Interest Rate Based Directional Exchange Rate Forecasting and Carry Trade Strategy Design: Kalman Filter Approach

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1 Abstract

Deriving from the uncovered interest rate parity theory and the assumption of nonreal-time reactions of median agents to fluctuations of the interest rate, the paper develops a state space model to estimate hidden states of interest rate differentials. The paper consequently constructs a carry trade strategy by identifying the surprises in interest rate differentials based on the Kalman filter, utilizing the persistence of directional changes in exchange rates to gain profit, and applying the carry trade strategy on real-world data to examine the profitability via statistical tests and performance metrics.
Keywords: exchange rate carry trade, state space model, Kalman filter, market surprise detection

2 Introduction

The systematical reactions of exchange rates on economic fundamentals is always a puzzle for both the academia and the industry. According to previous literature, the market could exist delayed overshooting problem according to Eichenbaum and Evans (1995) or could exist systematic under-reaction to current interest rate innovations as found in Gourinchas and Tornell (2002).

Traditional models such as productivity-based models or behavioral equilibrium exchange rate models are developed to perform point forecasts of the exchange rate. However, owing to the nature of strong noises and uncertainties in the exchange market, according to the research of Cheung, Chinn and Pascual (2005), the point estimate models cannot outperform the random walk model, as most of them could not pass the Diebold-Mariano test. Consequently, the paper develops a directional forecast on exchanges rate based on interest rates.

The paper will start with illustrating the relationship between interest rates and exchange rates by the uncovered interest rate parity, and inferring the potential of profitability based on the assumption that median agents in the market systematically underestimate the persistence of interest rate shocks on the exchange rate as proposed in Gourinchas and Tornell (2004). The paper will, hence, construct a carry trade strategy to identify the surprises in the interest rate and to gain profit from such systematical underestimations.

To distinguish the surprises in the interest rate, the hidden states of the interest rate will be characterized by a state space model and will be estimated by the Kalman filter. The paper then demonstrates the profitability by applying the carry trade strategy on two exchange pairs (the Japan / U.S. exchange rate pair and the U.K. / U.S. exchange rate pair). Both of the exchange pairs have satisfactory performances according to the statistical tests and the performance metrics.

3 Uncovered Interest Rate Parity

The influence of interest rates on exchange rate adjustments was illustrated by the uncovered interest rate parity theory as research such as Lothian and Wu (2011) supported. The theory claims the expected return of investing in different currencies should satisfy the non-arbitrage condition:

$$E_t[PV_t(1+i_t^{\alpha})] = E_t\left[PV_t(e_t)^{-1}\left(1+i_t^{\beta}\right)e_{t+1}\right] \quad \forall\, t \in N \tag{1}$$

The $E_t[\cdot]$ stands for the expected future (time $t+1$) value at present (time t). PV_t denotes the present value, e_t denotes the exchange rate of the number of currency of country α per currency of country β at time t. i_t^c denotes the interest rate of country c at time t, where $c \in \{\alpha, \beta\}$ in this case. Assume that one holds a unit of currency of country α at time t, the LHS of the equation (1) stands for the expected future value at time $t+1$ if one invests in the currency of country α, and the RHS stands for the expected future value at time $t+1$ if one invests in the currency of country β. The LHS equates RHS to satisfy the non-arbitrage condition.

As some information in equation (1) should be available at time t, it is truism that:

$$E_t[X_t] = X_t \quad \forall X_t \in \left\{PV_t, i_t^{\alpha}, i_t^{\beta}, e_t\right\} \tag{2}$$

Equation (2) can be simplified to:

$$(1+i_t^{\alpha}) = (e_t)^{-1}\left(1+i_t^{\beta}\right)E_t[e_{t+1}] \tag{3}$$

Rearrange equation (3) to get:

$$\frac{E_t[e_{t+1}]}{e_t} = \frac{(1 + i_t^{\alpha})}{\left(1 + i_t^{\beta}\right)} \tag{4}$$

The linear approximation of equation (4) is therefore:

$$\left(i_t^{\alpha} - i_t^{\beta}\right) = \frac{E_t[e_{t+1}] - e_t}{e_t} \tag{5}$$

Assume that agents in the market regards changes are transitory shocks and will not affect the future long run exchange rate, $E_t[e_{t+1}] = \overline{e}$. The equation (6) can be derived.

Theorem 1 Uncovered Interest Rate Parity

$$\left(i_t^{\alpha} - i_t^{\beta}\right) = \frac{\overline{e} - e_t}{e_t} \tag{6}$$

In this paper, $\left(i_t^{\alpha} - i_t^{\beta}\right)$ will be defined as the interest rate differential. As the interest rate differential increases, e_t also decreases, that is, an appreciation of currency α should take place at time t. Gourinchas and Tornell (2004) showed that, however, people in the exchange market would systematically underestimate the persistence of interest rate shocks. Although $\left(i_t^{\alpha} - i_t^{\beta}\right)$ increases, people will regard this change as a noise in the market in the beginning, and therefore the exchange rate e_t will not be affected immediately when $\left(i_t^{\alpha} - i_t^{\beta}\right)$ increases. After a while, people will recognize the persistence of the increase in $\left(i_t^{\alpha} - i_t^{\beta}\right)$, and the adjustment of $e_t < \overline{e}$ will be reached. There will be an appreciation in currency α.

To make a profit, one should long on currency α and short sell currency β before people detect the persistence of the increase in $\left(i_t^{\alpha} - i_t^{\beta}\right)$, and gain profit from the following appreciation of currency α when the exchange market adjusts. On the contrary, one should short sell currency α and long on currency β before people detect the persistence of the decrease in $\left(i_t^\alpha - i_t^\beta\right)$, and gain profit from the following depreciation of currency α. The carry trade transactions should be done at fast as the surprises in changes of $\left(i_t^\alpha - i_t^\beta\right)$ are detected by our strategy but not recognized yet by the median agents in the market.

To define a surprise, it is crucial to estimate what the median agents' belief in the interest rate level is after filtering out noises. The state space model would be an appropriate model to measure such hidden states.

4 Modeling the Interest Rate Differential

4.1 State Space Model and Kalman Filter

As it is assumed that people learn the policy shocks and interest rate changes slowly and believe the shocks are often merely noises, people's belief on the hidden true state of interest rate level will not be exactly the observed $\left(i_t^\alpha - i_t^\beta\right)$ but the hidden state s_t, as shown below:

$$\left(i_t^\alpha - i_t^\beta\right) = s_t + v_t \quad where\ v_t \sim N(0, \sigma_v^2)\ \ \forall t \in N \tag{7}$$

$$s_t = \emptyset(s_{t-1}) + w_t \quad where\ w_t \sim N(0, \sigma_w^2)\ \ \forall t \in N \tag{8}$$

In the above equations of the state space model, the v_t and w_t stands for white noises and s denotes the hidden state of the interest rate differential. Because the hidden state s and its updating procedure $\emptyset(s)$ are unobservable to us, we can consider the following Kalman filter to estimate the updating procedure and the value of $\hat{s}_t$:

$$\begin{gathered}\hat{s}_t = (1-k)\hat{s}_{t-1} + k\left(i_t^\alpha - i_t^\beta\right) \\ s.t.\ \ \hat{s}_0 \equiv \left(i_0^\alpha - i_0^\beta\right) \\ s.t.\ \ t \in N\ and\ \ k \in [0, \tfrac{1}{2}]\end{gathered} \tag{9}$$

In equation (9), k is the Kalman gain where $k \in [0, 1]$ to be well-defined. The Kalman gain stands for the weight we put on the current data (i.e., $i_t^\alpha - i_t^\beta$) versus the past information (i.e., $\hat{s}_{t-1}$) to update $\hat{s}_t$ at time t. The initial value of the hidden state $\hat{s}_0$ is defined as the initial value of the observed interest rate differential $(i_0^\alpha - i_0^\beta)$. Furthermore, to emphasize the assumed characteristic that the market adjustment mechanism is relatively slow, assume that $k \in [0, \frac{1}{2}]$ which implies more than half of the adjustment of hidden state s depends on the past information.

4.2 Surprises in Interest Rate Differential

The error of the Kalman filter at time t can be defined as the observed value $\left(i_t^\alpha - i_t^\beta\right)$ minus the hidden state $\hat{s}_t$, as shown below:

$$\varepsilon_t \equiv \left(i_t^\alpha - i_t^\beta\right) - \hat{s}_t \tag{10}$$

The value of ε_t represents the difference of the observed interest rate differential and the estimated hidden state of the interest rate differential. With the assumption that the exchange market adjustments are relatively slow, the persistence of the sign of ε_t would come into existence. Therefore, as the sign of ε_t changes, it will be a surprise to the state space model and time t will be the earliest moment that the state space model can possibly detect such surprise.

Theorem 2 Surprises in the State Space Model

$$\Delta\uparrow_t = I(\varepsilon_t < 0 \wedge \varepsilon_{t-1} > 0) \tag{11}$$

$$\Delta\downarrow_t = I(\varepsilon_t > 0 \wedge \varepsilon_{t-1} < 0) \tag{12}$$

The surprises in the interest rate differential at time t are defined below. $I(\cdot)$ is an indicator function where $I(\cdot) = 1$ when the proposition inside is true or else $I(\cdot) = 0$. $\Delta\uparrow_t = 1$ represents the surprise that the observed interest rate differential overpasses the estimated hidden state, and $\Delta\downarrow_t = 1$ represents the surprise that the estimated hidden state overpasses the observed interest rate differential.

5 Carry Trade Strategy

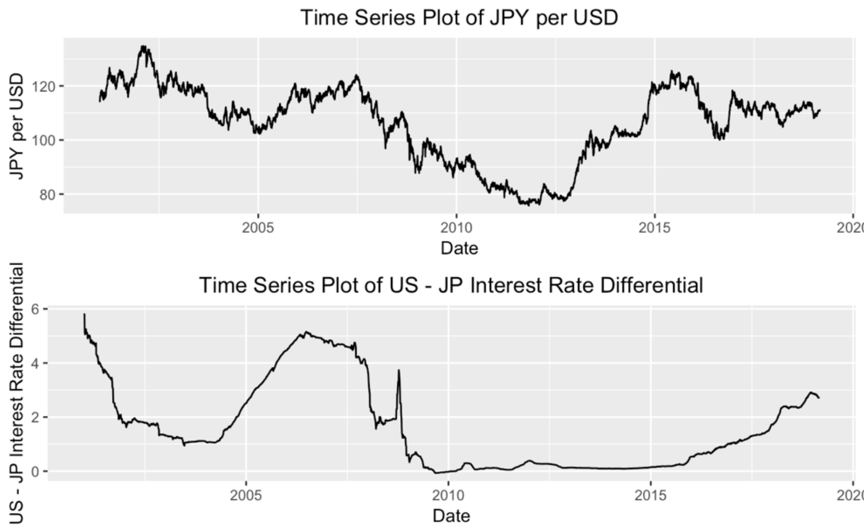
The strategy is to build a Kalman filter on the interest rate differential, and to trade basing on detected surprises of the interest rate differential. The trade rules will be when $\Delta\downarrow_t = 1$, exchange all the currency of country α into the currency of country β. On the contrary, when $\Delta\uparrow_t = 1$, exchange all the currency of country β into the currency of country α. Moreover, to reduce the impact of transitory noises σ_v^2 on the carry trade strategy, the strategy will be limited with a horizon period h after a transaction occurs. During the horizon period h, there will be no reversing transactions allowed. Thus, there will be two inputs in the carry trade strategy: the Kalman gain k and the horizon period h. A recursive algorithm will be implemented to maximize the return on historical data subject to the two inputs: $k \in [0, \frac{1}{2}]$ and $h \in Z^+$.

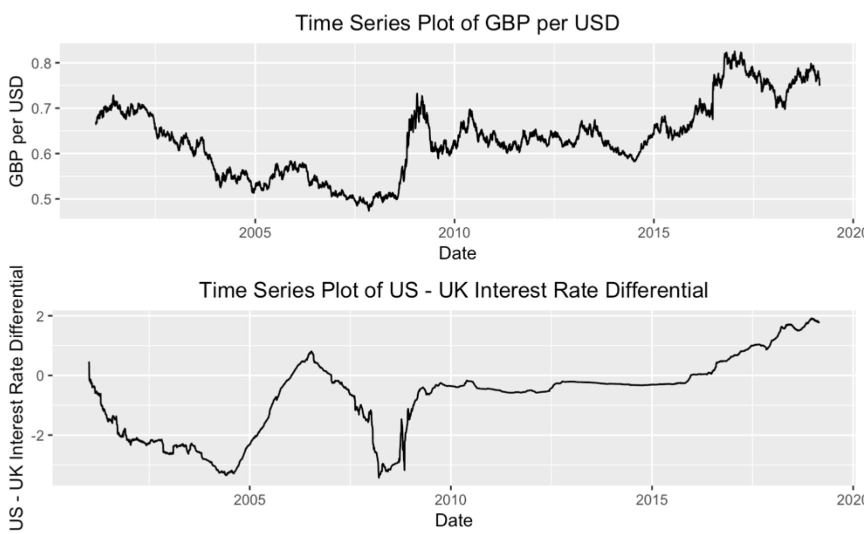
6 Application of the Carry Trade Strategy

To demonstrate the application of the carry trade strategy, the daily data from Jan/02/2001 to Feb/28/2019 of the following variables will be used in the analysis.

1. 3-month London interbank offered rate (LIBOR), based on British Pound.
   Unit: percent, not seasonally adjusted.
2. 3-month London interbank offered rate (LIBOR), based on Japanese Yen.
   Unit: percent, not seasonally adjusted.
3. 3-month London interbank offered rate (LIBOR), based on U.S. Dollar.
   Unit: percent, not seasonally adjusted.
4. Japan / U.S. Foreign Exchange Rate.
   Unit: Japanese yen to one U.S. dollar, not seasonally adjusted.
5. U.S. /U.K. Foreign Exchange Rate.
   Unit: U.S. dollar to one U.K. pound, not seasonally adjusted.

The data is extracted from the FRED Economic Data, Federal Reserve Bank of St. Louis. The interest rates will be used to construct the interest rate differential of Japanese Yen over U. S. Dollar and that of British Pound over U.S. Dollar. The visualized data is shown below.





The carry trade strategy will be implemented for the Japan / U.S. exchange rate pair and the U.S. /U.K. exchange rate pair. For the Japan / U.S. exchange rate pair, the strategy is to long on the Japanese Yen (and to short sell the U.S. Dollar) when the interest rate differential of the United States over Japan has a decreasing surprise. On the contrary, the strategy will short sell the Japanese Yen (and long on the U. S. Dollar) when the interest rate differential of the United States over Japan has an increasing surprise. The strategy for the U.S. /U.K. exchange rate pair is also the same.

To measure the return of the carry trade strategy, the holding period return will be used, which is defined as equation (13):

$$HPR = \frac{(V_1 - V_0)}{V_0} \tag{13}$$

The V_0 is defined as the equity value when a long/short position occurs, and V_1 is the equity value when a long/short position ended. Then term return in the following parts refers to the holding period return if not further specified.

To test the statistical significance of the profitability, the Jarque-Bera Test, the one-side unweighted binomial test as well as the one-side weighted binomial test are implemented on the vector of HPR.

Hypothesis Test 1 Jarque-Bera Test

$$\begin{aligned} &H_0: HPR \sim N(\mu_{HPR}, \sigma^2_{HPR}) \\ &H_1: HPR \nsim N(\mu_{HPR}, \sigma^2_{HPR}) \end{aligned} \tag{14}$$

Hypothesis Test 2 One-side Unweighted Binomial Test

$$\begin{aligned} &H_0: Prob(HPR > 0) = 0.5 \\ &H_1: Prob(HPR > 0) > 0.5 \end{aligned} \tag{15}$$

Hypothesis Test 3 One-side Weighted Binomial Test

$$\begin{aligned} &H_0: E[HPR] = 0 \\ &H_1: E[HPR] > 0 \end{aligned} \tag{16}$$

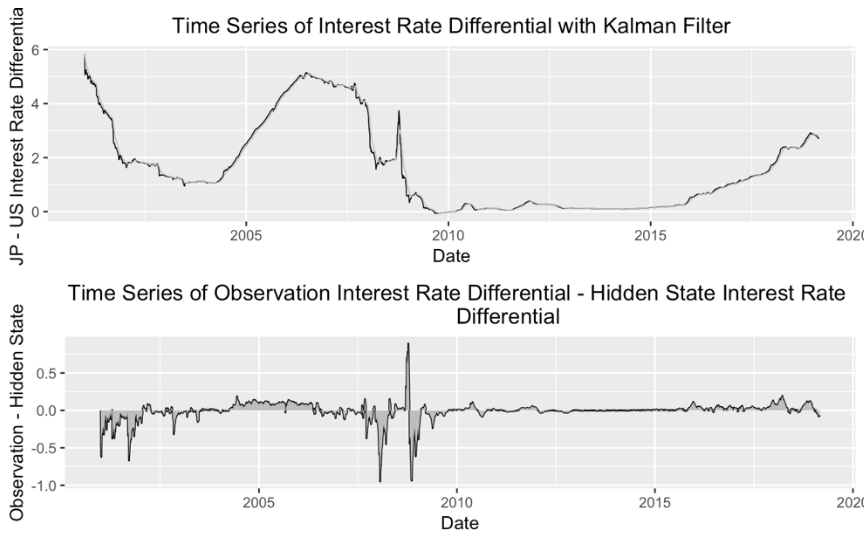
7 Results of the Japan / U.S. Exchange Rate Pair

7.1 Optimal Strategy

The optimal carry trade strategy is to optimize the profit subject to the Kalman gain and the horizon. The optimal inputs and outputs are shown below.

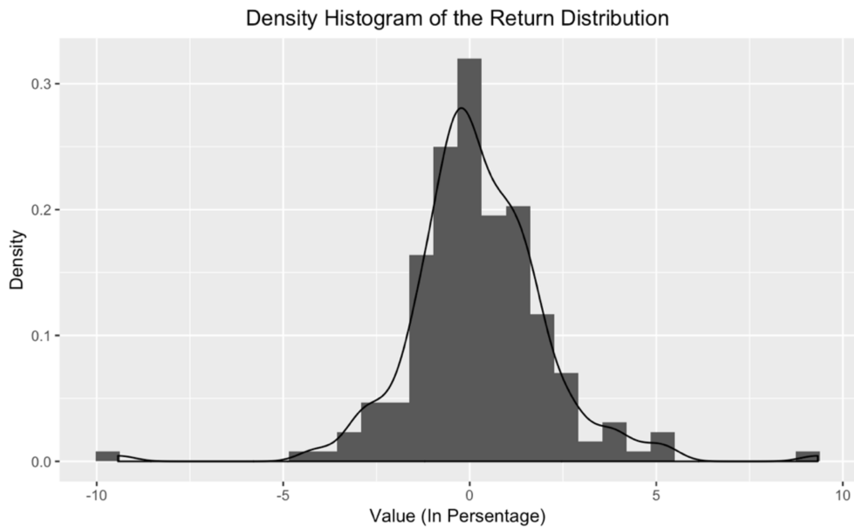
Kalman Gain	Horizon Period	Aggregate Return	Trade Counts
0.08	11 days	67.12%	76 times

There are 76 counts of trades during the past 18 years. Therefore, the strategy creates 38 times of change in long/short positions and therefore generates 38 holding period returns. Below is the visualized optimal Kalman filter for the interest rate differential between the United States and Japan. In the first plot, the black line stands for the true value of $\left(i_t^{\alpha} - i_t^{\beta}\right)$ and the gray line is the estimated hidden state $\hat{s}_t$. In the second plot, the error of the Kalman filter ε_t is presented.



7.2 Return Distribution and Statistical Tests

The histogram of the holding period returns is shown below. Most of the return are on the positive side, which indicates a good performance of the carry trade strategy. Moreover, the summary statistics of the holding period returns are also provided.



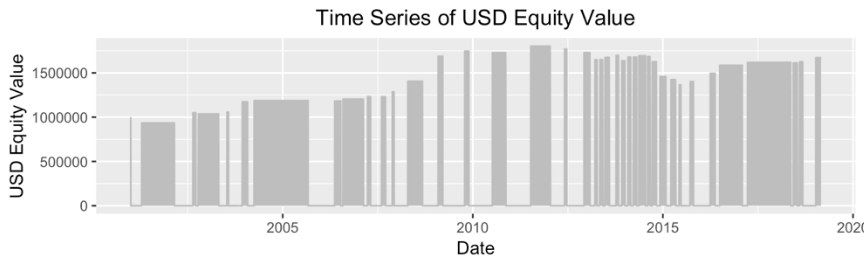
Minimum Return	- 10.25%
Maximum Return	20.06%
Mean of Returns	1.50%
Median of Returns	1.06%
Standard Deviation of Returns	5.41%
Skewness	1.04
Kurtosis	2.33

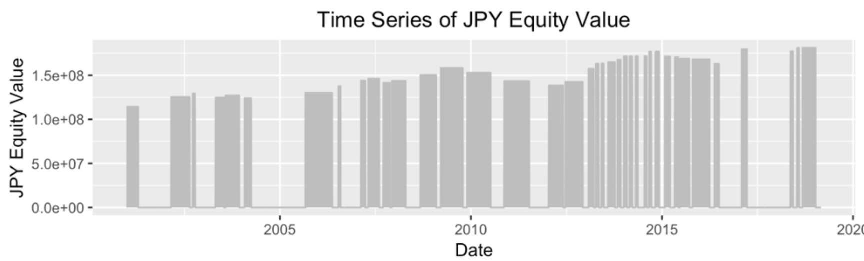
There exists positive mean of returns, positive median of returns, as well as positive skewness.

	P-Value of The Statistical Test
Jarque-Bera Test	<0.001
One-side Unweighted Binomial Test	0.2088
One-side Weighted Binomial Test	0.0484

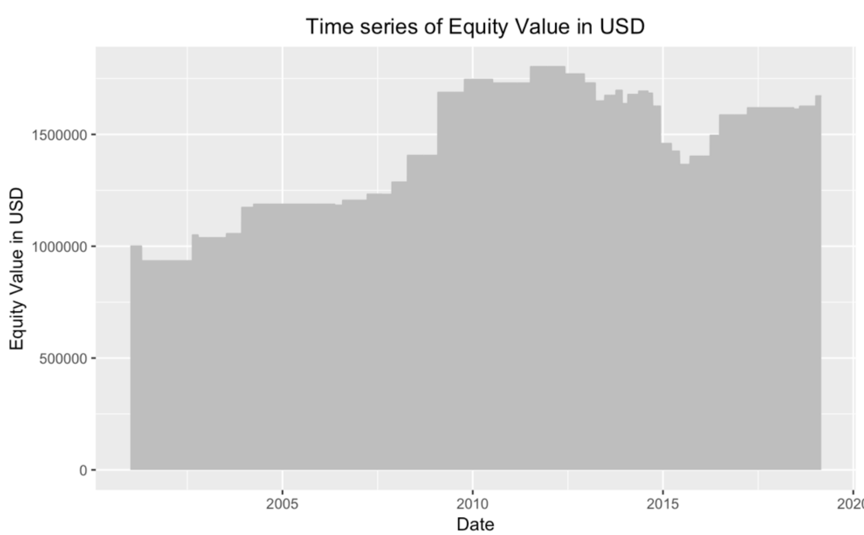
The p-value of the Jarque-Bera Test is below 0.0001, which indicates the return is not merely a normal distribution. Moreover, there are 22 out of 38 of the HPR have positive returns, and the p-value of the one-side unweighted binomial test is 0.2088. There is actually no significance shown in turns of generating positive HPR with more than 50% probability, however, there does exist a statistical significance that the expected HPR is greater than zero with a p-value smaller than 0.05 in the one-side weighted binomial test.

7.3 Equity Graph





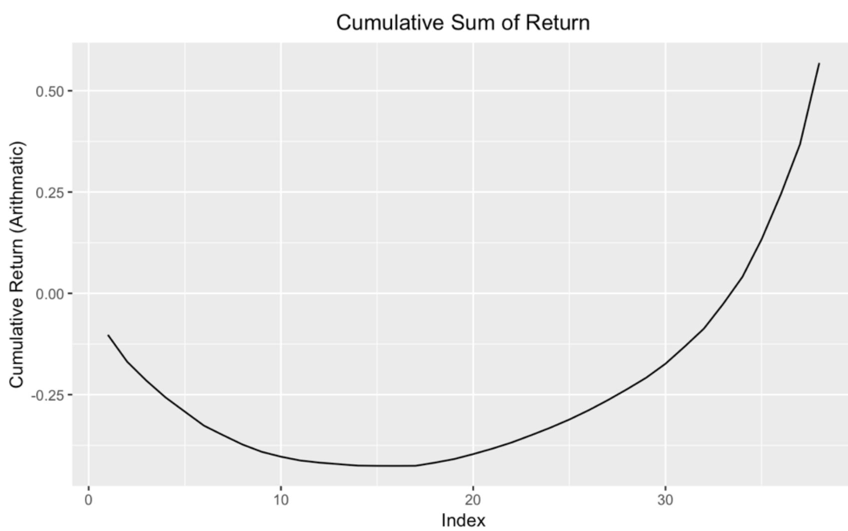
Above is the growth of equity values in the USD basket and the JPY basket of the strategy (starting with 1 million in USD). To make it more informative, the equity graph can be transformed into the present value of the equity in USD, as shown below.



7.4 Metrics for Performance Evaluation

Annualized Excess Return	2.86%
Sharpe Ratio	0.37
Skewness	1.05
Gini Index	0.71

To evaluate the performance of the carry trade strategy of the Japan / U.S. Exchange Rate Pair, annualized return, the Sharpe ratio, the skewness and the Gini index is reported. The carry trade strategy performs well on the Japan / U.S. exchange rate pair. Below is graph of the cumulative sum of return for the calculation of the Gini index.



8 Results of the U.K. / U.S. Exchange Rate Pair

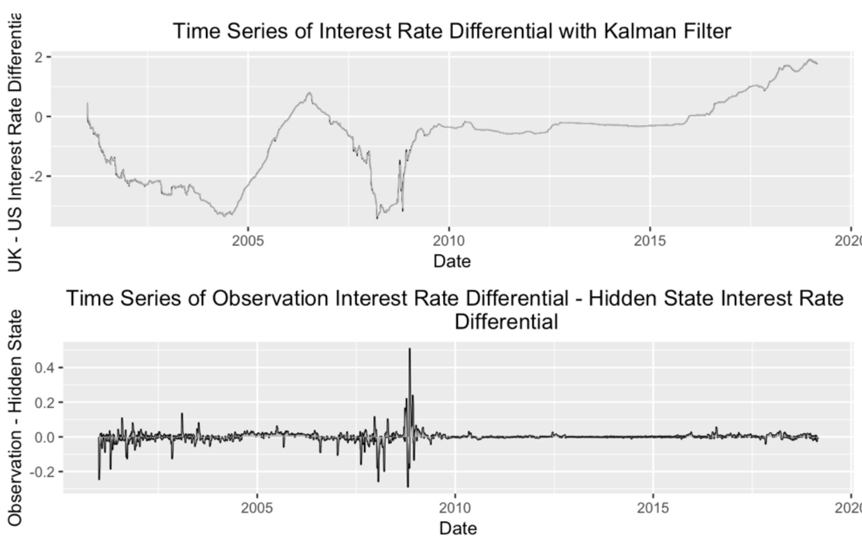
To evaluate a more generalized profitability of the carry trade strategy, another exchange rate pair (i.e., the U.K. / U.S. pair) will also be used.

8.1 Optimal Strategy

The optimal carry trade strategy is to optimize the profit subject to the Kalman gain and the horizon. The optimal inputs and outputs are shown below.

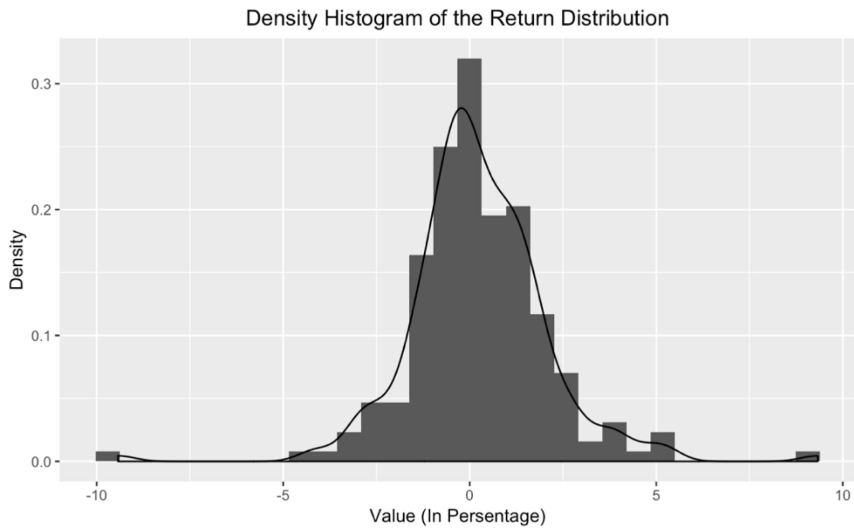
Kalman Gain	Horizon Period	Aggregate Return	Trade Counts
0.41	3 days	58.13%	398 times

There are 398 counts of trades during the past 18 years. Therefore, the strategy creates 199 times of change in long/short positions and therefore generates 199 holding period returns. The optimal strategy of the U.K./U.S. case is much more aggressive than the Japan/U.S. case, as the optimal Kalman gain is higher, the optimal horizon period is shorter, and the trade counts are more than the previous case. Below is the visualized optimal Kalman filter for the interest rate differential between the United States and the United Kingdom. In the first plot, the black line stands for the true value of $\left(i_t^\alpha - i_t^\beta\right)$ and the gray line is the estimated hidden state $\hat{s}_t$. In the second plot, the error of the Kalman filter ε_t is presented.



8.2 Return Distribution and Statistical Tests

The histogram of the holding period returns is shown below. Most of the return are on centered around zero. Moreover, the summary statistics of the holding period returns are also provided.



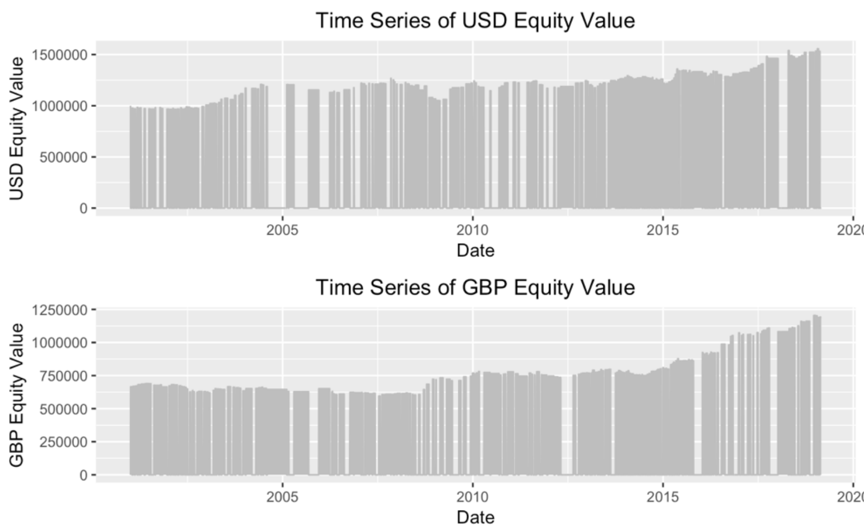
Minimum Return	- 9.43%
Maximum Return	9.33%
Mean of Returns	0.23%
Median of Returns	0.07%
Standard Deviation of Returns	1.87%
Skewness	0.09
Kurtosis	5.48

There exists positive mean of returns, positive median of returns, as well as positive skewness, however the summary statistics are not as profitable as in the previous case. The high kurtosis also shows the property that the return distribution centered around zero but being slightly positive.

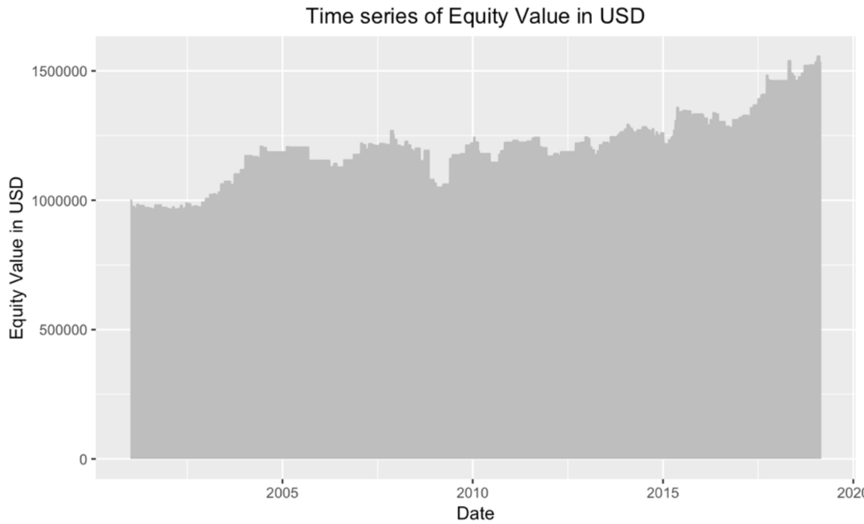
	P-Value of The Statistical Test
Jarque-Bera Test	<0.001
One-side Unweighted Binomial Test	0.2613
One-side Weighted Binomial Test	0.0409

The p-value of the Jarque-Bera Test is below 0.0001, which indicates the return is not merely a normal distribution. Moreover, there are 104 out of 198 of the HPR have positive returns, and the p-value of the one-side unweighted binomial test is 0.2613. There is actually no significance shown in turns of generating positive HPR with more than 50% probability, however, there does exist a statistical significance that the expected HPR is greater than zero with a p-value smaller than 0.05 in the one-side weighted binomial test.

8.3 Equity Graph



Above is the growth of equity values in the USD basket and the GBP basket of the strategy (starting with 1 million in USD). To make it more informative, the equity graph can be transformed into the present value of the equity in USD, as shown below. The growth is much steadier for the U.K./U.S. exchange rate pair because the trade has a higher frequency and the kurtosis of returns is higher.

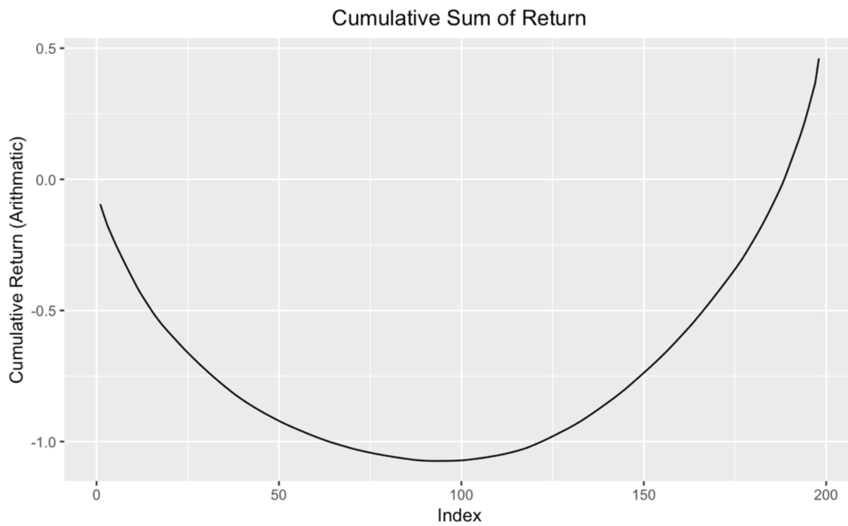


8.4 Metrics for Performance Evaluation

Annualized Excess Return	2.37%
Sharpe Ratio	0.38
Skewness	0.09
Gini Index	0.72

To evaluate the performance of the carry trade strategy of the U.K. / U.S. exchange rate pair, annualized return, the Sharpe ratio, the skewness and the Gini index is reported. Below is graph of the cumulative sum of return for the calculation of the Gini index.

Although the annual excess return of the U.K. / U.S. exchange rate pair is lower than that of the Japan / U.S. exchange rate pair, the standard error is also lower. Therefore, there is no much difference for the Sharpe ratio (i.e., the risk adjusted return) between the two currency pairs. Below is graph of the cumulative sum of return for the calculation of the Gini index.



9 Conclusion

Based on the economic theory of uncovered interest rate parity and the assumption of nonreal-time reactions of median agents in the market, surprises in interest rate differential changes can be identified prior to the median agents using Kalman filters on the state space model. A carry trade strategy can consequently be constructed by optimizing the profitability subject to the Kalman gain and the horizon period. By implementing the carry strategy on the Japan / U.S. exchange rate pair and the U.K. / U.S. exchange rate pair, the Sharpe ratios are approximately 0.37 for both cases. The return distributions for both cases have properties of either positive skewness or positive excess kurtosis with means and medians greater than zero. The expected return for both exchange rate pairs are significantly greater than zero according to the one-side weighted binomial test, which implies the potential feasibility to implement the carry trade strategy in the real world.

10 Appendix: R Code

The author provides the R code which develops all the statistical inferences and conclusions in the paper through GitHub Inc. on https://github.com/ShuChenTsao/UCLA-Masters-Thesis-2019.

11 References

1. Cheung, Y.W., Chinn, M. and Pascual, A.G. (2005). Empirical exchange rate models of the nineties: Are any fit to survive? *Journal of International Money and Finance.*
2. Eichenbaum, M. and Evans, C. (1995). Some empirical evidence on the effects of shocks to monetary policy on exchange rates. *The Quarterly Journal of Economics.*
3. Gourinchas, P.O. and Tornell, A. (2002). Exchange rate dynamics, learning and misperception. *National Bureau of Economic Research Working Paper.*
4. Gourinchas, P.O. and Tornell, A. (2004). Exchange rate puzzles and distorted beliefs. *Journal of International Economics.*
5. Lothian, J. and Wu, L. (2011). Uncovered interest-rate parity over the past two centuries. *Journal of International Money and Finance.*