Option Trading and Pricing

Jan 2, 2025

Contents

Options

- Derivatives are securities that get their value from the price of other securities.
- Can be powerful tools for hedging and speculation.
- Options are traded both on organized exchanges and OTC.

The Option Contract: Calls

- A call option gives its holder the right to buy an asset:
  - At the exercise or strike price
  - On or before the expiration date
- Exercise the option to buy the underlying asset if market value > strike price.

The Option Contract: Puts

- A put option gives its holder the right to sell an asset:
  - At the exercise or strike price
  - On or before the expiration date
- Exercise the option to sell the underlying asset if market value < strike price.

The Option Contract

- The purchase price of the option is called the <u>premium</u>.
- Sellers (writers) of options receive premium income.
- If holder exercises the option, the option writer must make (call) or take (put) delivery of the underlying asset.

Profit and Loss on a Call

- A January 2010 call on IBM with an exercise price of $130 was selling on December 2, 2009, for $2.18.
- The option expires on the third Friday of the month, or January 15, 2010.
- If IBM remains below $130, the call will expire worthless.

Profit and Loss on a Call

- Suppose IBM sells for $197 on the expiration date.
- Option value = stock price-exercise price
  $197- $195= $2
- Profit = Final value – Original investment
  $2.00 - $3.65 = -$1.65
- Option will be exercised to offset loss of premium.
- Call will not be strictly profitable unless IBM's price exceeds $198.65 (strike + premium) by expiration.

Profit and Loss on a Put

- Consider a February 2013 put on IBM with an exercise price of \$195, selling on January 18, for \$5.00.
- Option holder can sell a share of IBM for \$195 at any time until February 15.
- If IBM goes above \$195, the put is worthless.

Profit and Loss on a Put

- Suppose IBM's price at expiration is $188.
- Value at expiration = exercise price – stock price:
  $195 - $188 = $7
- Investor's profit:
  $7.00 - $5.00 = $2.00
- Holding period return = 40% over 28 days!

Market and Exercise Price Relationships

In the Money - exercise of the option produces a positive cash flow

Call: exercise price < asset price

Put: exercise price > asset price

Out of the Money - exercise of the option would not be profitable

Call: asset price < exercise price.

Put: asset price > exercise price.

At the Money - exercise price and asset price are equal

American vs. European Options

American - the option can be exercised at any time before expiration or maturity

European - the option can only be exercised on the expiration or maturity date

•In the U.S., most options are American style, except for currency and stock index options.

Different Types of Options

- Stock Options
- Index Options
- Futures Options
- Foreign Currency Options
- Interest Rate Options

Payoffs and Profits at Expiration - Calls

Notation

Stock Price = S_T Exercise Price = X

Payoff to Call Holder

$(S_T - X)$ if $S_T > X$

0 if $S_T \leq X$

Profit to Call Holder

Payoff - Purchase Price

Payoffs and Profits at Expiration - Calls

Payoff to Call Writer

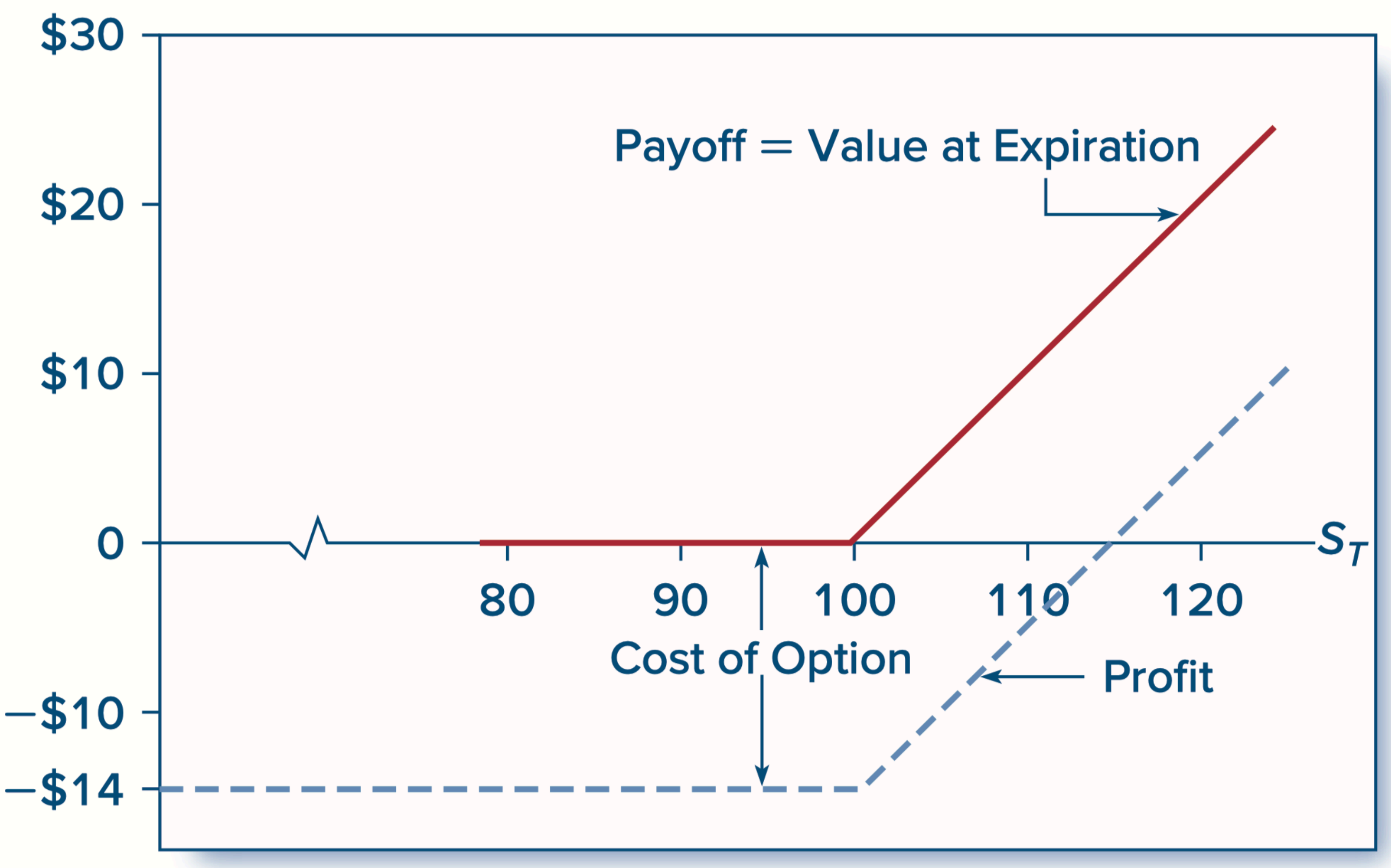
$-(S_T - X)$ if $S_T > X$

0 if $S_T \leq X$

Profit to Call Writer

Payoff + Premium

Payoff and Profit to Call Option at Expiration



Payoff and Profit to Call Writers at Expiration

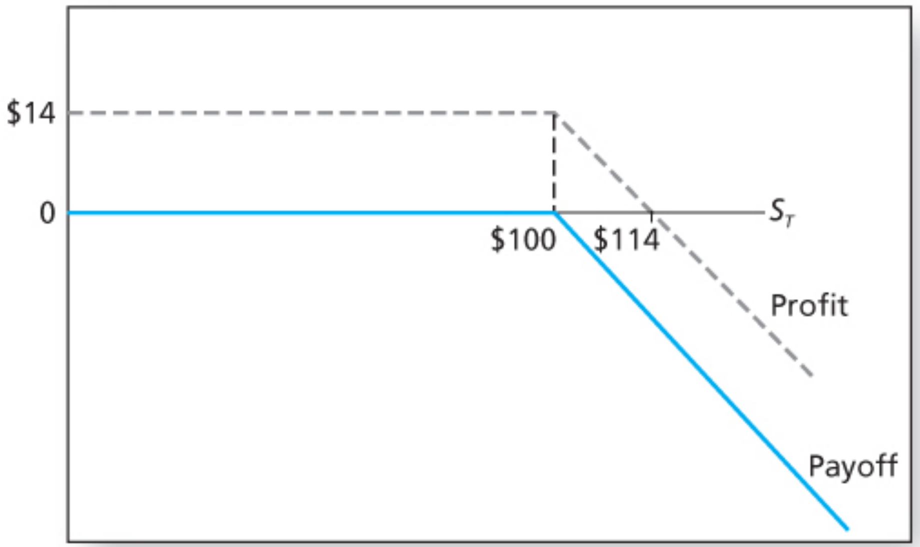


Figure 20.3 Payoff and profit to call writers at expiration

Payoffs and Profits at Expiration - Puts

<u>Payoffs to Put Holder</u>

0 if $S_T \geq X$

$(X - S_T)$ if $S_T < X$

<u>Profit to Put Holder</u>

Payoff - Premium

Payoffs and Profits at Expiration – Puts

Payoffs to Put Writer

0	if	$S_T \geq X$
$-(X - S_T)$	if	$S_T < X$

Profits to Put Writer

Payoff + Premium

Payoff and Profit to Put Option at Expiration

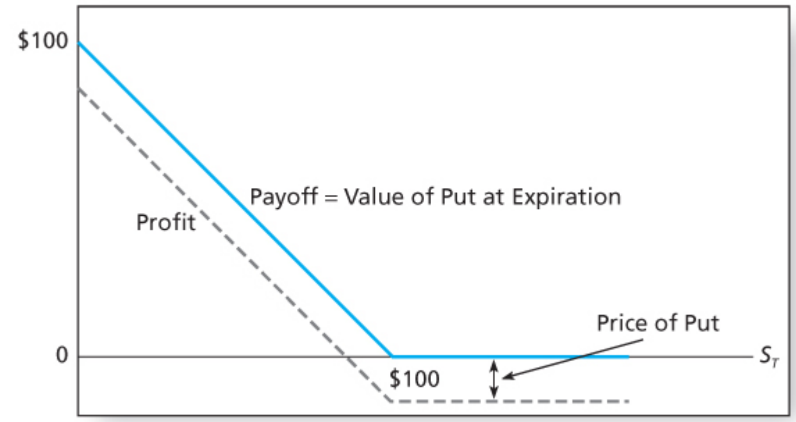


Figure 20.4 Payoff and profit to put option at expiration

Option versus Stock Investments

- Could a call option strategy be preferable to a direct stock purchase?
- Suppose you think a stock, currently selling for $100, will appreciate.
- A 6-month call costs $10 (contract size is 100 shares).
- You have $10,000 to invest.

Option versus Stock Investments

- Strategy A: Invest entirely in stock. Buy 100 shares, each selling for $100.
- Strategy B: Invest entirely in at-the-money call options. Buy 1,000 calls, each selling for $10. (This would require 10 contracts, each for 100 shares.)
- Strategy C: Purchase 100 call options for $1,000. Invest your remaining $9,000 in 6-month T-bills, to earn 3% interest. The bills will be worth $9,270 at expiration.

Option versus Stock Investment

Investment	Strategy		Investment
Equity only	Buy stock @ 100	100 shares	$10,000
Options only	Buy calls @ 10	1000 options	$10,000
Leveraged equity	Buy calls @ 10 Buy T-bills @ 3% Yield	100 options	$1,000 $9,000

Strategy Payoffs

Portfolio	Stock Price $95	$100	$105	$110	$115	$120
Portfolio *A*: All stock	$9,500	$10,000	$10,500	$11,000	$11,500	$12,000
Portfolio *B*: All options	0	0	5,000	10,000	15,000	20,000
Portfolio *C*: Call plus bills	9,270	9,270	9,770	10,270	10,770	11,270

Portfolio	Stock Price $95	$100	$105	$110	$115	$120
Portfolio *A*: All stock	−5.0%	0.0%	5.0%	10.0%	15.0%	20.0%
Portfolio *B*: All options	−100.0	−100.0	−50.0	0.0	50.0	100.0
Portfolio *C*: Call plus bills	−7.3	−7.3	−2.3	2.7	7.7	12.7

Rate of Return to Three Strategies

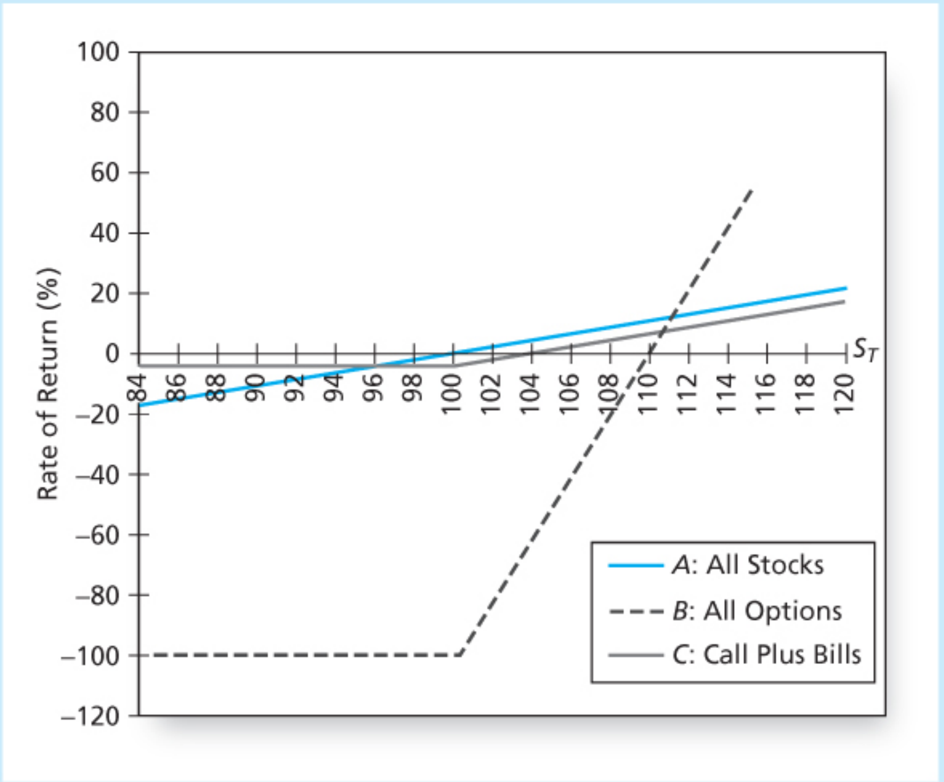


Figure 20.5 Rate of return to three strategies

Strategy Conclusions

- Figure 20.5 shows that the all-option portfolio, B, responds more than proportionately to changes in stock value; it is <u>levered</u>.
- Portfolio C, T-bills plus calls, shows the <u>insurance</u> value of options.
  - C ‘s T-bill position cannot be worth less than $9270.
  - Some return potential is sacrificed to limit downside risk.

Protective Put Conclusions

- Puts can be used as insurance against stock price declines.
- Protective puts lock in a minimum portfolio value.
- The cost of the insurance is the put premium.
- Options can be used for risk management, not just for speculation.

Covered Calls

- Purchase stock and write calls against it.
- Call writer gives up any stock value above X in return for the initial premium.
- If you planned to sell the stock when the price rises above X anyway, the call imposes “sell discipline.”

Value of a Covered Call Position at Expiration

Table 20.2

Value of a covered call position at option expiration

	$S_T \le X$	$S_T > X$
Payoff of stock	S_T	S_T
+ Payoff of written call	-0	$-(S_T - X)$
= *TOTAL*	S_T	X

Value of a Covered Call Position at Expiration

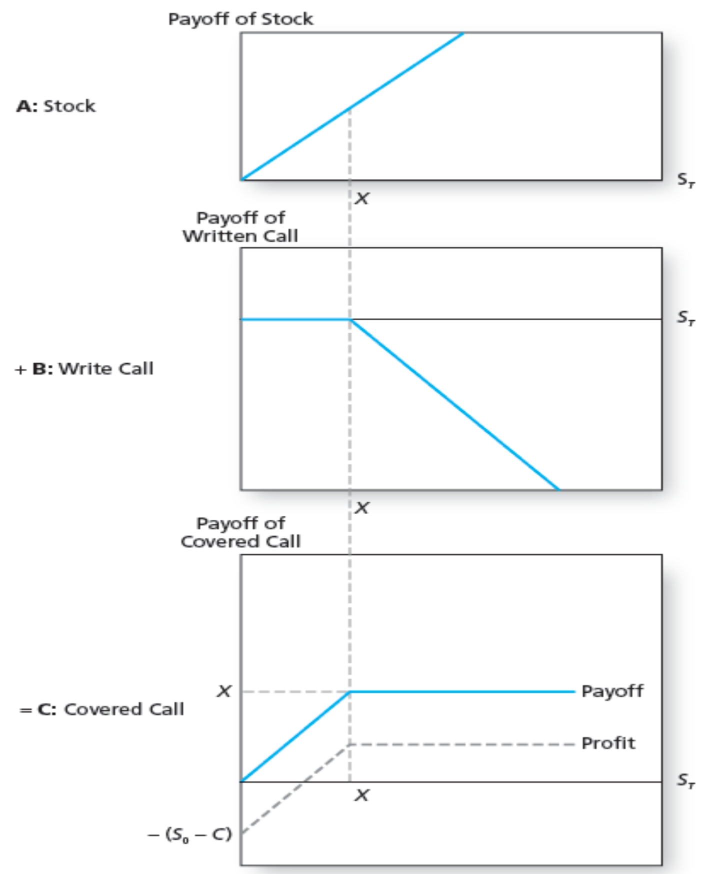


Figure 20.8 Value of a covered call position at expiration

Straddle

- Long straddle: Buy call and put with same exercise price and maturity.
- The straddle is a bet on volatility.
  - To make a profit, the change in stock price must exceed the cost of both options.
  - You need a strong change in stock price in either direction.
- The writer of a straddle is betting the stock price will not change much.

Value of a Straddle Position at Option Expiration

	$S_T < X$	$S_T \geq X$
Payoff of call	0	$S_T - X$
+ Payoff of put	$X - S_T$	0
= *TOTAL*	$X - S_T$	$S_T - X$

Table 20.3

Value of a straddle position at option expiration

Value of a Straddle at Expiration

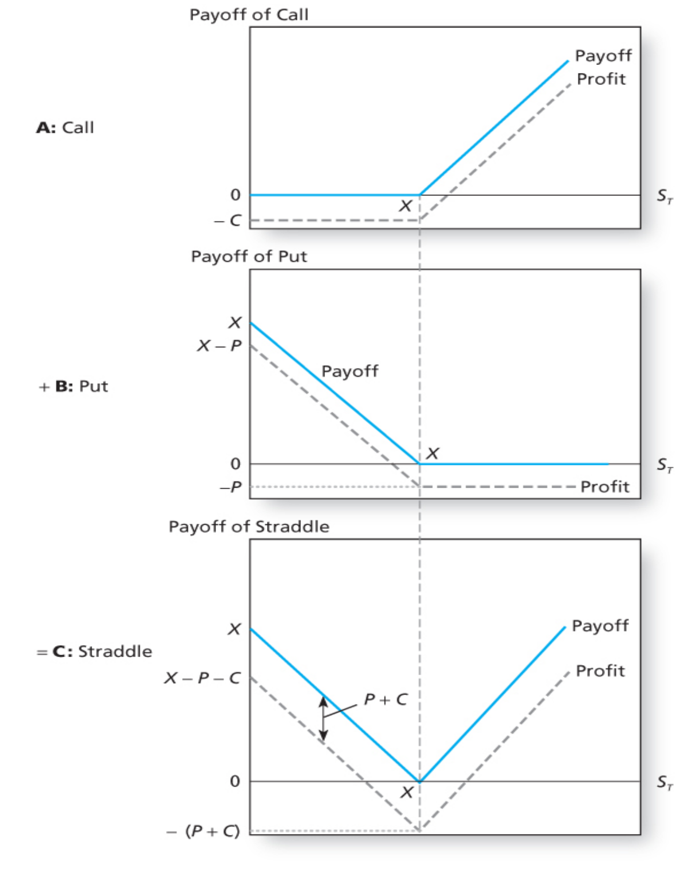


Figure 20.9 Value of a straddle at expiration

Spreads

- A spread is a combination of two or more calls (or two or more puts) on the same stock with differing exercise prices or times to maturity.

  - Some options are bought, whereas others are sold, or written.
  - A bullish spread is a way to profit from stock price increases.

Value of a Bullish Spread Position at Expiration

Table 20.4

Value of a bullish spread position at expiration

	$S_T \le X_1$	$X_1 < S_T \le X_2$	$S_T \ge X_2$
Payoff of purchased call, exercise price = X_1	0	$S_T - X_1$	$S_T - X_1$
+ Payoff of written call, exercise price = X_2	−0	−0	$-(S_T - X_2)$
= *TOTAL*	0	$S_T - X_1$	$X_2 - X_1$

Value of a Bullish Spread Position at Expiration

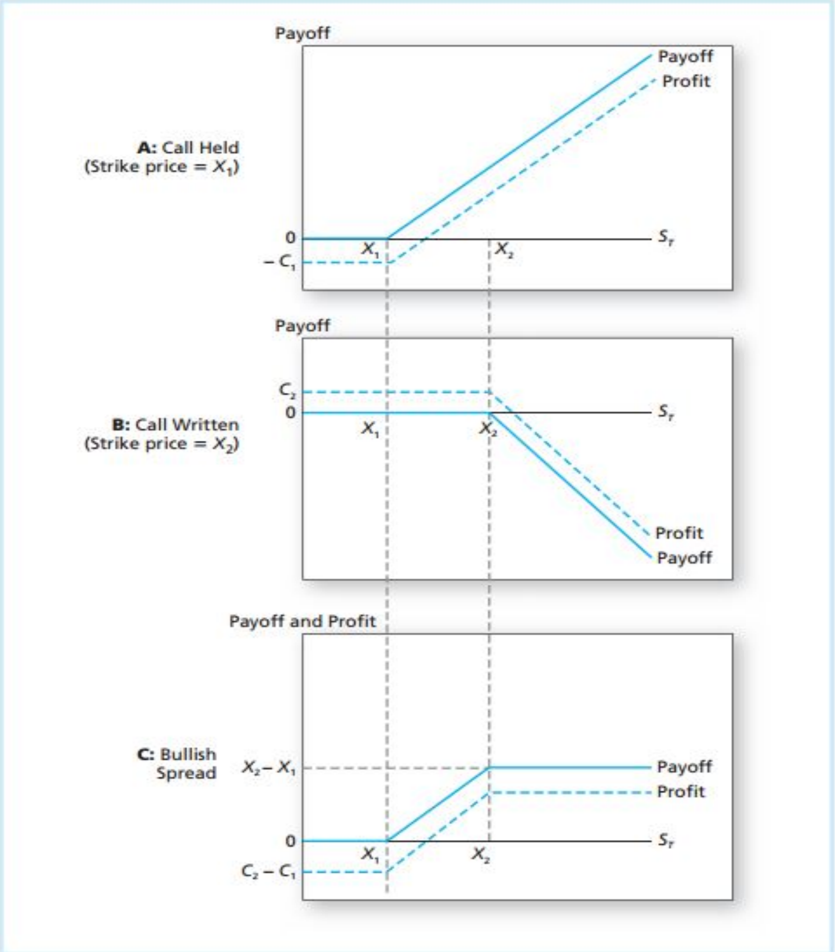


Figure 20.10 Value of a bullish spread position at expiration

Collars

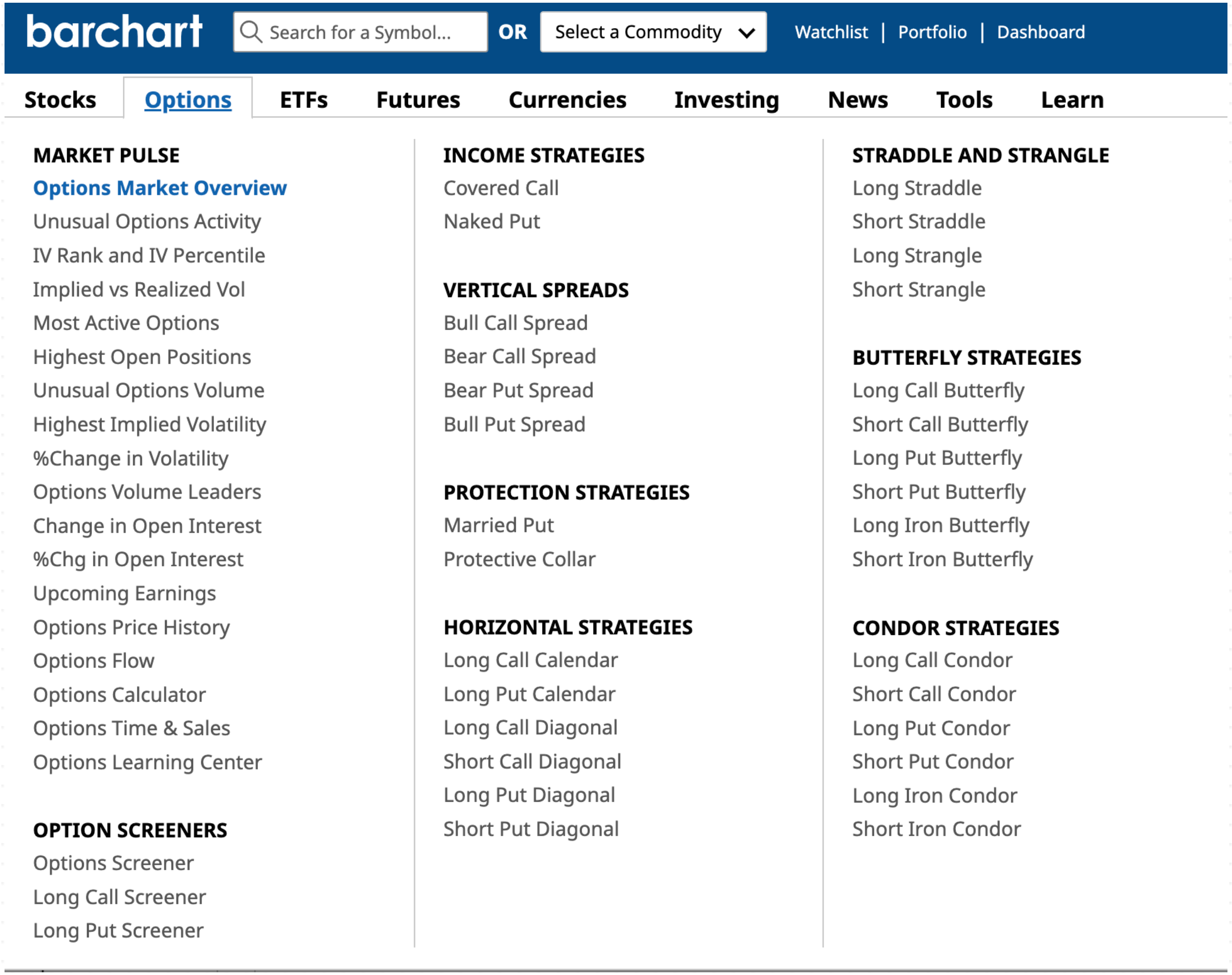
- A collar is an options strategy that brackets the value of a portfolio between two bounds.
- Limit downside risk by selling upside potential.
- Buy a protective put to limit downside risk of a position.
- Fund put purchase by writing a covered call.
  - Net outlay for options is approximately zero.

Arbitrage Strategy

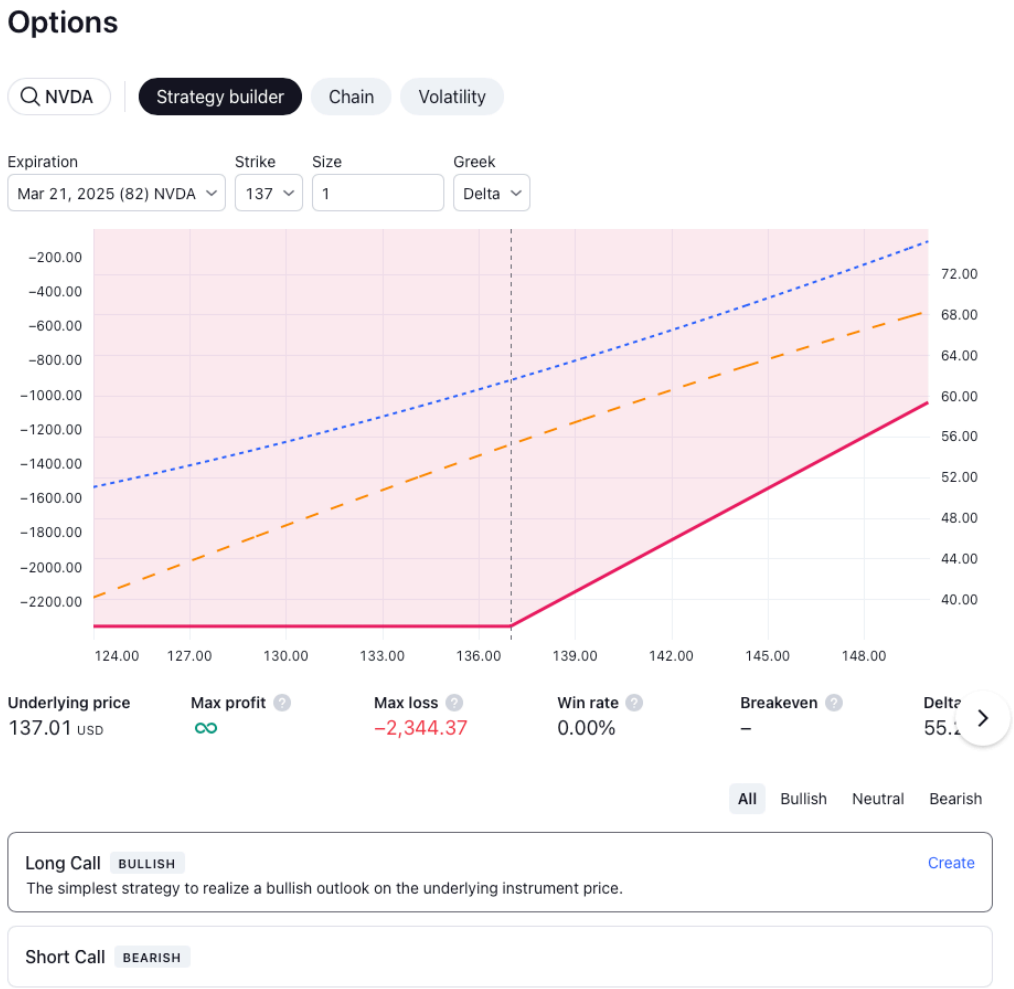
Position	Immediate Cash Flow	Cash Flow in 1 Year	
		$S_T < 105$	$S_T \geq 105$
Buy stock	−110	S_T	S_T
Borrow $105/1.05 = $100	+100	−105	−105
Sell call	+17	0	$-(S_T - 105)$
Buy put	−5	$105 - S_T$	0
TOTAL	2	0	0

Table 20.5 Arbitrage strategy

And a lot more…



Try it out!



https://www.tradingview.com/chart/FhQyJlQR/?symbol=NASDAQ:NVDA

Stochastic Processes

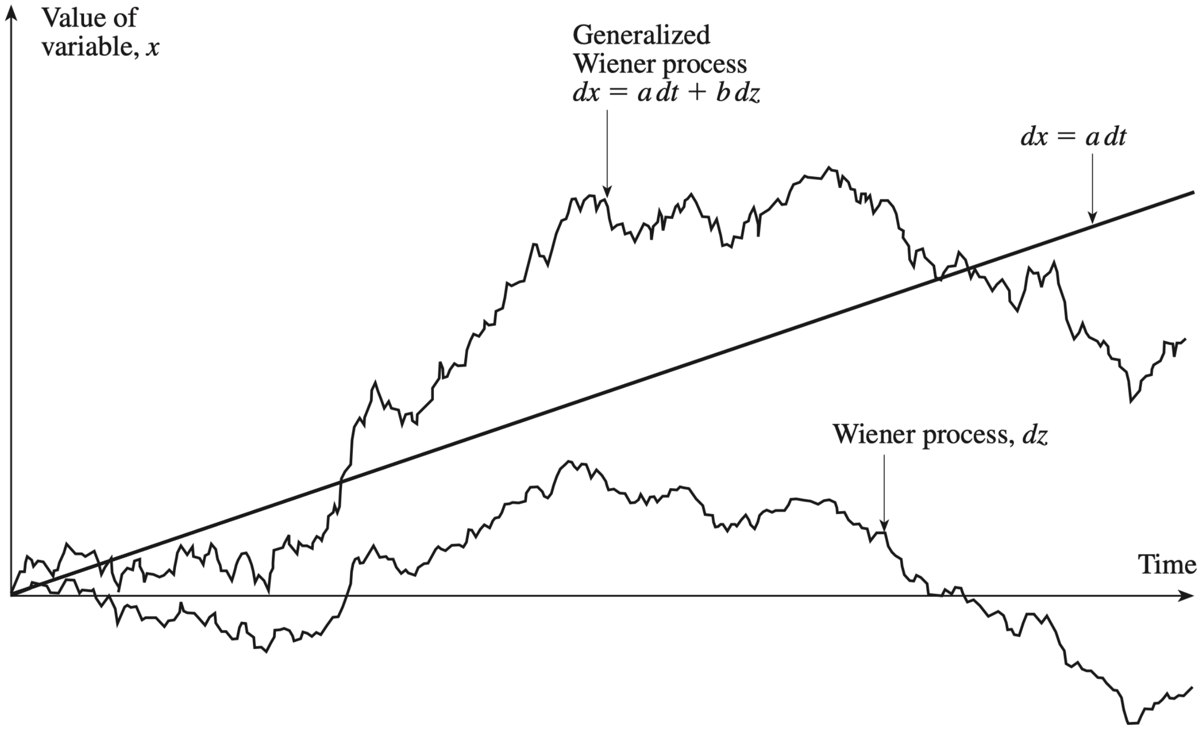
- How would you describe stock prices movements through time?
- Incorporates uncertainties

https://www.tradingview.com/chart/FhQyJlQR/?symbol=NASDAQ:NVDA

Wiener Process

$$dx = adt + bdz$$

Figure 14.2 Generalized Wiener process with $a = 0.3$ and $b = 1.5$.



An Ito Process for Stock Prices

Geometric Brownian motion

$$\frac{dS}{S} = \mu dt + \sigma dz$$

$$dS = \mu S dt + \sigma S dz$$

The discrete time equivalent:

$$\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t}$$

S: stock price

μ: the expected return

σ: volatility.

The Derivation of the Black-Scholes-Merton Differential Equation

$$\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t}$$

$$\Delta f = \left(\frac{\partial f}{\partial S} \mu S + \frac{\partial f}{\partial t} + ½ \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) \Delta t + \frac{\partial f}{\partial S} \sigma S \, \Delta z$$

We set up a portfolio consisting of

-1: derivative

$+\frac{\partial f}{\partial S}$: shares of stock

This gets rid of the dependence on Δz

The Derivation of the Black-Scholes-Merton Differential Equation

The value of the portfolio, Π, is given by

$$\Pi = -f + \frac{\partial f}{\partial S} S$$

The change in its value in time Δt is given by

$$\Delta \Pi = -\Delta f + \frac{\partial f}{\partial S} \Delta S$$

The Derivation of the Black-Scholes-Merton Differential Equation

The return on the portfolio must be risk-free rate. Hence

$$\Delta\Pi = r\,\Pi\Delta t$$

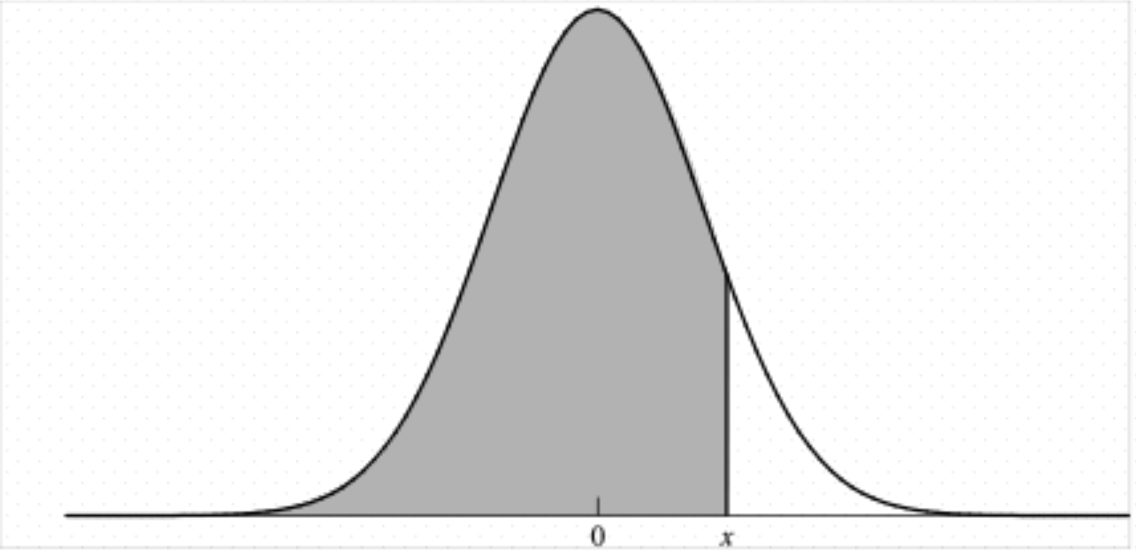
$$-\Delta f + \frac{\partial f}{\partial S}\Delta S = r\left(-f + \frac{\partial f}{\partial S}S\right)\Delta t$$

We substitute for Δf and ΔS in this equation to get the Black-Scholes differential equation

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + ½\,\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

The N(x) Function

- $N(x)$ is the probability that a normally distributed variable with a mean of zero and a standard deviation of 1 is less than x



The Black-Scholes-Merton Formulas for Options

Solving the BS differential equation for European calls and puts options (with proper boundary conditions), we get

$$c = S_0\, N(d_1) - K\, e^{-rT} N(d_2)$$

$$p = K\, e^{-rT}\, N(-d_2) - S_0\, N(-d_1)$$

where $$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Properties of Black-Scholes Formula

- As S_0 becomes very large c tends to $S_0 - Ke^{-rT}$ and p tends to zero
- As S_0 becomes very small c tends to zero and p tends to $Ke^{-rT} - S_0$
- What happens as σ becomes very large?
- What happens as T becomes very large?

Understanding Black-Scholes

$$c = e^{-rT} N(d_2)\left(S_0 e^{rT} N(d_1)/N(d_2) - K\right)$$

e^{-rT} : Present value factor

$N(d_2)$: Probability of exercise

$S_0 e^{rT} N(d_1)/N(d_2)$: Expected stock price in a risk - neutral world if option is exercised

K : Strike price paid if option is exercised

Implied Volatility

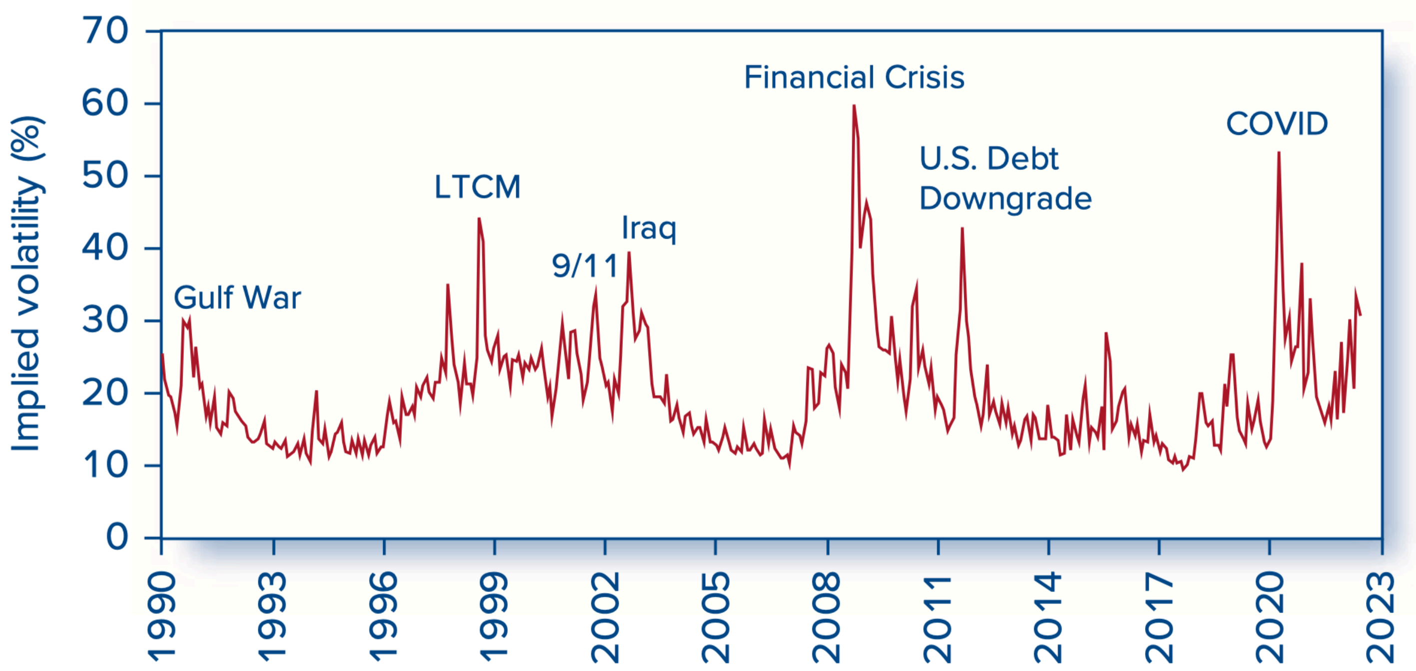


Figure 21.8 Implied volatility of the S&P 500 (VIX index)

Source: Chicago Board Options Exchange, **www.cboe.com.**

Using of Black-Scholes Option valuation

- Hedge Ratios and the Black-Scholes Formula: delta and gamma hedging
- Portfolio insurance
- Option pricing and the financial crisis
- Option pricing and portfolio theory
- Hedging bets on mispriced options

Programming project:

Simulate a delta-hedged portfolio with Monte Carlo simulations:

- Stock: NVDA
- Current stock price: $137
- Implied vol: 20%
- Interest rate: 5%
- Expiration: one year
- Portfolio: a short call option + Δ shares of NVDA
- Goal: simulate the portfolio value’s time evolution with 10,000 paths

Options Calculator

Generate fair value prices and Greeks for any U.S or Canadian equity or index options contract.

Sat, Dec 28th, 2024 Help

Customize your input parameters by entering the option type, strike price, days to expiration (DTE), and risk-free rate, volatility, and (optional) dividend yield% for equities. The calculator uses the latest price for the underlying symbol. Theoretical values and IV calculations are performed using the Black 76 Pricing model, which is different than the Binomial model used on the symbol's Volatility & Greeks page.

Enter an equity or index symbol: Search for a Symbol...

Apple Inc (AAPL)

Quick Links

255.59 -3.43 (-1.32%) 12/27/24 (NASDAQ)

Select the option for calculation. The Input Parameters may be overridden below.

Option Type Call **Expiration** 2025-03-21 (m) 83 DTE (days) **Strike Price** 255.00

Input Parameters

Underlying Price	255.59
Strike Price	255.00
DTE (days)	83
Risk-free rate%	4.20%
Volatility	22.15%
Dividend Yield% (equities)	0.39%

Calculated Theoretical Values

Theoretical Price	12.15
Delta	0.56227
Gamma	0.01460
Vega	0.48030
Theta	-0.07921
Rho	0.29888

IV Calculation

Option	Call
Market Option Price	Last Bid Ask 12.22
Implied Volatility	22.30%

https://www.barchart.com/options/options-calculator

Takeaways

- **Option trading strategies**
- **Stochastic process modeling of stock prices**
- **Black-Scholes model for option pricing**
- **Using the Black-Scholes formula**
- **Delta hedging simulation with Monte Carlo methods**

Reference

1. Bodie, Zvi, Alex Kane, and Alan J. Marcus. *Investments,* 13th edition. McGraw-hill, 2024.
2. Hull, John. *Options, futures and other derivatives, 11th edition*. Upper Saddle River, NJ: Prentice Hall, 2022.