Option Trading and Pricing

An Introduction for Aspiring Quants

Jan 2, 2025

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Payoffs and Profits at Expiration - Calls

Notation

Stock Price = S_T Exercise Price = X

Payoff to Call Holder

$(S_T - X)$ if $S_T > X$

0   if $S_T \le X$

Profit to Call Holder

Payoff - Purchase Price

Payoff to Call Writer

$-(S_T - X)$ if $S_T > X$

0   if $S_T \le X$

Profit to Call Writer

Payoff + Premium

The value at expiration of the call with exercise price $100 is given by the schedule:

Stock price	$90	$100	$110	$120	$130
Option value	0	0	10	20	30

Payoff and Profit to Call Option at Expiration

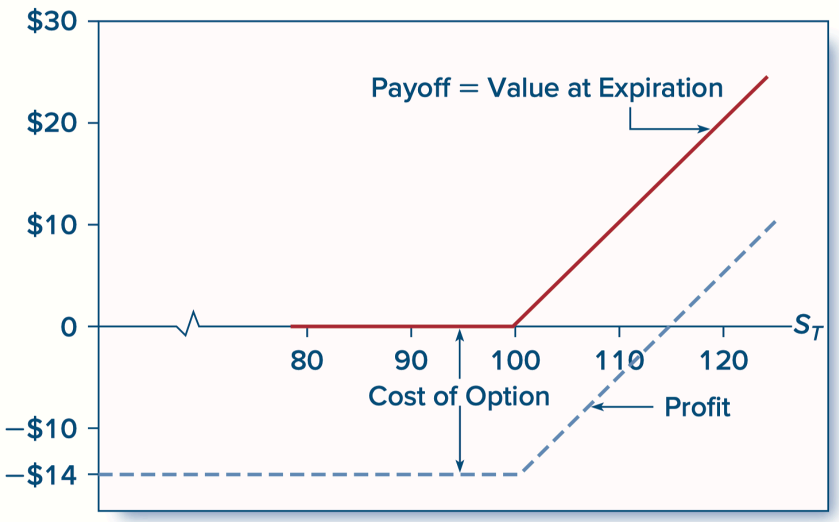

Figure 20.2 Payoff and profit to call option at expiration

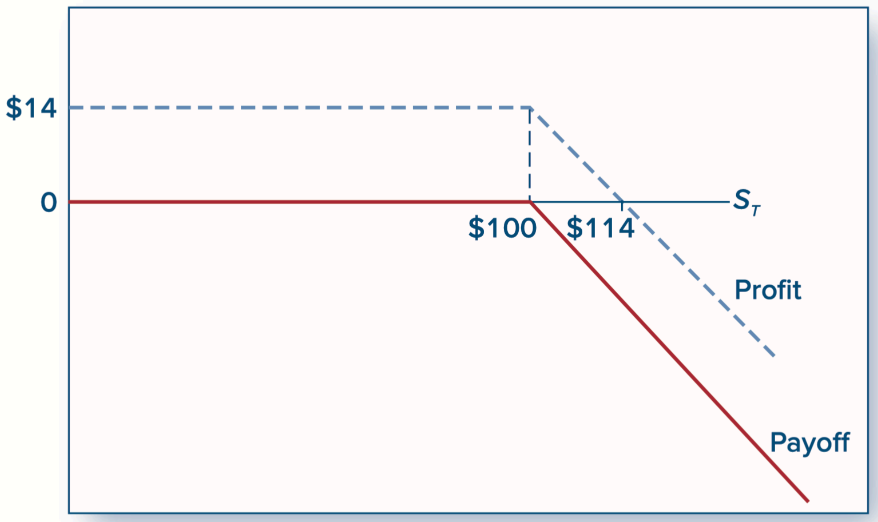

Figure 20.3 Payoff and profit to call writer at expiration

Payoffs and Profits at Expiration - Puts

Payoffs to Put Holder

0	if $S_T \geq X$
$(X - S_T)$	if $S_T < X$

Profit to Put Holder

Payoff - Premium

Payoffs to Put Writer

0	if $S_T \geq X$
$-(X - S_T)$	if $S_T < X$

Profits to Put Writer

Payoff + Premium

Payoff and Profit to Put Option at Expiration

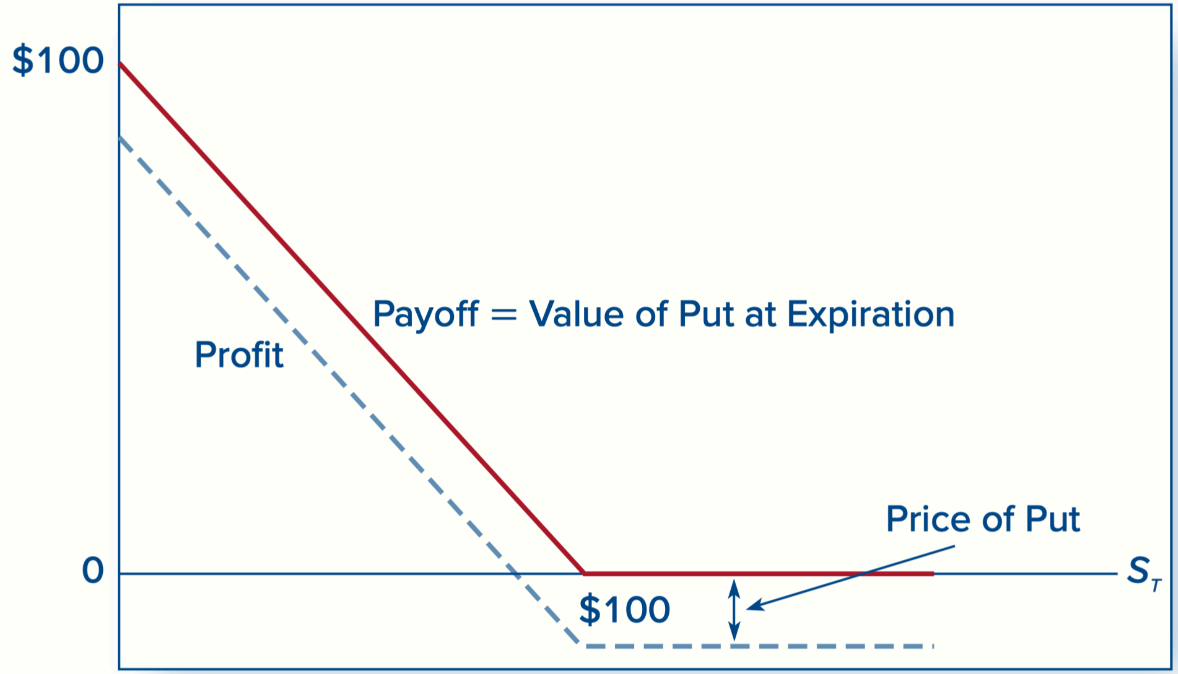

Figure 20.4 Payoff and profit to put option at expiration

Contents

Option versus Stock Investments

- Could a call option strategy be preferable to a direct stock purchase?
- Suppose you think a stock, currently selling for $100, will appreciate.
- A 12-month call costs $10 (contract size is 100 shares).
- You have $10,000 to invest.

Option versus Stock Investments

- Strategy A: Invest entirely in stock. Buy 100 shares, each selling for $100.
- Strategy B: Invest entirely in at-the-money call options. Buy 1,000 calls, each selling for $10. (This would require 10 contracts, each for 100 shares.)
- Strategy C: Purchase 100 call options for $1,000. Invest your remaining $9,000 in 12-month T-bills, to earn 3% interest. The bills will be worth $9,270 at expiration.

Option versus Stock Investment

Investment	Strategy		Investment
Equity only	Buy stock @ 100	100 shares	$10,000
Options only	Buy calls @ 10	1000 options	$10,000
Leveraged equity	Buy calls @ 10 Buy T-bills @ 3% Yield	100 options	$1,000 $9,000

Strategy Payoffs

Portfolio	Stock Price					
	$95	$100	$105	$110	$115	$120
Portfolio *A*: All stock	$9,500	$10,000	$10,500	$11,000	$11,500	$12,000
Portfolio *B*: All options	0	0	5,000	10,000	15,000	20,000
Portfolio *C*: Call plus bills	9,270	9,270	9,770	10,270	10,770	11,270

Portfolio	Stock Price					
	$95	$100	$105	$110	$115	$120
Portfolio *A*: All stock	−5.0%	0.0%	5.0%	10.0%	15.0%	20.0%
Portfolio *B*: All options	−100.0	−100.0	−50.0	0.0	50.0	100.0
Portfolio *C*: Call plus bills	−7.3	−7.3	−2.3	2.7	7.7	12.7

Rate of Return to Three Strategies

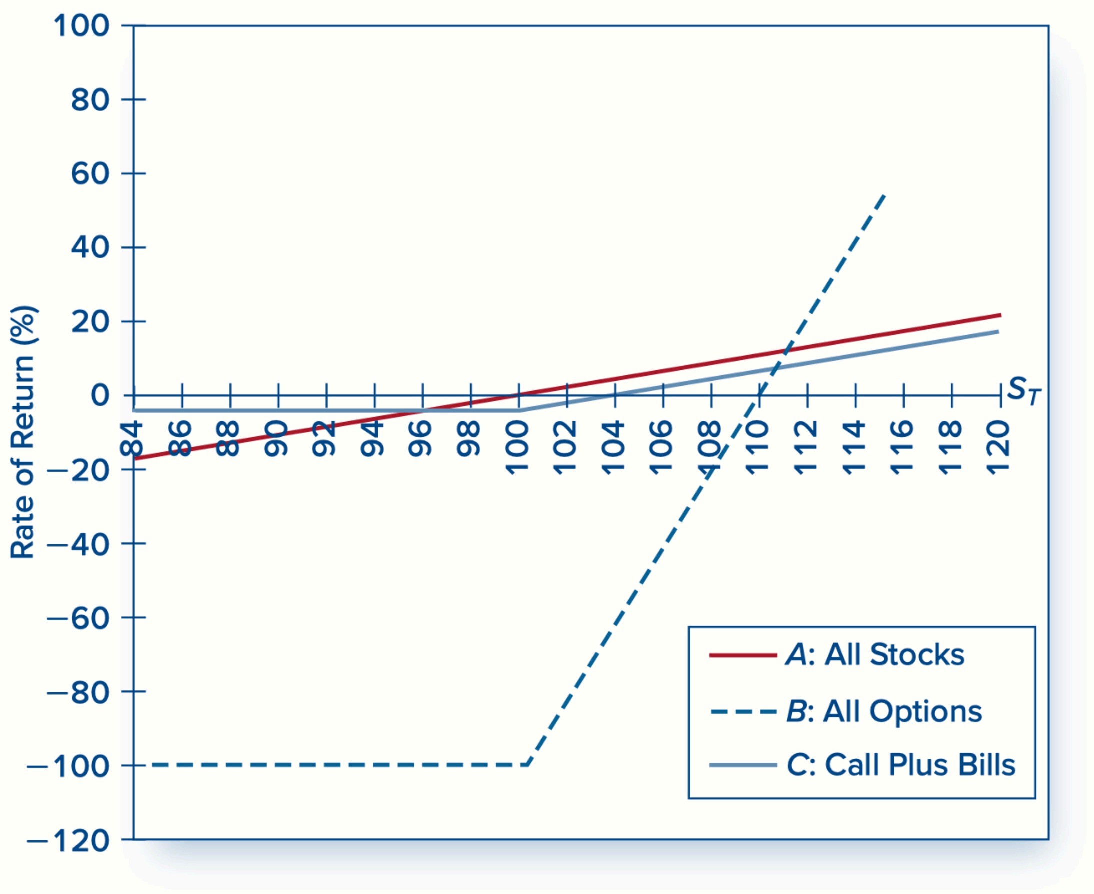

Figure 20.5 Rate of return to three strategies

Strategy Conclusions

- All-option portfolio, B, responds more than proportionately to changes in stock value; it is levered.

- Portfolio C, T-bills plus calls, shows the insurance value of options.
  - C ‘s T-bill position cannot be worth less than $9270.
  - Some return potential is sacrificed to limit downside risk.

- Option for speculation vs hedging

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Protective Put

- Puts can be used as insurance against stock price declines.
- Protective puts lock in a minimum portfolio value.
- The cost of the insurance is the put premium.

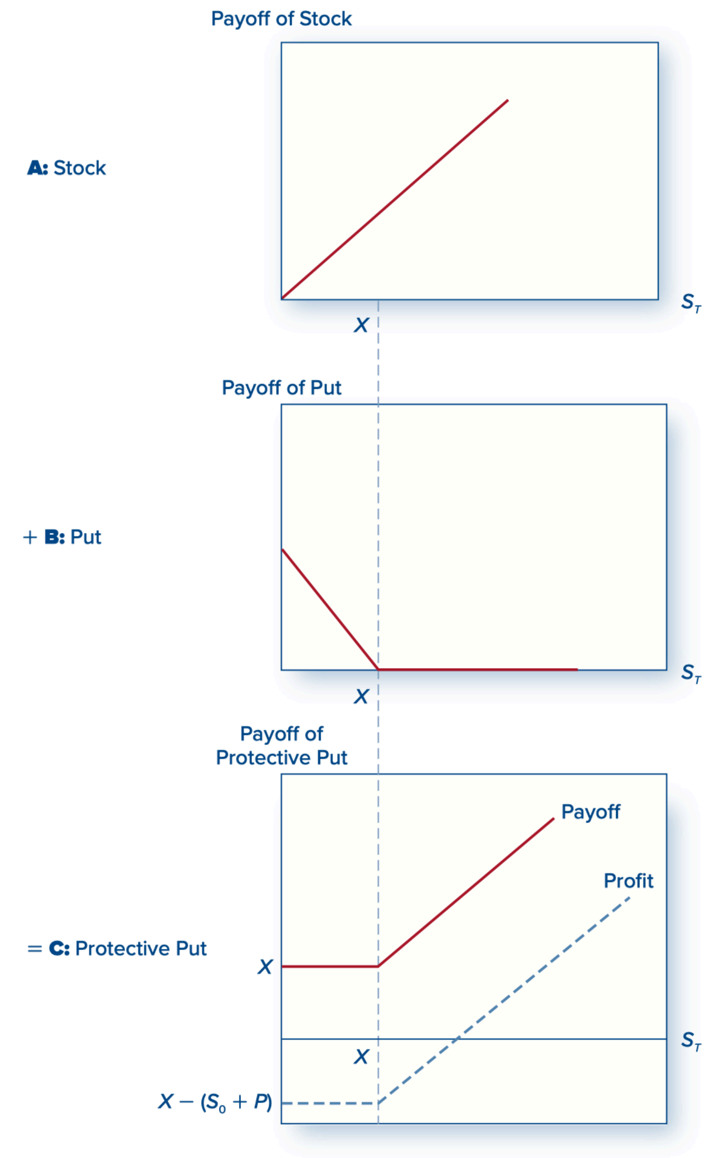

Figure 20.6 Value of a protective put position at option expiration

Protective Put

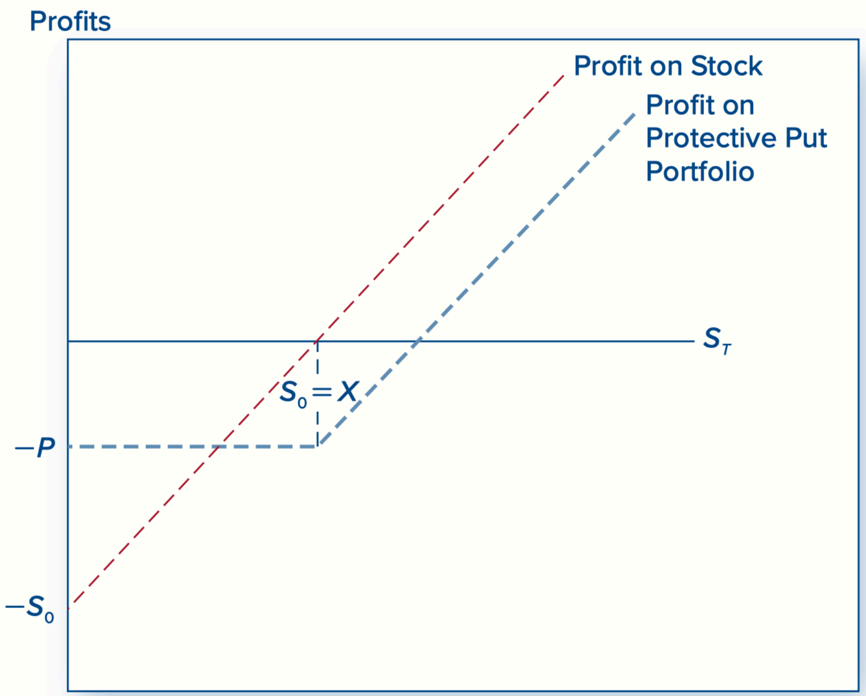

Figure 20.7 Protective put versus stock investment (at-the-money option)

Covered Calls

- Purchase stock and write calls against it.
- Call writer gives up any stock value above X in return for the initial premium.
- If you planned to sell the stock when the price rises above X anyway, the call imposes "sell discipline."

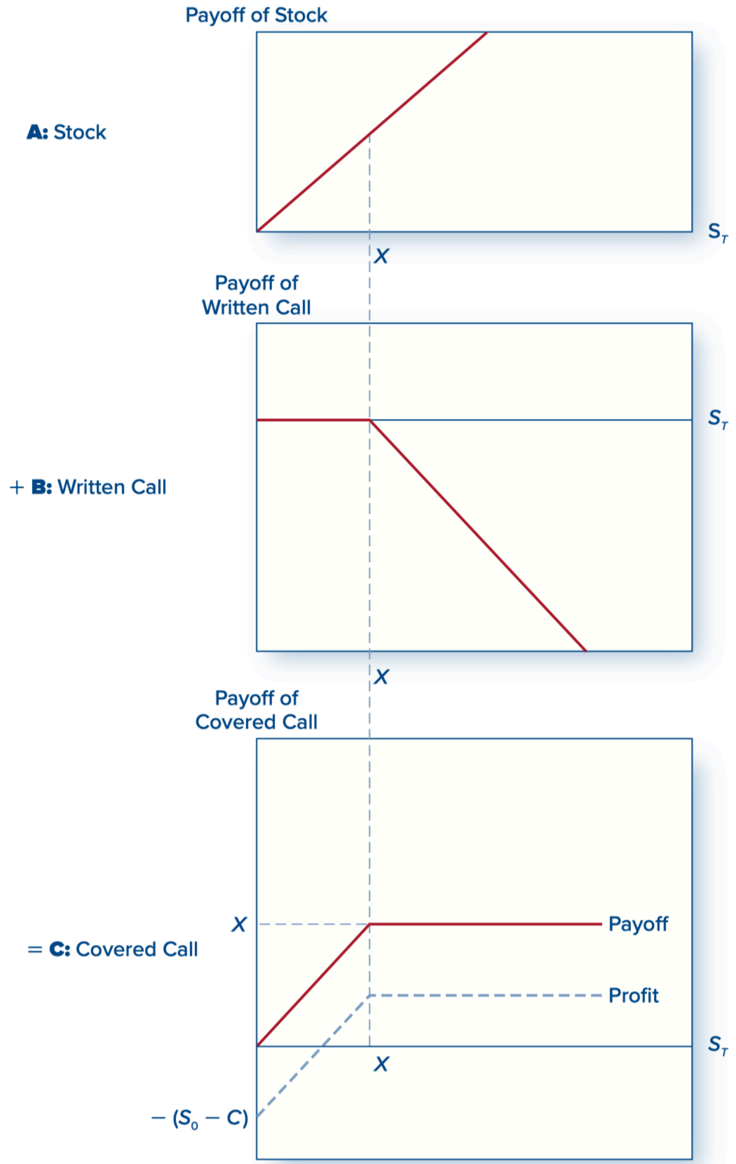

Figure 20.8 Value of a covered call position at expiration

Straddle

- Long straddle: Buy call and put with same exercise price and maturity.
- The straddle is a bet on volatility.
  - To make a profit, the change in stock price must exceed the cost of both options.
  - You need a strong change in stock price in either direction.
- The writer of a straddle is betting the stock price will not change much.

Value of a Straddle Position at Option Expiration

Table 20.3

Value of a straddle position at option expiration

	$S_T < X$	$S_T \geq X$
Payoff of call	0	$S_T - X$
+ Payoff of put	$X - S_T$	0
Total	$X - S_T$	$S_T - X$

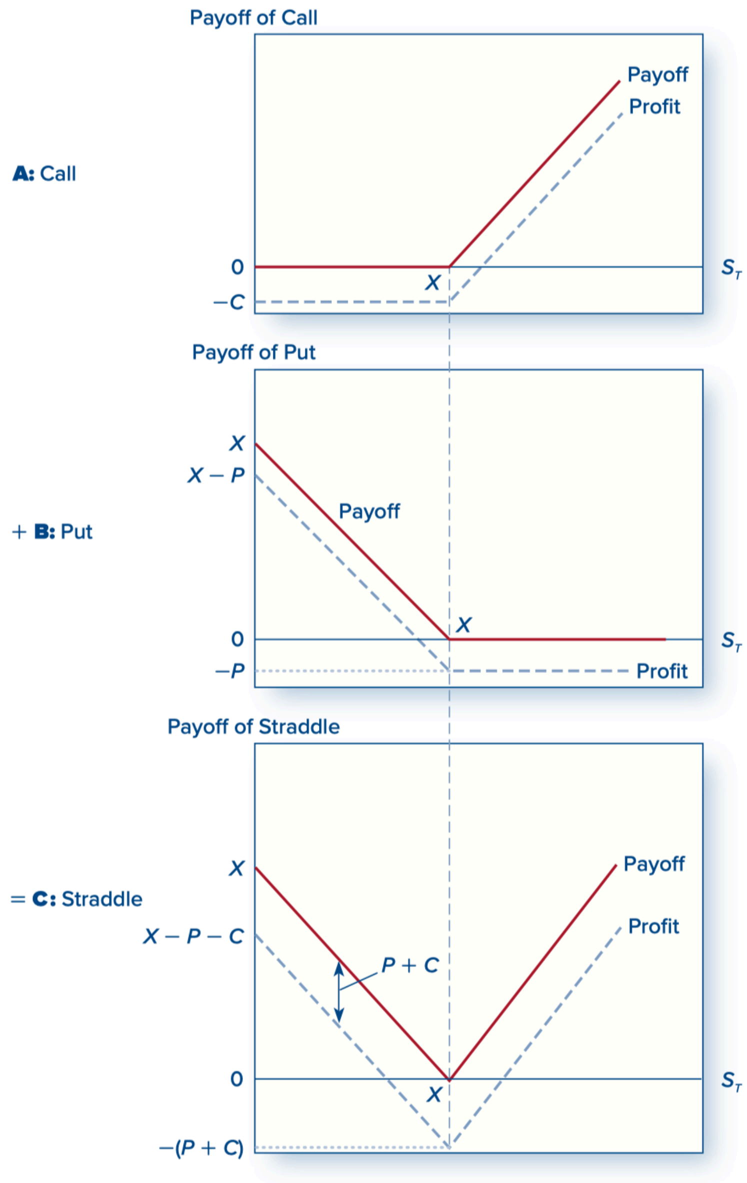

Figure 20.9 Value of a straddle at expiration

Spreads

- A spread is a combination of two or more calls (or two or more puts) on the same stock with differing exercise prices or times to maturity.

- Some options are bought, whereas others are sold, or written.
- A bullish spread is a way to profit from stock price increases.

Value of a Bullish Spread Position at Expiration

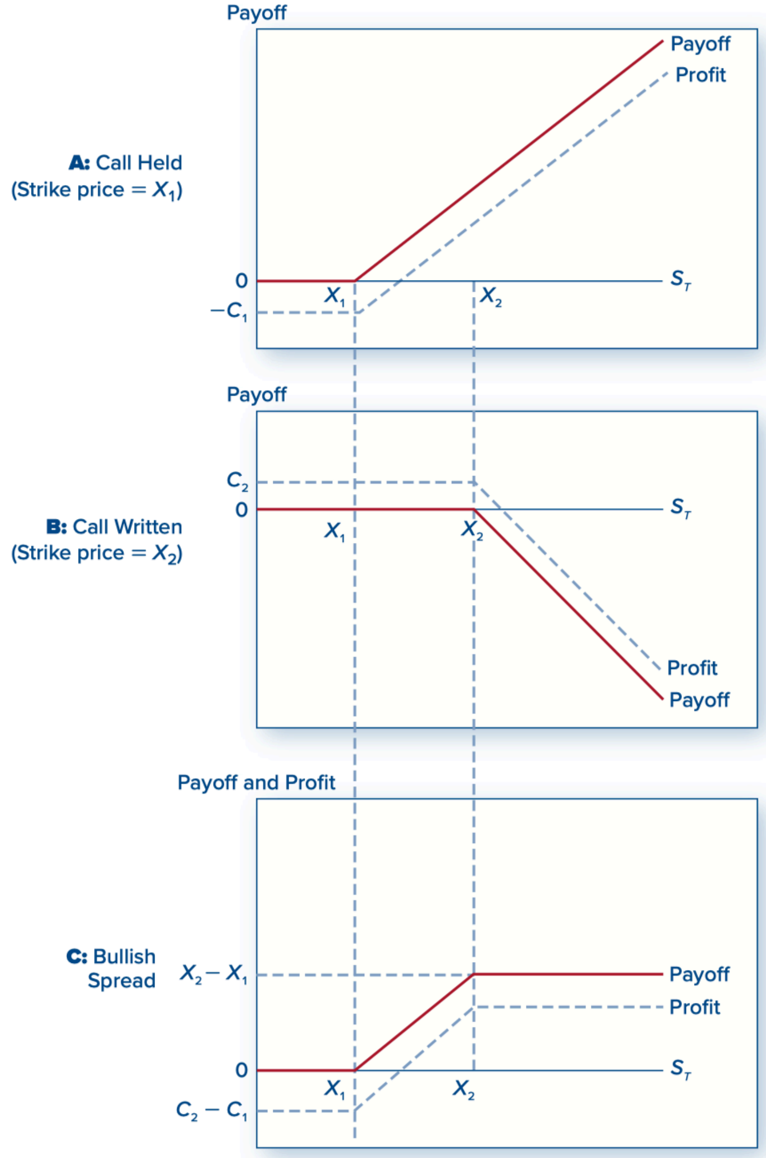

Figure 20.10 Value of a bullish spread position at expiration

Collars

- A collar is an options strategy that brackets the value of a portfolio between two bounds.
- Limit downside risk by selling upside potential.
- Buy a protective put to limit downside risk of a position.
- Fund put purchase by writing a covered call.
  - Net outlay for options is approximately zero.

Arbitrage Strategy

Table 20.5

Arbitrage strategy

Position	Immediate Cash Flow	Cash Flow in 1 Year	
		$S_T < 105$	$S_T \geq 105$
Buy stock	−110	S_T	S_T
Borrow \$105/1.05 = \$100	+100	−105	−105
Write call	+17	0	$-(S_T - 105)$
Buy put	−5	$105 - S_T$	0
Total	2	0	0

And a lot more…

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MARKET PULSE

- Options Market Overview
- Unusual Options Activity
- IV Rank and IV Percentile
- Implied vs Realized Vol
- Most Active Options
- Highest Open Positions
- Unusual Options Volume
- Highest Implied Volatility
- %Change in Volatility
- Options Volume Leaders
- Change in Open Interest
- %Chg in Open Interest
- Upcoming Earnings
- Options Price History
- Options Flow
- Options Calculator
- Options Time & Sales
- Options Learning Center

OPTION SCREENERS

- Options Screener
- Long Call Screener
- Long Put Screener

INCOME STRATEGIES

- Covered Call
- Naked Put

VERTICAL SPREADS

- Bull Call Spread
- Bear Call Spread
- Bear Put Spread
- Bull Put Spread

PROTECTION STRATEGIES

- Married Put
- Protective Collar

HORIZONTAL STRATEGIES

- Long Call Calendar
- Long Put Calendar
- Long Call Diagonal
- Short Call Diagonal
- Long Put Diagonal
- Short Put Diagonal

STRADDLE AND STRANGLE

- Long Straddle
- Short Straddle
- Long Strangle
- Short Strangle

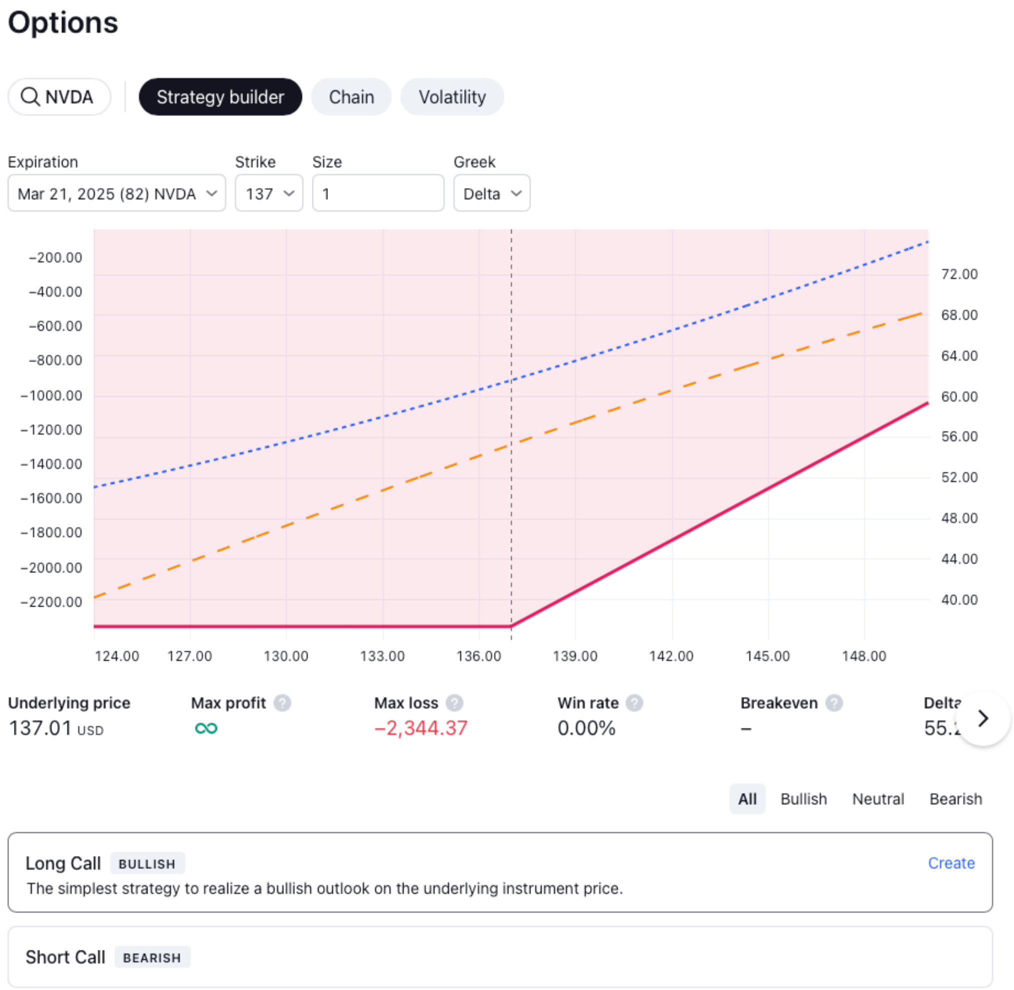
BUTTERFLY STRATEGIES

- Long Call Butterfly
- Short Call Butterfly
- Long Put Butterfly
- Short Put Butterfly
- Long Iron Butterfly
- Short Iron Butterfly

CONDOR STRATEGIES

- Long Call Condor
- Short Call Condor
- Long Put Condor
- Short Put Condor
- Long Iron Condor
- Short Iron Condor

Try it out!



https://www.tradingview.com/chart/FhQyJlQR/?symbol=NASDAQ:NVDA

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Stochastic Processes

- How would you describe stock prices movements through time?
- Incorporates uncertainties

https://www.tradingview.com/chart/FhQyJlQR/?symbol=NASDAQ:NVDA

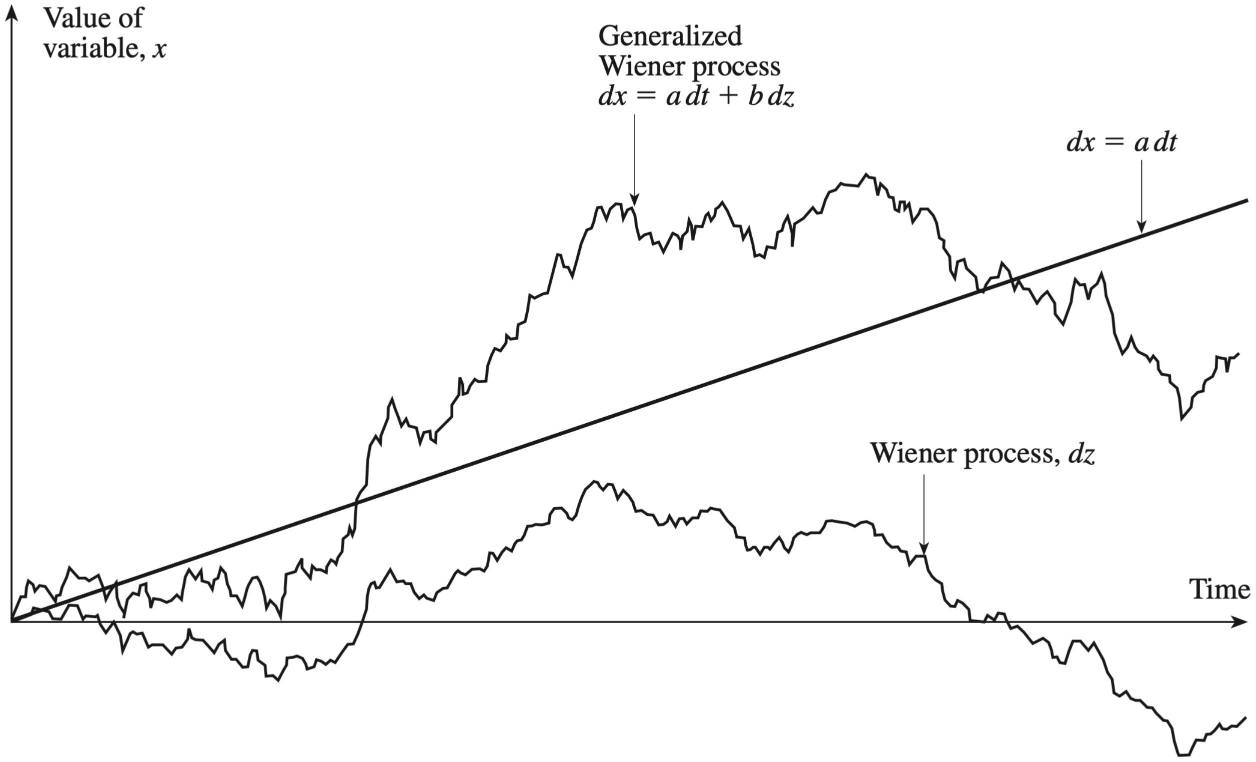
Brownian motion (Wiener Process)

$$dx = a\,dt + b\,dz$$

Drift

Diffusion

Figure 14.2 Generalized Wiener process with $a = 0.3$ and $b = 1.5$.



An Ito Process for Stock Prices

Geometric Brownian motion

$$\frac{dS}{S} = \mu dt + \sigma dz$$

$$dS = \mu S dt + \sigma S dz$$

The discrete time equivalent:

$$\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t}$$

S: stock price

μ: the expected return

σ: volatility.

The Derivation of the Black-Scholes-Merton Differential Equation

$$dS = \mu S dt + \sigma S dz$$

f is the price of derivative (i.e. an option)

$$f = f(t, S)$$

Ito's lemma (chain rule in stochastic world)

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}dS + \frac{1}{2}\frac{\partial^2 f}{\partial t^2}dS^2 = \left[\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2\frac{\partial^2 f}{\partial t^2}\right]dt + \frac{\partial f}{\partial S}dS$$

using $(dz)^2 = dt$ and ignore higher order terms

Another form of df

$$\boxed{df = \left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2\frac{\partial^2 f}{\partial t^2}\right]dt + \sigma\frac{\partial f}{\partial S}dz}$$

The Derivation of the Black-Scholes-Merton Differential Equation

We set up a portfolio P consisting of θ_1 derivative and θ_2 shares of stock

$$dP = \theta_1 df + \theta_2 dS = \theta_1 \left[\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial t^2}\right] dt + \theta_1 \frac{\partial f}{\partial S} dS + \theta_2 dS$$

We are the manager and free to choose the positions. Let $\theta_1 = 1$ and $\theta_2 = -\dfrac{\partial f}{\partial S}$

$$dP = \left[\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial t^2}\right] dt$$

Since there is no risk in P , its appreciation must equal the earnings of a risk-free investment

$$dP = rPdt = r(f - S\frac{\partial f}{\partial S})dt$$

Combine the two, we get Black-Scholes differential equation,

$$\boxed{\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial t^2} = rf}$$

The Black-Scholes-Merton Formulas for Options

European calls and puts options:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2\frac{\partial^2 f}{\partial t^2} = rf$$

$0 \leq S_T$ and $0 \leq t \leq T$

$f(T) = \max(S_T - K, 0)$ for an European call

$$\boxed{\begin{aligned} c &= S_0 N(d_1) - K e^{-rT} N(d_2) \\ p &= K e^{-rT} N(-d_2) - S_0 N(-d_1) \end{aligned}}$$

where $d_1 = \dfrac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$

$$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

$N(d)$ is cdf of a standard normal distribution,

$$N(d_i) = \int_{-\infty}^{d_i} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx$$

Understanding Black-Scholes

$$c = e^{-rT} N(d_2)\left(S_0 e^{rT} N(d_1)/N(d_2) - K\right)$$

e^{-rT} : Present value factor

$N(d_2)$: Probability of exercise

$S_0 e^{rT} N(d_1)/N(d_2)$: Expected stock price in a risk - neutral world if option is exercised

K : Strike price paid if option is exercised

Properties of Black-Scholes Formula

$$c = S_0 N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$

where $$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

- As S_0 becomes very large c tends to $S_0 - Ke^{-rT}$ and p tends to zero
- As S_0 becomes very small c tends to zero and p tends to $Ke^{-rT} - S_0$
- What happens as σ becomes very large?
- What happens as T becomes very large?

Implied Volatility

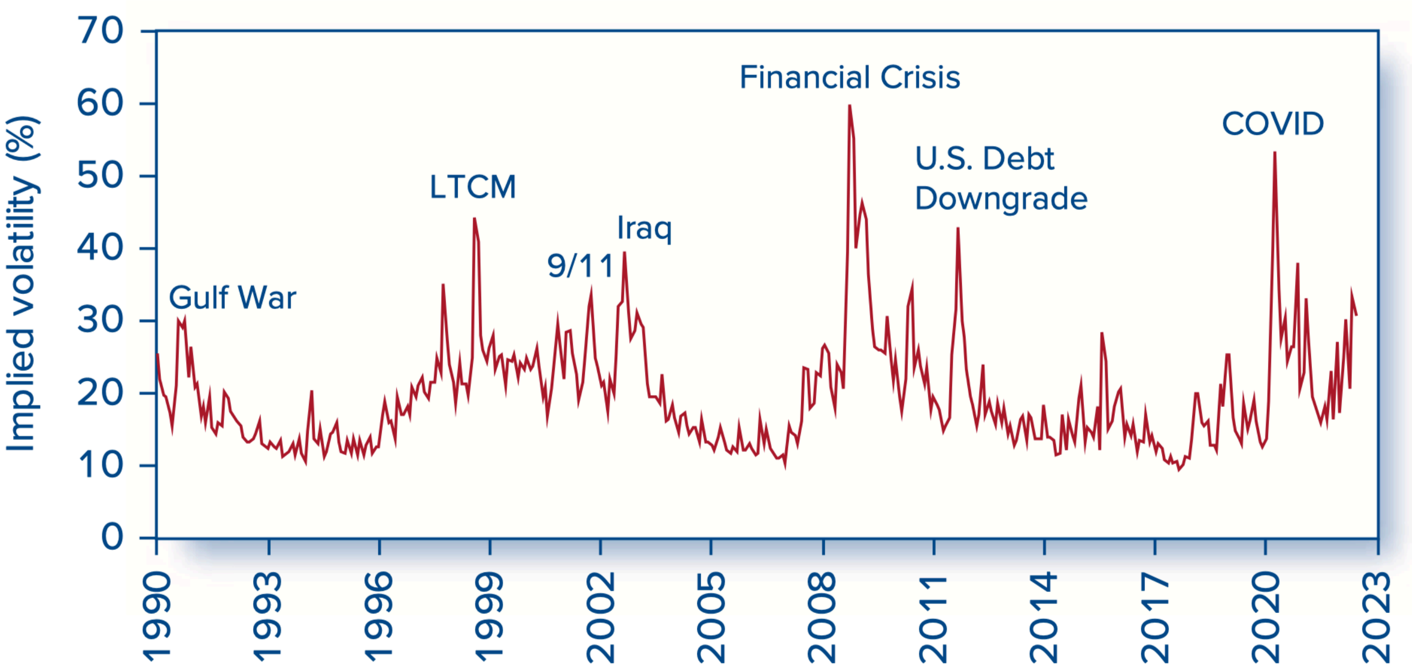


Figure 21.8 Implied volatility of the S&P 500 (VIX index)

Source: Chicago Board Options Exchange, **www.cboe.com.**

Using of Black-Scholes Option valuation

- Hedge Ratios and the Black-Scholes Formula: delta and gamma hedging
- Portfolio insurance
- Option pricing and the financial crisis
- Option pricing and portfolio theory
- Hedging bets on mispriced options (arbitrage)

Options Calculator

Generate fair value prices and Greeks for any U.S or Canadian equity or index options contract.

Sat, Dec 28th, 2024 Help ?

Customize your input parameters by entering the option type, strike price, days to expiration (DTE), and risk-free rate, volatility, and (optional) dividend yield% for equities. The calculator uses the latest price for the underlying symbol. Theoretical values and IV calculations are performed using the Black 76 Pricing model, which is different than the Binomial model used on the symbol's Volatility & Greeks page.

Enter an equity or index symbol: Search for a Symbol...

Apple Inc (AAPL)

Quick Links

255.59 -3.43 (-1.32%) 12/27/24 (NASDAQ)

Select the option for calculation. The Input Parameters may be overridden below.

Option Type Call **Expiration** 2025-03-21 (m) 83 DTE (days) **Strike Price** 255.00

Input Parameters

Underlying Price	255.59
Strike Price	255.00
DTE (days)	83
Risk-free rate%	4.20%
Volatility	22.15%
Dividend Yield% (equities)	0.39%

Calculated Theoretical Values

Theoretical Price	12.15
Delta	0.56227
Gamma	0.01460
Vega	0.48030
Theta	-0.07921
Rho	0.29888

IV Calculation

Option	Call
Market Option Price	Last Bid Ask
	12.22
Implied Volatility	22.30%

https://www.barchart.com/options/options-calculator

Contents

Project:

Simulate a delta-hedged portfolio with Monte Carlo simulations:

- Stock: NVDA
- Current stock price: $137
- Implied vol: 20%
- Interest rate: 5%
- Expiration: one year
- Portfolio: a short call option + Δ shares of NVDA
- Goal: simulate the portfolio value’s time evolution with 10,000 paths

Takeaways

- Valuations of options at expiration
- Options vs stock investments
- Trading strategies involving options
- Black-Scholes model for option pricing
- Delta hedging simulation with Monte Carlo methods

Q&A

Reference

1. Bodie, Zvi, Alex Kane, and Alan J. Marcus. *Investments,* 13th edition. McGraw-hill, 2024.
2. Hull, John. *Options, futures and other derivatives, 11th edition*. Upper Saddle River, NJ: Prentice Hall, 2022.

BS model and beyond?

1. If options are so good, why is not everyone using them?
   - High leverage and risk: you can lose them all
   - Complicated, you need permission
   - You pay a premium for the risk you are not taking
2. Is BS model realistic?
   - No: McDonald's p631 about assumptions and reasons: However, our goal is to have a thorough understanding of how
   - derivatives pricing and hedging works in this basic setting. This is a starting point for developing more realistic models - examples?
   - Numerical methods are important
3. What’s done in practice then?
   - Models beyond Black-Schole’s
   - Exotic options: Asian options, Bermudan options, cliquets, cms options, digital options
   - Beyond 1 stock and 1 option: multiple assets
   - Greek letters to hedge other risk factors
   - Numerical methods