached the decision trees.

The six *near-optimal planners* were best fit by a model with just two parameters. For each individual, the same two parameters were applied consistently across all stages, from planned decisions at DN(2) and DN(3) to executed decisions at all three DNs. To begin, ON values are converted into subjective utilities by first subtracting the value of ON(1) from ON(2) through ON(9) to center the scale on the sure payoff. Thus, everything greater than ON(1) becomes a gain and everything less, as loss. This proved critical, and is another common feature across all models. The utility of x points was x^{α} if x was positive, and $-|x|^{\beta}$ if x was negative. For powers less than 1, this produces an s-shaped function representing participants' diminishing sensitivity to differences as values become more extreme. The mean α and β values, across the six near-optimal planners, were 0.85 and 0.72, respectively. These are within the range of values commonly found in experiments, though it is worth noting that β is smaller. This produces a utility function that is flatter for losses than gains, meaning the large losses have less impact than large gains. One conclusion is that this group of individuals was slightly more concerned with pursuing large gains than with avoiding large loses. Over all, near-optimal planners appeared to exploit most of the available information, and made choices in accordance with their plans (with uncertainty), as described by DFT-D.

The next group of *myopic planners* included eight individuals who were best fit by a slightly more complex version of DFT-D that used different utility functions during the planning and execution phases to represent how information was used differently throughout the task. When planning ahead, the model uses the same α parameter for gains and losses. This is the utility function used for planned choices at DN(2) and DN(3), which in turn propagate expected utilities up to DN(1). The mean value for α was 0.63 producing a flatter utility function. Compared to near-optimal planners, these individuals' were less sensitive to outcome differences when planning ahead. After making an initial choice, based on these plans, and moving down the tree, the utility function changed to x^{β} for gains and $-\lambda|x|^{\beta}$ for losses. The mean β across participants was 0.84, producing a steeper curve and meaning that, when faced with later decisions between more immediate prospects, the myopic planners were more likely to choose the alternative with the higher expected value. λ is a parameter that is often used in decision models to produce *loss aversion*, or the overweighting of losses, relative to gains. Contrary to what is often found, the mean λ was 0.84, signifying *gain seeking*. These individuals evidently pursued large positive outcomes, more than they avoided large negative ones. α, β, and λ were simultaneously fit to each individual's entire dataset, with each parameter contributing to the model's predictions as indicated above. These simple modifications to the basic DFT-D framework reveal interesting behavioral differences across planning and execution phases. This is a first step toward a deeper understanding of the cognitive mechanisms underlying dynamic decision making and suggests fruitful avenues for further investigation.

The final group was *non-planners*. These eight individuals showed the greatest deviation from normative behavior and were best fit by a model that neglected to plan future choices at all. While deliberating over what to do at DN(1) this model did not calculate planned choice probabilities for DN(2) and DN(3). Instead, a 50/50 proportion was assumed for all future choices. This means that non-planners were unable or unwilling to predict their future behavior, treating it instead like a random coin flip. In addition, non-planners severely distorted outcome information when looking ahead. The utility function during the planning stage was x^{a} for gains and $-\lambda|x|^{\alpha}$ for losses. The average α was found to be very small, 0.44, indicating that participants were capable of distinguishing gains from losses, but made little use of outcome information to further differentiate payoffs. λ was 0.61, signifying a strong gain seeking bias. Thus, non-planners used no information about their future choices and very little information about future outcomes to make decisions at DN(1). After making an initial choice, the model used a higher utility exponent, 0.84, for executed choices at DN(2) and DN(3). Participants appeared to struggle when planning ahead, but made more reasonable decisions later on.

The first two models presented were perfectly accurate in their perception of the transition probabilities at each CN, but non-planners tended to under-weight extreme probabilities according to the weighting function:

$$\pi(p) = \frac{p^{\gamma}}{[p^{\gamma} + (1-p)^{\gamma}]^{\frac{1}{\gamma}}} \tag{6}$$

$\pi(p)$ is the subjective weight put on probability p. The mean γ value, across individuals, was 0.82, which effectively pulls high and low probabilities closer to 0.50, leading to less accurate expectations. Recall that in DFT-D probability weighting is an emergent property of attention, where events are sampled according to their likelihoods. Equation 6 shows that non-planners under-sampled high probability events and over-sampled low probability events. Without DFT-D these behavioral patterns, and their differences across stages, would be difficult to identify. Individuals may appear to make quite random choices, but applying the model illuminates an interesting mixture of misperceptions and simplifications that underlie decisions.

6 Concluding remarks

DFT-D provides a starting point for a new investigation into dynamic decision making and planning. The parameterized functions used so far are meant as formal stand-ins for models of how attention guides sequential simulation. For example, a flat utility curve suggests selective attention focusing on some nodes, while ignoring others. However, another interpretation of such functions involves risk aversion. The myopic planning model might reflect extreme caution in planning, but riskiness later on. Accounting for how, what, and when information is attended to remains a goal for future work. More controlled experimental manipulations focused on testing these and other hypotheses are the next step in developing DFT-D.

Since its formulation DFT has provided a means for investigating many well established empirical findings, some of which have been thought to suggest irrationality. The value of this model to researchers comes from its focus on the dynamic processes that produce choice behavior. This approach allows the model to account for results in myriad domains, including dynamic decision environments in which very little cognitive research exists. Even in its formative state, DFT-D provides new insight into how people cope with some of the real world complexities traditionally ignored in the laboratory. It offers a unified, coherent explanation for interesting individual differences, and provides a framework for future development as parameterized functions are replaced by mechanistic components based on cognitive principles. The model is a new tool for generating predictions and characterizing behavior across a vast expanse of under-explored decision environments.

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