# Are you Normal? The Problem of Confounded Residual Structures in Hierarchical Linear Models



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# Are you Normal? The Problem of Confounded Residual Structures in Hierarchical Linear Models

Adam Loy and Heike Hofmann *

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We encounter hierarchical data structures in a wide range of applications. Regular linear models are extended by random effects to address correlation between observations in the same group. Inference for random effects is sensitive to distributional mis-specifications of the model, making checks for (distributional) assumptions particularly important. The investigation of residual structures is complicated by the presence of different levels and corresponding dependencies. Ignoring these dependencies leads to erroneous conclusions using our familiar tools, such as Q-Q plots or normal tests. We first show the extent of the problem, then we introduce the *fraction of confounding* as a measure of the level of confounding in a model and finally introduce rotated random effects as a solution to assessing distributional model assumptions. This article has supplementary materials online.

**Keywords:** Diagnostic, Multilevel model, Q-Q plot, Random effects distribution

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## 1. INTRODUCTION

There are a wide range of application areas—from the biological and physical sciences to the social sciences—in which we encounter nested data. Whether it is quality control in a manufacturing process that involves the monitoring of a set of components over time or students' performances in different schools across the country, analysts have to account for the correlation between observations in the same group. Hierarchical linear models, or multilevel models, allow us to do exactly that—but they also require us to make distributional assumptions on both the error terms and the random effects. These assumptions must hold to ensure the validity of the model and all of its resulting conclusions. Inference for the fixed effects in linear mixed models is fairly robust against model mis-specification (Butler and Louis, 1992; Verbeke and Lesaffre, 1997). This is different for random effects (Verbeke and Lesaffre, 1996, 1997). Verbeke and Lesaffre (1996) have shown that the empirical Bayes estimates of the random effects can be severely distorted if the true distribution is not normal; thus, any distributional assumptions must be checked carefully when the random effects are central to the inferential goals. For example, in small area estimation the areas are often modeled as random effects and their distribution is critical in the calculation of prediction intervals when the within-group sample sizes are small (see Jiang, 2001, for a cogent discussion of this issue).

Extensions to the standard modeling framework have been suggested to address sensitivity to distributional mis-specifications, such as semiparametric, nonparametric methods (Shen and Louis, 1999; Zhang and Davidian, 2001; Ghidey et al., 2004) or finite mixtures of normal distributions for the random effects (Verbeke and Lesaffre, 1996).

We refer the reader to Ghidey et al. (2010) for a recent review comparing these methods. Semiparametric and nonparametric methods come at a cost: computational complexity is increased, and most of these methods are not widely implemented yet in standard statistical software, thus making them less accessible to the intended audience.

However, the situation is not quite as dire as it might seem: it turns out that a lot of tests for normality are overly sensitive in assessing deviations of random effects from normality (see discussion in section 4.2 and simulation results in the appendix). By addressing the issue of over-sensitivity in evaluating normality, we are able to extend the applicability of normal model methods in a wide range of situations.

Several methods to assess the distribution of random effects have been proposed. Formal tests have been proposed to to detect mixture distributions (Verbeke and Lesaffre, 1996) in the random effects and for overall goodness-of-fit tests for both the error terms and random effects (Jiang, 2001). In contrast, the approach on which we focus in this paper is of a graphical nature.

Quantile-quantile (Q-Q) plots (Wilk and Gnanadesikan, 1968) are an essential tool for visually evaluating a specific distributional assumption. In a Q-Q plot we plot the empirical distribution against the expected quantiles from the assumed distribution. The line of identity therefore represents the theoretical distribution and points show quantiles of the empirical distributions. Deviations from the theoretical distribution then manifest themselves as vertical differences between points and the line of identity. This difference is featured in a series of distributional tests. More formally, let $F_n$ be the empirical distribution function (ECDF) based on a sample size of $n$, and $F$ be the hypothesized/true distribution. The absolute difference between the two distribution functions for each sample point, $|F_n(x_i) - F(x_i)|$, is then the main contributor for the test statistics of the Kolmogorov-Smirnov (KS-test, Kolmogorov, 1933; Smirnov, 1948), the Anderson-Darling (AD-test, Anderson and Darling, 1954), and the Cramér-von-Mises test (CVM-test, Cramér, 1928; von Mises, 1928), as shown in table 1.

The KS test uses the maximal difference, which is shown as the maximal vertical extent between the line of identity and the data points in a Q-Q plot, regardless of the range of the sample. AD and CVM are both based on the total area between the line of identity and the empirical distribution function. Compared to the KS test, the CVM test downplays the effects in the tails of a (normal) distribution, while Anderson-Darling up-regulates the tail effect again due to its additional weighting of $1/\left(F(x)(1 - F(x)\right)$ across the range of the sample. We will be making use of these three tests and the Q-Q plots in our assessment of normality of random effects.

In hierarchical linear models the investigation of residual structures is complicated by the pres-

| Test | | Statistic |
|---|---|---|
| Kolmogorov-Smirnov | $D =$ | $\sup_{1\leq i\leq n} \lvert F_n(x_i) - F(x_i)\rvert$ |
| Anderson-Darling | $A =$ | $n \int_{-\infty}^{+\infty} \lvert F_n(x) - F(x)\rvert^2 / (F(x)(1 - F(x))\, dF(x)$ |
| Cramér-von-Mises | $C =$ | $n \int_{-\infty}^{+\infty} \lvert F_n(x) - F(x)\rvert^2\, dF(x)$ |

Table 1: Three prominent tests for normality based on the difference between empirical and hypothesized distribution function. An overview of the performance and power of these tests can be found in Stephens (1974).

ence of different levels. The nested structure of the data is reflected in the residual structure, and just as there is dependence between different levels in the data, we can expect dependencies between different levels in the residual structure. Q-Q plots, in their weighted (Dempster and Ryan, 1985; Lange and Ryan, 1989) or unweighted form, show univariate distributions, so they are sensitive to distributional deviations due to hidden dependencies, which might lead to erroneous conclusions in evaluating normality, in particular, when the dependency between levels is high due to an unbalanced design. Data imbalances lead to higher degrees of shrinkage, i.e. in the prediction of the random effects the mean structure is considered to a higher degree in a trade-off to prevent overfitting of the raw data. Data imbalances commonly occur in practice, but are often overlooked in the literature. For example, Eberly and Thackeray (2005) explored properties of Lange and Ryan's weighted Q-Q plots for a balanced longitudinal data set and found that, for a properly specified mean structure, weighted Q-Q plots are effective in assessing the distribution of the random effects. The same cannot be said for unbalanced data (with properly specified mean structure).

In this paper, we address the problem of distributional assessment due to confounding in residual structures. Section 2 illustrates the inadequacy of existing methods based on the predicted random effects. We introduce the concept of rotating the random effects into a reduced-dimensional subspace that is less confounded in Section 3, and illustrate how to obtain rotated random effects at all levels. In Section 4 we evaluate the sensitivity and specificity of tests of normality for the rotated random effects in a simulation study to investigate the behavior of Q-Q plots constructed from the rotated random effects. Finally, we demonstrate how this enables an appropriate graphical assessment of the distributional assumptions in Section 5.

## 2. MOTIVATING EXAMPLE

To illustrate the effect of confounding between different levels of residuals, we consider the data set discussed by Gelman and Pardoe (2006). This data set consists of a stratified random sample of 919 owner-occupied homes in 85 counties in Minnesota. Gelman and Pardoe (2006) suggest a hierarchical model of the form

$$\log(y_{ij}) = \beta_0 + \beta_1 x_{1ij} + \beta_2 x_{2i} + b_{0i} + b_{1i} x_{1ij} + \varepsilon_{ij} \tag{1}$$

where $\log(y_{ij})$ denotes the radon measurement (in log $pCi/L$, i.e log picoCurie per liter) for house $j$ within county $i$ ($1 \leq j \leq n_i, 1 \leq i \leq 85$). $x_{1ij}$ is a binary variable describing the level at which the measurement was taken (0 for the basement and 1 for a higher level), and $x_{2i}$ denotes the average soil uranium content for county $i$. We assume i.i.d. normal errors $\varepsilon_{ij} \sim \mathcal{N}(0,\ \sigma_\varepsilon^2)$ and $\boldsymbol{b}_i \sim \mathcal{N}(\mathbf{0},\ \boldsymbol{D})$, where $\boldsymbol{D}$ allows for correlation between random effects within the same county $i$. Further, we assume independence between random effects and error terms.

Figure 1 shows a map of counties in Minnesota. The color shading represents average radon activity in a county. For two counties no data are available. Generally, more southern locations exhibit higher levels of radon activity. Figure 2 focuses on the counties of Hennepin (home to Minneapolis) and Winona (home to the city of the same name), plotting radon level by floor level. Radon levels are usually the highest at the basement level of a house. The within-county sample sizes, $n_i$, are extremely unbalanced, ranging from one house to 116 houses, with 50% of the counties having between three and ten houses. Such unbalanced designs are common in applications, and result in a high degree of pooling in the predicted random effects, which results in quantities for many counties that are highly shrunken toward the global mean. It is this high degree of shrinkage that leads to dependence between predicted random effects and error terms (cf. eqns. 3 and 4), which in turn can lead us to draw erroneous conclusions for corresponding residual quantities.

In this example, Q-Q plots (Figure 3) for the residuals show that normality seems to be violated for the error terms and random intercepts. But is this cause for concern? As there is little pooling

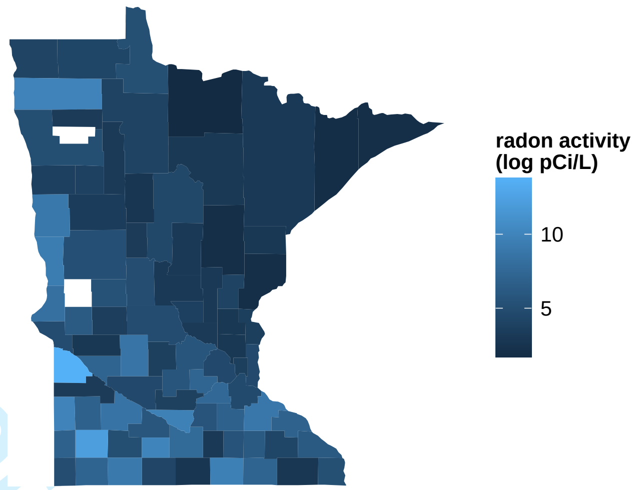



Figure 1: Map of the counties in Minnesota. The color shading represents average radon activity (in log $pCi/L$, i.e log picoCurie per liter).

at the observation level (level 1) we expect the distributional assessment of the error terms to be reliable, but the high degree of pooling for the random effects casts doubt on the reliability of their Q-Q plots. The lineup protocol (Buja et al., 2009) allows us to assess the reliability of the Q-Q plots: Figure 4 shows a lineup of 20 Q-Q plots for the predicted random slope. The Q-Q plot of the observed random slopes is placed among 19 decoy plots created from parametric bootstrap samples based on model (1) satisfying the normal distribution assumptions. The simulation parameters were set to the maximum likelihood estimates of model (1). If we can identify the real Q-Q plot in the lineup, this provides evidence that the distribution of the observed random slopes is not normal. However, the observed Q-Q plot (panel $3^2 + 7$) is virtually indistinguishable from the field of null plots. This suggests that the predicted random slopes from the data do not deviate significantly from model (1). Note that in practice we must blind ourselves from the true plot for proper use of lineups. In order to not violate this, we did not show the Q-Q plot of random slopes earlier.

What becomes apparent from the lineup is that *most* of the null plots in the lineup do not conform to normality. To further investigate the apparent non-normal behavior of predicted random effects we conducted a small simulation study: We generated 1000 parametric bootstrap samples from model (1) assuming a normal distribution for all random effects, but varying distributions for level-1 residuals: level-1 residuals were generated as normal ($\varepsilon^*_{ij} \overset{iid}{\sim} \mathcal{N}(\mathbf{0},\ \widehat{\sigma}^2_\varepsilon)$), heavy tailed

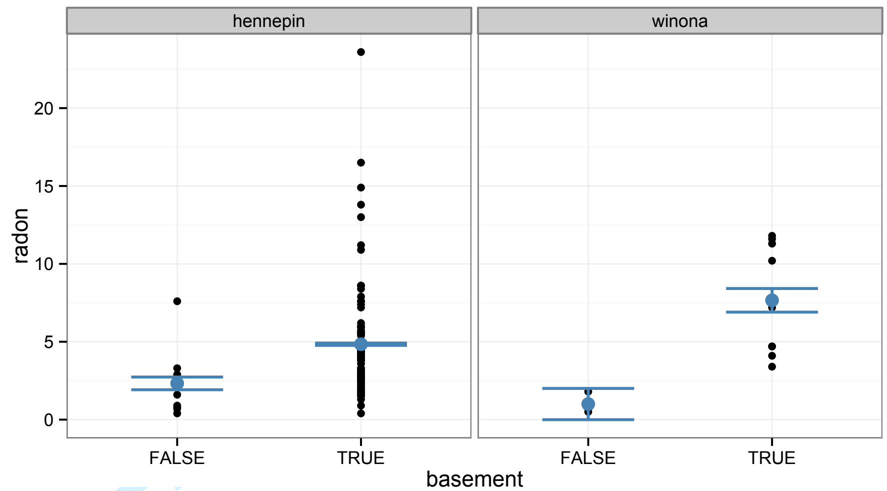
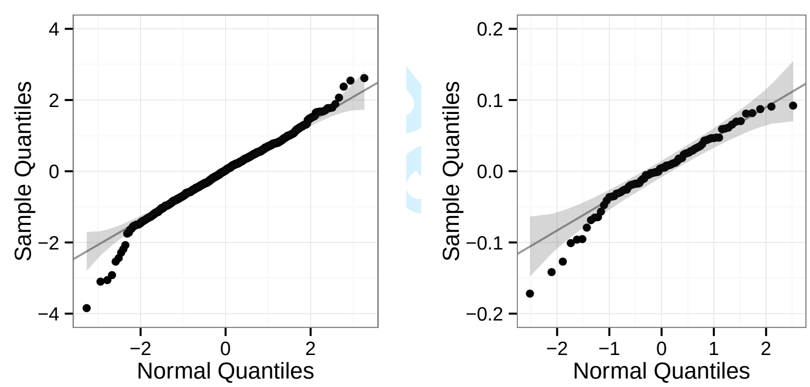



Figure 2: Activity of radon levels for Hennepin and Winona counties at basement (basement = TRUE) or higher in the residence. The bigger points indicate the sample means with 95% confidence intervals given by the error bars. Radon levels at the basement level are usually higher.




(a) Predicted error terms (b) Random intercepts

Figure 3: Q-Q plots of predicted residuals at different levels for model (1). Both plots suggest a deviation of residuals from a normal distribution. Note that random slopes (see figure 4) exhibit the largest deviation from normality.

$(\varepsilon^*_{ij} \stackrel{iid}{\sim} (\widehat{\sigma}_\varepsilon/\sqrt{3})\, t_3)$, and skewed $(\varepsilon^*_{ij} \stackrel{iid}{\sim} \widehat{\sigma}_\varepsilon \{\text{Exp}(1) - 1\})$. After fitting model (1) to each simulated data set, we evaluated the assumption of normality for both the error terms and the random effects using the Anderson-Darling, Cramér-von-Mises, and Kolmogorov-Smirnov tests for normality. All tests were expected to reject normality at a nominal 5% rate, however, all type I error rates are hugely inflated for both random effects, which suggests, that an assessment of normality based on the empirical distribution is not possible. Table 2 shows the percentage of these tests rejecting

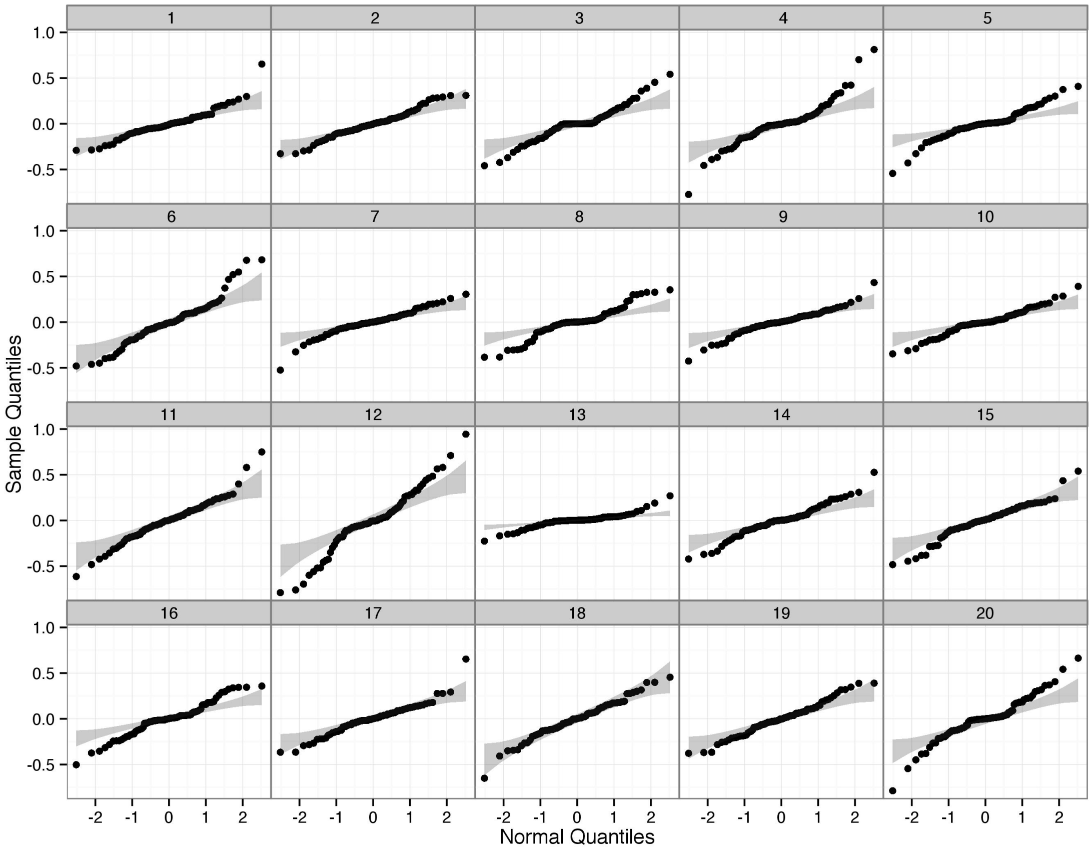



Figure 4: Lineup of normal Q-Q plots for the random slope term in model (1). The 19 null plots are obtained by simulation from the model. Can you identify the observed Q-Q plot? In a human subject study 15 out of 23 participants identified plot #$3^2 + 3$ as the least normal, plot #9+7 shows the actual data, which none of the participants identified as the least normal.

the null hypothesis of normality at the 5% significance level. Even when level-1 residuals are normally distributed, normality for random effects is rejected almost half the time (KS-test). For heavy-tailed or skewed level-1 residuals, the distribution of random effects is affected even worse. For example, 84.1% of the AD tests rejected normality of the random intercept distribution when the level-1 residuals were skewed. Use of standardized random effects and the weighted cumulative distribution function proposed by Lange and Ryan (1989) reduce the type I errors to the nominal level when the error terms are normal, but the type I errors remain inflated for non-normal error terms (this is based on the results of a larger simulation study which we include in Section 1.5 of the supplement). Similarly, tests of non-normal random effects often failed to reject if the error terms were normally distributed. This inability to assess distributional assumptions correctly is a

Table 2: Percentage of tests rejecting the null hypothesis of normality of the predicted random effects at a 5% significance level when the error terms are normal, heavy tailed, and skewed. The percentages are hugely inflated under each setting compared to the nominal type I error rate.

(a) Random intercept

| | Test | | |
|---|---|---|---|
| | AD | CVM | KS |
| Normal | 65.5 | 61.5 | 49.4 |
| Heavy tailed | 89.0 | 87.8 | 78.5 |
| Skewed | 84.1 | 83.0 | 71.5 |

(b) Random slope

| | Test | | |
|---|---|---|---|
| | AD | CVM | KS |
| Normal | 87.4 | 86.9 | 81.5 |
| Heavy tailed | 96.5 | 96.7 | 92.7 |
| Skewed | 95.3 | 95.6 | 90.9 |

symptom typical of confounding between levels of residuals.

In situations with a large amount of pooling, confounding also affects the error terms, which in this particular example were the least affected and did not exhibit signs of deviation from normality.

In the remainder of this paper we investigate the root of concern that leads to the distributional deviations, and derive residuals that address the issues introduced by pooling, allowing again for a familiar graphical assessment of these distributions.

## 3. ASSESSING THE DISTRIBUTION OF THE RANDOM EFFECTS

### 3.1 Model notation and residuals

The general stacked representation of the hierarchical linear model is given by

$$\begin{aligned} &\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{Z}\boldsymbol{b} + \boldsymbol{\varepsilon}, \\ &\mathrm{E}\begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{\varepsilon} \end{bmatrix} = \mathbf{0}, \ \mathrm{Cov}\begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{\varepsilon} \end{bmatrix} = \begin{bmatrix} \boldsymbol{D} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{R} \end{bmatrix} \end{aligned} \tag{2}$$

where $\boldsymbol{y}$ is an $n \times 1$ vector of observed responses, $\boldsymbol{X}$ $(n \times p)$ and $\boldsymbol{Z}$ $(n \times q)$ are design matrices, $\boldsymbol{\beta}$ is a $p \times 1$ vector of unknown fixed effects, $\boldsymbol{b}$ is a $q \times 1$ vector of unobserved random effects, $\boldsymbol{\varepsilon}$ is an $n \times 1$ vector of unobserved errors, and $\boldsymbol{R}$ and $\boldsymbol{D}$ are positive definite covariance matrices.

Using this specification, the predicted error terms and random effects are given by

$$\widehat{\boldsymbol{\varepsilon}} = \boldsymbol{RPy} = \boldsymbol{RPZb} + \boldsymbol{RP\varepsilon} \tag{3}$$

$$\widehat{\boldsymbol{b}} = \boldsymbol{DZ'Py} = \boldsymbol{DZ'PZb} + \boldsymbol{DZ'P\varepsilon} \tag{4}$$

where $\boldsymbol{P} = \boldsymbol{V}^{-1}(\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X'V}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X'V}^{-1})$ and $\boldsymbol{V} = \boldsymbol{ZDZ'} + \boldsymbol{R}$. Equations (3) and (4) reveal that the distributions of the predicted random effects and error terms are interrelated—i.e., the distribution of the predicted random effects depends on the distribution of the error terms and vice versa. As each residual quantity depends on the other, the predicted error terms and random effects are said to be confounded (Hilden-Minton, 1995). Additionally, it is easily seen that both (3) and (4) lead to the analysis of correlated and potentially heteroscedastic disturbances as $\mathrm{Var}(\widehat{\boldsymbol{\varepsilon}}) = \boldsymbol{RPR}$ and $\mathrm{Var}(\widehat{\boldsymbol{b}}) = \boldsymbol{DZ'PZD}$. The use of standardized residuals can correct the latter issue, but does not address the fact that the residuals are interrelated. While problems may be expected at both levels of the model based on (3) and (4), we have found that the interpretation of Q-Q plots of the standardized predicted error terms

$$\boldsymbol{z}_{\varepsilon} = \mathrm{diag}\,(\boldsymbol{RPR})^{-1/2}\,\widehat{\boldsymbol{\varepsilon}}$$

is unaffected by this interrelationship in the situations that we have considered (see section 1 of the supplement for simulation results). This is not the case with the standardized random effects. When the degree of pooling is high—as it is in the above radon example, and often is in practice—interpretation of the predicted random effects cannot be separated from the distribution of the error terms.

## 3.2 Rotating the random effects

In section 2 we have seen that confounding between the different levels of a hierarchical linear model as specified in (2) leads to erroneous conclusions with respect to the distributions of error terms and random effects. In order to reduce confounding between the levels, we propose an assessment of the residual structure based on an $s$-dimensional rotation of the form $\boldsymbol{W}'\widehat{\boldsymbol{b}}$. This rotation is chosen such that it substantially reduces the amount of confounding. The resulting rotated random effects are standardized, uncorrelated, and homoscedastic residual quantities, which makes them easily assessable with standard tools. In our discussion (and notation) we focus on a two-level model with a single random effect for ease of explanation. However, this approach is not restricted to a two-level model with a single random effect; an extension of the method is given at the end of the section.

As a measure of confounding between the levels of a hierarchical model of the form (2), we introduce FC, the *fraction of confounding*:

**Definition 1** (Fraction of confounding)**.** *Let* $\boldsymbol{b}$ *denote a vector of* $q$ *random effects and* $\widehat{\boldsymbol{b}}$ *its predictions as defined in* (4)*. For a full rank matrix* $\boldsymbol{W} \in \mathbb{R}^{q\times s}$*, where* $s \leq q$*, the fraction of confounding in the* $s$*-dimensional space spanned by* $\boldsymbol{W}$ *is defined as*

$$FC(\boldsymbol{W};\widehat{\boldsymbol{b}}) = \frac{1}{q}\,\mathrm{Trace}\left(\left(\boldsymbol{W'BW}\right)^{-}\left(\boldsymbol{W'AW}\right)\right), \tag{5}$$

*where* $\boldsymbol{B}$ *is the covariance structure of* $\boldsymbol{b}$*,* $\boldsymbol{B} = \mathrm{Var}(\widehat{\boldsymbol{b}})$*, and* $\boldsymbol{A}$ *is the conditional covariance structure of* $\widehat{\boldsymbol{b}}$ *given* $\boldsymbol{b}$*, i.e.* $\boldsymbol{A} = \mathrm{Var}(\widehat{\boldsymbol{b}}|\boldsymbol{b})$*.*

Note that both $\boldsymbol{A}$ and $\boldsymbol{B}$ are positive semidefinite matrices. While it is not immediately obvious that FC is well defined, the following discussion will clarify this point.

Generally, FC $\in [0, 1]$, where 1 indicates that, due to confounding, the predicted random effects contain no information in addition to that found in the error terms, while a value of FC = 0 indicates that no confounding is present.

Notice that if $\boldsymbol{W}$ is the identity, then (5) measures the fraction of confounding in the original vector of predicted random effects as proposed by Hilden-Minton (1995). As noted by a reviewer, there are many ways of defining a fraction of confounding using different symmetric functions of the relative eigenvalues of $\boldsymbol{WAW}'$ to $\boldsymbol{WBW}'$. We chose the trace of the ratio to generalize the results in Hilden-Minton (1995), but other symmetric functions could be investigated. For example, the ratio of the traces could be examined, however, there is no closed form solution to analog of the optimization problem we discuss below (cf., Jia et al., 2009).

For any given dimension $s$, the linear combination $\boldsymbol{W} \in \mathbb{R}^{q\times s}$ that minimizes (5) also minimizes

$$J_1(s) = \text{Trace}\left( (\boldsymbol{W'BW})^{-1} (\boldsymbol{W'AW}) \right) \tag{6}$$

Mathematically, this problem is solved using the generalized eigenvalue decomposition

$$\boldsymbol{A}\boldsymbol{w}_k = \gamma_k \boldsymbol{B}\boldsymbol{w}_k \tag{7}$$

where $\gamma_k$ and $\boldsymbol{w}_k$ are the $k$-th smallest eigenvalues and eigenvectors, respectively (Fukunaga, 1990). Simultaneous diagonalization of $\boldsymbol{A}$ and $\boldsymbol{B}$ can be used to solve this problem, given that the kernel of $\boldsymbol{B}$ is a subspace of the kernel of $\boldsymbol{A}$ (de Leeuw, 1982). Equivalent to that, we have to show that the difference $\boldsymbol{B} - \boldsymbol{A}$ is nonnegative definite:

$$\boldsymbol{A} = \text{Var}(\widehat{\boldsymbol{b}}|\boldsymbol{b}) = \boldsymbol{RPZDZ'PR}$$

and

$$\boldsymbol{B} = \text{Var}(\widehat{\boldsymbol{b}}) = \boldsymbol{RPR} = \boldsymbol{RPVPR} = \boldsymbol{RPRPR} + \boldsymbol{RPZDZ'PR},$$

so $\boldsymbol{B} - \boldsymbol{A} = \boldsymbol{RPRPR}$ is nonnegative definite.

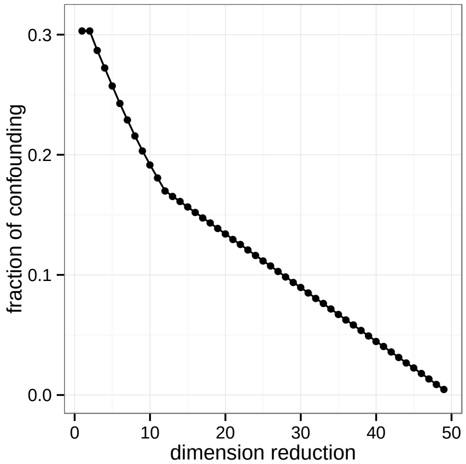
Simultaneous diagonalization of $\boldsymbol{A}$ and $\boldsymbol{B}$ requires $\boldsymbol{W}$ to be $\boldsymbol{B}$-orthogonal, so the optimal $\boldsymbol{W}$ is found to be

$$\boldsymbol{W}^*(s) = \underset{\boldsymbol{W}\in\mathbb{R}^{q\times s},\ \boldsymbol{W'BW}=\boldsymbol{I}}{\arg\min} \text{Trace}\left(\boldsymbol{W'AW}\right). \tag{8}$$

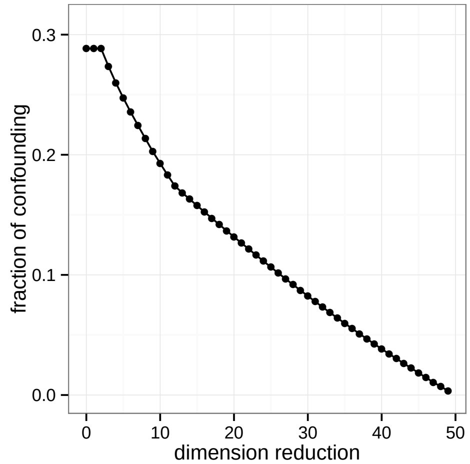
The rotated random effects are then given by $\boldsymbol{W}^{*\prime}\widehat{\boldsymbol{b}}$, which are standardized, uncorrelated, and homoscedastic (see the appendix for a proof and a detailed overview of simultaneous diagonalization). Simultaneous diagonalization will be familiar to many as the standard canonical transformation used in multivariate regression.

**Selection of the dimension of the subspace** spanned by the rotated residuals is central to our proposed method. Ideally, we would select the dimension such that the fraction of confounding is reduced to zero; however, this is not realistic in practice. Alternatively, we propose choosing the dimension that provides a substantial reduction in the fraction of confounding. As our ultimate objective is an assessment of the distribution, we must balance this reduction in the fraction of confounding with the loss in power of a test of the empirical distribution function associated with dimension reduction. To guide this choice we suggest plotting the reduction in the fraction of confounding against the reduction in dimension, which is similar to the concept of a scree plot used to select the number of principal components. To illustrate the use of this plot we simulate two simple random intercept models with a group-level predictor: model $M1$ has 40 groups of 30 observations and 10 groups of 5 observations; model $M2$ also has 50 groups, with group sizes determined as random draws from either a Poisson(30) distribution (40 groups) or a Poisson(5) distribution (10 groups). Figure 5 shows two examples of such plots constructed for the simulated models. For neither model the fraction of confounding for a one-dimensional reduction decreases because this one-dimensional reduction simply adjusts for the rank deficiency of the covariance matrix, which can be thought of in terms of adjusting for the effective degrees of freedom. Both figures have an "elbow" in the plot corresponding to a reduction in the dimension of the subspace of 11; thus we would choose $s = 39$. Additionally, we see that the elbow in the plot for model $M2$ is less pronounced. This occurs because the groups sizes are less balanced so the difference between the large and small group sizes is reduced.

**Correcting for supernormality.** The transformation of the random effects results in a vector where each component is a linear combination of elements of $\widehat{\boldsymbol{b}}$. Consequently, the rotated residuals will appear more normal than the underlying distribution, if the underlying distribution is not normal. This issue is often referred to as supernormality (Atkinson, 1985). One approach to address supernormality in this context is to reduce the number of elements in the linear combinations. Similar to factor loadings in factor analysis, let us therefore use an orthogonal rotation of




(a) Model $M1$

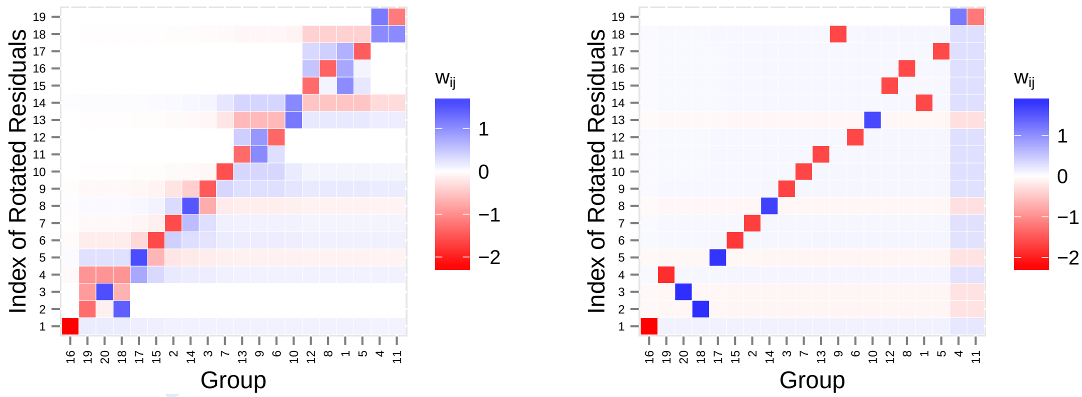



(b) Model $M2$

Figure 5: Plots of the fraction of confounding for each reduction in the dimension of subspace spanned by the rotated random intercepts from two simulation models. Model $M1$ has 50 groups: 40 groups of size 30, and 10 groups of size 5. Model $M2$ has 50 groups with group sizes determined as random draws from either a Poisson(30) distribution (40 groups) or a Poisson(5) distribution (10 groups). Based on these plots we choose $s = 39$, corresponding to a dimension reduction of 11.

$\boldsymbol{W}^*$, denoted as $\boldsymbol{Q}$. The rotated residuals are then obtained as $\boldsymbol{Q}'\boldsymbol{W}^{*\prime}\widehat{\boldsymbol{b}}$. Different choices for $\boldsymbol{Q}$ will produce different optimal properties in the residuals: One rotation that will produce rotated residuals comprised of a small number of raw residuals is the raw varimax rotation (Johnson and Wichern, 2007). Figure 6 displays heat maps of $\boldsymbol{W}^{*\prime}$ (left) and $\boldsymbol{Q}'\boldsymbol{W}^{*\prime}$ (right) for a simulated random intercept model with 20 groups, and demonstrates that the raw varimax rotation reduces the number of groups loading highly on each rotated residual. Other orthogonal rotations could be used, but the varimax rotation is familiar to a wide range of analysts and is widely implemented in statistical software packages. A similar approach was used by Jensen and Ramirez (1999), who used the raw varimax rotation to produce recovered errors for distributional assessment in the ordinary regression model.

**Extension to multiple random effects.** Up to this point our discussion has ignored that a model may (and often will) contain numerous random effects. In this case, the assumptions made on each random effect should be checked; thus, we propose assessing each random effect sepa-




(a) $\boldsymbol{W}^{*\prime}$ (b) $\boldsymbol{Q}'\boldsymbol{W}^{*\prime}$

Figure 6: Heat map of $\boldsymbol{W}^{*\prime}$ and $\boldsymbol{Q}'\boldsymbol{W}^{*\prime}$ for a simulated random intercept model with 20 groups. Applying the raw varimax rotation, $\boldsymbol{Q}$, reduces the number of groups loading on a given rotated residual.

rately. To this end we must define linear combinations $\boldsymbol{L}_k$ such that $\boldsymbol{L}_k'\widehat{\boldsymbol{b}}$ produces the $k$th marginal random effect. For example, in a model with a random intercept and random slope, if $\boldsymbol{Z}$ is organized as a block diagonal matrix—that is $\boldsymbol{Z} = \bigoplus_{i=1}^{m} \boldsymbol{Z}_i$ where $\bigoplus$ denotes the direct sum (Gentle, 2007, page 47)—then $\boldsymbol{L}_0 = \boldsymbol{I}_m \bigotimes (1,\ 0)$ produces the random intercepts and $\boldsymbol{L}_1 = \boldsymbol{I}_m \bigotimes (0,\ 1)$ produces the random slopes. The methodology presented in this section can be generalized to models with numerous random effects by substituting $\boldsymbol{L}_k'\widehat{\boldsymbol{b}}$ for $\widehat{\boldsymbol{b}}$.

## 4. SIMULATION STUDY

We conducted a simulation study to assess the specificity and sensitivity of tests of normality based on the two rotated residuals proposed in the previous section.

### 4.1 Design

We want to examine situations in which we correctly and incorrectly reject the null hypothesis of normality—that is, power and type I error, respectively. We compute the percentage of Anderson-Darling (AD), Cramér von Mises (CVM), and Kolmogorov-Smirnov (KS) tests that rejected the null hypothesis of normality.

The design matrices from model (1) were used as templates for realistic data generation; for simplicity of the simulation design, only the 60 counties with full rank $\boldsymbol{Z}$ matrices were included. Normal, heavy-tailed, and skewed distributions were used to generate the simulated errors and random effects. We used a rescaled $t$ distribution with 3 degrees of freedom to generate heavy tailed residuals, and a centered and rescaled exponential distribution with a rate parameter of 1 to generate skewed residuals. For simplicity we required the distributions of the random slope and intercept to the be same and assumed independence between the random effects. The nine distributional settings considered in the simulation study are summarized in Table 3. The fixed

Table 3: A summary of the nine distributional settings considered in the simulation study.

| Distributions of | | Random effects, $F_2$ | | |
|---|---|---|---|---|
| | | $\mathcal{N}(0,\ \sigma_b^2)$ | $(\sigma_b/\sqrt{3})\, t_3$ | $\sigma_b\,\{\mathrm{Exp}(1) - 1\}$ |
| Error terms, $F_1$ | $\mathcal{N}(0,\ \sigma_\varepsilon^2)$ | | | |
| | $(\sigma_\varepsilon/\sqrt{3})\, t_3$ | | $\varepsilon_{ij}^* \overset{iid}{\sim} F_1,\ \ b_{0i}^*, b_{1i}^* \overset{iid}{\sim} F_2$ | |
| | $\sigma_\varepsilon\,\{\mathrm{Exp}(1) - 1\}$ | | | |

effects coefficients were set to the maximum likelihood estimates.

To investigate the effect that pooling has on the rotated random effects we considered three variance structures to represent different degrees of confounding for the random effects:

**high:** $\sigma_\varepsilon^2 = 4$ and $\sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$

**moderate:** $\sigma_\varepsilon^2 = 1$ and $\sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$

**low:** $\sigma_\varepsilon^2 = 1$ and $\sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$

Under each simulation setting 1000 data sets were generated for each model and the rotated residuals were obtained using $s = \mathrm{rank}(\boldsymbol{B})$ (which is 58 and 59 for the random intercept and slope, respectively) as well as $s = 55$, 50, 45, 40, 35, and 30.

We ran the above simulation both in the case where $\boldsymbol{D}$ and $\boldsymbol{R}$ were known and where $\boldsymbol{D}$ and $\boldsymbol{R}$ were estimated. The results were quite similar between these two simulation studies, so we present the results from the simulation study for estimated $\boldsymbol{D}$ and $\boldsymbol{R}$, as this reflects more closely what is done in practice. Full simulation results are available in the supplement.

## 4.2 Results

Figure 7 shows the average fraction of confounding for the rotated random intercept (left) and random slope (right) over the different values for $s$ for each variance structure. As $s$ is reduced, the fraction of confounding is reduced, which aligns with the expectation that smaller choices of $s$ reduce the contributions of more highly confounded groups.

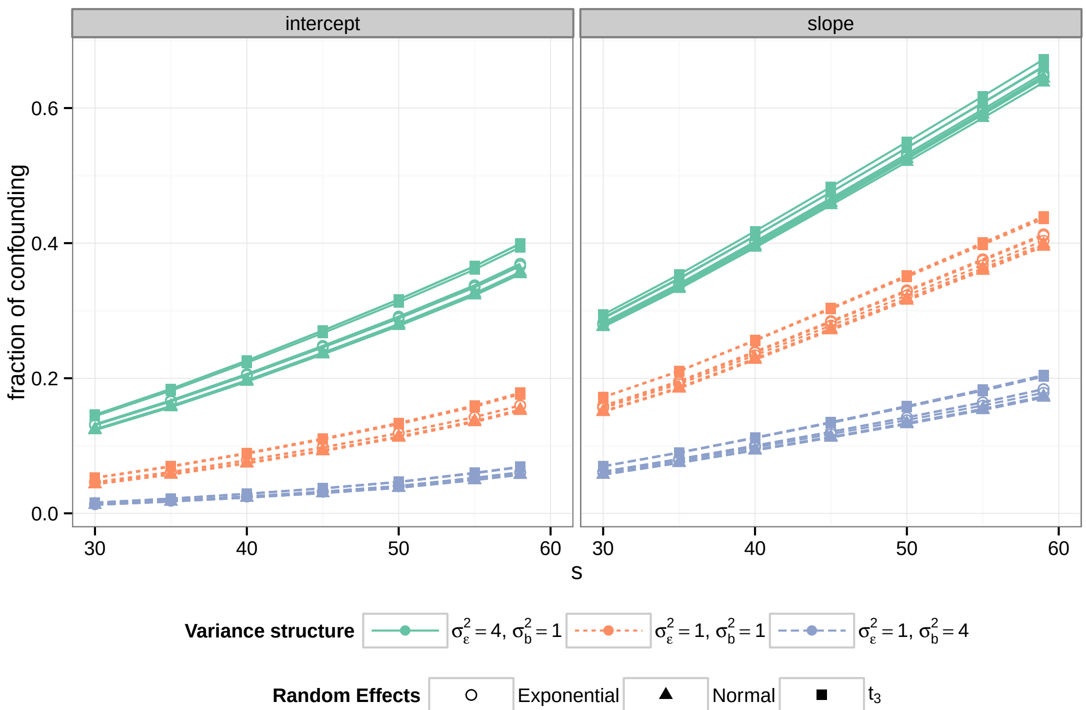



Figure 7: Change in the fraction of confounding (FC) as the dimension of the rotated random effects vector, $s$, is reduced for the three variance structures considered in the simulation study.

Table 4 and Figure 8 display the estimated type I error rates using the AD normality test ($\alpha = 0.05$) on the rotated and varimax rotated random intercepts and random slopes, respectively, when $\sigma^2_\varepsilon = 4$ and $\sigma^2_{b_0} = \sigma^2_{b_1} = 1$. The CVM and KS tests performed similarly and are omitted for brevity (full simulation results can be found in the supplementary material). Both figures show that the type I error rate is stabilized close to the nominal level with the appropriate choice of $s$. For the random intercept most choices of $s$ perform reasonably well, with the type I error rate closest to the nominal level for all error distributions between 30 and 40. For the random slope, $s$ must be chosen to be 30 for type I error to be near the nominal level; however, $s$ may need to be even smaller to achieve the nominal rate.

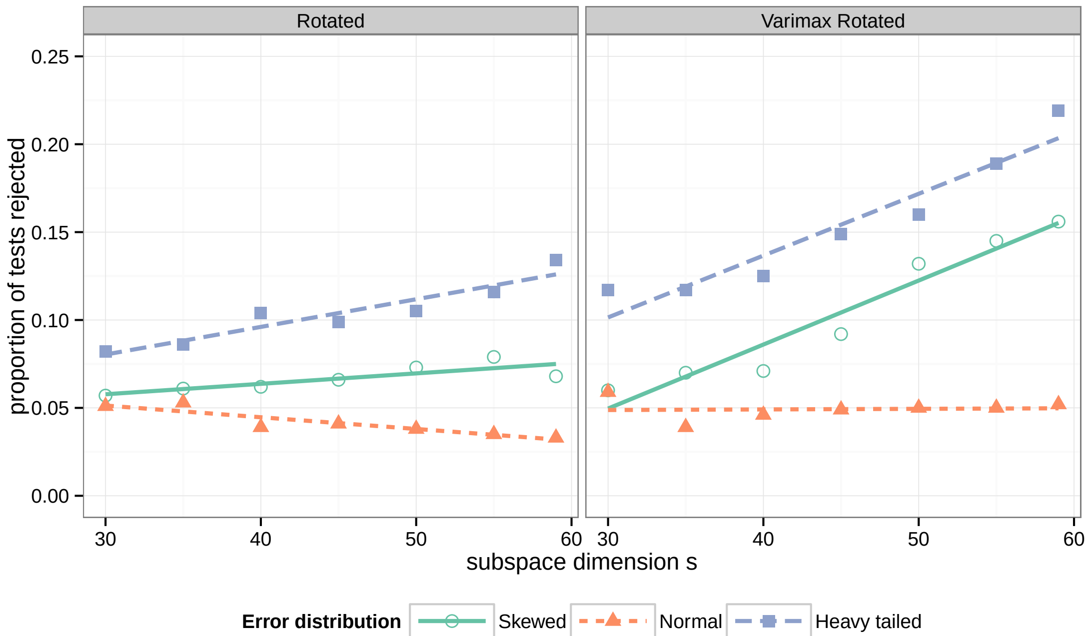



Figure 8: Estimated type I error rate using the Anderson-Darling normality test ($\alpha = 0.05$) on the rotated random slopes (left) and varimax rotated random slopes (right) by the distribution of the error terms and $s$.

Table 4 and Figure 9 show the estimated power of the AD test ($\alpha = 0.05$) on the rotated and varimax rotated random intercepts and random slopes, respectively, for the highly confounded variance structure. The estimated power to detect non-normal random effects distributions is amplified by the varimax rotation and larger choices of $s$. We also find that the estimated power is lower than would be expected from randomly sampled values from an exponential or $t_3$ distribution (what we will refer to as the “gold standard”). For example, when $s = 30$, simulations indicate the power of the AD test to detect a $t_3$ distribution to be approximately 0.4, whereas our simulations indicate nearly half the power, with the random slope generally having lower power than the random intercept. Interestingly, there is higher power to detect a heavy tailed distribution than a skewed distribution. Additional simulations (not shown) using a model with a continuous variable defining the random slope showed results similar to the random intercept (Table 4).

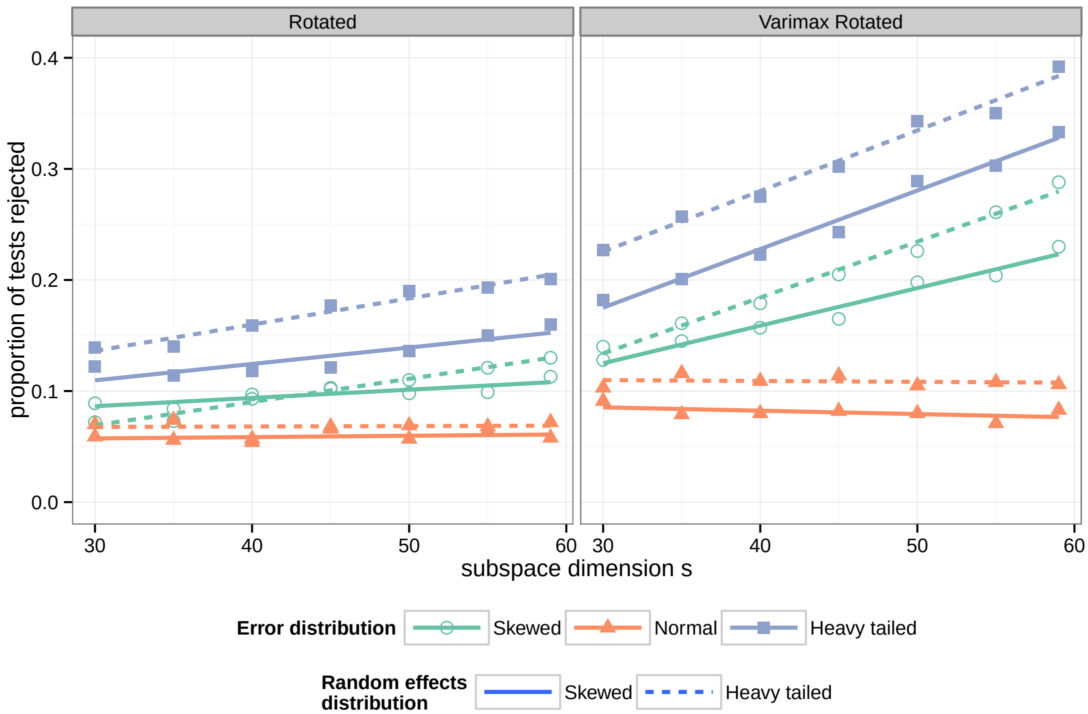



Figure 9: Linear smoother of the estimated power using the Anderson-Darling normality test ($\alpha = 0.05$) on the rotated random slopes (left) and varimax rotated random slopes (right) by $s$. The color denotes the distribution of the errors and the line type denotes the distribution of the random slope.

While the estimated power is lower than the gold standard, the fact that the type I error rate can be stabilized indicates that distributional problems detected using the rotated random effects will truly be problems; thus, providing more diagnostic information than the (unrotated) predicted random effects.

## 5. RADON DATA: REVISITED

Recall that in Section 2 we determined that the error terms were not normally distributed. Consequently, examination of Q-Q plots of the predicted random effects will likely lead to erroneous conclusions due to the high degree of shrinkage.

In order to construct Q-Q plots of the rotated random effects we first consider the choice of $s$. For model (1) the high degree of shrinkage leads to a large fraction of confounding for each random term: 0.72 for the random intercept and 0.70 for the random slope. In choosing $s$ we

Table 4: Percentages of AD tests rejecting the null hypothesis of normality at the 5% significance level for the rotated (a) and varimax rotated (b) random intercepts by $s$. The gray shading indicates the situations where the random intercept is normal (i.e., type I error).

(a) Rotated random intercept.

| | | $s$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Random intercept | Error term | 58 | 55 | 50 | 45 | 40 | 35 | 30 |
| Normal | Normal | 4.4 | 4.0 | 4.6 | 4.3 | 4.4 | 5.0 | 5.7 |
| | Heavy tailed | 7.5 | 7.6 | 6.7 | 6.2 | 5.0 | 4.2 | 5.0 |
| | Skewed | 5.1 | 4.7 | 5.7 | 5.8 | 5.6 | 5.5 | 5.6 |
| Heavy tailed | Normal | 13.9 | 13.6 | 13.1 | 13.4 | 13.0 | 13.1 | 12.1 |
| | Heavy tailed | 19.0 | 18.6 | 16.7 | 16.1 | 16.0 | 14.8 | 13.9 |
| | Skewed | 15.5 | 15.1 | 14.2 | 13.6 | 13.2 | 12.7 | 11.9 |
| Skewed | Normal | 9.6 | 8.7 | 9.5 | 9.7 | 10.0 | 11.0 | 10.0 |
| | Heavy tailed | 12.6 | 12.5 | 12.0 | 11.3 | 10.1 | 11.3 | 11.0 |
| | Skewed | 13.4 | 13.4 | 12.2 | 12.2 | 11.0 | 11.3 | 10.8 |

(b) Varimax rotated random intercept.

| | | $s$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Random intercept | Error term | 58 | 55 | 50 | 45 | 40 | 35 | 30 |
| Normal | Normal | 4.9 | 5.3 | 5.2 | 5.3 | 5.3 | 5.2 | 5.5 |
| | Heavy tailed | 9.0 | 9.1 | 8.0 | 7.1 | 6.0 | 5.1 | 5.2 |
| | Skewed | 5.2 | 5.1 | 4.4 | 5.5 | 6.1 | 5.1 | 6.1 |
| Heavy tailed | Normal | 22.1 | 22.3 | 23.3 | 22.9 | 23.3 | 22.3 | 21.6 |
| | Heavy tailed | 34.4 | 33.3 | 32.1 | 31.6 | 30.1 | 27.0 | 26.6 |
| | Skewed | 27.8 | 26.7 | 25.6 | 27.0 | 24.4 | 23.1 | 21.8 |
| Skewed | Normal | 19.7 | 21.2 | 21.3 | 22.1 | 22.7 | 21.1 | 22.3 |
| | Heavy tailed | 29.7 | 28.4 | 27.1 | 25.0 | 25.5 | 25.0 | 23.8 |
| | Skewed | 22.2 | 23.5 | 21.7 | 23.1 | 21.1 | 22.9 | 21.1 |

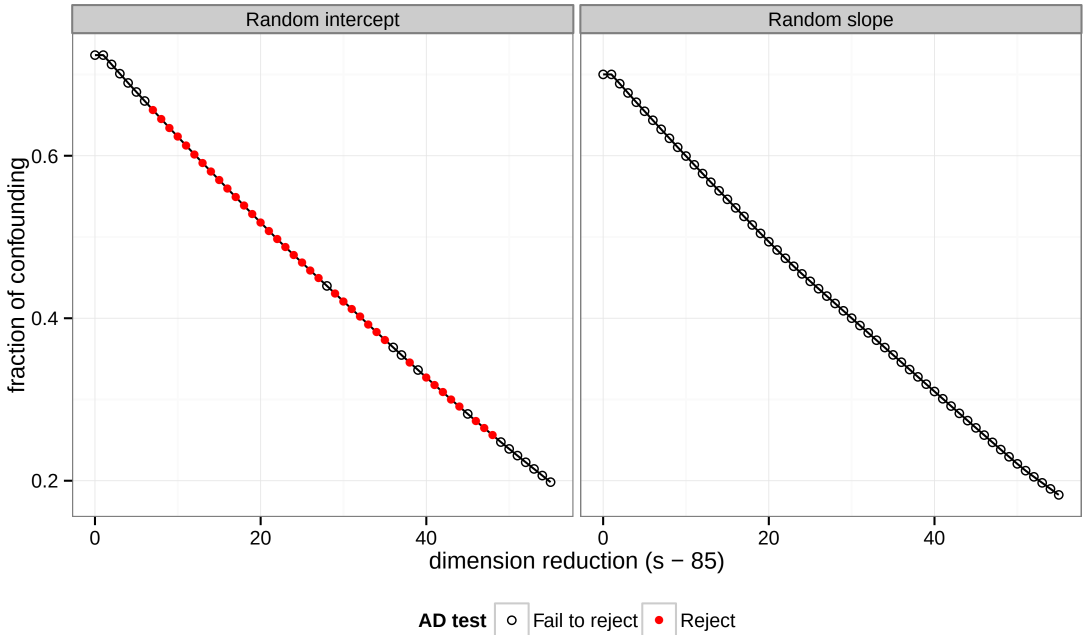
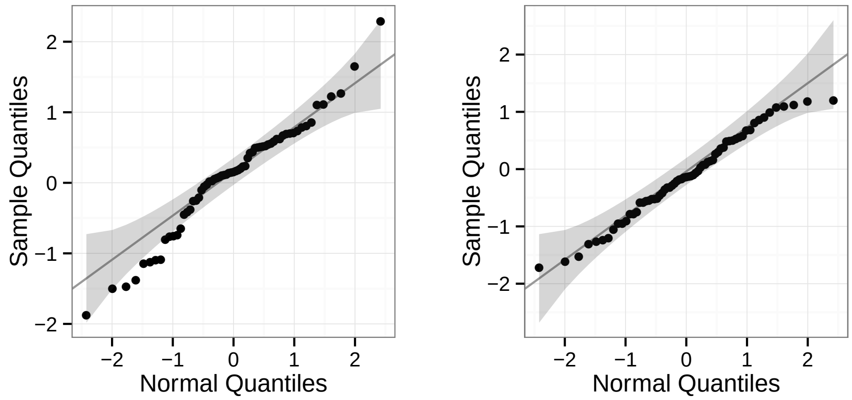



Figure 10: Plots of the fraction of confounding for each reduction in the dimension of subspace spanned by the rotated random intercept (left) and random slope (right) for the radon example. Filled points represent situations in which the AD test rejects normality of the varimax rotated random effects.

wish to substantially reduce the fraction of confounding, but restrict attention to $s \geq 30$ so as not to decrease the maximum possible power of a normality test too severely. Figure 10 shows the fraction confounding for $s \geq 30$.

We do not find elbows for either random effect in plots of the fraction of confounding against the reduction in dimensionality (Figure 10) making the choice of the dimension more difficult; however, because of the large number of counties with few observations, the smoothness of the plots is not unexpected. For a decision on the distribution, the exact choice of the subspace dimension $s$ is also not particularly critical. The color (fill) of the points in figure 10 shows the results of the AD test at the 5% significance level for each dimension. The red (solid) points denote a rejection of the null hypothesis of normality. The results are stable until the dimensionality is reduced to the point where the AD test lacks power. To show an example of the Q-Q plots produced from the rotated random effects Figure 11 shows Q-Q plots of the rotated random effects for subspace

$s = 65$. We see that the random intercept significantly deviates from the assumption of normality while the random slope does not.




(a) Random intercept (b) Random slope

Figure 11: Normal Q-Q plots with point-wise 95% confidence bands of the marginals rotated random effects. The deviations from normality are much less pronounced than before resulting in the failure to reject the null hypothesis of normality.

## 6. DISCUSSION

In this paper we have discussed two graphical approaches to assess the distributional assumptions made on the random effects in hierarchical linear models. The first approach used the lineup protocol to compare the predicted random effects produced in estimating the observed model to those produced when estimating a properly specified model. This method only assumes a distributional specification for the random effects and does not directly compare the predicted random effects to their distribution. Consequently, the conclusions that are drawn from this approach relate to evidence that the predicted random effects either are or are not consistent with what is expected under a correctly specified random effects distribution. The second approach rotates the predicted random effects so as to be able to compare them directly to the hypothesized distribution using a Q-Q plot. We have shown that the rotated random effects are standardized, uncorrelated, and homoscedastic, and that the rotation addresses the confounding present allowing for the random effects to be targeted separately from the error terms.

Under either approach, a misspecified covariance structure may lead to erroneous rejection of the null hypothesis. Therefore, in practice we recommend an assessment of the structure of the within- and between-group covariance matrices prior to distributional assessment. An alternative approach would be the use of robust covariance estimation techniques to protect against such misspecification; however, it is not clear how this impacts the diagnostic tools. We will leave this investigation for future study.

It is important to note that formal tests have been proposed to detect mixture distributions in the random effects (Verbeke and Lesaffre, 1996) and for overall goodness-of-fit tests for both the error terms and random effects (Jiang, 2001); however, these methods do not lend themselves to graphical inspection and have not been implemented in statistical software. Our method, on the other hand, requires only byproducts of the model fitting procedure and the use of matrix decompositions for simultaneous diagonalization, which are widely accessible in standard software. All of the methods and graphics discussed in this paper are implemented in `R` (R Core Team, 2013). In particular, the rotated residuals are part of the package `HLMdiag` (Loy, 2013; Loy and Hofmann, 2014).

Simulation has revealed that tests of normality using the rotated random effects achieve approximately nominal type I error rates with appropriate choice of the dimension, $s$. This indicates that assessment of the rotated residuals can target the distribution of the random effects in the presence of pooling, which the predicted random effects cannot. The power to detect non-normal random effects distributions is lower than the gold standard, which is to be expected as the rotated residuals consist of sums of predicted random effects, resulting in a total distribution that is closer to a normal distribution than its individuals. The varimax rotation reduces the impact of this supernormality effect. While we do think that the loss in power is troubling, the inflated type I error rates resulting from high levels of confounding is of a much bigger concern. Unlike before, any detection of a distributional deviation can now be trusted even in situations with high amounts of confounding between the levels of residuals.

# 7. APPENDIX: ADDITIONAL TECHNICAL DETAILS

## 7.1 Simultaneous diagonalization

Below, we outline the procedure used to simultaneously diagonalize $\boldsymbol{A}$ and $\boldsymbol{B}$ for reference.

**Algorithm 1** (Simultaneous diagonalization)**.** *Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be two positive semidefinite matrices such that the kernel of B is a subspace of the kernel of $\boldsymbol{A}$. The transformation that simultaneously diagonalizes both matrices can be found through the following procedure:*

1. *Find a transformation that whitens $\boldsymbol{B}$. Such a transformation is given by $\boldsymbol{T_r}\boldsymbol{\Lambda_r}^{-1/2}$, where $\boldsymbol{T}_r$ and $\boldsymbol{\Lambda}_r$ are the first $r$ eigenvectors and eigenvalues of $\boldsymbol{B}$, where $r = \operatorname{rank}(\boldsymbol{B})$.*

2. *Transform $\boldsymbol{A}$ and $\boldsymbol{B}$ to*

$$\boldsymbol{\Lambda_r}^{-1/2}\boldsymbol{T}_r'\boldsymbol{A}\boldsymbol{T_r}\boldsymbol{\Lambda_r}^{-1/2} = \boldsymbol{A}^* \tag{9}$$

$$\boldsymbol{\Lambda_r}^{-1/2}\boldsymbol{T}_r'\boldsymbol{B}\boldsymbol{T_r}\boldsymbol{\Lambda_r}^{-1/2} = \boldsymbol{I} \tag{10}$$

3. *Find an orthonormal transformation that diagonalizes $\boldsymbol{A}^*$. Such a transformation is given by the eigenvectors of $\boldsymbol{A}^*$, which we denote $\boldsymbol{U}$.*

*Based on the above three steps, the transformation that simultaneously diagonalizes $\boldsymbol{A}$ and $\boldsymbol{B}$ is $\boldsymbol{T_r}\boldsymbol{\Lambda_r}^{-1/2}\boldsymbol{U}$.*

## 7.2 Rotated random effects are standardized, uncorrelated, and homoscedastic

We present the proof of the claim that the rotated residuals, $\boldsymbol{W}^{*\prime}\widehat{\boldsymbol{b}}$, are standardized, uncorrelated, and homoscedastic. Following the developments presented in Section 3.2, we present this discussion for the random effects assuming that there is only a random intercept. Generalization to the situation with multiple random effects follows as previously discussed.

*Proof.* Let $\boldsymbol{A} = \text{Var}(\widehat{\boldsymbol{b}}|\boldsymbol{b})$, $\boldsymbol{B} = \text{Var}(\widehat{\boldsymbol{b}})$, $r = \text{rank}(\boldsymbol{B})$, and $q =$ the number of elements in $\widehat{\boldsymbol{b}}$. Recall that $\boldsymbol{A}$ and $\boldsymbol{B}$ are symmetric and positive semidefinite. Following from above, $\boldsymbol{T}_r$ and $\boldsymbol{\Lambda}_r$ follow from the spectral (or eigenvalue) decomposition of $\boldsymbol{B} = \boldsymbol{T}_r\boldsymbol{\Lambda}_r\boldsymbol{T}_r'$, and $\boldsymbol{U}$ follows from the spectral decomposition of $\boldsymbol{A}^* = \boldsymbol{\Lambda}_r^{-1/2}\boldsymbol{T}_r'\boldsymbol{A}\boldsymbol{T}_r\boldsymbol{\Lambda}_r^{-1/2} = \boldsymbol{U}\boldsymbol{\Gamma}\boldsymbol{U}'$. Then,

$$\begin{aligned}
\text{Var}(\boldsymbol{W}^{*\prime}\widehat{\boldsymbol{b}}) &= \text{Var}(\boldsymbol{U}'\boldsymbol{\Lambda}_r^{-1/2}\boldsymbol{T}_r'\widehat{\boldsymbol{b}}) \\
&= (\boldsymbol{U}'\boldsymbol{\Lambda}_r^{-1/2}\boldsymbol{T}_r')\text{Var}(\widehat{\boldsymbol{b}})(\boldsymbol{T}_r\boldsymbol{\Lambda}_r^{-1/2}\boldsymbol{U}) \\
&= (\boldsymbol{U}'\boldsymbol{\Lambda}_r^{-1/2}\boldsymbol{T}_r')\boldsymbol{B}(\boldsymbol{T}_r\boldsymbol{\Lambda}_r^{-1/2}\boldsymbol{U}) \\
&= \boldsymbol{I}
\end{aligned}$$

proving that the rotated random effects are standardized, uncorrelated, and homoscedastic. □

## SUPPLEMENTARY MATERIALS

The following supplemental materials can be obtained online:

**Simulation results:** The supplementary materials include the full simulation study discussed in Section 4. Additionally, the results of a simulation supporting the small simulation study discussed in Section 2 are presented and further show the need for alternative procedures to assess the distribution of the random effects.

**`R` script for figures and simulations:** The `R` code and data used to generate results discussed in this paper are available in the file `code_supplement.zip`.

**`R` package `HLMdiag`:** We have included the function to calculate the rotated random effects in the `R` package `HLMdiag`. The stable version of `HLMdiag` is available from the Comprehensive R Archive Network (CRAN, `http://cran.r-project.org/`) and the developmental version is available of github (`https://github.com/aloy/HLMdiag`).

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# Supplement to "Are you Normal? The Problem of Confounded Residual Structures in Hierarchical Linear Models"

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The materials in this document supplement the information presented in "Are you Normal? The Problem of Confounded Residual Structures in Hierarchical Linear Models". Section 1 presents a review of existing proposals for residual analysis for hierarchical linear models and a simulation study evaluating the performance of each. Section 2 presents the complete results for all simulation settings considered in the paper: Section 2.1 presents the results of the simulation study where the covariance matrices, $\boldsymbol{D}$ and $\boldsymbol{R}$, were estimated and Section 2.2 presents the results of the simulation study where the covariance matrices were known.

## 1 Evaluations of existing proposals

### 1.1 Model notation

Recall that the stacked representation of the hierarchical linear model is given by

$$\underset{(n\times 1)}{\boldsymbol{y}} = \underset{(n\times p)}{\boldsymbol{X}} \underset{(p\times 1)}{\boldsymbol{\beta}} + \underset{(n\times q)}{\boldsymbol{Z}} \underset{(q\times 1)}{\boldsymbol{b}} + \underset{(n\times 1)}{\boldsymbol{\varepsilon}}, \tag{1}$$
$$\boldsymbol{\varepsilon} \overset{\text{iid}}{\sim} \mathcal{N}(\boldsymbol{0},\ \boldsymbol{R}), \qquad \boldsymbol{b} \overset{\text{iid}}{\sim} \mathcal{N}(\boldsymbol{0},\ \boldsymbol{D})$$

where $\boldsymbol{y}$ is a vector of responses, $\boldsymbol{X}$ and $\boldsymbol{Z}$ are design matrices for the fixed and random effects, respectively, $\boldsymbol{\beta}$ is a vector of fixed effects, $\boldsymbol{b}$ is a vector of random effects, $\boldsymbol{\varepsilon}$ is a vector of error terms, and $\boldsymbol{R}$ and $\boldsymbol{D}$ are positive definite covariance matrices. Further, we assume that $\text{Cov}(\boldsymbol{\varepsilon},\ \boldsymbol{b}) = \boldsymbol{0}$. The above assumptions imply that, marginally, $\boldsymbol{y} \sim \mathcal{N}(\boldsymbol{X\beta},\ \boldsymbol{V})$ where $\boldsymbol{V} = \boldsymbol{ZDZ}' + \boldsymbol{R}$.

### 1.2 Residuals

In this section we consider residuals that are commonly used to check the distributional assumptions in a hierarchical linear model. For more general discussions of residual analysis for hierarchical linear models we refer the reader to Haslett and Haslett (2007) and Nobre and Singer (2007).

**Marginal residuals.** The marginal distribution of $\boldsymbol{y}$ leads to the marginal residuals which are defined as

$$\widehat{\boldsymbol{\zeta}} = \boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}} = \boldsymbol{V}\boldsymbol{P}\boldsymbol{y} \tag{2}$$

where $\boldsymbol{P} = \boldsymbol{V^{-1}} - \boldsymbol{V^{-1}X}\left(\boldsymbol{X'V^{-1}X}\right)\boldsymbol{X'V^{-1}}$, and reveal how the observations deviate from the global trend. The use of these residuals for distributional assessment provides an omnibus assessment of goodness-of-fit as the marginal residuals are a linear combination of the other residual quantities; however, this assessment requires the empirical distribution of the marginal residuals to resemble true distribution. Asymptotically, the variance of the marginal residuals is $\mathrm{Var}(\widehat{\zeta}) = \boldsymbol{V}$ leading to correlated residuals. To obtain asymptotically uncorrelated residuals the marginal residuals can be scaled by the Cholesky root of $\boldsymbol{V}$ (Houseman et al., 2004), $\boldsymbol{C}$, yielding

$$\boldsymbol{z}_{\zeta} = \boldsymbol{C}^{-1}\widehat{\boldsymbol{\zeta}} \tag{3}$$

**Level-1 residuals.** The distribution of $\boldsymbol{y}$ conditional on the random effects, $\boldsymbol{b}$, is given by

$$\boldsymbol{y}|\boldsymbol{b} \sim \mathcal{N}(\boldsymbol{X\beta} + \boldsymbol{Zb},\ \boldsymbol{R}), \tag{4}$$

and leads to the level-1 residuals, commonly referred to as the error terms, which are defined as

$$\widehat{\boldsymbol{\varepsilon}} = \boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}} + \boldsymbol{Z}\widehat{\boldsymbol{b}} = \boldsymbol{RPy} \tag{5}$$

and reveal the deviations of the observations from the conditional model. The variance of the level-1 residuals is given by $\mathrm{Var}(\widehat{\varepsilon}) = \boldsymbol{RPR}$, so studentized level-1 residuals can be obtained by

$$\boldsymbol{z}_{\varepsilon} = \mathrm{diag}\left(\boldsymbol{RPR}\right)^{-1/2}\widehat{\boldsymbol{\varepsilon}} \tag{6}$$

which have been recommended for distributional assessment (Nobre and Singer, 2007). An alternative approach is recommended by Pinheiro and Bates (2000, Section 4.3) uses the Pearson residuals, which are obtained by dividing the predicted residuals by the estimated within-group standard deviation, $\widehat{\sigma}_{\varepsilon}$.

**Level-2 residuals.** The final type of residual we consider is the the best linear unbiased predictor (BLUP) of the random effects (i.e., predicted random effects), providing insight into the differences between the marginal (global) and conditional models. By definition, the BLUP of $\boldsymbol{b}$ is

$$\widehat{\boldsymbol{b}} = \boldsymbol{D}\boldsymbol{Z}'\boldsymbol{V}^{-1}\left(\boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}}\right) = \boldsymbol{D}\boldsymbol{Z}'\boldsymbol{P}\boldsymbol{y} \tag{7}$$

which has variance $\mathrm{Var}(\widehat{\boldsymbol{b}}) = \boldsymbol{D}\boldsymbol{Z}'\boldsymbol{P}\boldsymbol{Z}\boldsymbol{D}$. For distributional assessment of the BLUPs it makes sense to examine each random effect individually, though Lange and Ryan (1989) suggest the examination of linear combinations of standardized BLUPs. Rewritting the definition of $\mathrm{Var}(\widehat{\boldsymbol{b}})$

$$\boldsymbol{D}\boldsymbol{Z}'\boldsymbol{P}\boldsymbol{Z}\boldsymbol{D} = \boldsymbol{D}\boldsymbol{Z}'\boldsymbol{V}^{-1}\left(\boldsymbol{V} - \boldsymbol{X}\left(\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X}\right)\boldsymbol{X}'\right)\boldsymbol{V}^{-1}\boldsymbol{Z}\boldsymbol{D} \tag{8}$$

leads to two similar standardizations of the BLUPs. The first utilizes the fact that when the number of groups is large $\boldsymbol{X}\left(\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X}\right)$ will be small (Goldstein, 2011), so for a large number of groups standardized BLUPs can be calculated by

$$\boldsymbol{z}_b = \mathrm{diag}\left(\boldsymbol{D}\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z}\boldsymbol{D}\right)^{-1/2}\widehat{\boldsymbol{b}} \tag{9}$$

This formulation is the same used by Lange and Ryan (1989) (discussed below). The second standardization applies for all sample sizes and is given by

$$\boldsymbol{z}_b = \mathrm{diag}\left(\boldsymbol{D}\boldsymbol{Z}'\boldsymbol{P}\boldsymbol{Z}\boldsymbol{D}\right)^{-1/2}\widehat{\boldsymbol{b}} \tag{10}$$

### 1.3 Weighted Q-Q plots

As an alternative to Q-Q plots constructed from the BLUPs Lange and Ryan (1989) propose using weighted Q-Q plots of standardized linear combinations of the BLUPs, $\boldsymbol{L}'\widehat{\boldsymbol{b}}$,

$$z_b = \text{diag}\left(\boldsymbol{L}'\boldsymbol{D}\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z}\boldsymbol{D}\boldsymbol{L}\right)^{-1/2}\boldsymbol{L}'\widehat{\boldsymbol{b}} \tag{11}$$

The specific form of $\boldsymbol{L}$ chosen highlights different departures from distributional assumptions—for example, $\boldsymbol{L}$s can be chosen to extract the random slope and the random intercept terms individually. When the random effects may be correlated, Lange and Ryan suggest examining a range of additional linear combinations in-between the two marginal random effects either through manual specification of $\boldsymbol{L}$ or projection pursuit. After choosing $\boldsymbol{L}$ a weighted Q-Q plot is constructed by comparing the weighted empirical cumulative distribution function

$$F_m^*(x) = \sum_{i=1}^{m} I(x - z_{b_i} \geq 0)w_i \Big/ \sum_{i=1}^{m} w_i, \tag{12}$$

to $\Phi^{-1}\left(F_m^*(z_{b_i})\right)$. Here, $w_i$ is the $i$th element of $\boldsymbol{L}'\boldsymbol{D}\boldsymbol{Z}'\boldsymbol{V}^{-1}\boldsymbol{Z}\boldsymbol{D}\boldsymbol{L}$. For balanced group sizes this simplifies to the unweighted Q-Q plot of $\boldsymbol{z}_b$.

### 1.4 Simulation-based approaches

All of the above approaches to checking the distributional assumptions rely on the use of interrelated residuals, which has been reported to be problematic (Hilden-Minton, 1995; Verbeke and Lesaffre, 1996). One alternative that has been proposed to overcome this problem is the use of the parametric bootstrap to develop point-wise and simultaneous confidence bands for Q-Q plots. We evaluate the potential of this method using bootstrap tests of normality.

### 1.5 Simulation study

To evaluate the above proposals we carried out a simulation study under the same settings as in the paper, with the only difference being that the original $\boldsymbol{Z}$ was used for data generation. To evaluate the bootstrap tests of normality, a null distribution of 5000 simulated test statistics for each situation was used.

Tables 1–4 present the results of using standard normality tests to assess the distributional assumptions of the residuals from a hierarchical model. The gray background on the table indicates which simulation settings estimate type I error, with the other rows estimating power. Tables 5–8 present the results of the bootstrap tests for normality. Table 9 presents the results of using a weighted CDF to evaluate the normality of the random effects, in this case the null distribution was obtained using the parametric bootstrap.

The simulation study reveals that none of the residual-based diagnostics for assessing distributional assumptions are appropriate in all situations. The error terms can be targeted either by the use of studentized residuals or a parametric bootstrap; however, the assessment of this assumption is less critical. The random effects, on which predictive inference relies, cannot be targeted by the current methods when the residual variance is larger than the variance component associated with the random effects—that is, situations with higher degrees of shrinkage. Such situations are often encountered in practice. Additionally, use of the parametric bootstrap—to construct simulation envelopes for Q-Q plots, for example—does not appear to remedy this situation based on the performance of the bootstrap tests. Finally, we can see that Lange and Ryan's weighted Q-Q plots cannot target the random effects distribution when the residual variance is large, as the distribution of the error terms overly influences tests for the random slope, resulting in inflated type I error rates for both random effects.

Table 1: Proportion of tests rejecting the null hypothesis of normality of the error terms.

| Distributions: Errors | Distributions: Random effects | Nominal $\alpha$ | Raw residuals: AD | Raw residuals: CVM | Raw residuals: KS | Pearson residuals: AD | Pearson residuals: CVM | Pearson residuals: KS | Studentized residuals: AD | Studentized residuals: CVM | Studentized residuals: KS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | | |
| Normal | Normal | 0.05 | 0.07 | 0.06 | 0.06 | 0.07 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.13 | 0.12 | 0.12 | 0.13 | 0.12 | 0.12 | 0.09 | 0.09 | 0.10 |
| | Heavy tailed | 0.05 | 0.07 | 0.08 | 0.06 | 0.07 | 0.08 | 0.06 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.14 | 0.13 | 0.14 | 0.14 | 0.13 | 0.14 | 0.11 | 0.11 | 0.10 |
| | Skewed | 0.05 | 0.07 | 0.06 | 0.06 | 0.07 | 0.06 | 0.06 | 0.04 | 0.04 | 0.05 |
| | | 0.10 | 0.13 | 0.12 | 0.13 | 0.13 | 0.12 | 0.13 | 0.09 | 0.09 | 0.10 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.11 | 0.10 | 0.09 | 0.11 | 0.10 | 0.09 | 0.09 | 0.09 | 0.08 |
| | Heavy tailed | 0.05 | 0.07 | 0.06 | 0.06 | 0.07 | 0.06 | 0.06 | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.13 | 0.12 | 0.11 | 0.13 | 0.12 | 0.11 | 0.11 | 0.11 | 0.10 |
| | Skewed | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.10 | 0.09 | 0.11 | 0.10 | 0.09 | 0.11 | 0.08 | 0.09 | 0.10 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | | |
| Normal | Normal | 0.05 | 0.10 | 0.09 | 0.07 | 0.10 | 0.09 | 0.07 | 0.05 | 0.04 | 0.04 |
| | | 0.10 | 0.17 | 0.17 | 0.15 | 0.17 | 0.17 | 0.15 | 0.09 | 0.09 | 0.09 |
| | Heavy tailed | 0.05 | 0.11 | 0.11 | 0.10 | 0.11 | 0.11 | 0.10 | 0.06 | 0.06 | 0.05 |
| | | 0.10 | 0.19 | 0.19 | 0.19 | 0.19 | 0.19 | 0.19 | 0.12 | 0.11 | 0.12 |
| | Skewed | 0.05 | 0.10 | 0.10 | 0.09 | 0.10 | 0.10 | 0.09 | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 0.18 | 0.18 | 0.17 | 0.18 | 0.18 | 0.17 | 0.11 | 0.11 | 0.11 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

Table 2: Proportion of tests rejecting the null hypothesis of normality of the random intercept.

| Distributions | | Nominal | Raw residuals | | | Pearson residuals | | | Studentized residuals | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS | AD | CVM | KS |
| | | | | | $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.11 | 0.10 | 0.12 | 0.11 | 0.10 | 0.12 |
| | Heavy tailed | 0.05 | 0.15 | 0.13 | 0.12 | 0.17 | 0.15 | 0.13 | 0.17 | 0.15 | 0.13 |
| | | 0.10 | 0.22 | 0.20 | 0.20 | 0.26 | 0.23 | 0.21 | 0.26 | 0.23 | 0.20 |
| | Skewed | 0.05 | 0.16 | 0.15 | 0.13 | 0.18 | 0.17 | 0.14 | 0.18 | 0.17 | 0.14 |
| | | 0.10 | 0.25 | 0.23 | 0.21 | 0.28 | 0.25 | 0.22 | 0.28 | 0.25 | 0.21 |
| Heavy tailed | Normal | 0.05 | 0.27 | 0.24 | 0.19 | 0.28 | 0.26 | 0.21 | 0.28 | 0.26 | 0.21 |
| | | 0.10 | 0.35 | 0.31 | 0.28 | 0.36 | 0.34 | 0.30 | 0.36 | 0.33 | 0.30 |
| | Heavy tailed | 0.05 | 0.49 | 0.45 | 0.35 | 0.51 | 0.46 | 0.36 | 0.50 | 0.46 | 0.36 |
| | | 0.10 | 0.58 | 0.54 | 0.46 | 0.60 | 0.55 | 0.47 | 0.60 | 0.55 | 0.47 |
| | Skewed | 0.05 | 0.52 | 0.48 | 0.36 | 0.55 | 0.50 | 0.40 | 0.55 | 0.50 | 0.40 |
| | | 0.10 | 0.62 | 0.59 | 0.51 | 0.65 | 0.60 | 0.53 | 0.65 | 0.60 | 0.52 |
| Skewed | Normal | 0.05 | 0.51 | 0.48 | 0.39 | 0.51 | 0.49 | 0.38 | 0.51 | 0.49 | 0.39 |
| | | 0.10 | 0.61 | 0.58 | 0.51 | 0.61 | 0.59 | 0.52 | 0.60 | 0.58 | 0.52 |
| | Heavy tailed | 0.05 | 0.73 | 0.69 | 0.58 | 0.73 | 0.70 | 0.59 | 0.73 | 0.70 | 0.59 |
| | | 0.10 | 0.80 | 0.77 | 0.69 | 0.81 | 0.78 | 0.70 | 0.80 | 0.78 | 0.70 |
| | Skewed | 0.05 | 0.87 | 0.83 | 0.70 | 0.87 | 0.82 | 0.69 | 0.86 | 0.83 | 0.69 |
| | | 0.10 | 0.92 | 0.89 | 0.80 | 0.91 | 0.88 | 0.80 | 0.91 | 0.88 | 0.80 |
| | | | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.09 | 0.09 | 0.08 | 0.09 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 |
| | Heavy tailed | 0.05 | 0.07 | 0.07 | 0.06 | 0.07 | 0.07 | 0.06 | 0.07 | 0.07 | 0.06 |
| | | 0.10 | 0.12 | 0.12 | 0.11 | 0.12 | 0.11 | 0.11 | 0.12 | 0.11 | 0.11 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.07 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 | 0.11 | 0.12 |
| Heavy tailed | Normal | 0.05 | 0.55 | 0.50 | 0.42 | 0.52 | 0.48 | 0.40 | 0.52 | 0.48 | 0.40 |
| | | 0.10 | 0.63 | 0.60 | 0.51 | 0.60 | 0.56 | 0.51 | 0.60 | 0.56 | 0.52 |
| | Heavy tailed | 0.05 | 0.63 | 0.58 | 0.49 | 0.60 | 0.56 | 0.47 | 0.60 | 0.56 | 0.47 |
| | | 0.10 | 0.71 | 0.68 | 0.60 | 0.68 | 0.63 | 0.56 | 0.68 | 0.63 | 0.56 |
| | Skewed | 0.05 | 0.62 | 0.57 | 0.47 | 0.61 | 0.55 | 0.46 | 0.61 | 0.55 | 0.46 |
| | | 0.10 | 0.71 | 0.66 | 0.58 | 0.69 | 0.65 | 0.57 | 0.69 | 0.64 | 0.57 |
| Skewed | Normal | 0.05 | 0.93 | 0.92 | 0.86 | 0.93 | 0.91 | 0.86 | 0.93 | 0.91 | 0.85 |
| | | 0.10 | 0.96 | 0.94 | 0.91 | 0.96 | 0.94 | 0.90 | 0.95 | 0.94 | 0.90 |
| | Heavy tailed | 0.05 | 0.97 | 0.96 | 0.89 | 0.97 | 0.96 | 0.88 | 0.97 | 0.96 | 0.88 |
| | | 0.10 | 0.99 | 0.98 | 0.94 | 0.99 | 0.98 | 0.94 | 0.99 | 0.98 | 0.94 |
| | Skewed | 0.05 | 0.98 | 0.96 | 0.90 | 0.98 | 0.97 | 0.90 | 0.98 | 0.97 | 0.91 |
| | | 0.10 | 0.99 | 0.97 | 0.95 | 0.99 | 0.98 | 0.94 | 0.99 | 0.98 | 0.95 |
| | | | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.10 | 0.09 | 0.09 | 0.10 | 0.09 | 0.09 | 0.10 | 0.09 | 0.09 |
| | Heavy tailed | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 |
| | Skewed | 0.05 | 0.04 | 0.04 | 0.03 | 0.04 | 0.04 | 0.03 | 0.04 | 0.04 | 0.03 |
| | | 0.10 | 0.09 | 0.09 | 0.08 | 0.10 | 0.09 | 0.08 | 0.09 | 0.09 | 0.08 |
| Heavy tailed | Normal | 0.05 | 0.68 | 0.63 | 0.54 | 0.68 | 0.63 | 0.54 | 0.68 | 0.63 | 0.54 |
| | | 0.10 | 0.75 | 0.71 | 0.63 | 0.75 | 0.70 | 0.64 | 0.75 | 0.71 | 0.64 |
| | Heavy tailed | 0.05 | 0.71 | 0.67 | 0.57 | 0.71 | 0.67 | 0.58 | 0.71 | 0.67 | 0.57 |
| | | 0.10 | 0.78 | 0.76 | 0.68 | 0.79 | 0.76 | 0.67 | 0.79 | 0.75 | 0.67 |
| | Skewed | 0.05 | 0.70 | 0.68 | 0.57 | 0.70 | 0.67 | 0.56 | 0.70 | 0.67 | 0.56 |
| | | 0.10 | 0.78 | 0.74 | 0.68 | 0.78 | 0.74 | 0.68 | 0.78 | 0.74 | 0.67 |
| Skewed | Normal | 0.05 | 1.00 | 0.99 | 0.97 | 1.00 | 0.99 | 0.97 | 1.00 | 0.99 | 0.97 |
| | | 0.10 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 0.98 | 1.00 | 1.00 | 0.98 | 1.00 | 1.00 | 0.98 |
| | | 0.10 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 |
| | Skewed | 0.05 | 1.00 | 1.00 | 0.98 | 1.00 | 1.00 | 0.98 | 1.00 | 1.00 | 0.98 |
| | | 0.10 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 |

Table 3: Proportion of tests rejecting the null hypothesis of normality of the random slope.

| Distributions | | Nominal | Raw residuals | | | Pearson residuals | | | Studentized residuals | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS | AD | CVM | KS |
| | | | | | $\sigma_\varepsilon^2 = 4$, | $\sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 0.05 | 0.05 | 0.06 | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 0.26 | 0.24 | 0.19 | 0.26 | 0.24 | 0.19 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.35 | 0.32 | 0.27 | 0.35 | 0.32 | 0.27 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 0.33 | 0.31 | 0.24 | 0.33 | 0.31 | 0.24 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.41 | 0.38 | 0.34 | 0.41 | 0.38 | 0.34 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 0.13 | 0.12 | 0.09 | 0.13 | 0.12 | 0.09 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.18 | 0.19 | 0.17 | 0.18 | 0.19 | 0.17 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 0.40 | 0.36 | 0.29 | 0.40 | 0.36 | 0.29 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.49 | 0.44 | 0.37 | 0.49 | 0.45 | 0.37 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 0.49 | 0.46 | 0.37 | 0.49 | 0.46 | 0.37 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.59 | 0.56 | 0.50 | 0.59 | 0.56 | 0.49 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 0.12 | 0.11 | 0.10 | 0.12 | 0.11 | 0.10 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.17 | 0.16 | 0.16 | 0.18 | 0.16 | 0.16 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 0.41 | 0.37 | 0.30 | 0.40 | 0.37 | 0.30 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.51 | 0.47 | 0.39 | 0.51 | 0.47 | 0.39 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 0.59 | 0.56 | 0.46 | 0.59 | 0.56 | 0.46 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.70 | 0.66 | 0.58 | 0.70 | 0.66 | 0.58 |
| | | | | | $\sigma_\varepsilon^2 = 1$, | $\sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 0.05 | 0.05 | 0.06 | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.12 | 0.11 | 0.11 | 0.12 | 0.11 | 0.11 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 0.13 | 0.12 | 0.11 | 0.13 | 0.12 | 0.11 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.22 | 0.20 | 0.17 | 0.22 | 0.20 | 0.17 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 0.14 | 0.12 | 0.10 | 0.14 | 0.12 | 0.11 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.22 | 0.20 | 0.17 | 0.22 | 0.20 | 0.17 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 0.27 | 0.24 | 0.19 | 0.27 | 0.24 | 0.19 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.35 | 0.32 | 0.27 | 0.35 | 0.31 | 0.27 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 0.44 | 0.40 | 0.33 | 0.44 | 0.40 | 0.33 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.50 | 0.48 | 0.42 | 0.50 | 0.48 | 0.42 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 0.41 | 0.38 | 0.31 | 0.41 | 0.38 | 0.31 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.51 | 0.48 | 0.42 | 0.51 | 0.48 | 0.42 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 0.46 | 0.42 | 0.34 | 0.46 | 0.42 | 0.34 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.57 | 0.52 | 0.46 | 0.57 | 0.52 | 0.46 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 0.65 | 0.60 | 0.51 | 0.65 | 0.60 | 0.51 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.73 | 0.69 | 0.62 | 0.73 | 0.69 | 0.62 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 0.75 | 0.70 | 0.57 | 0.75 | 0.70 | 0.57 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.83 | 0.78 | 0.70 | 0.83 | 0.78 | 0.70 |
| | | | | | $\sigma_\varepsilon^2 = 1$, | $\sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 0.04 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.11 | 0.10 | 0.10 | 0.11 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.12 | 0.12 | 0.13 | 0.12 | 0.12 | 0.13 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 0.06 | 0.06 | 0.05 | 0.06 | 0.06 | 0.05 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 0.46 | 0.41 | 0.34 | 0.46 | 0.41 | 0.34 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.56 | 0.52 | 0.44 | 0.56 | 0.52 | 0.44 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 0.55 | 0.50 | 0.43 | 0.55 | 0.50 | 0.42 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.63 | 0.60 | 0.53 | 0.63 | 0.60 | 0.53 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 0.48 | 0.46 | 0.37 | 0.48 | 0.46 | 0.37 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.57 | 0.53 | 0.48 | 0.57 | 0.53 | 0.48 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 0.90 | 0.87 | 0.74 | 0.90 | 0.87 | 0.74 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.94 | 0.93 | 0.83 | 0.94 | 0.93 | 0.84 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 0.92 | 0.91 | 0.80 | 0.93 | 0.91 | 0.80 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.96 | 0.95 | 0.90 | 0.96 | 0.95 | 0.90 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 0.92 | 0.90 | 0.79 | 0.92 | 0.90 | 0.79 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.95 | 0.93 | 0.89 | 0.95 | 0.93 | 0.89 |

Table 4: Proportion of tests rejecting the null hypothesis of normality of the marginal residuals.

| Distributions | | Nominal | Raw residuals | | | Cholesky residuals | | |
|---|---|---|---|---|---|---|---|---|
| Errors | Random effects | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.06 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.12 | 0.12 | 0.13 | 0.11 | 0.11 | 0.12 |
| | Heavy tailed | 0.05 | 0.20 | 0.19 | 0.14 | 0.09 | 0.08 | 0.05 |
| | | 0.10 | 0.28 | 0.24 | 0.22 | 0.14 | 0.13 | 0.12 |
| | Skewed | 0.05 | 0.30 | 0.27 | 0.22 | 0.05 | 0.04 | 0.05 |
| | | 0.10 | 0.39 | 0.36 | 0.32 | 0.10 | 0.10 | 0.10 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | |
| Normal | Normal | 0.05 | 0.32 | 0.30 | 0.25 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.41 | 0.38 | 0.34 | 0.09 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.65 | 0.61 | 0.52 | 0.10 | 0.09 | 0.06 |
| | | 0.10 | 0.72 | 0.68 | 0.61 | 0.16 | 0.14 | 0.13 |
| | Skewed | 0.05 | 0.93 | 0.90 | 0.87 | 0.11 | 0.10 | 0.09 |
| | | 0.10 | 0.94 | 0.93 | 0.91 | 0.18 | 0.17 | 0.16 |
| Heavy tailed | Normal | 0.05 | 0.95 | 0.91 | 0.85 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.97 | 0.94 | 0.90 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 1.00 | 0.99 | 0.98 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | |
| Normal | Normal | 0.05 | 0.82 | 0.80 | 0.71 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.87 | 0.85 | 0.80 | 0.10 | 0.09 | 0.08 |
| | Heavy tailed | 0.05 | 0.96 | 0.94 | 0.91 | 0.16 | 0.14 | 0.10 |
| | | 0.10 | 0.97 | 0.95 | 0.93 | 0.26 | 0.23 | 0.20 |
| | Skewed | 0.05 | 1.00 | 1.00 | 0.99 | 0.31 | 0.29 | 0.23 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 0.43 | 0.40 | 0.34 |
| Heavy tailed | Normal | 0.05 | 0.98 | 0.96 | 0.92 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.99 | 0.97 | 0.96 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 0.99 | 0.98 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 0.99 | 0.99 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 0.98 | 0.97 | 0.95 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.99 | 0.98 | 0.98 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

Table 5: Proportion of bootstrap tests rejecting the null hypothesis of normality of the error terms.

| Distributions | | Nominal | Raw residuals | | | Pearson residuals | | | Studentized residuals | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Errors | Random effects | $\alpha$ | AD | CVM | KS | AD | CVM | KS | AD | CVM | KS |
| | | | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.10 | 0.10 | 0.09 | 0.10 | 0.10 | 0.09 | 0.09 | 0.09 | 0.09 |
| | Heavy tailed | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.11 | 0.10 | 0.11 | 0.11 | 0.10 | 0.11 | 0.11 | 0.11 | 0.09 |
| | Skewed | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.05 | 0.04 | 0.04 | 0.05 |
| | | 0.10 | 0.10 | 0.09 | 0.09 | 0.10 | 0.09 | 0.09 | 0.09 | 0.09 | 0.10 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.04 | 0.04 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.10 | 0.09 | 0.08 | 0.10 | 0.09 | 0.08 | 0.09 | 0.09 | 0.08 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.05 |
| | | 0.10 | 0.12 | 0.12 | 0.11 | 0.12 | 0.12 | 0.11 | 0.11 | 0.12 | 0.09 |
| | Skewed | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.05 | 0.04 |
| | | 0.10 | 0.09 | 0.08 | 0.09 | 0.09 | 0.08 | 0.09 | 0.08 | 0.09 | 0.09 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.09 |
| | Heavy tailed | 0.05 | 0.07 | 0.06 | 0.07 | 0.07 | 0.06 | 0.07 | 0.06 | 0.06 | 0.05 |
| | | 0.10 | 0.12 | 0.13 | 0.14 | 0.12 | 0.13 | 0.14 | 0.13 | 0.12 | 0.11 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.11 | 0.10 | 0.12 | 0.11 | 0.10 | 0.12 | 0.11 | 0.11 | 0.11 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

Table 6: Proportion of bootstrap tests rejecting the null hypothesis of normality of the random intercept.

| Distributions | | Nominal | Raw residuals | | | Pearson residuals | | | Studentized residuals | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS | AD | CVM | KS |
| $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.05 |
| | | 0.10 | 0.11 | 0.10 | 0.09 | 0.10 | 0.10 | 0.11 | 0.10 | 0.10 | 0.11 |
| | Heavy tailed | 0.05 | 0.15 | 0.13 | 0.11 | 0.16 | 0.14 | 0.12 | 0.16 | 0.14 | 0.12 |
| | | 0.10 | 0.22 | 0.20 | 0.18 | 0.25 | 0.23 | 0.20 | 0.24 | 0.23 | 0.19 |
| | Skewed | 0.05 | 0.16 | 0.15 | 0.13 | 0.18 | 0.16 | 0.13 | 0.18 | 0.16 | 0.13 |
| | | 0.10 | 0.26 | 0.23 | 0.20 | 0.27 | 0.24 | 0.20 | 0.27 | 0.24 | 0.21 |
| Heavy tailed | Normal | 0.05 | 0.27 | 0.24 | 0.19 | 0.28 | 0.25 | 0.20 | 0.28 | 0.25 | 0.20 |
| | | 0.10 | 0.36 | 0.31 | 0.26 | 0.35 | 0.33 | 0.28 | 0.35 | 0.33 | 0.27 |
| | Heavy tailed | 0.05 | 0.49 | 0.45 | 0.34 | 0.50 | 0.45 | 0.35 | 0.49 | 0.45 | 0.35 |
| | | 0.10 | 0.59 | 0.54 | 0.45 | 0.59 | 0.55 | 0.45 | 0.58 | 0.55 | 0.45 |
| | Skewed | 0.05 | 0.52 | 0.48 | 0.35 | 0.54 | 0.48 | 0.38 | 0.54 | 0.48 | 0.38 |
| | | 0.10 | 0.63 | 0.59 | 0.49 | 0.64 | 0.60 | 0.51 | 0.64 | 0.60 | 0.51 |
| Skewed | Normal | 0.05 | 0.51 | 0.48 | 0.38 | 0.51 | 0.47 | 0.37 | 0.51 | 0.47 | 0.37 |
| | | 0.10 | 0.62 | 0.59 | 0.49 | 0.59 | 0.58 | 0.50 | 0.59 | 0.58 | 0.50 |
| | Heavy tailed | 0.05 | 0.73 | 0.70 | 0.57 | 0.73 | 0.69 | 0.57 | 0.73 | 0.69 | 0.57 |
| | | 0.10 | 0.80 | 0.77 | 0.67 | 0.80 | 0.77 | 0.69 | 0.80 | 0.77 | 0.69 |
| | Skewed | 0.05 | 0.87 | 0.83 | 0.68 | 0.86 | 0.81 | 0.67 | 0.86 | 0.82 | 0.67 |
| | | 0.10 | 0.92 | 0.89 | 0.79 | 0.91 | 0.87 | 0.79 | 0.90 | 0.88 | 0.80 |
| $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.04 | 0.04 | 0.06 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.09 | 0.09 | 0.08 | 0.09 | 0.09 | 0.08 | 0.09 | 0.09 | 0.08 |
| | Heavy tailed | 0.05 | 0.07 | 0.07 | 0.06 | 0.07 | 0.07 | 0.06 | 0.07 | 0.07 | 0.06 |
| | | 0.10 | 0.12 | 0.11 | 0.11 | 0.13 | 0.12 | 0.10 | 0.13 | 0.12 | 0.10 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.07 | 0.07 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.54 | 0.49 | 0.41 | 0.52 | 0.48 | 0.40 | 0.52 | 0.48 | 0.40 |
| | | 0.10 | 0.63 | 0.59 | 0.50 | 0.61 | 0.57 | 0.51 | 0.61 | 0.56 | 0.51 |
| | Heavy tailed | 0.05 | 0.62 | 0.57 | 0.48 | 0.60 | 0.55 | 0.46 | 0.60 | 0.55 | 0.47 |
| | | 0.10 | 0.70 | 0.67 | 0.59 | 0.68 | 0.64 | 0.56 | 0.68 | 0.64 | 0.55 |
| | Skewed | 0.05 | 0.61 | 0.56 | 0.46 | 0.61 | 0.55 | 0.45 | 0.61 | 0.55 | 0.45 |
| | | 0.10 | 0.71 | 0.65 | 0.57 | 0.70 | 0.65 | 0.57 | 0.70 | 0.65 | 0.56 |
| Skewed | Normal | 0.05 | 0.92 | 0.91 | 0.85 | 0.93 | 0.91 | 0.85 | 0.93 | 0.91 | 0.85 |
| | | 0.10 | 0.95 | 0.94 | 0.91 | 0.96 | 0.94 | 0.90 | 0.96 | 0.94 | 0.90 |
| | Heavy tailed | 0.05 | 0.97 | 0.95 | 0.89 | 0.97 | 0.96 | 0.88 | 0.97 | 0.96 | 0.88 |
| | | 0.10 | 0.99 | 0.98 | 0.94 | 0.99 | 0.98 | 0.94 | 0.99 | 0.98 | 0.94 |
| | Skewed | 0.05 | 0.98 | 0.96 | 0.90 | 0.98 | 0.97 | 0.90 | 0.98 | 0.97 | 0.91 |
| | | 0.10 | 0.99 | 0.97 | 0.95 | 0.99 | 0.98 | 0.94 | 0.99 | 0.98 | 0.94 |
| $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.09 | 0.10 | 0.09 | 0.09 | 0.10 | 0.09 | 0.09 |
| | Heavy tailed | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.09 | 0.09 | 0.08 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 |
| | Skewed | 0.05 | 0.04 | 0.04 | 0.03 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 |
| | | 0.10 | 0.09 | 0.09 | 0.08 | 0.10 | 0.09 | 0.08 | 0.10 | 0.09 | 0.08 |
| Heavy tailed | Normal | 0.05 | 0.68 | 0.63 | 0.55 | 0.68 | 0.63 | 0.55 | 0.68 | 0.63 | 0.55 |
| | | 0.10 | 0.76 | 0.72 | 0.63 | 0.75 | 0.71 | 0.65 | 0.75 | 0.71 | 0.64 |
| | Heavy tailed | 0.05 | 0.72 | 0.67 | 0.58 | 0.71 | 0.67 | 0.58 | 0.71 | 0.67 | 0.58 |
| | | 0.10 | 0.79 | 0.77 | 0.68 | 0.79 | 0.76 | 0.67 | 0.79 | 0.76 | 0.68 |
| | Skewed | 0.05 | 0.71 | 0.68 | 0.57 | 0.70 | 0.67 | 0.57 | 0.70 | 0.68 | 0.57 |
| | | 0.10 | 0.78 | 0.75 | 0.68 | 0.78 | 0.74 | 0.68 | 0.78 | 0.74 | 0.68 |
| Skewed | Normal | 0.05 | 1.00 | 0.99 | 0.97 | 1.00 | 0.99 | 0.97 | 1.00 | 0.99 | 0.98 |
| | | 0.10 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 0.98 | 1.00 | 1.00 | 0.98 | 1.00 | 1.00 | 0.98 |
| | | 0.10 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 |
| | Skewed | 0.05 | 1.00 | 1.00 | 0.98 | 1.00 | 1.00 | 0.98 | 1.00 | 1.00 | 0.98 |
| | | 0.10 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 0.99 |

Table 7: Proportion of bootstrap tests rejecting the null hypothesis of normality of the random slope.

| Distributions | | Nominal | Raw residuals | | | Pearson residuals | | | Studentized residuals | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS | AD | CVM | KS |
| | | | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | | |
| Normal | Normal | 0.05 | 0.06 | 0.06 | 0.06 | 0.05 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.10 | 0.11 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.31 | 0.29 | 0.12 | 0.27 | 0.25 | 0.19 | 0.26 | 0.25 | 0.18 |
| | | 0.10 | 0.41 | 0.38 | 0.19 | 0.35 | 0.32 | 0.27 | 0.35 | 0.32 | 0.27 |
| | Skewed | 0.05 | 0.23 | 0.20 | 0.21 | 0.33 | 0.32 | 0.24 | 0.33 | 0.31 | 0.24 |
| | | 0.10 | 0.34 | 0.32 | 0.31 | 0.41 | 0.38 | 0.34 | 0.41 | 0.38 | 0.34 |
| Heavy tailed | Normal | 0.05 | 0.11 | 0.10 | 0.09 | 0.13 | 0.13 | 0.09 | 0.13 | 0.13 | 0.09 |
| | | 0.10 | 0.18 | 0.17 | 0.15 | 0.18 | 0.18 | 0.16 | 0.18 | 0.18 | 0.16 |
| | Heavy tailed | 0.05 | 0.49 | 0.45 | 0.20 | 0.40 | 0.37 | 0.29 | 0.40 | 0.37 | 0.29 |
| | | 0.10 | 0.61 | 0.58 | 0.28 | 0.49 | 0.44 | 0.37 | 0.49 | 0.44 | 0.37 |
| | Skewed | 0.05 | 0.41 | 0.35 | 0.31 | 0.49 | 0.48 | 0.36 | 0.49 | 0.48 | 0.36 |
| | | 0.10 | 0.54 | 0.50 | 0.44 | 0.59 | 0.55 | 0.49 | 0.59 | 0.55 | 0.49 |
| Skewed | Normal | 0.05 | 0.10 | 0.10 | 0.10 | 0.12 | 0.12 | 0.10 | 0.12 | 0.12 | 0.10 |
| | | 0.10 | 0.18 | 0.17 | 0.16 | 0.17 | 0.16 | 0.16 | 0.17 | 0.16 | 0.16 |
| | Heavy tailed | 0.05 | 0.41 | 0.38 | 0.22 | 0.41 | 0.37 | 0.30 | 0.41 | 0.37 | 0.30 |
| | | 0.10 | 0.55 | 0.50 | 0.31 | 0.51 | 0.47 | 0.38 | 0.51 | 0.47 | 0.38 |
| | Skewed | 0.05 | 0.31 | 0.25 | 0.41 | 0.59 | 0.57 | 0.46 | 0.60 | 0.57 | 0.46 |
| | | 0.10 | 0.43 | 0.38 | 0.53 | 0.70 | 0.66 | 0.57 | 0.70 | 0.65 | 0.57 |
| | | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | | |
| Normal | Normal | 0.05 | 0.06 | 0.06 | 0.05 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.12 | 0.12 | 0.10 | 0.12 | 0.12 | 0.11 | 0.12 | 0.12 | 0.11 |
| | Heavy tailed | 0.05 | 0.17 | 0.16 | 0.08 | 0.15 | 0.14 | 0.11 | 0.15 | 0.14 | 0.12 |
| | | 0.10 | 0.25 | 0.23 | 0.14 | 0.23 | 0.20 | 0.18 | 0.23 | 0.21 | 0.18 |
| | Skewed | 0.05 | 0.12 | 0.12 | 0.10 | 0.16 | 0.14 | 0.11 | 0.16 | 0.14 | 0.11 |
| | | 0.10 | 0.20 | 0.19 | 0.16 | 0.23 | 0.21 | 0.18 | 0.23 | 0.21 | 0.18 |
| Heavy tailed | Normal | 0.05 | 0.28 | 0.25 | 0.13 | 0.28 | 0.24 | 0.20 | 0.28 | 0.24 | 0.20 |
| | | 0.10 | 0.36 | 0.33 | 0.20 | 0.36 | 0.32 | 0.28 | 0.36 | 0.32 | 0.28 |
| | Heavy tailed | 0.05 | 0.50 | 0.47 | 0.20 | 0.45 | 0.41 | 0.34 | 0.45 | 0.41 | 0.34 |
| | | 0.10 | 0.58 | 0.56 | 0.28 | 0.51 | 0.49 | 0.43 | 0.51 | 0.49 | 0.43 |
| | Skewed | 0.05 | 0.44 | 0.40 | 0.20 | 0.43 | 0.40 | 0.32 | 0.43 | 0.40 | 0.32 |
| | | 0.10 | 0.52 | 0.50 | 0.30 | 0.53 | 0.48 | 0.43 | 0.53 | 0.48 | 0.43 |
| Skewed | Normal | 0.05 | 0.30 | 0.25 | 0.30 | 0.48 | 0.43 | 0.35 | 0.48 | 0.43 | 0.35 |
| | | 0.10 | 0.39 | 0.35 | 0.39 | 0.58 | 0.54 | 0.47 | 0.58 | 0.54 | 0.47 |
| | Heavy tailed | 0.05 | 0.46 | 0.39 | 0.45 | 0.66 | 0.61 | 0.52 | 0.66 | 0.62 | 0.52 |
| | | 0.10 | 0.53 | 0.49 | 0.56 | 0.74 | 0.71 | 0.63 | 0.74 | 0.71 | 0.63 |
| | Skewed | 0.05 | 0.35 | 0.28 | 0.51 | 0.76 | 0.71 | 0.59 | 0.76 | 0.71 | 0.59 |
| | | 0.10 | 0.45 | 0.37 | 0.63 | 0.83 | 0.79 | 0.70 | 0.83 | 0.79 | 0.70 |
| | | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 |
| | | 0.10 | 0.11 | 0.11 | 0.11 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.07 | 0.07 | 0.05 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 |
| | | 0.10 | 0.12 | 0.12 | 0.11 | 0.11 | 0.12 | 0.13 | 0.11 | 0.12 | 0.13 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.04 | 0.06 | 0.06 | 0.05 | 0.06 | 0.06 | 0.05 |
| | | 0.10 | 0.10 | 0.11 | 0.09 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.49 | 0.45 | 0.18 | 0.46 | 0.41 | 0.34 | 0.46 | 0.41 | 0.34 |
| | | 0.10 | 0.58 | 0.54 | 0.28 | 0.56 | 0.51 | 0.44 | 0.56 | 0.51 | 0.44 |
| | Heavy tailed | 0.05 | 0.57 | 0.54 | 0.24 | 0.54 | 0.50 | 0.43 | 0.54 | 0.50 | 0.43 |
| | | 0.10 | 0.66 | 0.62 | 0.32 | 0.63 | 0.60 | 0.52 | 0.63 | 0.60 | 0.52 |
| | Skewed | 0.05 | 0.52 | 0.48 | 0.22 | 0.49 | 0.46 | 0.37 | 0.48 | 0.46 | 0.37 |
| | | 0.10 | 0.61 | 0.57 | 0.32 | 0.56 | 0.53 | 0.47 | 0.56 | 0.53 | 0.47 |
| Skewed | Normal | 0.05 | 0.58 | 0.49 | 0.69 | 0.90 | 0.87 | 0.74 | 0.90 | 0.87 | 0.74 |
| | | 0.10 | 0.68 | 0.59 | 0.79 | 0.94 | 0.92 | 0.83 | 0.94 | 0.92 | 0.83 |
| | Heavy tailed | 0.05 | 0.61 | 0.52 | 0.76 | 0.92 | 0.91 | 0.80 | 0.93 | 0.91 | 0.80 |
| | | 0.10 | 0.71 | 0.61 | 0.84 | 0.96 | 0.95 | 0.89 | 0.96 | 0.95 | 0.89 |
| | Skewed | 0.05 | 0.59 | 0.47 | 0.72 | 0.92 | 0.90 | 0.79 | 0.92 | 0.90 | 0.79 |
| | | 0.10 | 0.69 | 0.58 | 0.83 | 0.95 | 0.93 | 0.88 | 0.95 | 0.93 | 0.88 |

Table 8: Bootstrap tests for normality of marginal residuals.

| Distributions | | Nominal | Raw residuals | | | Cholesky residuals | | |
|---|---|---|---|---|---|---|---|---|
| Errors | Random effects | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.09 | 0.10 | 0.10 | 0.11 | 0.10 | 0.11 |
| | Heavy tailed | 0.05 | 0.20 | 0.17 | 0.13 | 0.09 | 0.07 | 0.05 |
| | | 0.10 | 0.26 | 0.23 | 0.19 | 0.14 | 0.12 | 0.10 |
| | Skewed | 0.05 | 0.28 | 0.25 | 0.21 | 0.04 | 0.04 | 0.05 |
| | | 0.10 | 0.36 | 0.34 | 0.30 | 0.09 | 0.09 | 0.09 |
| Heavy tailed | Normal | 0.05 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.10 | 0.11 | 0.11 | 0.10 | 0.10 | 0.09 |
| | Heavy tailed | 0.05 | 0.72 | 0.66 | 0.58 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.81 | 0.77 | 0.71 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 0.94 | 0.90 | 0.86 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.97 | 0.95 | 0.94 | 1.00 | 1.00 | 1.00 |
| Heavy tailed | Normal | 0.05 | 0.35 | 0.31 | 0.27 | 0.10 | 0.09 | 0.07 |
| | | 0.10 | 0.44 | 0.40 | 0.36 | 0.16 | 0.14 | 0.13 |
| | Heavy tailed | 0.05 | 0.97 | 0.96 | 0.92 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.99 | 0.98 | 0.96 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 0.72 | 0.70 | 0.64 | 0.12 | 0.10 | 0.09 |
| | | 0.10 | 0.81 | 0.78 | 0.74 | 0.19 | 0.17 | 0.16 |
| | Heavy tailed | 0.05 | 1.00 | 0.99 | 0.97 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.08 |
| | Heavy tailed | 0.05 | 0.40 | 0.36 | 0.33 | 0.17 | 0.14 | 0.11 |
| | | 0.10 | 0.49 | 0.44 | 0.41 | 0.26 | 0.24 | 0.20 |
| | Skewed | 0.05 | 0.81 | 0.79 | 0.74 | 0.33 | 0.29 | 0.24 |
| | | 0.10 | 0.88 | 0.86 | 0.83 | 0.44 | 0.40 | 0.34 |
| Heavy tailed | Normal | 0.05 | 0.17 | 0.15 | 0.16 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.27 | 0.27 | 0.30 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 0.64 | 0.60 | 0.54 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.74 | 0.70 | 0.66 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 0.93 | 0.91 | 0.89 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.96 | 0.95 | 0.94 | 1.00 | 1.00 | 1.00 |
| Skewed | Normal | 0.05 | 0.15 | 0.16 | 0.18 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.26 | 0.27 | 0.30 | 1.00 | 1.00 | 1.00 |
| | Heavy tailed | 0.05 | 0.66 | 0.62 | 0.61 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.77 | 0.75 | 0.74 | 1.00 | 1.00 | 1.00 |
| | Skewed | 0.05 | 0.96 | 0.95 | 0.93 | 1.00 | 1.00 | 1.00 |
| | | 0.10 | 0.98 | 0.97 | 0.97 | 1.00 | 1.00 | 1.00 |

Table 9: Bootstrap tests of the weighted Q-Q plots for the random effects.

| Distributions: Random effects | Distributions: Errors | Nominal $\alpha$ | $b_0$ | $b_1$ |
|---|---|---|---|---|
| | | | $\sigma_\varepsilon^2 = 4,$ | $\sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ |
| Normal | Normal | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.10 | 0.11 |
| | Heavy tailed | 0.05 | 0.12 | 0.13 |
| | | 0.10 | 0.18 | 0.18 |
| | Skewed | 0.05 | 0.15 | 0.14 |
| | | 0.10 | 0.23 | 0.22 |
| Heavy tailed | Normal | 0.05 | 0.20 | 0.06 |
| | | 0.10 | 0.30 | 0.13 |
| | Heavy tailed | 0.05 | 0.34 | 0.18 |
| | | 0.10 | 0.45 | 0.26 |
| | Skewed | 0.05 | 0.41 | 0.22 |
| | | 0.10 | 0.53 | 0.32 |
| Skewed | Normal | 0.05 | 0.46 | 0.07 |
| | | 0.10 | 0.59 | 0.14 |
| | Heavy tailed | 0.05 | 0.64 | 0.15 |
| | | 0.10 | 0.73 | 0.26 |
| | Skewed | 0.05 | 0.72 | 0.24 |
| | | 0.10 | 0.82 | 0.36 |
| | | | $\sigma_\varepsilon^2 = 1,$ | $\sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ |
| Normal | Normal | 0.05 | 0.04 | 0.04 |
| | | 0.10 | 0.09 | 0.10 |
| | Heavy tailed | 0.05 | 0.06 | 0.07 |
| | | 0.10 | 0.10 | 0.12 |
| | Skewed | 0.05 | 0.07 | 0.08 |
| | | 0.10 | 0.13 | 0.14 |
| Heavy tailed | Normal | 0.05 | 0.40 | 0.12 |
| | | 0.10 | 0.48 | 0.19 |
| | Heavy tailed | 0.05 | 0.46 | 0.18 |
| | | 0.10 | 0.55 | 0.27 |
| | Skewed | 0.05 | 0.45 | 0.18 |
| | | 0.10 | 0.56 | 0.27 |
| Skewed | Normal | 0.05 | 0.89 | 0.20 |
| | | 0.10 | 0.93 | 0.30 |
| | Heavy tailed | 0.05 | 0.92 | 0.27 |
| | | 0.10 | 0.96 | 0.38 |
| | Skewed | 0.05 | 0.93 | 0.31 |
| | | 0.10 | 0.96 | 0.41 |
| | | | $\sigma_\varepsilon^2 = 1,$ | $\sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ |
| Normal | Normal | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 0.09 | 0.10 |
| | Skewed | 0.05 | 0.04 | 0.06 |
| | | 0.10 | 0.09 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.55 | 0.19 |
| | | 0.10 | 0.63 | 0.28 |
| | Heavy tailed | 0.05 | 0.58 | 0.24 |
| | | 0.10 | 0.68 | 0.35 |
| | Skewed | 0.05 | 0.58 | 0.23 |
| | | 0.10 | 0.66 | 0.33 |
| Skewed | Normal | 0.05 | 0.99 | 0.46 |
| | | 0.10 | 1.00 | 0.59 |
| | Heavy tailed | 0.05 | 0.99 | 0.49 |
| | | 0.10 | 1.00 | 0.64 |
| | Skewed | 0.05 | 0.99 | 0.47 |
| | | 0.10 | 1.00 | 0.60 |

# 2 Full results from the simulation study

## 2.1 Estimated covariance matrices

In the paper we described a simulation study and only presented results for the the Anderson-Darling test under one variance structure ($\sigma_\varepsilon^2 = 4$ and $\sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$). In this section we present the results from the full simulation study where the covariance matrices were estimated. Tables 10–16 present the results for the rotated random intercept and Tables 17–23 present the results for the rotated random slope. We use a gray background to highlight the simulation settings under which the tests should fail to reject the null hypothesis of normality.

Table 10: Proportion of tests rejecting normality of the random intercept using two rotations and $s = \mathrm{rank}(\boldsymbol{B})$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma^2_\varepsilon = 4, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.11 | 0.11 | 0.11 |
| | Heavy tailed | 0.05 | 0.07 | 0.07 | 0.07 | 0.09 | 0.08 | 0.07 |
| | | 0.10 | 0.13 | 0.13 | 0.13 | 0.16 | 0.15 | 0.14 |
| | Skewed | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.09 | 0.09 | 0.10 | 0.10 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.14 | 0.13 | 0.10 | 0.22 | 0.20 | 0.17 |
| | | 0.10 | 0.20 | 0.19 | 0.18 | 0.31 | 0.28 | 0.24 |
| | Heavy tailed | 0.05 | 0.19 | 0.17 | 0.15 | 0.34 | 0.32 | 0.26 |
| | | 0.10 | 0.26 | 0.24 | 0.21 | 0.44 | 0.41 | 0.35 |
| | Skewed | 0.05 | 0.15 | 0.14 | 0.13 | 0.28 | 0.23 | 0.20 |
| | | 0.10 | 0.21 | 0.18 | 0.18 | 0.37 | 0.32 | 0.28 |
| Skewed | Normal | 0.05 | 0.10 | 0.09 | 0.08 | 0.20 | 0.17 | 0.12 |
| | | 0.10 | 0.17 | 0.15 | 0.15 | 0.29 | 0.25 | 0.19 |
| | Heavy tailed | 0.05 | 0.13 | 0.11 | 0.11 | 0.30 | 0.24 | 0.19 |
| | | 0.10 | 0.22 | 0.19 | 0.17 | 0.39 | 0.34 | 0.28 |
| | Skewed | 0.05 | 0.13 | 0.12 | 0.09 | 0.22 | 0.19 | 0.15 |
| | | 0.10 | 0.19 | 0.17 | 0.16 | 0.33 | 0.28 | 0.25 |
| | | | $\sigma^2_\varepsilon = 1, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.09 | 0.09 | 0.09 | 0.11 | 0.11 | 0.12 |
| | Heavy tailed | 0.05 | 0.06 | 0.07 | 0.06 | 0.06 | 0.06 | 0.05 |
| | | 0.10 | 0.11 | 0.10 | 0.11 | 0.11 | 0.11 | 0.11 |
| | Skewed | 0.05 | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.09 | 0.09 | 0.09 | 0.10 | 0.10 | 0.09 |
| Heavy tailed | Normal | 0.05 | 0.21 | 0.20 | 0.16 | 0.40 | 0.35 | 0.28 |
| | | 0.10 | 0.29 | 0.27 | 0.23 | 0.49 | 0.45 | 0.39 |
| | Heavy tailed | 0.05 | 0.25 | 0.22 | 0.17 | 0.47 | 0.42 | 0.34 |
| | | 0.10 | 0.32 | 0.30 | 0.25 | 0.54 | 0.50 | 0.44 |
| | Skewed | 0.05 | 0.25 | 0.22 | 0.17 | 0.42 | 0.38 | 0.30 |
| | | 0.10 | 0.33 | 0.30 | 0.27 | 0.50 | 0.46 | 0.40 |
| Skewed | Normal | 0.05 | 0.17 | 0.16 | 0.13 | 0.36 | 0.28 | 0.21 |
| | | 0.10 | 0.26 | 0.24 | 0.20 | 0.47 | 0.38 | 0.31 |
| | Heavy tailed | 0.05 | 0.20 | 0.18 | 0.14 | 0.42 | 0.34 | 0.25 |
| | | 0.10 | 0.28 | 0.26 | 0.23 | 0.51 | 0.44 | 0.35 |
| | Skewed | 0.05 | 0.18 | 0.17 | 0.12 | 0.40 | 0.30 | 0.21 |
| | | 0.10 | 0.26 | 0.23 | 0.22 | 0.51 | 0.39 | 0.33 |
| | | | $\sigma^2_\varepsilon = 1, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.10 | 0.09 | 0.10 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.06 | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.12 | 0.12 | 0.11 | 0.10 | 0.10 | 0.10 |
| | Skewed | 0.05 | 0.04 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.10 | 0.10 | 0.11 | 0.08 | 0.08 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.28 | 0.25 | 0.21 | 0.43 | 0.38 | 0.29 |
| | | 0.10 | 0.34 | 0.32 | 0.29 | 0.53 | 0.49 | 0.41 |
| | Heavy tailed | 0.05 | 0.30 | 0.28 | 0.21 | 0.44 | 0.40 | 0.33 |
| | | 0.10 | 0.37 | 0.34 | 0.29 | 0.54 | 0.50 | 0.42 |
| | Skewed | 0.05 | 0.28 | 0.26 | 0.21 | 0.44 | 0.40 | 0.31 |
| | | 0.10 | 0.37 | 0.34 | 0.29 | 0.53 | 0.47 | 0.40 |
| Skewed | Normal | 0.05 | 0.24 | 0.22 | 0.15 | 0.37 | 0.30 | 0.23 |
| | | 0.10 | 0.33 | 0.30 | 0.24 | 0.49 | 0.41 | 0.33 |
| | Heavy tailed | 0.05 | 0.26 | 0.23 | 0.18 | 0.38 | 0.29 | 0.22 |
| | | 0.10 | 0.33 | 0.33 | 0.26 | 0.48 | 0.39 | 0.32 |
| | Skewed | 0.05 | 0.24 | 0.21 | 0.15 | 0.38 | 0.29 | 0.23 |
| | | 0.10 | 0.33 | 0.31 | 0.26 | 0.50 | 0.39 | 0.33 |

Table 11: Proportion of tests rejecting normality of the random intercept using two rotations and $s = 55$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.09 | 0.09 | 0.10 | 0.10 | 0.10 | 0.11 |
| | Heavy tailed | 0.05 | 0.08 | 0.08 | 0.06 | 0.09 | 0.08 | 0.08 |
| | | 0.10 | 0.13 | 0.14 | 0.12 | 0.17 | 0.15 | 0.13 |
| | Skewed | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.09 | 0.09 | 0.10 | 0.09 | 0.09 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.14 | 0.12 | 0.11 | 0.22 | 0.20 | 0.17 |
| | | 0.10 | 0.20 | 0.20 | 0.18 | 0.30 | 0.27 | 0.23 |
| | Heavy tailed | 0.05 | 0.19 | 0.17 | 0.14 | 0.33 | 0.32 | 0.25 |
| | | 0.10 | 0.25 | 0.23 | 0.21 | 0.45 | 0.40 | 0.34 |
| | Skewed | 0.05 | 0.15 | 0.14 | 0.11 | 0.27 | 0.21 | 0.17 |
| | | 0.10 | 0.21 | 0.19 | 0.19 | 0.36 | 0.32 | 0.27 |
| Skewed | Normal | 0.05 | 0.09 | 0.08 | 0.08 | 0.21 | 0.17 | 0.12 |
| | | 0.10 | 0.16 | 0.14 | 0.14 | 0.29 | 0.24 | 0.20 |
| | Heavy tailed | 0.05 | 0.12 | 0.11 | 0.10 | 0.28 | 0.24 | 0.18 |
| | | 0.10 | 0.20 | 0.18 | 0.17 | 0.38 | 0.33 | 0.27 |
| | Skewed | 0.05 | 0.13 | 0.12 | 0.10 | 0.23 | 0.19 | 0.15 |
| | | 0.10 | 0.18 | 0.17 | 0.15 | 0.32 | 0.28 | 0.24 |
| | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.09 | 0.09 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.05 | 0.06 | 0.05 | 0.04 |
| | | 0.10 | 0.10 | 0.10 | 0.11 | 0.11 | 0.10 | 0.10 |
| | Skewed | 0.05 | 0.04 | 0.03 | 0.03 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.09 | 0.08 | 0.07 | 0.10 | 0.09 | 0.09 |
| Heavy tailed | Normal | 0.05 | 0.19 | 0.18 | 0.15 | 0.39 | 0.36 | 0.30 |
| | | 0.10 | 0.28 | 0.25 | 0.23 | 0.49 | 0.45 | 0.39 |
| | Heavy tailed | 0.05 | 0.24 | 0.23 | 0.18 | 0.44 | 0.40 | 0.33 |
| | | 0.10 | 0.31 | 0.30 | 0.26 | 0.53 | 0.49 | 0.42 |
| | Skewed | 0.05 | 0.24 | 0.21 | 0.17 | 0.41 | 0.36 | 0.28 |
| | | 0.10 | 0.32 | 0.30 | 0.25 | 0.49 | 0.45 | 0.38 |
| Skewed | Normal | 0.05 | 0.17 | 0.17 | 0.14 | 0.36 | 0.28 | 0.22 |
| | | 0.10 | 0.27 | 0.24 | 0.20 | 0.47 | 0.38 | 0.32 |
| | Heavy tailed | 0.05 | 0.20 | 0.17 | 0.14 | 0.41 | 0.34 | 0.25 |
| | | 0.10 | 0.29 | 0.27 | 0.22 | 0.51 | 0.42 | 0.34 |
| | Skewed | 0.05 | 0.18 | 0.16 | 0.12 | 0.40 | 0.30 | 0.21 |
| | | 0.10 | 0.24 | 0.22 | 0.21 | 0.52 | 0.40 | 0.33 |
| | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.03 |
| | | 0.10 | 0.10 | 0.11 | 0.11 | 0.09 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.07 | 0.07 | 0.06 | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 0.13 | 0.12 | 0.11 | 0.10 | 0.10 | 0.10 |
| | Skewed | 0.05 | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.27 | 0.24 | 0.20 | 0.42 | 0.37 | 0.29 |
| | | 0.10 | 0.33 | 0.31 | 0.28 | 0.51 | 0.47 | 0.39 |
| | Heavy tailed | 0.05 | 0.28 | 0.25 | 0.22 | 0.43 | 0.39 | 0.31 |
| | | 0.10 | 0.37 | 0.35 | 0.28 | 0.52 | 0.48 | 0.42 |
| | Skewed | 0.05 | 0.27 | 0.24 | 0.20 | 0.42 | 0.37 | 0.28 |
| | | 0.10 | 0.35 | 0.32 | 0.28 | 0.51 | 0.47 | 0.39 |
| Skewed | Normal | 0.05 | 0.23 | 0.21 | 0.15 | 0.37 | 0.29 | 0.21 |
| | | 0.10 | 0.31 | 0.29 | 0.24 | 0.46 | 0.38 | 0.31 |
| | Heavy tailed | 0.05 | 0.23 | 0.21 | 0.17 | 0.35 | 0.27 | 0.21 |
| | | 0.10 | 0.33 | 0.32 | 0.26 | 0.44 | 0.37 | 0.30 |
| | Skewed | 0.05 | 0.23 | 0.21 | 0.16 | 0.38 | 0.28 | 0.21 |
| | | 0.10 | 0.32 | 0.30 | 0.26 | 0.47 | 0.38 | 0.31 |

Table 12: Proportion of tests rejecting normality of the random intercept using two rotations and $s = 50$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 | 0.06 |
| | | 0.10 | 0.11 | 0.10 | 0.10 | 0.11 | 0.11 | 0.11 |
| | Heavy tailed | 0.05 | 0.07 | 0.07 | 0.07 | 0.08 | 0.08 | 0.07 |
| | | 0.10 | 0.12 | 0.13 | 0.12 | 0.14 | 0.13 | 0.13 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.05 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.11 | 0.10 | 0.10 | 0.09 | 0.09 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.13 | 0.12 | 0.10 | 0.23 | 0.20 | 0.15 |
| | | 0.10 | 0.20 | 0.18 | 0.17 | 0.31 | 0.27 | 0.24 |
| | Heavy tailed | 0.05 | 0.17 | 0.16 | 0.13 | 0.32 | 0.28 | 0.22 |
| | | 0.10 | 0.24 | 0.23 | 0.20 | 0.41 | 0.38 | 0.32 |
| | Skewed | 0.05 | 0.14 | 0.13 | 0.12 | 0.26 | 0.22 | 0.18 |
| | | 0.10 | 0.21 | 0.19 | 0.18 | 0.35 | 0.31 | 0.28 |
| Skewed | Normal | 0.05 | 0.10 | 0.08 | 0.08 | 0.21 | 0.17 | 0.12 |
| | | 0.10 | 0.16 | 0.15 | 0.14 | 0.31 | 0.27 | 0.19 |
| | Heavy tailed | 0.05 | 0.12 | 0.11 | 0.09 | 0.27 | 0.22 | 0.17 |
| | | 0.10 | 0.20 | 0.17 | 0.16 | 0.37 | 0.31 | 0.25 |
| | Skewed | 0.05 | 0.12 | 0.11 | 0.09 | 0.22 | 0.18 | 0.14 |
| | | 0.10 | 0.19 | 0.17 | 0.15 | 0.31 | 0.26 | 0.23 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 |
| | | 0.10 | 0.09 | 0.09 | 0.08 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.05 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.10 | 0.11 | 0.11 | 0.12 | 0.12 | 0.10 |
| | Skewed | 0.05 | 0.04 | 0.04 | 0.03 | 0.05 | 0.04 | 0.04 |
| | | 0.10 | 0.09 | 0.08 | 0.08 | 0.08 | 0.08 | 0.09 |
| Heavy tailed | Normal | 0.05 | 0.18 | 0.17 | 0.14 | 0.39 | 0.36 | 0.30 |
| | | 0.10 | 0.27 | 0.24 | 0.23 | 0.47 | 0.43 | 0.40 |
| | Heavy tailed | 0.05 | 0.23 | 0.21 | 0.17 | 0.43 | 0.38 | 0.31 |
| | | 0.10 | 0.31 | 0.28 | 0.25 | 0.52 | 0.48 | 0.41 |
| | Skewed | 0.05 | 0.24 | 0.21 | 0.16 | 0.41 | 0.37 | 0.29 |
| | | 0.10 | 0.31 | 0.29 | 0.24 | 0.50 | 0.45 | 0.38 |
| Skewed | Normal | 0.05 | 0.18 | 0.16 | 0.12 | 0.34 | 0.28 | 0.20 |
| | | 0.10 | 0.25 | 0.23 | 0.21 | 0.46 | 0.38 | 0.31 |
| | Heavy tailed | 0.05 | 0.19 | 0.18 | 0.15 | 0.40 | 0.31 | 0.24 |
| | | 0.10 | 0.28 | 0.26 | 0.23 | 0.49 | 0.42 | 0.35 |
| | Skewed | 0.05 | 0.17 | 0.15 | 0.11 | 0.37 | 0.27 | 0.22 |
| | | 0.10 | 0.24 | 0.22 | 0.19 | 0.47 | 0.38 | 0.33 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 0.06 | 0.06 | 0.06 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.11 | 0.11 | 0.10 | 0.11 | 0.10 | 0.10 |
| | Skewed | 0.05 | 0.05 | 0.05 | 0.06 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.11 | 0.10 | 0.11 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.25 | 0.23 | 0.20 | 0.38 | 0.34 | 0.26 |
| | | 0.10 | 0.33 | 0.31 | 0.26 | 0.47 | 0.43 | 0.37 |
| | Heavy tailed | 0.05 | 0.27 | 0.25 | 0.19 | 0.42 | 0.39 | 0.31 |
| | | 0.10 | 0.35 | 0.33 | 0.28 | 0.51 | 0.46 | 0.41 |
| | Skewed | 0.05 | 0.25 | 0.24 | 0.19 | 0.39 | 0.34 | 0.26 |
| | | 0.10 | 0.33 | 0.31 | 0.27 | 0.47 | 0.44 | 0.36 |
| Skewed | Normal | 0.05 | 0.23 | 0.20 | 0.15 | 0.36 | 0.28 | 0.23 |
| | | 0.10 | 0.31 | 0.29 | 0.24 | 0.47 | 0.38 | 0.32 |
| | Heavy tailed | 0.05 | 0.22 | 0.21 | 0.17 | 0.35 | 0.27 | 0.21 |
| | | 0.10 | 0.32 | 0.32 | 0.25 | 0.46 | 0.37 | 0.30 |
| | Skewed | 0.05 | 0.23 | 0.21 | 0.16 | 0.36 | 0.27 | 0.21 |
| | | 0.10 | 0.31 | 0.29 | 0.26 | 0.46 | 0.37 | 0.30 |

Table 13: Proportion of tests rejecting normality of the random intercept using two rotations and $s = 45$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.11 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.06 | 0.07 | 0.07 | 0.06 |
| | | 0.10 | 0.12 | 0.12 | 0.12 | 0.13 | 0.12 | 0.10 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.05 | 0.06 | 0.05 | 0.04 |
| | | 0.10 | 0.10 | 0.10 | 0.09 | 0.11 | 0.10 | 0.09 |
| Heavy tailed | Normal | 0.05 | 0.13 | 0.12 | 0.10 | 0.23 | 0.21 | 0.15 |
| | | 0.10 | 0.20 | 0.18 | 0.18 | 0.30 | 0.27 | 0.24 |
| | Heavy tailed | 0.05 | 0.16 | 0.15 | 0.13 | 0.32 | 0.27 | 0.22 |
| | | 0.10 | 0.23 | 0.22 | 0.19 | 0.40 | 0.37 | 0.32 |
| | Skewed | 0.05 | 0.14 | 0.13 | 0.11 | 0.27 | 0.24 | 0.18 |
| | | 0.10 | 0.21 | 0.19 | 0.17 | 0.34 | 0.31 | 0.28 |
| Skewed | Normal | 0.05 | 0.10 | 0.10 | 0.08 | 0.22 | 0.20 | 0.14 |
| | | 0.10 | 0.18 | 0.16 | 0.14 | 0.31 | 0.26 | 0.22 |
| | Heavy tailed | 0.05 | 0.11 | 0.10 | 0.10 | 0.25 | 0.20 | 0.16 |
| | | 0.10 | 0.18 | 0.17 | 0.15 | 0.35 | 0.30 | 0.25 |
| | Skewed | 0.05 | 0.12 | 0.11 | 0.10 | 0.23 | 0.20 | 0.14 |
| | | 0.10 | 0.19 | 0.18 | 0.16 | 0.32 | 0.27 | 0.20 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 |
| | | 0.10 | 0.09 | 0.09 | 0.09 | 0.10 | 0.10 | 0.11 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.05 | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.12 | 0.11 | 0.10 |
| | Skewed | 0.05 | 0.04 | 0.04 | 0.04 | 0.06 | 0.06 | 0.05 |
| | | 0.10 | 0.09 | 0.08 | 0.08 | 0.10 | 0.10 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.18 | 0.17 | 0.14 | 0.37 | 0.33 | 0.28 |
| | | 0.10 | 0.26 | 0.24 | 0.22 | 0.45 | 0.41 | 0.37 |
| | Heavy tailed | 0.05 | 0.21 | 0.20 | 0.14 | 0.40 | 0.36 | 0.27 |
| | | 0.10 | 0.28 | 0.26 | 0.23 | 0.49 | 0.45 | 0.38 |
| | Skewed | 0.05 | 0.22 | 0.19 | 0.16 | 0.39 | 0.34 | 0.27 |
| | | 0.10 | 0.28 | 0.26 | 0.23 | 0.47 | 0.43 | 0.37 |
| Skewed | Normal | 0.05 | 0.17 | 0.15 | 0.12 | 0.36 | 0.28 | 0.23 |
| | | 0.10 | 0.25 | 0.24 | 0.19 | 0.45 | 0.38 | 0.31 |
| | Heavy tailed | 0.05 | 0.19 | 0.17 | 0.14 | 0.38 | 0.32 | 0.24 |
| | | 0.10 | 0.27 | 0.24 | 0.22 | 0.48 | 0.41 | 0.34 |
| | Skewed | 0.05 | 0.15 | 0.13 | 0.10 | 0.34 | 0.26 | 0.20 |
| | | 0.10 | 0.22 | 0.20 | 0.16 | 0.47 | 0.38 | 0.30 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.09 | 0.11 | 0.10 | 0.11 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.12 | 0.12 | 0.11 | 0.09 | 0.09 | 0.09 |
| | Skewed | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 | 0.06 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.11 | 0.11 | 0.11 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.22 | 0.20 | 0.18 | 0.36 | 0.32 | 0.25 |
| | | 0.10 | 0.30 | 0.28 | 0.24 | 0.48 | 0.43 | 0.35 |
| | Heavy tailed | 0.05 | 0.24 | 0.23 | 0.17 | 0.37 | 0.35 | 0.29 |
| | | 0.10 | 0.32 | 0.31 | 0.26 | 0.46 | 0.43 | 0.37 |
| | Skewed | 0.05 | 0.24 | 0.23 | 0.17 | 0.37 | 0.34 | 0.26 |
| | | 0.10 | 0.33 | 0.30 | 0.26 | 0.46 | 0.44 | 0.35 |
| Skewed | Normal | 0.05 | 0.21 | 0.20 | 0.14 | 0.34 | 0.27 | 0.21 |
| | | 0.10 | 0.30 | 0.30 | 0.23 | 0.44 | 0.36 | 0.29 |
| | Heavy tailed | 0.05 | 0.22 | 0.22 | 0.16 | 0.33 | 0.25 | 0.20 |
| | | 0.10 | 0.31 | 0.30 | 0.26 | 0.43 | 0.36 | 0.28 |
| | Skewed | 0.05 | 0.21 | 0.21 | 0.17 | 0.33 | 0.24 | 0.19 |
| | | 0.10 | 0.29 | 0.27 | 0.25 | 0.43 | 0.34 | 0.28 |

Table 14: Proportion of tests rejecting normality of the random intercept using two rotations and $s = 40$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma^2_\varepsilon = 4,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.05 | 0.04 | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 0.11 | 0.10 | 0.09 | 0.12 | 0.12 | 0.11 |
| | Heavy tailed | 0.05 | 0.05 | 0.05 | 0.06 | 0.06 | 0.06 | 0.04 |
| | | 0.10 | 0.11 | 0.11 | 0.12 | 0.12 | 0.12 | 0.10 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.05 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.10 | 0.12 | 0.11 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.13 | 0.12 | 0.10 | 0.23 | 0.20 | 0.17 |
| | | 0.10 | 0.21 | 0.19 | 0.18 | 0.31 | 0.29 | 0.25 |
| | Heavy tailed | 0.05 | 0.16 | 0.15 | 0.11 | 0.30 | 0.27 | 0.22 |
| | | 0.10 | 0.23 | 0.21 | 0.20 | 0.38 | 0.35 | 0.30 |
| | Skewed | 0.05 | 0.13 | 0.12 | 0.10 | 0.24 | 0.22 | 0.17 |
| | | 0.10 | 0.20 | 0.17 | 0.16 | 0.31 | 0.29 | 0.26 |
| Skewed | Normal | 0.05 | 0.10 | 0.08 | 0.08 | 0.23 | 0.18 | 0.15 |
| | | 0.10 | 0.18 | 0.17 | 0.14 | 0.32 | 0.27 | 0.23 |
| | Heavy tailed | 0.05 | 0.10 | 0.09 | 0.08 | 0.26 | 0.21 | 0.16 |
| | | 0.10 | 0.18 | 0.16 | 0.14 | 0.35 | 0.29 | 0.25 |
| | Skewed | 0.05 | 0.11 | 0.11 | 0.10 | 0.21 | 0.17 | 0.11 |
| | | 0.10 | 0.18 | 0.17 | 0.16 | 0.31 | 0.25 | 0.19 |
| | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.05 | 0.04 | 0.07 | 0.06 | 0.05 |
| | | 0.10 | 0.09 | 0.11 | 0.09 | 0.11 | 0.11 | 0.10 |
| | Heavy tailed | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.12 | 0.10 | 0.10 |
| | Skewed | 0.05 | 0.05 | 0.05 | 0.03 | 0.05 | 0.04 | 0.05 |
| | | 0.10 | 0.09 | 0.09 | 0.09 | 0.10 | 0.10 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.17 | 0.16 | 0.14 | 0.36 | 0.33 | 0.26 |
| | | 0.10 | 0.24 | 0.22 | 0.21 | 0.45 | 0.41 | 0.35 |
| | Heavy tailed | 0.05 | 0.19 | 0.17 | 0.14 | 0.38 | 0.34 | 0.28 |
| | | 0.10 | 0.26 | 0.24 | 0.20 | 0.46 | 0.42 | 0.37 |
| | Skewed | 0.05 | 0.21 | 0.19 | 0.13 | 0.36 | 0.33 | 0.26 |
| | | 0.10 | 0.27 | 0.25 | 0.21 | 0.46 | 0.41 | 0.35 |
| Skewed | Normal | 0.05 | 0.16 | 0.15 | 0.10 | 0.33 | 0.27 | 0.21 |
| | | 0.10 | 0.25 | 0.23 | 0.18 | 0.44 | 0.36 | 0.32 |
| | Heavy tailed | 0.05 | 0.17 | 0.17 | 0.13 | 0.35 | 0.29 | 0.22 |
| | | 0.10 | 0.26 | 0.24 | 0.21 | 0.46 | 0.39 | 0.33 |
| | Skewed | 0.05 | 0.15 | 0.14 | 0.10 | 0.35 | 0.28 | 0.20 |
| | | 0.10 | 0.22 | 0.20 | 0.17 | 0.43 | 0.37 | 0.30 |
| | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.12 | 0.12 | 0.10 |
| | Heavy tailed | 0.05 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.10 |
| | Skewed | 0.05 | 0.05 | 0.05 | 0.06 | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.11 | 0.11 | 0.11 | 0.10 | 0.10 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.20 | 0.18 | 0.16 | 0.33 | 0.29 | 0.24 |
| | | 0.10 | 0.28 | 0.26 | 0.23 | 0.42 | 0.39 | 0.33 |
| | Heavy tailed | 0.05 | 0.23 | 0.20 | 0.16 | 0.36 | 0.34 | 0.26 |
| | | 0.10 | 0.31 | 0.27 | 0.24 | 0.45 | 0.41 | 0.36 |
| | Skewed | 0.05 | 0.21 | 0.20 | 0.17 | 0.35 | 0.31 | 0.22 |
| | | 0.10 | 0.29 | 0.27 | 0.25 | 0.43 | 0.41 | 0.33 |
| Skewed | Normal | 0.05 | 0.20 | 0.19 | 0.12 | 0.32 | 0.25 | 0.20 |
| | | 0.10 | 0.27 | 0.27 | 0.21 | 0.44 | 0.35 | 0.29 |
| | Heavy tailed | 0.05 | 0.19 | 0.19 | 0.15 | 0.30 | 0.25 | 0.19 |
| | | 0.10 | 0.29 | 0.28 | 0.23 | 0.42 | 0.35 | 0.29 |
| | Skewed | 0.05 | 0.21 | 0.19 | 0.17 | 0.29 | 0.21 | 0.19 |
| | | 0.10 | 0.29 | 0.27 | 0.25 | 0.38 | 0.31 | 0.27 |

Table 15: Proportion of tests rejecting normality of the random intercept using two rotations and $s = 35$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | | $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.10 | 0.11 | 0.11 | 0.11 |
| | Heavy tailed | 0.05 | 0.04 | 0.04 | 0.06 | 0.05 | 0.06 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.11 | 0.11 | 0.10 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.06 | 0.05 | 0.06 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.11 | 0.12 | 0.12 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.13 | 0.11 | 0.10 | 0.22 | 0.20 | 0.17 |
| | | 0.10 | 0.21 | 0.19 | 0.17 | 0.30 | 0.27 | 0.23 |
| | Heavy tailed | 0.05 | 0.15 | 0.14 | 0.12 | 0.27 | 0.24 | 0.19 |
| | | 0.10 | 0.21 | 0.20 | 0.18 | 0.35 | 0.31 | 0.28 |
| | Skewed | 0.05 | 0.13 | 0.12 | 0.10 | 0.23 | 0.21 | 0.17 |
| | | 0.10 | 0.19 | 0.16 | 0.16 | 0.30 | 0.28 | 0.26 |
| Skewed | Normal | 0.05 | 0.11 | 0.10 | 0.07 | 0.21 | 0.17 | 0.13 |
| | | 0.10 | 0.17 | 0.16 | 0.15 | 0.30 | 0.26 | 0.22 |
| | Heavy tailed | 0.05 | 0.11 | 0.10 | 0.09 | 0.25 | 0.21 | 0.16 |
| | | 0.10 | 0.18 | 0.18 | 0.15 | 0.33 | 0.29 | 0.24 |
| | Skewed | 0.05 | 0.11 | 0.11 | 0.09 | 0.23 | 0.19 | 0.14 |
| | | 0.10 | 0.18 | 0.17 | 0.17 | 0.31 | 0.26 | 0.21 |
| | | | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | |
| Normal | Normal | 0.05 | 0.04 | 0.05 | 0.04 | 0.06 | 0.06 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.09 | 0.11 | 0.11 | 0.10 |
| | Heavy tailed | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 |
| | | 0.10 | 0.09 | 0.10 | 0.08 | 0.09 | 0.09 | 0.09 |
| | Skewed | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.15 | 0.15 | 0.13 | 0.33 | 0.30 | 0.26 |
| | | 0.10 | 0.23 | 0.21 | 0.19 | 0.41 | 0.38 | 0.34 |
| | Heavy tailed | 0.05 | 0.19 | 0.17 | 0.13 | 0.34 | 0.31 | 0.25 |
| | | 0.10 | 0.25 | 0.23 | 0.19 | 0.43 | 0.39 | 0.33 |
| | Skewed | 0.05 | 0.19 | 0.17 | 0.14 | 0.34 | 0.30 | 0.24 |
| | | 0.10 | 0.27 | 0.25 | 0.20 | 0.42 | 0.39 | 0.34 |
| Skewed | Normal | 0.05 | 0.15 | 0.13 | 0.10 | 0.33 | 0.28 | 0.21 |
| | | 0.10 | 0.23 | 0.20 | 0.18 | 0.42 | 0.36 | 0.31 |
| | Heavy tailed | 0.05 | 0.15 | 0.14 | 0.12 | 0.35 | 0.29 | 0.23 |
| | | 0.10 | 0.23 | 0.21 | 0.19 | 0.45 | 0.39 | 0.32 |
| | Skewed | 0.05 | 0.14 | 0.12 | 0.10 | 0.31 | 0.26 | 0.19 |
| | | 0.10 | 0.20 | 0.19 | 0.15 | 0.41 | 0.34 | 0.29 |
| | | | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | |
| Normal | Normal | 0.05 | 0.06 | 0.05 | 0.05 | 0.04 | 0.05 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.10 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.11 | 0.11 | 0.12 | 0.10 | 0.11 | 0.11 |
| | Skewed | 0.05 | 0.04 | 0.04 | 0.05 | 0.04 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.11 | 0.10 | 0.10 | 0.09 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.17 | 0.16 | 0.13 | 0.32 | 0.28 | 0.21 |
| | | 0.10 | 0.26 | 0.24 | 0.22 | 0.41 | 0.38 | 0.33 |
| | Heavy tailed | 0.05 | 0.21 | 0.19 | 0.14 | 0.35 | 0.31 | 0.24 |
| | | 0.10 | 0.29 | 0.27 | 0.22 | 0.43 | 0.41 | 0.36 |
| | Skewed | 0.05 | 0.20 | 0.19 | 0.15 | 0.32 | 0.29 | 0.23 |
| | | 0.10 | 0.29 | 0.26 | 0.23 | 0.40 | 0.37 | 0.32 |
| Skewed | Normal | 0.05 | 0.17 | 0.16 | 0.12 | 0.29 | 0.22 | 0.18 |
| | | 0.10 | 0.26 | 0.24 | 0.21 | 0.40 | 0.33 | 0.28 |
| | Heavy tailed | 0.05 | 0.20 | 0.20 | 0.15 | 0.31 | 0.25 | 0.18 |
| | | 0.10 | 0.27 | 0.26 | 0.22 | 0.40 | 0.34 | 0.30 |
| | Skewed | 0.05 | 0.18 | 0.16 | 0.14 | 0.29 | 0.22 | 0.17 |
| | | 0.10 | 0.26 | 0.25 | 0.22 | 0.39 | 0.32 | 0.27 |

Table 16: Proportion of tests rejecting normality of the random intercept using two rotations and $s = 30$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma^2_\varepsilon = 4, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.06 | 0.06 | 0.05 | 0.06 | 0.05 | 0.05 |
| | | 0.10 | 0.11 | 0.10 | 0.12 | 0.11 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.09 | 0.09 | 0.11 | 0.10 | 0.10 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.12 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.12 | 0.11 | 0.09 | 0.22 | 0.20 | 0.15 |
| | | 0.10 | 0.18 | 0.17 | 0.15 | 0.29 | 0.28 | 0.22 |
| | Heavy tailed | 0.05 | 0.14 | 0.13 | 0.11 | 0.27 | 0.24 | 0.19 |
| | | 0.10 | 0.21 | 0.19 | 0.17 | 0.35 | 0.31 | 0.28 |
| | Skewed | 0.05 | 0.12 | 0.11 | 0.09 | 0.22 | 0.20 | 0.16 |
| | | 0.10 | 0.19 | 0.16 | 0.15 | 0.29 | 0.27 | 0.24 |
| Skewed | Normal | 0.05 | 0.10 | 0.09 | 0.08 | 0.22 | 0.18 | 0.12 |
| | | 0.10 | 0.17 | 0.15 | 0.14 | 0.30 | 0.27 | 0.21 |
| | Heavy tailed | 0.05 | 0.11 | 0.09 | 0.09 | 0.24 | 0.21 | 0.16 |
| | | 0.10 | 0.17 | 0.17 | 0.16 | 0.32 | 0.29 | 0.24 |
| | Skewed | 0.05 | 0.11 | 0.10 | 0.09 | 0.21 | 0.18 | 0.12 |
| | | 0.10 | 0.18 | 0.17 | 0.16 | 0.31 | 0.26 | 0.19 |
| | | | $\sigma^2_\varepsilon = 1, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.09 | 0.10 | 0.11 | 0.10 |
| | Heavy tailed | 0.05 | 0.05 | 0.05 | 0.04 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.09 | 0.10 | 0.08 | 0.12 | 0.12 | 0.11 |
| | Skewed | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.09 | 0.09 | 0.10 | 0.08 | 0.10 | 0.08 |
| Heavy tailed | Normal | 0.05 | 0.14 | 0.14 | 0.12 | 0.29 | 0.28 | 0.22 |
| | | 0.10 | 0.21 | 0.20 | 0.18 | 0.39 | 0.35 | 0.32 |
| | Heavy tailed | 0.05 | 0.17 | 0.15 | 0.13 | 0.34 | 0.30 | 0.24 |
| | | 0.10 | 0.24 | 0.22 | 0.19 | 0.41 | 0.39 | 0.33 |
| | Skewed | 0.05 | 0.17 | 0.15 | 0.12 | 0.31 | 0.29 | 0.23 |
| | | 0.10 | 0.25 | 0.23 | 0.20 | 0.39 | 0.36 | 0.31 |
| Skewed | Normal | 0.05 | 0.14 | 0.12 | 0.09 | 0.30 | 0.25 | 0.20 |
| | | 0.10 | 0.22 | 0.21 | 0.17 | 0.39 | 0.33 | 0.28 |
| | Heavy tailed | 0.05 | 0.14 | 0.13 | 0.12 | 0.32 | 0.26 | 0.21 |
| | | 0.10 | 0.22 | 0.21 | 0.18 | 0.44 | 0.36 | 0.32 |
| | Skewed | 0.05 | 0.13 | 0.11 | 0.09 | 0.30 | 0.23 | 0.18 |
| | | 0.10 | 0.19 | 0.18 | 0.14 | 0.40 | 0.34 | 0.28 |
| | | | $\sigma^2_\varepsilon = 1, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.11 | 0.10 | 0.09 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.05 | 0.05 | 0.06 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.11 | 0.11 | 0.10 | 0.11 | 0.11 |
| | Skewed | 0.05 | 0.04 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.10 | 0.09 | 0.10 | 0.09 | 0.09 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.17 | 0.17 | 0.14 | 0.28 | 0.26 | 0.19 |
| | | 0.10 | 0.24 | 0.23 | 0.21 | 0.37 | 0.33 | 0.30 |
| | Heavy tailed | 0.05 | 0.19 | 0.17 | 0.13 | 0.31 | 0.28 | 0.23 |
| | | 0.10 | 0.28 | 0.25 | 0.20 | 0.40 | 0.38 | 0.34 |
| | Skewed | 0.05 | 0.17 | 0.16 | 0.14 | 0.29 | 0.26 | 0.21 |
| | | 0.10 | 0.24 | 0.22 | 0.19 | 0.38 | 0.34 | 0.30 |
| Skewed | Normal | 0.05 | 0.16 | 0.16 | 0.13 | 0.28 | 0.21 | 0.15 |
| | | 0.10 | 0.26 | 0.24 | 0.19 | 0.39 | 0.32 | 0.27 |
| | Heavy tailed | 0.05 | 0.19 | 0.18 | 0.13 | 0.29 | 0.23 | 0.17 |
| | | 0.10 | 0.26 | 0.25 | 0.22 | 0.39 | 0.33 | 0.27 |
| | Skewed | 0.05 | 0.16 | 0.14 | 0.12 | 0.26 | 0.21 | 0.16 |
| | | 0.10 | 0.25 | 0.23 | 0.21 | 0.36 | 0.30 | 0.26 |

Table 17: Proportion of tests rejecting normality of the random slope using two rotations and $s = \text{rank}(\boldsymbol{B})$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma^2_\varepsilon = 4, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.03 | 0.03 | 0.04 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.08 | 0.08 | 0.10 | 0.09 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.13 | 0.12 | 0.11 | 0.22 | 0.20 | 0.16 |
| | | 0.10 | 0.20 | 0.19 | 0.17 | 0.29 | 0.27 | 0.23 |
| | Skewed | 0.05 | 0.07 | 0.07 | 0.07 | 0.16 | 0.12 | 0.10 |
| | | 0.10 | 0.14 | 0.13 | 0.13 | 0.24 | 0.20 | 0.17 |
| Heavy tailed | Normal | 0.05 | 0.07 | 0.07 | 0.07 | 0.11 | 0.09 | 0.08 |
| | | 0.10 | 0.12 | 0.12 | 0.13 | 0.17 | 0.15 | 0.15 |
| | Heavy tailed | 0.05 | 0.20 | 0.19 | 0.14 | 0.39 | 0.35 | 0.27 |
| | | 0.10 | 0.28 | 0.27 | 0.22 | 0.47 | 0.44 | 0.38 |
| | Skewed | 0.05 | 0.13 | 0.11 | 0.10 | 0.29 | 0.24 | 0.18 |
| | | 0.10 | 0.20 | 0.18 | 0.16 | 0.38 | 0.33 | 0.28 |
| Skewed | Normal | 0.05 | 0.06 | 0.06 | 0.06 | 0.08 | 0.07 | 0.06 |
| | | 0.10 | 0.12 | 0.11 | 0.12 | 0.15 | 0.14 | 0.11 |
| | Heavy tailed | 0.05 | 0.16 | 0.15 | 0.13 | 0.33 | 0.28 | 0.23 |
| | | 0.10 | 0.22 | 0.21 | 0.20 | 0.42 | 0.37 | 0.33 |
| | Skewed | 0.05 | 0.11 | 0.10 | 0.08 | 0.23 | 0.18 | 0.13 |
| | | 0.10 | 0.19 | 0.16 | 0.16 | 0.34 | 0.28 | 0.22 |
| | | | $\sigma^2_\varepsilon = 1, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.10 | 0.10 | 0.11 | 0.08 | 0.09 | 0.09 |
| | Heavy tailed | 0.05 | 0.09 | 0.09 | 0.09 | 0.14 | 0.12 | 0.10 |
| | | 0.10 | 0.16 | 0.15 | 0.14 | 0.20 | 0.18 | 0.16 |
| | Skewed | 0.05 | 0.07 | 0.07 | 0.06 | 0.07 | 0.07 | 0.06 |
| | | 0.10 | 0.13 | 0.12 | 0.10 | 0.13 | 0.12 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.12 | 0.11 | 0.11 | 0.21 | 0.19 | 0.15 |
| | | 0.10 | 0.19 | 0.18 | 0.16 | 0.28 | 0.26 | 0.21 |
| | Heavy tailed | 0.05 | 0.21 | 0.21 | 0.15 | 0.39 | 0.36 | 0.30 |
| | | 0.10 | 0.31 | 0.29 | 0.25 | 0.47 | 0.45 | 0.40 |
| | Skewed | 0.05 | 0.14 | 0.13 | 0.12 | 0.32 | 0.27 | 0.21 |
| | | 0.10 | 0.21 | 0.20 | 0.19 | 0.40 | 0.35 | 0.31 |
| Skewed | Normal | 0.05 | 0.10 | 0.09 | 0.07 | 0.19 | 0.16 | 0.11 |
| | | 0.10 | 0.18 | 0.16 | 0.14 | 0.27 | 0.23 | 0.19 |
| | Heavy tailed | 0.05 | 0.18 | 0.16 | 0.12 | 0.33 | 0.28 | 0.21 |
| | | 0.10 | 0.26 | 0.24 | 0.20 | 0.44 | 0.38 | 0.30 |
| | Skewed | 0.05 | 0.13 | 0.11 | 0.09 | 0.25 | 0.20 | 0.14 |
| | | 0.10 | 0.21 | 0.19 | 0.16 | 0.35 | 0.29 | 0.23 |
| | | | $\sigma^2_\varepsilon = 1, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | | | |
| Normal | Normal | 0.05 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.11 | 0.12 | 0.10 | 0.10 | 0.09 | 0.09 |
| | Heavy tailed | 0.05 | 0.08 | 0.07 | 0.07 | 0.08 | 0.07 | 0.07 |
| | | 0.10 | 0.13 | 0.13 | 0.12 | 0.13 | 0.12 | 0.12 |
| | Skewed | 0.05 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.09 | 0.10 | 0.10 | 0.09 | 0.09 | 0.09 |
| Heavy tailed | Normal | 0.05 | 0.23 | 0.20 | 0.16 | 0.41 | 0.36 | 0.29 |
| | | 0.10 | 0.30 | 0.28 | 0.24 | 0.50 | 0.47 | 0.40 |
| | Heavy tailed | 0.05 | 0.29 | 0.26 | 0.19 | 0.49 | 0.45 | 0.37 |
| | | 0.10 | 0.36 | 0.33 | 0.28 | 0.57 | 0.53 | 0.49 |
| | Skewed | 0.05 | 0.21 | 0.20 | 0.16 | 0.47 | 0.44 | 0.34 |
| | | 0.10 | 0.31 | 0.28 | 0.25 | 0.56 | 0.51 | 0.45 |
| Skewed | Normal | 0.05 | 0.20 | 0.17 | 0.12 | 0.38 | 0.29 | 0.22 |
| | | 0.10 | 0.30 | 0.26 | 0.21 | 0.49 | 0.39 | 0.32 |
| | Heavy tailed | 0.05 | 0.25 | 0.22 | 0.17 | 0.45 | 0.36 | 0.26 |
| | | 0.10 | 0.36 | 0.33 | 0.27 | 0.55 | 0.46 | 0.39 |
| | Skewed | 0.05 | 0.20 | 0.17 | 0.13 | 0.43 | 0.33 | 0.24 |
| | | 0.10 | 0.29 | 0.26 | 0.22 | 0.54 | 0.42 | 0.36 |

Table 18: Proportion of tests rejecting normality of the random slope using two rotations and $s = 55$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | $\sigma^2_\varepsilon = 4,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 0.07 | 0.08 | 0.09 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.12 | 0.11 | 0.10 | 0.19 | 0.17 | 0.13 |
| | | 0.10 | 0.18 | 0.17 | 0.15 | 0.26 | 0.24 | 0.21 |
| | Skewed | 0.05 | 0.08 | 0.07 | 0.07 | 0.14 | 0.12 | 0.09 |
| | | 0.10 | 0.14 | 0.14 | 0.13 | 0.22 | 0.19 | 0.16 |
| Heavy tailed | Normal | 0.05 | 0.07 | 0.07 | 0.07 | 0.11 | 0.10 | 0.08 |
| | | 0.10 | 0.12 | 0.11 | 0.12 | 0.16 | 0.15 | 0.14 |
| | Heavy tailed | 0.05 | 0.19 | 0.18 | 0.15 | 0.35 | 0.32 | 0.27 |
| | | 0.10 | 0.28 | 0.26 | 0.22 | 0.43 | 0.39 | 0.36 |
| | Skewed | 0.05 | 0.12 | 0.11 | 0.08 | 0.26 | 0.22 | 0.17 |
| | | 0.10 | 0.19 | 0.17 | 0.14 | 0.34 | 0.30 | 0.25 |
| Skewed | Normal | 0.05 | 0.06 | 0.06 | 0.05 | 0.07 | 0.07 | 0.06 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.15 | 0.14 | 0.12 |
| | Heavy tailed | 0.05 | 0.15 | 0.14 | 0.12 | 0.30 | 0.26 | 0.23 |
| | | 0.10 | 0.22 | 0.21 | 0.19 | 0.40 | 0.36 | 0.32 |
| | Skewed | 0.05 | 0.10 | 0.09 | 0.07 | 0.20 | 0.15 | 0.11 |
| | | 0.10 | 0.16 | 0.15 | 0.14 | 0.30 | 0.24 | 0.19 |
| | | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.06 | 0.04 | 0.05 | 0.05 |
| | | 0.10 | 0.09 | 0.11 | 0.11 | 0.08 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.09 | 0.09 | 0.08 | 0.12 | 0.11 | 0.10 |
| | | 0.10 | 0.16 | 0.14 | 0.13 | 0.20 | 0.19 | 0.16 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.06 | 0.08 | 0.07 | 0.06 |
| | | 0.10 | 0.12 | 0.11 | 0.12 | 0.14 | 0.12 | 0.12 |
| Heavy tailed | Normal | 0.05 | 0.12 | 0.11 | 0.11 | 0.21 | 0.18 | 0.16 |
| | | 0.10 | 0.19 | 0.19 | 0.17 | 0.29 | 0.27 | 0.23 |
| | Heavy tailed | 0.05 | 0.21 | 0.20 | 0.15 | 0.37 | 0.35 | 0.28 |
| | | 0.10 | 0.29 | 0.27 | 0.22 | 0.45 | 0.42 | 0.37 |
| | Skewed | 0.05 | 0.14 | 0.13 | 0.11 | 0.28 | 0.26 | 0.20 |
| | | 0.10 | 0.22 | 0.20 | 0.18 | 0.39 | 0.34 | 0.29 |
| Skewed | Normal | 0.05 | 0.10 | 0.09 | 0.07 | 0.18 | 0.15 | 0.12 |
| | | 0.10 | 0.18 | 0.16 | 0.13 | 0.26 | 0.23 | 0.22 |
| | Heavy tailed | 0.05 | 0.17 | 0.16 | 0.13 | 0.32 | 0.27 | 0.20 |
| | | 0.10 | 0.24 | 0.22 | 0.20 | 0.42 | 0.36 | 0.30 |
| | Skewed | 0.05 | 0.14 | 0.12 | 0.09 | 0.23 | 0.18 | 0.14 |
| | | 0.10 | 0.21 | 0.20 | 0.16 | 0.33 | 0.26 | 0.22 |
| | | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | | |
| Normal | Normal | 0.05 | 0.06 | 0.06 | 0.05 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.11 | 0.11 | 0.11 | 0.09 | 0.09 | 0.08 |
| | Heavy tailed | 0.05 | 0.07 | 0.07 | 0.07 | 0.08 | 0.07 | 0.06 |
| | | 0.10 | 0.14 | 0.13 | 0.13 | 0.12 | 0.12 | 0.11 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.12 | 0.11 | 0.09 | 0.09 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.23 | 0.20 | 0.15 | 0.40 | 0.36 | 0.31 |
| | | 0.10 | 0.31 | 0.28 | 0.24 | 0.50 | 0.46 | 0.40 |
| | Heavy tailed | 0.05 | 0.27 | 0.24 | 0.19 | 0.46 | 0.41 | 0.34 |
| | | 0.10 | 0.36 | 0.33 | 0.27 | 0.54 | 0.50 | 0.45 |
| | Skewed | 0.05 | 0.22 | 0.20 | 0.17 | 0.44 | 0.40 | 0.31 |
| | | 0.10 | 0.31 | 0.28 | 0.24 | 0.54 | 0.49 | 0.42 |
| Skewed | Normal | 0.05 | 0.18 | 0.16 | 0.11 | 0.37 | 0.29 | 0.22 |
| | | 0.10 | 0.29 | 0.25 | 0.21 | 0.47 | 0.38 | 0.32 |
| | Heavy tailed | 0.05 | 0.23 | 0.20 | 0.15 | 0.45 | 0.35 | 0.26 |
| | | 0.10 | 0.33 | 0.30 | 0.25 | 0.54 | 0.45 | 0.35 |
| | Skewed | 0.05 | 0.18 | 0.17 | 0.13 | 0.39 | 0.30 | 0.22 |
| | | 0.10 | 0.29 | 0.25 | 0.22 | 0.52 | 0.41 | 0.34 |

Table 19: Proportion of tests rejecting normality of the random slope using two rotations and $s = 50$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.06 |
| | | 0.10 | 0.08 | 0.08 | 0.09 | 0.10 | 0.10 | 0.11 |
| | Heavy tailed | 0.05 | 0.10 | 0.11 | 0.09 | 0.16 | 0.15 | 0.12 |
| | | 0.10 | 0.16 | 0.15 | 0.15 | 0.24 | 0.21 | 0.20 |
| | Skewed | 0.05 | 0.07 | 0.06 | 0.05 | 0.13 | 0.11 | 0.09 |
| | | 0.10 | 0.12 | 0.12 | 0.12 | 0.20 | 0.18 | 0.15 |
| Heavy tailed | Normal | 0.05 | 0.07 | 0.07 | 0.06 | 0.10 | 0.09 | 0.08 |
| | | 0.10 | 0.12 | 0.12 | 0.11 | 0.16 | 0.15 | 0.15 |
| | Heavy tailed | 0.05 | 0.19 | 0.17 | 0.14 | 0.34 | 0.32 | 0.26 |
| | | 0.10 | 0.26 | 0.24 | 0.21 | 0.43 | 0.40 | 0.35 |
| | Skewed | 0.05 | 0.11 | 0.11 | 0.08 | 0.23 | 0.19 | 0.15 |
| | | 0.10 | 0.19 | 0.17 | 0.14 | 0.29 | 0.26 | 0.23 |
| Skewed | Normal | 0.05 | 0.06 | 0.06 | 0.06 | 0.08 | 0.08 | 0.07 |
| | | 0.10 | 0.11 | 0.12 | 0.11 | 0.14 | 0.13 | 0.13 |
| | Heavy tailed | 0.05 | 0.14 | 0.12 | 0.11 | 0.29 | 0.25 | 0.19 |
| | | 0.10 | 0.19 | 0.19 | 0.17 | 0.37 | 0.33 | 0.29 |
| | Skewed | 0.05 | 0.10 | 0.09 | 0.08 | 0.20 | 0.15 | 0.11 |
| | | 0.10 | 0.15 | 0.15 | 0.14 | 0.29 | 0.23 | 0.20 |
| | | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 |
| | | 0.10 | 0.10 | 0.11 | 0.10 | 0.10 | 0.10 | 0.09 |
| | Heavy tailed | 0.05 | 0.10 | 0.09 | 0.08 | 0.12 | 0.10 | 0.09 |
| | | 0.10 | 0.15 | 0.15 | 0.12 | 0.19 | 0.17 | 0.14 |
| | Skewed | 0.05 | 0.06 | 0.05 | 0.05 | 0.06 | 0.05 | 0.05 |
| | | 0.10 | 0.12 | 0.11 | 0.10 | 0.13 | 0.12 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.13 | 0.12 | 0.11 | 0.22 | 0.18 | 0.16 |
| | | 0.10 | 0.20 | 0.19 | 0.16 | 0.30 | 0.27 | 0.24 |
| | Heavy tailed | 0.05 | 0.20 | 0.18 | 0.14 | 0.36 | 0.34 | 0.27 |
| | | 0.10 | 0.28 | 0.26 | 0.21 | 0.42 | 0.41 | 0.35 |
| | Skewed | 0.05 | 0.14 | 0.12 | 0.11 | 0.27 | 0.24 | 0.19 |
| | | 0.10 | 0.21 | 0.20 | 0.17 | 0.37 | 0.33 | 0.29 |
| Skewed | Normal | 0.05 | 0.10 | 0.08 | 0.07 | 0.19 | 0.15 | 0.10 |
| | | 0.10 | 0.17 | 0.15 | 0.14 | 0.27 | 0.23 | 0.18 |
| | Heavy tailed | 0.05 | 0.17 | 0.15 | 0.13 | 0.30 | 0.26 | 0.19 |
| | | 0.10 | 0.24 | 0.23 | 0.19 | 0.40 | 0.35 | 0.28 |
| | Skewed | 0.05 | 0.12 | 0.12 | 0.09 | 0.23 | 0.17 | 0.13 |
| | | 0.10 | 0.20 | 0.18 | 0.17 | 0.31 | 0.25 | 0.20 |
| | | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.06 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.10 | 0.10 | 0.11 | 0.10 | 0.10 | 0.09 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.05 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.12 | 0.11 | 0.12 | 0.13 | 0.12 | 0.10 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.11 | 0.10 | 0.10 | 0.09 | 0.09 |
| Heavy tailed | Normal | 0.05 | 0.22 | 0.19 | 0.15 | 0.38 | 0.34 | 0.28 |
| | | 0.10 | 0.30 | 0.26 | 0.24 | 0.48 | 0.43 | 0.38 |
| | Heavy tailed | 0.05 | 0.25 | 0.23 | 0.19 | 0.43 | 0.40 | 0.32 |
| | | 0.10 | 0.33 | 0.30 | 0.26 | 0.50 | 0.47 | 0.42 |
| | Skewed | 0.05 | 0.20 | 0.19 | 0.15 | 0.42 | 0.38 | 0.30 |
| | | 0.10 | 0.29 | 0.27 | 0.23 | 0.49 | 0.46 | 0.39 |
| Skewed | Normal | 0.05 | 0.18 | 0.15 | 0.13 | 0.36 | 0.30 | 0.22 |
| | | 0.10 | 0.29 | 0.25 | 0.21 | 0.48 | 0.39 | 0.32 |
| | Heavy tailed | 0.05 | 0.22 | 0.19 | 0.15 | 0.41 | 0.33 | 0.26 |
| | | 0.10 | 0.33 | 0.30 | 0.24 | 0.53 | 0.43 | 0.36 |
| | Skewed | 0.05 | 0.19 | 0.16 | 0.12 | 0.39 | 0.30 | 0.23 |
| | | 0.10 | 0.28 | 0.25 | 0.21 | 0.49 | 0.40 | 0.34 |

Table 20: Proportion of tests rejecting normality of the random slope using two rotations and $s = 45$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma^2_\varepsilon = 4, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.03 | 0.04 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.07 | 0.08 | 0.08 | 0.10 | 0.10 | 0.09 |
| | Heavy tailed | 0.05 | 0.10 | 0.10 | 0.09 | 0.15 | 0.13 | 0.10 |
| | | 0.10 | 0.16 | 0.16 | 0.14 | 0.21 | 0.18 | 0.15 |
| | Skewed | 0.05 | 0.07 | 0.07 | 0.05 | 0.09 | 0.09 | 0.06 |
| | | 0.10 | 0.13 | 0.12 | 0.11 | 0.17 | 0.14 | 0.13 |
| Heavy tailed | Normal | 0.05 | 0.07 | 0.07 | 0.05 | 0.11 | 0.11 | 0.09 |
| | | 0.10 | 0.11 | 0.11 | 0.10 | 0.18 | 0.16 | 0.14 |
| | Heavy tailed | 0.05 | 0.18 | 0.17 | 0.14 | 0.30 | 0.28 | 0.23 |
| | | 0.10 | 0.25 | 0.23 | 0.21 | 0.38 | 0.35 | 0.31 |
| | Skewed | 0.05 | 0.10 | 0.10 | 0.07 | 0.20 | 0.17 | 0.14 |
| | | 0.10 | 0.19 | 0.16 | 0.14 | 0.28 | 0.25 | 0.20 |
| Skewed | Normal | 0.05 | 0.07 | 0.06 | 0.06 | 0.08 | 0.07 | 0.07 |
| | | 0.10 | 0.12 | 0.12 | 0.11 | 0.14 | 0.13 | 0.12 |
| | Heavy tailed | 0.05 | 0.12 | 0.11 | 0.10 | 0.24 | 0.21 | 0.16 |
| | | 0.10 | 0.19 | 0.18 | 0.17 | 0.33 | 0.30 | 0.25 |
| | Skewed | 0.05 | 0.10 | 0.10 | 0.08 | 0.17 | 0.13 | 0.11 |
| | | 0.10 | 0.17 | 0.15 | 0.14 | 0.25 | 0.22 | 0.19 |
| | | | $\sigma^2_\varepsilon = 1, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.09 |
| | Heavy tailed | 0.05 | 0.10 | 0.08 | 0.07 | 0.11 | 0.10 | 0.09 |
| | | 0.10 | 0.15 | 0.14 | 0.13 | 0.16 | 0.14 | 0.14 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.05 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.10 | 0.13 | 0.12 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.14 | 0.12 | 0.10 | 0.21 | 0.18 | 0.15 |
| | | 0.10 | 0.21 | 0.19 | 0.15 | 0.28 | 0.25 | 0.21 |
| | Heavy tailed | 0.05 | 0.18 | 0.17 | 0.14 | 0.32 | 0.29 | 0.24 |
| | | 0.10 | 0.26 | 0.24 | 0.22 | 0.40 | 0.36 | 0.32 |
| | Skewed | 0.05 | 0.14 | 0.13 | 0.12 | 0.25 | 0.23 | 0.19 |
| | | 0.10 | 0.20 | 0.18 | 0.18 | 0.35 | 0.31 | 0.28 |
| Skewed | Normal | 0.05 | 0.10 | 0.09 | 0.07 | 0.18 | 0.15 | 0.13 |
| | | 0.10 | 0.17 | 0.15 | 0.14 | 0.27 | 0.23 | 0.19 |
| | Heavy tailed | 0.05 | 0.16 | 0.15 | 0.13 | 0.28 | 0.23 | 0.17 |
| | | 0.10 | 0.23 | 0.22 | 0.19 | 0.37 | 0.33 | 0.25 |
| | Skewed | 0.05 | 0.12 | 0.11 | 0.09 | 0.22 | 0.17 | 0.12 |
| | | 0.10 | 0.20 | 0.18 | 0.16 | 0.33 | 0.26 | 0.21 |
| | | | $\sigma^2_\varepsilon = 1, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.11 | 0.11 | 0.10 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.05 | 0.05 | 0.06 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.11 | 0.11 | 0.12 | 0.11 | 0.10 | 0.10 |
| | Skewed | 0.05 | 0.05 | 0.06 | 0.05 | 0.06 | 0.05 | 0.05 |
| | | 0.10 | 0.09 | 0.09 | 0.10 | 0.10 | 0.10 | 0.09 |
| Heavy tailed | Normal | 0.05 | 0.20 | 0.19 | 0.13 | 0.36 | 0.32 | 0.24 |
| | | 0.10 | 0.28 | 0.26 | 0.22 | 0.45 | 0.41 | 0.35 |
| | Heavy tailed | 0.05 | 0.24 | 0.22 | 0.17 | 0.42 | 0.39 | 0.32 |
| | | 0.10 | 0.32 | 0.30 | 0.26 | 0.52 | 0.47 | 0.41 |
| | Skewed | 0.05 | 0.19 | 0.17 | 0.14 | 0.40 | 0.37 | 0.28 |
| | | 0.10 | 0.28 | 0.26 | 0.24 | 0.48 | 0.44 | 0.38 |
| Skewed | Normal | 0.05 | 0.18 | 0.15 | 0.12 | 0.35 | 0.29 | 0.21 |
| | | 0.10 | 0.27 | 0.24 | 0.21 | 0.47 | 0.37 | 0.32 |
| | Heavy tailed | 0.05 | 0.20 | 0.18 | 0.15 | 0.40 | 0.32 | 0.24 |
| | | 0.10 | 0.29 | 0.27 | 0.22 | 0.48 | 0.41 | 0.33 |
| | Skewed | 0.05 | 0.18 | 0.16 | 0.13 | 0.36 | 0.28 | 0.21 |
| | | 0.10 | 0.27 | 0.24 | 0.21 | 0.47 | 0.38 | 0.32 |

Table 21: Proportion of tests rejecting normality of the random slope using two rotations and $s = 40$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | $\sigma^2_\varepsilon = 4,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | |
| Normal | Normal | 0.05 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.09 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.10 | 0.10 | 0.08 | 0.12 | 0.12 | 0.10 |
| | | 0.10 | 0.16 | 0.16 | 0.14 | 0.18 | 0.17 | 0.15 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.05 | 0.07 | 0.06 | 0.05 |
| | | 0.10 | 0.12 | 0.11 | 0.10 | 0.14 | 0.13 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.06 | 0.05 | 0.06 | 0.11 | 0.10 | 0.08 |
| | | 0.10 | 0.11 | 0.10 | 0.10 | 0.17 | 0.15 | 0.13 |
| | Heavy tailed | 0.05 | 0.16 | 0.15 | 0.12 | 0.28 | 0.24 | 0.19 |
| | | 0.10 | 0.22 | 0.21 | 0.19 | 0.36 | 0.33 | 0.28 |
| | Skewed | 0.05 | 0.09 | 0.08 | 0.07 | 0.18 | 0.16 | 0.13 |
| | | 0.10 | 0.15 | 0.14 | 0.12 | 0.27 | 0.23 | 0.19 |
| Skewed | Normal | 0.05 | 0.05 | 0.06 | 0.05 | 0.08 | 0.08 | 0.07 |
| | | 0.10 | 0.10 | 0.09 | 0.10 | 0.15 | 0.14 | 0.12 |
| | Heavy tailed | 0.05 | 0.12 | 0.11 | 0.08 | 0.22 | 0.18 | 0.15 |
| | | 0.10 | 0.17 | 0.17 | 0.15 | 0.30 | 0.27 | 0.23 |
| | Skewed | 0.05 | 0.10 | 0.09 | 0.07 | 0.16 | 0.13 | 0.09 |
| | | 0.10 | 0.15 | 0.14 | 0.14 | 0.24 | 0.20 | 0.17 |
| | | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.09 | 0.09 | 0.07 | 0.08 | 0.08 | 0.07 |
| | | 0.10 | 0.15 | 0.14 | 0.11 | 0.14 | 0.13 | 0.12 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.05 | 0.06 | 0.05 | 0.05 |
| | | 0.10 | 0.12 | 0.12 | 0.11 | 0.11 | 0.12 | 0.11 |
| Heavy tailed | Normal | 0.05 | 0.13 | 0.12 | 0.09 | 0.19 | 0.17 | 0.13 |
| | | 0.10 | 0.19 | 0.19 | 0.16 | 0.26 | 0.24 | 0.21 |
| | Heavy tailed | 0.05 | 0.18 | 0.16 | 0.12 | 0.31 | 0.29 | 0.22 |
| | | 0.10 | 0.24 | 0.23 | 0.21 | 0.39 | 0.36 | 0.32 |
| | Skewed | 0.05 | 0.13 | 0.12 | 0.09 | 0.24 | 0.22 | 0.18 |
| | | 0.10 | 0.18 | 0.17 | 0.17 | 0.34 | 0.30 | 0.27 |
| Skewed | Normal | 0.05 | 0.10 | 0.09 | 0.08 | 0.18 | 0.14 | 0.12 |
| | | 0.10 | 0.18 | 0.16 | 0.14 | 0.27 | 0.24 | 0.19 |
| | Heavy tailed | 0.05 | 0.14 | 0.14 | 0.11 | 0.26 | 0.22 | 0.18 |
| | | 0.10 | 0.22 | 0.20 | 0.17 | 0.36 | 0.32 | 0.26 |
| | Skewed | 0.05 | 0.13 | 0.11 | 0.07 | 0.21 | 0.16 | 0.12 |
| | | 0.10 | 0.18 | 0.18 | 0.16 | 0.30 | 0.25 | 0.21 |
| | | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.11 | 0.11 | 0.10 | 0.11 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.05 |
| | | 0.10 | 0.11 | 0.12 | 0.11 | 0.11 | 0.11 | 0.10 |
| | Skewed | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 |
| | | 0.10 | 0.09 | 0.09 | 0.08 | 0.09 | 0.10 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.19 | 0.17 | 0.13 | 0.36 | 0.32 | 0.27 |
| | | 0.10 | 0.27 | 0.24 | 0.21 | 0.46 | 0.41 | 0.35 |
| | Heavy tailed | 0.05 | 0.23 | 0.21 | 0.16 | 0.38 | 0.34 | 0.29 |
| | | 0.10 | 0.30 | 0.28 | 0.24 | 0.47 | 0.43 | 0.38 |
| | Skewed | 0.05 | 0.19 | 0.17 | 0.12 | 0.37 | 0.34 | 0.28 |
| | | 0.10 | 0.26 | 0.23 | 0.22 | 0.45 | 0.43 | 0.36 |
| Skewed | Normal | 0.05 | 0.17 | 0.15 | 0.11 | 0.34 | 0.28 | 0.21 |
| | | 0.10 | 0.26 | 0.24 | 0.20 | 0.44 | 0.37 | 0.31 |
| | Heavy tailed | 0.05 | 0.20 | 0.19 | 0.15 | 0.36 | 0.27 | 0.21 |
| | | 0.10 | 0.28 | 0.26 | 0.23 | 0.47 | 0.38 | 0.31 |
| | Skewed | 0.05 | 0.17 | 0.14 | 0.12 | 0.35 | 0.27 | 0.20 |
| | | 0.10 | 0.28 | 0.24 | 0.21 | 0.46 | 0.37 | 0.29 |

Table 22: Proportion of tests rejecting normality of the random slope using two rotations and $s = 35$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.06 | 0.05 | 0.04 | 0.04 | 0.05 |
| | | 0.10 | 0.10 | 0.11 | 0.10 | 0.09 | 0.09 | 0.09 |
| | Heavy tailed | 0.05 | 0.09 | 0.08 | 0.07 | 0.12 | 0.10 | 0.10 |
| | | 0.10 | 0.15 | 0.14 | 0.13 | 0.19 | 0.17 | 0.16 |
| | Skewed | 0.05 | 0.06 | 0.05 | 0.04 | 0.07 | 0.07 | 0.05 |
| | | 0.10 | 0.10 | 0.11 | 0.10 | 0.12 | 0.11 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.07 | 0.06 | 0.07 | 0.12 | 0.11 | 0.09 |
| | | 0.10 | 0.12 | 0.11 | 0.11 | 0.18 | 0.16 | 0.15 |
| | Heavy tailed | 0.05 | 0.14 | 0.14 | 0.11 | 0.26 | 0.23 | 0.19 |
| | | 0.10 | 0.21 | 0.20 | 0.16 | 0.33 | 0.31 | 0.27 |
| | Skewed | 0.05 | 0.07 | 0.07 | 0.06 | 0.16 | 0.13 | 0.12 |
| | | 0.10 | 0.15 | 0.13 | 0.12 | 0.23 | 0.21 | 0.19 |
| Skewed | Normal | 0.05 | 0.06 | 0.06 | 0.06 | 0.08 | 0.07 | 0.07 |
| | | 0.10 | 0.10 | 0.11 | 0.11 | 0.14 | 0.13 | 0.12 |
| | Heavy tailed | 0.05 | 0.11 | 0.10 | 0.08 | 0.20 | 0.18 | 0.14 |
| | | 0.10 | 0.19 | 0.17 | 0.14 | 0.28 | 0.26 | 0.22 |
| | Skewed | 0.05 | 0.08 | 0.08 | 0.06 | 0.14 | 0.13 | 0.10 |
| | | 0.10 | 0.16 | 0.14 | 0.12 | 0.22 | 0.19 | 0.18 |
| | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.09 | 0.08 | 0.09 | 0.09 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.08 | 0.07 | 0.07 | 0.08 | 0.07 | 0.07 |
| | | 0.10 | 0.14 | 0.14 | 0.12 | 0.14 | 0.13 | 0.12 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.05 | 0.04 | 0.05 | 0.04 |
| | | 0.10 | 0.11 | 0.10 | 0.10 | 0.10 | 0.11 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.13 | 0.11 | 0.10 | 0.22 | 0.19 | 0.16 |
| | | 0.10 | 0.19 | 0.19 | 0.17 | 0.29 | 0.27 | 0.23 |
| | Heavy tailed | 0.05 | 0.17 | 0.15 | 0.12 | 0.29 | 0.28 | 0.22 |
| | | 0.10 | 0.23 | 0.22 | 0.20 | 0.36 | 0.35 | 0.31 |
| | Skewed | 0.05 | 0.10 | 0.10 | 0.09 | 0.24 | 0.21 | 0.18 |
| | | 0.10 | 0.17 | 0.15 | 0.15 | 0.33 | 0.30 | 0.26 |
| Skewed | Normal | 0.05 | 0.12 | 0.10 | 0.08 | 0.19 | 0.15 | 0.12 |
| | | 0.10 | 0.18 | 0.16 | 0.15 | 0.28 | 0.24 | 0.20 |
| | Heavy tailed | 0.05 | 0.12 | 0.10 | 0.09 | 0.22 | 0.20 | 0.16 |
| | | 0.10 | 0.18 | 0.17 | 0.14 | 0.31 | 0.27 | 0.23 |
| | Skewed | 0.05 | 0.12 | 0.11 | 0.08 | 0.20 | 0.17 | 0.13 |
| | | 0.10 | 0.18 | 0.17 | 0.15 | 0.28 | 0.24 | 0.21 |
| | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 0.05 | 0.06 | 0.06 | 0.06 | 0.07 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 |
| | Heavy tailed | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.10 | 0.11 | 0.10 | 0.09 | 0.10 | 0.10 |
| | Skewed | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 |
| | | 0.10 | 0.08 | 0.08 | 0.09 | 0.11 | 0.11 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.17 | 0.15 | 0.12 | 0.34 | 0.31 | 0.26 |
| | | 0.10 | 0.24 | 0.24 | 0.18 | 0.43 | 0.40 | 0.35 |
| | Heavy tailed | 0.05 | 0.21 | 0.20 | 0.16 | 0.37 | 0.34 | 0.29 |
| | | 0.10 | 0.27 | 0.27 | 0.21 | 0.45 | 0.41 | 0.38 |
| | Skewed | 0.05 | 0.17 | 0.16 | 0.12 | 0.35 | 0.32 | 0.25 |
| | | 0.10 | 0.25 | 0.23 | 0.20 | 0.43 | 0.39 | 0.34 |
| Skewed | Normal | 0.05 | 0.17 | 0.16 | 0.12 | 0.32 | 0.26 | 0.20 |
| | | 0.10 | 0.25 | 0.23 | 0.20 | 0.42 | 0.35 | 0.29 |
| | Heavy tailed | 0.05 | 0.17 | 0.16 | 0.12 | 0.35 | 0.27 | 0.19 |
| | | 0.10 | 0.26 | 0.25 | 0.20 | 0.44 | 0.38 | 0.30 |
| | Skewed | 0.05 | 0.17 | 0.14 | 0.11 | 0.33 | 0.27 | 0.19 |
| | | 0.10 | 0.24 | 0.22 | 0.20 | 0.44 | 0.36 | 0.29 |

Table 23: Proportion of tests rejecting normality of the random slope using two rotations and $s = 30$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | | $\sigma^2_\varepsilon = 4, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.04 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.11 | 0.11 | 0.10 | 0.12 | 0.12 | 0.10 |
| | Heavy tailed | 0.05 | 0.08 | 0.07 | 0.06 | 0.12 | 0.10 | 0.09 |
| | | 0.10 | 0.14 | 0.14 | 0.12 | 0.17 | 0.16 | 0.15 |
| | Skewed | 0.05 | 0.06 | 0.05 | 0.04 | 0.06 | 0.06 | 0.06 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.12 | 0.12 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.07 | 0.06 | 0.06 | 0.10 | 0.09 | 0.07 |
| | | 0.10 | 0.12 | 0.12 | 0.11 | 0.17 | 0.14 | 0.14 |
| | Heavy tailed | 0.05 | 0.14 | 0.13 | 0.11 | 0.23 | 0.19 | 0.17 |
| | | 0.10 | 0.20 | 0.19 | 0.17 | 0.30 | 0.28 | 0.24 |
| | Skewed | 0.05 | 0.07 | 0.06 | 0.06 | 0.14 | 0.13 | 0.11 |
| | | 0.10 | 0.13 | 0.13 | 0.12 | 0.25 | 0.21 | 0.19 |
| Skewed | Normal | 0.05 | 0.06 | 0.06 | 0.05 | 0.09 | 0.08 | 0.08 |
| | | 0.10 | 0.12 | 0.11 | 0.10 | 0.15 | 0.14 | 0.13 |
| | Heavy tailed | 0.05 | 0.12 | 0.11 | 0.08 | 0.18 | 0.15 | 0.11 |
| | | 0.10 | 0.17 | 0.18 | 0.15 | 0.27 | 0.23 | 0.19 |
| | Skewed | 0.05 | 0.09 | 0.08 | 0.07 | 0.13 | 0.11 | 0.09 |
| | | 0.10 | 0.14 | 0.13 | 0.13 | 0.20 | 0.17 | 0.15 |
| | | | | | $\sigma^2_\varepsilon = 1, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | |
| Normal | Normal | 0.05 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.10 |
| | Heavy tailed | 0.05 | 0.07 | 0.07 | 0.06 | 0.07 | 0.07 | 0.06 |
| | | 0.10 | 0.13 | 0.13 | 0.12 | 0.14 | 0.13 | 0.12 |
| | Skewed | 0.05 | 0.06 | 0.06 | 0.05 | 0.04 | 0.05 | 0.04 |
| | | 0.10 | 0.11 | 0.10 | 0.11 | 0.10 | 0.09 | 0.09 |
| Heavy tailed | Normal | 0.05 | 0.13 | 0.12 | 0.10 | 0.22 | 0.19 | 0.15 |
| | | 0.10 | 0.18 | 0.18 | 0.16 | 0.28 | 0.26 | 0.23 |
| | Heavy tailed | 0.05 | 0.14 | 0.14 | 0.11 | 0.27 | 0.24 | 0.21 |
| | | 0.10 | 0.21 | 0.19 | 0.17 | 0.35 | 0.33 | 0.28 |
| | Skewed | 0.05 | 0.11 | 0.09 | 0.08 | 0.22 | 0.19 | 0.16 |
| | | 0.10 | 0.17 | 0.16 | 0.14 | 0.30 | 0.28 | 0.24 |
| Skewed | Normal | 0.05 | 0.12 | 0.10 | 0.09 | 0.19 | 0.17 | 0.12 |
| | | 0.10 | 0.19 | 0.18 | 0.15 | 0.27 | 0.24 | 0.20 |
| | Heavy tailed | 0.05 | 0.11 | 0.10 | 0.09 | 0.23 | 0.20 | 0.14 |
| | | 0.10 | 0.18 | 0.16 | 0.14 | 0.32 | 0.28 | 0.22 |
| | Skewed | 0.05 | 0.12 | 0.10 | 0.08 | 0.20 | 0.16 | 0.12 |
| | | 0.10 | 0.19 | 0.17 | 0.13 | 0.28 | 0.24 | 0.20 |
| | | | | | $\sigma^2_\varepsilon = 1, \quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | |
| Normal | Normal | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 |
| | | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| | Heavy tailed | 0.05 | 0.04 | 0.04 | 0.04 | 0.05 | 0.06 | 0.06 |
| | | 0.10 | 0.10 | 0.09 | 0.09 | 0.11 | 0.10 | 0.10 |
| | Skewed | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 |
| | | 0.10 | 0.08 | 0.08 | 0.07 | 0.10 | 0.10 | 0.10 |
| Heavy tailed | Normal | 0.05 | 0.17 | 0.16 | 0.12 | 0.30 | 0.28 | 0.23 |
| | | 0.10 | 0.24 | 0.23 | 0.18 | 0.40 | 0.36 | 0.33 |
| | Heavy tailed | 0.05 | 0.19 | 0.17 | 0.14 | 0.34 | 0.31 | 0.25 |
| | | 0.10 | 0.26 | 0.24 | 0.22 | 0.43 | 0.40 | 0.33 |
| | Skewed | 0.05 | 0.16 | 0.14 | 0.12 | 0.32 | 0.29 | 0.23 |
| | | 0.10 | 0.24 | 0.23 | 0.20 | 0.40 | 0.37 | 0.33 |
| Skewed | Normal | 0.05 | 0.17 | 0.15 | 0.12 | 0.31 | 0.25 | 0.19 |
| | | 0.10 | 0.25 | 0.23 | 0.19 | 0.40 | 0.34 | 0.29 |
| | Heavy tailed | 0.05 | 0.15 | 0.14 | 0.12 | 0.30 | 0.25 | 0.17 |
| | | 0.10 | 0.23 | 0.20 | 0.18 | 0.41 | 0.34 | 0.28 |
| | Skewed | 0.05 | 0.15 | 0.14 | 0.13 | 0.30 | 0.22 | 0.17 |
| | | 0.10 | 0.24 | 0.22 | 0.20 | 0.40 | 0.33 | 0.28 |

### 2.2 Known covariance matrices

In this section we present the results of a simulations where the covariance matrices, $\boldsymbol{R}$ and $\boldsymbol{D}$ were fixed/known. Tables 24–30 present the results for the rotated random intercept and Tables 31–37 present the results for the rotated random slope. We use a gray background to highlight the simulation settings under which the tests should fail to reject the null hypothesis of normality.

Table 24: Percentage of tests rejecting normality of the random intercept using two rotations and $s = \text{rank}(\boldsymbol{B})$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 4.7 | 4.9 | 4.7 | 4.3 | 4.7 | 4.6 |
| | | 0.10 | 10.6 | 10.9 | 10.6 | 8.7 | 9.1 | 9.9 |
| | Heavy tailed | 0.05 | 6.2 | 5.5 | 4.7 | 9.4 | 8.3 | 6.9 |
| | | 0.10 | 11.4 | 11.3 | 9.9 | 14.9 | 14.0 | 12.0 |
| | Skewed | 0.05 | 4.8 | 5.2 | 4.7 | 5.7 | 6.0 | 5.0 |
| | | 0.10 | 10.1 | 9.8 | 9.2 | 11.1 | 11.0 | 11.2 |
| Heavy tailed | Normal | 0.05 | 14.8 | 13.6 | 10.6 | 24.0 | 21.6 | 17.7 |
| | | 0.10 | 20.6 | 18.6 | 16.7 | 32.0 | 28.8 | 24.9 |
| | Heavy tailed | 0.05 | 20.3 | 17.8 | 14.1 | 38.2 | 33.8 | 28.0 |
| | | 0.10 | 27.8 | 25.6 | 22.0 | 47.1 | 42.6 | 37.7 |
| | Skewed | 0.05 | 15.4 | 14.9 | 12.0 | 27.6 | 24.5 | 20.0 |
| | | 0.10 | 23.8 | 21.8 | 19.5 | 36.7 | 33.5 | 27.7 |
| Skewed | Normal | 0.05 | 11.7 | 9.9 | 7.8 | 22.3 | 17.3 | 13.1 |
| | | 0.10 | 19.1 | 16.8 | 14.3 | 29.9 | 26.6 | 21.9 |
| | Heavy tailed | 0.05 | 14.6 | 13.1 | 10.6 | 27.8 | 23.2 | 17.5 |
| | | 0.10 | 22.1 | 20.5 | 17.3 | 36.9 | 30.6 | 26.1 |
| | Skewed | 0.05 | 13.5 | 11.7 | 9.4 | 22.6 | 16.0 | 12.2 |
| | | 0.10 | 20.9 | 18.3 | 16.8 | 31.4 | 24.6 | 20.5 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 3.4 | 3.6 | 3.8 | 5.5 | 5.7 | 5.1 |
| | | 0.10 | 6.8 | 7.4 | 7.9 | 10.9 | 10.6 | 12.2 |
| | Heavy tailed | 0.05 | 5.4 | 5.3 | 5.2 | 5.6 | 5.2 | 4.9 |
| | | 0.10 | 9.6 | 9.0 | 9.4 | 11.3 | 10.7 | 9.1 |
| | Skewed | 0.05 | 4.2 | 4.1 | 4.1 | 3.7 | 4.4 | 4.5 |
| | | 0.10 | 8.9 | 9.2 | 8.7 | 8.8 | 8.4 | 10.1 |
| Heavy tailed | Normal | 0.05 | 21.8 | 20.2 | 16.4 | 40.9 | 37.7 | 31.7 |
| | | 0.10 | 29.1 | 26.3 | 24.2 | 50.2 | 46.0 | 39.7 |
| | Heavy tailed | 0.05 | 25.9 | 23.8 | 19.4 | 47.6 | 43.5 | 35.3 |
| | | 0.10 | 34.8 | 31.4 | 27.7 | 57.5 | 52.8 | 45.8 |
| | Skewed | 0.05 | 26.4 | 24.4 | 18.4 | 43.7 | 38.8 | 31.6 |
| | | 0.10 | 34.8 | 31.3 | 26.1 | 53.5 | 48.0 | 41.7 |
| Skewed | Normal | 0.05 | 19.9 | 16.2 | 13.5 | 37.8 | 28.9 | 20.4 |
| | | 0.10 | 29.8 | 25.1 | 21.8 | 47.6 | 39.6 | 31.4 |
| | Heavy tailed | 0.05 | 19.9 | 17.1 | 12.9 | 42.2 | 34.4 | 27.3 |
| | | 0.10 | 28.7 | 26.3 | 22.1 | 52.7 | 43.9 | 36.6 |
| | Skewed | 0.05 | 17.3 | 15.5 | 11.8 | 38.2 | 29.1 | 21.5 |
| | | 0.10 | 26.5 | 23.5 | 21.6 | 50.0 | 38.7 | 30.9 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 4.5 | 4.7 | 5.5 | 5.3 | 5.2 | 5.6 |
| | | 0.10 | 9.3 | 9.5 | 9.8 | 11.2 | 11.3 | 11.0 |
| | Heavy tailed | 0.05 | 5.6 | 5.4 | 5.9 | 6.9 | 6.8 | 6.6 |
| | | 0.10 | 10.6 | 9.8 | 11.2 | 13.0 | 12.4 | 11.2 |
| | Skewed | 0.05 | 5.5 | 6.0 | 6.2 | 5.1 | 4.8 | 5.6 |
| | | 0.10 | 9.9 | 10.7 | 11.7 | 10.3 | 9.2 | 10.7 |
| Heavy tailed | Normal | 0.05 | 29.5 | 26.4 | 19.5 | 20.5 | 19.6 | 15.9 |
| | | 0.10 | 39.1 | 36.0 | 32.1 | 29.4 | 27.3 | 23.6 |
| | Heavy tailed | 0.05 | 31.6 | 28.4 | 23.9 | 22.8 | 21.1 | 16.5 |
| | | 0.10 | 38.5 | 37.1 | 33.2 | 32.1 | 28.5 | 25.9 |
| | Skewed | 0.05 | 30.6 | 28.1 | 23.8 | 22.1 | 20.5 | 16.1 |
| | | 0.10 | 39.0 | 36.3 | 31.8 | 29.2 | 27.0 | 23.4 |
| Skewed | Normal | 0.05 | 24.6 | 22.6 | 17.5 | 15.4 | 13.2 | 10.3 |
| | | 0.10 | 33.8 | 31.4 | 27.0 | 24.6 | 20.8 | 17.4 |
| | Heavy tailed | 0.05 | 26.7 | 25.1 | 20.0 | 16.0 | 13.9 | 11.3 |
| | | 0.10 | 37.4 | 34.5 | 29.9 | 23.5 | 21.9 | 17.9 |
| | Skewed | 0.05 | 28.1 | 25.3 | 19.1 | 16.6 | 13.8 | 11.1 |
| | | 0.10 | 37.7 | 35.5 | 29.2 | 24.2 | 21.2 | 18.8 |

Table 25: Percentage of tests rejecting normality of the random intercept using two rotations and $s = 55$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 5.2 | 5.5 | 5.4 | 4.8 | 4.8 | 4.6 |
| | | 0.10 | 9.9 | 11.1 | 10.5 | 8.8 | 9.2 | 8.8 |
| | Heavy tailed | 0.05 | 5.6 | 5.2 | 5.5 | 7.9 | 7.3 | 7.3 |
| | | 0.10 | 11.0 | 10.2 | 9.2 | 13.3 | 12.3 | 12.1 |
| | Skewed | 0.05 | 4.9 | 5.4 | 4.6 | 5.8 | 5.6 | 5.3 |
| | | 0.10 | 10.3 | 9.6 | 8.6 | 11.4 | 10.8 | 10.7 |
| Heavy tailed | Normal | 0.05 | 15.2 | 13.2 | 10.6 | 24.2 | 21.8 | 17.1 |
| | | 0.10 | 21.6 | 20.0 | 17.4 | 31.9 | 28.5 | 24.3 |
| | Heavy tailed | 0.05 | 18.9 | 16.3 | 14.5 | 36.8 | 32.6 | 27.3 |
| | | 0.10 | 25.5 | 24.2 | 21.1 | 46.9 | 42.3 | 37.3 |
| | Skewed | 0.05 | 16.0 | 14.9 | 11.4 | 28.2 | 24.8 | 20.1 |
| | | 0.10 | 22.3 | 20.9 | 18.6 | 35.7 | 32.0 | 28.0 |
| Skewed | Normal | 0.05 | 11.5 | 10.1 | 7.6 | 21.8 | 17.3 | 12.8 |
| | | 0.10 | 19.5 | 16.7 | 15.1 | 29.4 | 25.5 | 21.8 |
| | Heavy tailed | 0.05 | 13.7 | 12.2 | 9.7 | 27.4 | 22.9 | 18.0 |
| | | 0.10 | 21.0 | 19.0 | 16.5 | 36.1 | 30.7 | 26.3 |
| | Skewed | 0.05 | 13.5 | 11.6 | 9.0 | 23.4 | 15.6 | 12.5 |
| | | 0.10 | 20.7 | 18.5 | 16.8 | 31.3 | 24.7 | 20.6 |
| | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 4.1 | 4.1 | 3.9 | 4.9 | 6.1 | 4.9 |
| | | 0.10 | 8.0 | 7.6 | 8.7 | 10.2 | 11.1 | 11.8 |
| | Heavy tailed | 0.05 | 5.8 | 5.4 | 4.7 | 5.4 | 5.4 | 4.6 |
| | | 0.10 | 9.9 | 10.0 | 9.0 | 10.7 | 9.8 | 9.4 |
| | Skewed | 0.05 | 4.2 | 4.5 | 4.2 | 4.3 | 4.6 | 4.5 |
| | | 0.10 | 8.7 | 9.4 | 8.8 | 8.5 | 9.0 | 10.5 |
| Heavy tailed | Normal | 0.05 | 21.5 | 19.8 | 16.3 | 40.3 | 36.3 | 29.4 |
| | | 0.10 | 29.8 | 26.7 | 23.9 | 49.2 | 44.8 | 40.1 |
| | Heavy tailed | 0.05 | 26.1 | 23.5 | 19.2 | 45.7 | 41.7 | 34.8 |
| | | 0.10 | 32.9 | 30.9 | 27.4 | 55.1 | 51.2 | 44.3 |
| | Skewed | 0.05 | 25.2 | 23.0 | 16.9 | 40.9 | 37.7 | 31.4 |
| | | 0.10 | 32.6 | 29.6 | 26.1 | 51.7 | 44.7 | 40.2 |
| Skewed | Normal | 0.05 | 19.5 | 15.7 | 12.7 | 39.0 | 29.8 | 20.6 |
| | | 0.10 | 28.6 | 25.2 | 21.9 | 47.6 | 40.6 | 30.4 |
| | Heavy tailed | 0.05 | 19.2 | 17.1 | 12.6 | 43.5 | 36.2 | 26.9 |
| | | 0.10 | 28.2 | 26.0 | 21.8 | 52.3 | 44.5 | 38.3 |
| | Skewed | 0.05 | 17.9 | 15.7 | 12.2 | 39.4 | 29.5 | 21.3 |
| | | 0.10 | 25.4 | 24.0 | 20.8 | 51.0 | 39.4 | 32.5 |
| | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 4.4 | 4.5 | 5.3 | 4.3 | 4.4 | 4.3 |
| | | 0.10 | 9.0 | 9.8 | 10.6 | 9.0 | 10.1 | 10.4 |
| | Heavy tailed | 0.05 | 5.8 | 5.2 | 4.9 | 5.5 | 5.1 | 5.1 |
| | | 0.10 | 9.7 | 9.7 | 10.5 | 11.2 | 10.1 | 11.3 |
| | Skewed | 0.05 | 4.7 | 5.7 | 6.0 | 5.5 | 4.4 | 5.3 |
| | | 0.10 | 9.9 | 10.6 | 10.3 | 10.1 | 10.8 | 9.7 |
| Heavy tailed | Normal | 0.05 | 28.2 | 24.2 | 19.3 | 20.4 | 17.6 | 14.1 |
| | | 0.10 | 37.4 | 34.6 | 29.5 | 28.2 | 25.7 | 21.5 |
| | Heavy tailed | 0.05 | 29.7 | 26.9 | 23.3 | 21.8 | 21.0 | 16.3 |
| | | 0.10 | 37.9 | 36.1 | 31.9 | 31.1 | 29.4 | 24.0 |
| | Skewed | 0.05 | 29.1 | 26.1 | 21.1 | 21.3 | 20.0 | 16.0 |
| | | 0.10 | 37.1 | 34.9 | 31.0 | 30.3 | 27.4 | 24.4 |
| Skewed | Normal | 0.05 | 22.5 | 22.1 | 15.8 | 15.0 | 12.4 | 9.1 |
| | | 0.10 | 32.0 | 30.4 | 26.6 | 22.1 | 19.6 | 16.8 |
| | Heavy tailed | 0.05 | 24.5 | 22.2 | 17.6 | 16.2 | 14.0 | 9.9 |
| | | 0.10 | 35.5 | 34.1 | 27.9 | 23.9 | 21.0 | 17.6 |
| | Skewed | 0.05 | 26.0 | 23.9 | 18.0 | 17.0 | 14.2 | 11.1 |
| | | 0.10 | 37.7 | 34.5 | 28.1 | 24.9 | 21.3 | 19.2 |

Table 26: Percentage of tests rejecting normality of the random intercept using two rotations and $s = 50$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | $\sigma^2_\varepsilon = 4,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | |
| Normal | Normal | 0.05 | 4.1 | 4.7 | 4.4 | 4.0 | 4.4 | 4.4 |
| | | 0.10 | 9.6 | 9.4 | 9.5 | 8.3 | 8.5 | 9.1 |
| | Heavy tailed | 0.05 | 5.2 | 4.4 | 5.0 | 7.7 | 7.5 | 7.2 |
| | | 0.10 | 10.5 | 9.6 | 8.8 | 13.5 | 12.8 | 12.2 |
| | Skewed | 0.05 | 5.3 | 5.7 | 4.6 | 4.9 | 5.4 | 5.0 |
| | | 0.10 | 9.6 | 9.1 | 10.2 | 11.3 | 10.7 | 9.8 |
| Heavy tailed | Normal | 0.05 | 14.2 | 12.7 | 10.7 | 23.8 | 21.1 | 17.6 |
| | | 0.10 | 20.8 | 18.8 | 17.0 | 31.7 | 29.7 | 24.4 |
| | Heavy tailed | 0.05 | 18.5 | 17.2 | 14.0 | 33.7 | 30.8 | 24.0 |
| | | 0.10 | 27.0 | 23.3 | 21.4 | 42.6 | 38.7 | 32.6 |
| | Skewed | 0.05 | 16.3 | 14.4 | 11.7 | 26.8 | 24.3 | 18.6 |
| | | 0.10 | 23.0 | 22.2 | 18.7 | 34.6 | 31.6 | 26.8 |
| Skewed | Normal | 0.05 | 12.1 | 10.2 | 8.1 | 21.5 | 17.3 | 12.6 |
| | | 0.10 | 20.1 | 18.2 | 14.3 | 29.6 | 24.5 | 20.3 |
| | Heavy tailed | 0.05 | 13.8 | 12.2 | 9.1 | 27.1 | 21.8 | 17.1 |
| | | 0.10 | 19.9 | 19.0 | 16.8 | 34.8 | 30.9 | 25.5 |
| | Skewed | 0.05 | 12.7 | 11.9 | 8.8 | 21.3 | 16.2 | 12.6 |
| | | 0.10 | 19.1 | 18.0 | 16.8 | 30.5 | 24.0 | 20.0 |
| | | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | |
| Normal | Normal | 0.05 | 4.1 | 4.1 | 4.3 | 4.6 | 4.4 | 4.7 |
| | | 0.10 | 7.7 | 8.6 | 8.5 | 8.5 | 9.2 | 9.8 |
| | Heavy tailed | 0.05 | 5.4 | 5.5 | 5.0 | 6.2 | 6.1 | 4.5 |
| | | 0.10 | 9.3 | 9.3 | 8.8 | 10.3 | 10.6 | 10.9 |
| | Skewed | 0.05 | 5.0 | 4.8 | 4.2 | 4.0 | 4.0 | 4.5 |
| | | 0.10 | 10.2 | 10.4 | 9.7 | 9.1 | 9.7 | 10.0 |
| Heavy tailed | Normal | 0.05 | 20.8 | 18.4 | 14.9 | 37.8 | 35.8 | 28.3 |
| | | 0.10 | 27.1 | 25.2 | 22.2 | 47.4 | 44.1 | 38.7 |
| | Heavy tailed | 0.05 | 23.7 | 20.9 | 17.5 | 43.3 | 38.7 | 31.2 |
| | | 0.10 | 31.6 | 29.5 | 25.0 | 51.9 | 48.7 | 41.4 |
| | Skewed | 0.05 | 23.2 | 21.1 | 15.5 | 39.2 | 36.7 | 28.8 |
| | | 0.10 | 31.4 | 27.3 | 25.2 | 48.3 | 43.4 | 37.5 |
| Skewed | Normal | 0.05 | 17.9 | 14.5 | 11.7 | 37.3 | 29.0 | 22.3 |
| | | 0.10 | 27.4 | 24.5 | 20.4 | 48.4 | 39.1 | 32.1 |
| | Heavy tailed | 0.05 | 18.2 | 16.2 | 12.4 | 41.2 | 33.0 | 25.7 |
| | | 0.10 | 27.0 | 25.0 | 20.4 | 50.6 | 43.9 | 34.9 |
| | Skewed | 0.05 | 16.2 | 14.7 | 11.9 | 37.4 | 28.7 | 20.2 |
| | | 0.10 | 25.4 | 23.3 | 19.3 | 48.3 | 39.2 | 31.8 |
| | | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | | |
| Normal | Normal | 0.05 | 4.5 | 4.5 | 5.5 | 5.5 | 5.2 | 4.6 |
| | | 0.10 | 9.4 | 9.0 | 9.5 | 10.0 | 10.2 | 10.0 |
| | Heavy tailed | 0.05 | 5.8 | 5.9 | 5.5 | 6.0 | 6.0 | 5.4 |
| | | 0.10 | 10.1 | 10.7 | 10.4 | 11.2 | 10.7 | 10.5 |
| | Skewed | 0.05 | 4.1 | 5.0 | 5.1 | 6.2 | 6.2 | 5.9 |
| | | 0.10 | 10.4 | 11.1 | 10.7 | 12.0 | 12.7 | 11.3 |
| Heavy tailed | Normal | 0.05 | 25.3 | 22.8 | 19.8 | 16.9 | 14.7 | 13.3 |
| | | 0.10 | 34.5 | 32.5 | 27.4 | 26.1 | 22.9 | 20.8 |
| | Heavy tailed | 0.05 | 28.6 | 26.9 | 22.3 | 19.3 | 17.8 | 13.5 |
| | | 0.10 | 37.2 | 34.4 | 31.1 | 27.8 | 24.6 | 21.1 |
| | Skewed | 0.05 | 28.0 | 25.8 | 20.1 | 18.6 | 17.0 | 12.9 |
| | | 0.10 | 35.8 | 33.1 | 28.3 | 26.3 | 24.8 | 20.6 |
| Skewed | Normal | 0.05 | 21.4 | 20.3 | 14.9 | 14.3 | 11.0 | 9.1 |
| | | 0.10 | 31.5 | 29.9 | 25.9 | 22.0 | 18.3 | 16.5 |
| | Heavy tailed | 0.05 | 22.9 | 21.1 | 16.4 | 14.2 | 12.1 | 9.9 |
| | | 0.10 | 33.4 | 31.3 | 26.5 | 22.2 | 18.4 | 15.4 |
| | Skewed | 0.05 | 24.8 | 22.4 | 18.0 | 16.5 | 13.7 | 11.0 |
| | | 0.10 | 34.7 | 32.4 | 27.4 | 22.3 | 20.7 | 18.5 |

Table 27: Percentage of tests rejecting normality of the random intercept using two rotations and $s = 45$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | |
| Normal | Normal | 0.05 | 5.0 | 5.2 | 4.3 | 5.2 | 5.3 | 4.4 |
| | | 0.10 | 10.1 | 10.1 | 9.2 | 9.3 | 9.5 | 9.6 |
| | Heavy tailed | 0.05 | 5.0 | 4.7 | 3.9 | 6.8 | 7.2 | 7.1 |
| | | 0.10 | 10.0 | 9.4 | 8.3 | 12.4 | 11.9 | 12.4 |
| | Skewed | 0.05 | 6.1 | 5.9 | 4.8 | 4.6 | 4.4 | 4.0 |
| | | 0.10 | 9.8 | 10.5 | 9.1 | 9.9 | 8.9 | 9.3 |
| Heavy tailed | Normal | 0.05 | 13.5 | 12.4 | 9.2 | 24.1 | 21.6 | 18.5 |
| | | 0.10 | 19.8 | 17.6 | 15.7 | 31.3 | 28.6 | 27.0 |
| | Heavy tailed | 0.05 | 16.4 | 15.1 | 13.0 | 33.4 | 30.4 | 23.3 |
| | | 0.10 | 24.8 | 22.4 | 19.4 | 43.0 | 39.6 | 34.6 |
| | Skewed | 0.05 | 14.8 | 14.8 | 11.6 | 24.6 | 22.6 | 18.4 |
| | | 0.10 | 22.3 | 20.5 | 19.2 | 33.1 | 31.4 | 26.1 |
| Skewed | Normal | 0.05 | 11.9 | 10.7 | 8.6 | 21.9 | 18.8 | 13.6 |
| | | 0.10 | 20.6 | 18.0 | 15.3 | 30.4 | 24.7 | 22.5 |
| | Heavy tailed | 0.05 | 12.9 | 11.7 | 9.2 | 26.1 | 21.0 | 16.6 |
| | | 0.10 | 20.0 | 18.4 | 16.4 | 35.4 | 30.0 | 25.6 |
| | Skewed | 0.05 | 11.6 | 10.9 | 8.9 | 22.3 | 15.9 | 12.6 |
| | | 0.10 | 18.5 | 18.1 | 15.0 | 31.7 | 25.2 | 20.8 |
| | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | |
| Normal | Normal | 0.05 | 4.7 | 4.9 | 4.6 | 5.4 | 5.5 | 5.5 |
| | | 0.10 | 8.3 | 8.5 | 8.5 | 10.8 | 10.8 | 9.9 |
| | Heavy tailed | 0.05 | 5.6 | 5.4 | 4.7 | 4.5 | 4.4 | 3.7 |
| | | 0.10 | 10.3 | 10.6 | 9.6 | 9.1 | 8.9 | 8.9 |
| | Skewed | 0.05 | 4.7 | 4.6 | 4.6 | 5.6 | 5.4 | 5.1 |
| | | 0.10 | 10.2 | 10.0 | 10.1 | 10.0 | 9.7 | 9.4 |
| Heavy tailed | Normal | 0.05 | 20.0 | 17.3 | 14.1 | 38.1 | 34.5 | 29.1 |
| | | 0.10 | 27.7 | 24.5 | 21.3 | 45.3 | 43.1 | 38.9 |
| | Heavy tailed | 0.05 | 23.0 | 20.4 | 16.3 | 39.5 | 35.3 | 28.5 |
| | | 0.10 | 30.3 | 27.6 | 24.7 | 48.5 | 44.8 | 38.7 |
| | Skewed | 0.05 | 21.8 | 19.5 | 16.4 | 38.2 | 34.4 | 27.1 |
| | | 0.10 | 31.5 | 27.8 | 25.2 | 46.9 | 42.8 | 36.1 |
| Skewed | Normal | 0.05 | 16.5 | 14.3 | 10.5 | 36.9 | 29.3 | 21.7 |
| | | 0.10 | 27.4 | 24.2 | 20.0 | 47.0 | 39.6 | 33.1 |
| | Heavy tailed | 0.05 | 17.2 | 15.8 | 12.0 | 39.4 | 32.6 | 23.9 |
| | | 0.10 | 25.2 | 24.0 | 19.4 | 49.4 | 42.6 | 34.5 |
| | Skewed | 0.05 | 16.3 | 13.7 | 11.1 | 35.8 | 27.6 | 20.1 |
| | | 0.10 | 22.2 | 21.4 | 18.9 | 46.0 | 37.7 | 30.0 |
| | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | |
| Normal | Normal | 0.05 | 5.6 | 5.6 | 4.9 | 5.0 | 6.4 | 5.7 |
| | | 0.10 | 10.5 | 10.0 | 9.6 | 10.4 | 10.4 | 10.1 |
| | Heavy tailed | 0.05 | 4.9 | 5.1 | 4.8 | 4.6 | 5.7 | 5.0 |
| | | 0.10 | 10.0 | 8.8 | 9.1 | 10.1 | 10.1 | 9.7 |
| | Skewed | 0.05 | 4.5 | 5.7 | 5.7 | 4.0 | 4.2 | 4.4 |
| | | 0.10 | 9.8 | 9.4 | 10.9 | 10.1 | 9.4 | 9.8 |
| Heavy tailed | Normal | 0.05 | 24.1 | 21.9 | 16.3 | 18.6 | 17.7 | 13.3 |
| | | 0.10 | 33.2 | 31.3 | 26.3 | 26.1 | 25.0 | 22.0 |
| | Heavy tailed | 0.05 | 26.0 | 24.4 | 21.3 | 21.9 | 19.7 | 16.2 |
| | | 0.10 | 34.2 | 32.5 | 28.0 | 30.1 | 29.1 | 25.2 |
| | Skewed | 0.05 | 24.6 | 23.7 | 18.5 | 21.0 | 19.6 | 14.5 |
| | | 0.10 | 32.6 | 31.8 | 26.6 | 29.2 | 27.3 | 22.9 |
| Skewed | Normal | 0.05 | 21.4 | 19.2 | 15.7 | 15.3 | 13.7 | 9.7 |
| | | 0.10 | 29.8 | 28.8 | 24.5 | 23.0 | 19.9 | 16.7 |
| | Heavy tailed | 0.05 | 20.7 | 18.3 | 14.4 | 14.9 | 12.6 | 11.1 |
| | | 0.10 | 30.1 | 28.1 | 24.3 | 23.8 | 20.7 | 16.8 |
| | Skewed | 0.05 | 22.6 | 20.4 | 17.7 | 17.5 | 15.2 | 11.5 |
| | | 0.10 | 31.6 | 30.6 | 25.0 | 25.9 | 23.0 | 18.6 |

Table 28: Percentage of tests rejecting normality of the random intercept using two rotations and $s = 40$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 4.4 | 4.3 | 4.8 | 4.9 | 4.3 | 4.9 |
| | | 0.10 | 9.2 | 9.8 | 8.8 | 9.3 | 9.7 | 10.8 |
| | Heavy tailed | 0.05 | 4.3 | 4.9 | 4.4 | 6.8 | 6.8 | 6.2 |
| | | 0.10 | 9.7 | 9.3 | 8.3 | 11.7 | 11.1 | 10.4 |
| | Skewed | 0.05 | 5.6 | 5.5 | 5.0 | 4.6 | 4.8 | 4.2 |
| | | 0.10 | 11.0 | 11.5 | 9.7 | 9.9 | 9.6 | 9.1 |
| Heavy tailed | Normal | 0.05 | 13.3 | 11.7 | 10.0 | 24.6 | 21.5 | 19.2 |
| | | 0.10 | 20.1 | 18.1 | 16.2 | 31.5 | 29.6 | 26.3 |
| | Heavy tailed | 0.05 | 16.2 | 15.7 | 12.8 | 32.0 | 28.7 | 22.4 |
| | | 0.10 | 23.7 | 21.5 | 19.4 | 41.1 | 37.2 | 30.8 |
| | Skewed | 0.05 | 14.5 | 13.5 | 12.0 | 26.3 | 23.9 | 19.1 |
| | | 0.10 | 21.9 | 20.5 | 18.8 | 33.7 | 31.0 | 25.9 |
| Skewed | Normal | 0.05 | 13.0 | 11.3 | 8.8 | 19.7 | 15.9 | 13.0 |
| | | 0.10 | 19.7 | 18.0 | 16.3 | 29.1 | 23.9 | 19.7 |
| | Heavy tailed | 0.05 | 11.4 | 10.6 | 8.5 | 24.2 | 20.3 | 15.7 |
| | | 0.10 | 17.8 | 17.2 | 15.7 | 34.0 | 29.9 | 25.4 |
| | Skewed | 0.05 | 10.7 | 9.3 | 8.4 | 23.0 | 17.3 | 12.8 |
| | | 0.10 | 17.8 | 17.0 | 15.5 | 31.3 | 26.7 | 20.9 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 4.6 | 4.5 | 4.2 | 4.7 | 5.2 | 5.2 |
| | | 0.10 | 9.1 | 9.4 | 8.8 | 10.3 | 10.1 | 10.2 |
| | Heavy tailed | 0.05 | 4.9 | 5.6 | 4.3 | 4.7 | 5.1 | 4.3 |
| | | 0.10 | 10.6 | 11.5 | 10.3 | 9.9 | 9.7 | 8.7 |
| | Skewed | 0.05 | 5.5 | 5.4 | 5.1 | 4.8 | 6.0 | 6.1 |
| | | 0.10 | 10.3 | 10.0 | 10.0 | 11.2 | 11.0 | 11.0 |
| Heavy tailed | Normal | 0.05 | 17.4 | 15.8 | 13.2 | 36.3 | 34.0 | 28.0 |
| | | 0.10 | 23.7 | 22.0 | 20.0 | 46.0 | 41.3 | 37.5 |
| | Heavy tailed | 0.05 | 21.7 | 19.1 | 15.6 | 38.7 | 34.8 | 26.9 |
| | | 0.10 | 29.9 | 26.7 | 23.8 | 47.2 | 44.3 | 37.7 |
| | Skewed | 0.05 | 20.2 | 18.2 | 15.4 | 35.8 | 32.1 | 27.1 |
| | | 0.10 | 29.6 | 26.3 | 23.2 | 45.5 | 42.3 | 35.7 |
| Skewed | Normal | 0.05 | 16.8 | 15.4 | 10.6 | 36.6 | 29.5 | 23.5 |
| | | 0.10 | 26.3 | 23.3 | 18.9 | 47.5 | 40.5 | 35.7 |
| | Heavy tailed | 0.05 | 16.6 | 15.3 | 10.9 | 38.3 | 32.7 | 25.5 |
| | | 0.10 | 24.7 | 23.0 | 20.7 | 47.9 | 40.6 | 34.5 |
| | Skewed | 0.05 | 14.0 | 11.7 | 9.7 | 36.6 | 29.0 | 21.7 |
| | | 0.10 | 22.4 | 20.3 | 16.2 | 46.0 | 39.3 | 31.9 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 4.6 | 4.8 | 4.5 | 5.4 | 5.5 | 4.9 |
| | | 0.10 | 8.7 | 9.3 | 9.8 | 9.7 | 10.9 | 9.9 |
| | Heavy tailed | 0.05 | 5.1 | 5.1 | 5.4 | 5.0 | 5.3 | 4.0 |
| | | 0.10 | 10.5 | 10.2 | 9.9 | 10.9 | 10.3 | 9.4 |
| | Skewed | 0.05 | 4.7 | 5.7 | 5.6 | 4.9 | 5.2 | 4.4 |
| | | 0.10 | 10.8 | 10.6 | 11.0 | 11.1 | 11.7 | 10.6 |
| Heavy tailed | Normal | 0.05 | 22.7 | 21.3 | 16.7 | 15.7 | 14.4 | 12.4 |
| | | 0.10 | 32.0 | 29.7 | 25.0 | 22.8 | 22.3 | 19.6 |
| | Heavy tailed | 0.05 | 24.9 | 23.7 | 20.5 | 15.3 | 13.7 | 11.1 |
| | | 0.10 | 32.2 | 31.3 | 27.7 | 22.1 | 21.0 | 18.2 |
| | Skewed | 0.05 | 23.8 | 21.1 | 17.9 | 17.2 | 15.0 | 11.7 |
| | | 0.10 | 30.8 | 29.2 | 24.8 | 24.2 | 22.6 | 18.6 |
| Skewed | Normal | 0.05 | 19.5 | 18.0 | 14.3 | 14.0 | 12.3 | 8.4 |
| | | 0.10 | 28.0 | 26.1 | 24.2 | 21.1 | 18.6 | 16.0 |
| | Heavy tailed | 0.05 | 19.9 | 18.2 | 15.3 | 13.4 | 12.5 | 9.3 |
| | | 0.10 | 28.2 | 26.1 | 24.3 | 20.3 | 19.7 | 16.1 |
| | Skewed | 0.05 | 21.9 | 20.1 | 14.9 | 12.0 | 10.7 | 9.4 |
| | | 0.10 | 30.0 | 29.1 | 24.7 | 20.3 | 18.4 | 15.9 |

Table 29: Percentage of tests rejecting normality of the random intercept using two rotations and $s = 35$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | | $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | |
| Normal | Normal | 0.05 | 5.4 | 5.1 | 4.7 | 5.3 | 5.5 | 5.3 |
| | | 0.10 | 10.2 | 9.4 | 9.1 | 11.1 | 11.1 | 10.7 |
| | Heavy tailed | 0.05 | 5.1 | 4.8 | 4.3 | 6.5 | 7.0 | 6.2 |
| | | 0.10 | 10.9 | 10.4 | 10.4 | 11.5 | 11.6 | 11.2 |
| | Skewed | 0.05 | 5.8 | 5.6 | 5.0 | 3.1 | 2.9 | 3.7 |
| | | 0.10 | 10.1 | 10.6 | 8.8 | 9.5 | 8.2 | 8.1 |
| Heavy tailed | Normal | 0.05 | 12.8 | 11.7 | 9.2 | 22.4 | 20.7 | 18.8 |
| | | 0.10 | 19.6 | 18.1 | 15.8 | 29.8 | 27.9 | 26.4 |
| | Heavy tailed | 0.05 | 15.9 | 14.1 | 12.1 | 27.4 | 25.0 | 19.0 |
| | | 0.10 | 21.5 | 20.0 | 18.1 | 36.0 | 33.0 | 28.2 |
| | Skewed | 0.05 | 14.9 | 12.8 | 10.4 | 23.0 | 21.4 | 18.0 |
| | | 0.10 | 20.5 | 18.9 | 18.0 | 30.8 | 27.7 | 25.4 |
| Skewed | Normal | 0.05 | 10.8 | 9.2 | 8.4 | 18.1 | 15.4 | 12.6 |
| | | 0.10 | 20.2 | 18.1 | 15.9 | 29.7 | 24.0 | 20.0 |
| | Heavy tailed | 0.05 | 11.2 | 11.9 | 9.2 | 23.8 | 21.1 | 16.7 |
| | | 0.10 | 18.5 | 16.9 | 15.2 | 31.5 | 26.8 | 24.3 |
| | Skewed | 0.05 | 11.2 | 9.9 | 7.9 | 20.4 | 15.3 | 11.7 |
| | | 0.10 | 17.8 | 16.0 | 13.8 | 28.9 | 23.5 | 19.9 |
| | | | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | |
| Normal | Normal | 0.05 | 4.5 | 5.1 | 4.2 | 4.7 | 5.0 | 5.1 |
| | | 0.10 | 9.1 | 8.7 | 8.7 | 9.7 | 9.0 | 10.4 |
| | Heavy tailed | 0.05 | 4.9 | 5.4 | 3.9 | 5.2 | 4.7 | 5.3 |
| | | 0.10 | 10.4 | 10.6 | 9.2 | 9.9 | 9.8 | 8.8 |
| | Skewed | 0.05 | 5.3 | 5.9 | 4.5 | 5.3 | 5.5 | 5.0 |
| | | 0.10 | 10.8 | 10.9 | 9.9 | 10.2 | 9.2 | 10.1 |
| Heavy tailed | Normal | 0.05 | 15.6 | 14.6 | 11.8 | 31.5 | 29.7 | 24.9 |
| | | 0.10 | 23.3 | 20.7 | 17.7 | 42.6 | 39.5 | 33.7 |
| | Heavy tailed | 0.05 | 20.8 | 18.0 | 15.0 | 35.8 | 32.7 | 27.1 |
| | | 0.10 | 28.6 | 26.3 | 21.8 | 44.8 | 41.1 | 37.5 |
| | Skewed | 0.05 | 18.7 | 16.5 | 13.3 | 33.0 | 29.1 | 25.0 |
| | | 0.10 | 25.9 | 24.3 | 21.6 | 41.8 | 38.4 | 33.6 |
| Skewed | Normal | 0.05 | 15.3 | 14.3 | 9.7 | 36.5 | 30.1 | 24.8 |
| | | 0.10 | 24.0 | 21.7 | 17.6 | 45.2 | 39.4 | 32.8 |
| | Heavy tailed | 0.05 | 16.2 | 14.0 | 11.5 | 37.6 | 32.1 | 25.2 |
| | | 0.10 | 24.6 | 22.6 | 19.8 | 48.6 | 41.4 | 34.8 |
| | Skewed | 0.05 | 13.8 | 12.4 | 9.3 | 35.9 | 28.4 | 22.0 |
| | | 0.10 | 20.9 | 19.7 | 16.0 | 46.6 | 38.0 | 32.2 |
| | | | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | |
| Normal | Normal | 0.05 | 4.5 | 4.5 | 4.4 | 3.7 | 3.9 | 4.4 |
| | | 0.10 | 9.6 | 9.2 | 9.9 | 9.6 | 9.2 | 9.0 |
| | Heavy tailed | 0.05 | 5.3 | 5.2 | 5.0 | 4.6 | 4.3 | 4.0 |
| | | 0.10 | 9.1 | 9.3 | 8.2 | 9.9 | 9.8 | 9.6 |
| | Skewed | 0.05 | 5.3 | 5.7 | 5.4 | 6.2 | 6.0 | 5.2 |
| | | 0.10 | 9.6 | 10.1 | 9.8 | 10.8 | 11.3 | 10.7 |
| Heavy tailed | Normal | 0.05 | 21.6 | 19.9 | 15.5 | 16.1 | 14.8 | 12.9 |
| | | 0.10 | 30.7 | 28.9 | 24.3 | 23.8 | 23.0 | 20.2 |
| | Heavy tailed | 0.05 | 22.8 | 22.5 | 18.0 | 15.9 | 15.0 | 12.2 |
| | | 0.10 | 32.6 | 30.5 | 26.8 | 23.3 | 21.9 | 18.8 |
| | Skewed | 0.05 | 21.5 | 19.3 | 14.8 | 15.5 | 13.8 | 11.4 |
| | | 0.10 | 29.1 | 26.7 | 23.0 | 23.6 | 21.7 | 18.4 |
| Skewed | Normal | 0.05 | 18.3 | 16.9 | 14.3 | 12.1 | 11.1 | 8.4 |
| | | 0.10 | 26.7 | 24.9 | 22.0 | 18.9 | 17.3 | 15.1 |
| | Heavy tailed | 0.05 | 18.0 | 17.3 | 15.4 | 12.7 | 11.8 | 9.0 |
| | | 0.10 | 26.5 | 24.8 | 22.5 | 21.6 | 20.1 | 16.8 |
| | Skewed | 0.05 | 19.4 | 18.2 | 14.8 | 13.6 | 12.3 | 9.2 |
| | | 0.10 | 27.2 | 26.1 | 22.3 | 21.4 | 18.6 | 16.6 |

Table 30: Percentage of tests rejecting normality of the random intercept using two rotations and $s = 30$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 4.8 | 4.7 | 4.8 | 5.5 | 5.8 | 5.8 |
| | | 0.10 | 10.0 | 9.4 | 8.7 | 10.8 | 10.7 | 11.3 |
| | Heavy tailed | 0.05 | 5.2 | 5.1 | 5.2 | 6.4 | 5.9 | 6.0 |
| | | 0.10 | 10.9 | 11.2 | 9.2 | 10.7 | 11.2 | 10.9 |
| | Skewed | 0.05 | 5.0 | 4.9 | 5.0 | 3.7 | 3.8 | 4.0 |
| | | 0.10 | 10.3 | 9.9 | 9.5 | 9.4 | 8.7 | 9.4 |
| Heavy tailed | Normal | 0.05 | 12.0 | 10.8 | 9.2 | 21.7 | 19.8 | 16.4 |
| | | 0.10 | 17.7 | 17.4 | 14.4 | 28.7 | 27.4 | 24.8 |
| | Heavy tailed | 0.05 | 15.0 | 14.7 | 11.4 | 24.6 | 22.5 | 18.1 |
| | | 0.10 | 21.0 | 19.3 | 18.6 | 33.5 | 30.5 | 27.3 |
| | Skewed | 0.05 | 13.2 | 12.5 | 10.8 | 24.0 | 22.0 | 17.5 |
| | | 0.10 | 20.6 | 19.2 | 18.5 | 30.5 | 29.5 | 25.5 |
| Skewed | Normal | 0.05 | 10.7 | 9.6 | 8.7 | 21.4 | 18.3 | 14.4 |
| | | 0.10 | 18.8 | 16.5 | 14.3 | 30.1 | 25.8 | 22.8 |
| | Heavy tailed | 0.05 | 11.1 | 11.0 | 8.3 | 22.0 | 20.0 | 14.6 |
| | | 0.10 | 17.7 | 16.7 | 14.9 | 30.2 | 26.9 | 23.4 |
| | Skewed | 0.05 | 10.4 | 9.7 | 7.4 | 22.1 | 17.1 | 13.3 |
| | | 0.10 | 15.8 | 14.4 | 15.0 | 29.5 | 25.7 | 20.3 |
| | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 4.7 | 4.6 | 4.3 | 5.3 | 4.9 | 4.9 |
| | | 0.10 | 9.3 | 9.3 | 7.8 | 9.7 | 10.4 | 10.6 |
| | Heavy tailed | 0.05 | 6.2 | 6.2 | 4.7 | 5.8 | 6.1 | 5.7 |
| | | 0.10 | 10.6 | 10.1 | 9.2 | 11.6 | 11.8 | 10.5 |
| | Skewed | 0.05 | 4.8 | 4.8 | 5.1 | 5.1 | 5.2 | 4.7 |
| | | 0.10 | 10.3 | 9.9 | 8.6 | 9.2 | 9.7 | 9.5 |
| Heavy tailed | Normal | 0.05 | 14.9 | 13.8 | 11.2 | 30.1 | 28.1 | 23.1 |
| | | 0.10 | 21.5 | 20.3 | 18.0 | 40.2 | 35.3 | 32.9 |
| | Heavy tailed | 0.05 | 17.6 | 15.5 | 14.0 | 34.5 | 31.7 | 25.3 |
| | | 0.10 | 24.1 | 22.6 | 20.5 | 43.4 | 39.3 | 33.5 |
| | Skewed | 0.05 | 16.4 | 15.2 | 12.1 | 30.2 | 28.0 | 20.5 |
| | | 0.10 | 24.5 | 22.2 | 18.8 | 38.4 | 34.6 | 29.6 |
| Skewed | Normal | 0.05 | 14.7 | 13.0 | 8.4 | 31.9 | 25.6 | 18.2 |
| | | 0.10 | 22.1 | 20.3 | 16.5 | 40.8 | 34.8 | 28.8 |
| | Heavy tailed | 0.05 | 13.4 | 13.2 | 11.3 | 31.6 | 25.2 | 17.5 |
| | | 0.10 | 21.6 | 21.0 | 17.6 | 42.3 | 35.2 | 28.6 |
| | Skewed | 0.05 | 12.5 | 10.4 | 7.8 | 29.8 | 24.1 | 17.9 |
| | | 0.10 | 19.5 | 18.2 | 15.0 | 39.8 | 34.2 | 27.5 |
| | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 6.0 | 6.3 | 5.7 | 4.5 | 4.7 | 4.8 |
| | | 0.10 | 10.8 | 11.2 | 10.7 | 10.5 | 9.4 | 9.9 |
| | Heavy tailed | 0.05 | 4.9 | 5.3 | 4.2 | 5.2 | 5.7 | 5.7 |
| | | 0.10 | 10.5 | 9.7 | 9.1 | 9.9 | 9.7 | 9.6 |
| | Skewed | 0.05 | 4.6 | 5.1 | 4.9 | 5.3 | 5.4 | 4.3 |
| | | 0.10 | 9.6 | 10.5 | 10.1 | 10.5 | 10.3 | 10.7 |
| Heavy tailed | Normal | 0.05 | 20.6 | 18.8 | 13.9 | 16.5 | 15.7 | 13.4 |
| | | 0.10 | 29.4 | 28.8 | 22.1 | 24.1 | 22.9 | 20.7 |
| | Heavy tailed | 0.05 | 22.8 | 22.2 | 18.7 | 16.0 | 14.7 | 12.3 |
| | | 0.10 | 30.4 | 30.1 | 25.2 | 24.5 | 22.9 | 20.1 |
| | Skewed | 0.05 | 19.1 | 17.1 | 14.6 | 14.9 | 14.1 | 10.7 |
| | | 0.10 | 26.4 | 24.5 | 23.0 | 21.3 | 20.2 | 18.0 |
| Skewed | Normal | 0.05 | 17.5 | 16.3 | 13.6 | 14.4 | 13.3 | 10.1 |
| | | 0.10 | 25.8 | 24.8 | 21.1 | 21.8 | 20.1 | 17.6 |
| | Heavy tailed | 0.05 | 16.0 | 16.5 | 14.4 | 12.9 | 10.8 | 7.8 |
| | | 0.10 | 25.6 | 23.6 | 21.2 | 19.2 | 18.1 | 16.7 |
| | Skewed | 0.05 | 18.2 | 16.9 | 12.6 | 12.9 | 11.8 | 9.4 |
| | | 0.10 | 25.6 | 24.0 | 21.4 | 20.4 | 17.5 | 15.8 |

Table 31: Percentage of tests rejecting normality of the random slope using two rotations and $s = \text{rank}(\boldsymbol{B})$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 5.7 | 5.5 | 5.0 | 4.7 | 5.0 | 4.2 |
| | | 0.10 | 10.6 | 10.1 | 10.0 | 10.3 | 10.0 | 10.6 |
| | Heavy tailed | 0.05 | 14.2 | 12.7 | 10.4 | 24.0 | 20.6 | 16.9 |
| | | 0.10 | 19.7 | 18.3 | 17.0 | 33.4 | 29.3 | 26.5 |
| | Skewed | 0.05 | 8.2 | 7.8 | 7.1 | 17.2 | 14.7 | 11.0 |
| | | 0.10 | 14.6 | 13.7 | 12.9 | 24.4 | 21.5 | 17.2 |
| Heavy tailed | Normal | 0.05 | 9.7 | 9.5 | 8.5 | 12.5 | 10.3 | 9.3 |
| | | 0.10 | 15.9 | 15.3 | 15.1 | 18.1 | 16.8 | 15.4 |
| | Heavy tailed | 0.05 | 20.6 | 18.3 | 14.3 | 40.3 | 36.6 | 28.3 |
| | | 0.10 | 28.8 | 25.2 | 21.6 | 49.2 | 44.7 | 39.0 |
| | Skewed | 0.05 | 15.2 | 13.2 | 11.2 | 32.4 | 27.3 | 21.5 |
| | | 0.10 | 23.1 | 19.9 | 17.9 | 41.4 | 37.9 | 31.1 |
| Skewed | Normal | 0.05 | 5.3 | 5.6 | 4.8 | 7.6 | 6.8 | 5.5 |
| | | 0.10 | 10.8 | 11.1 | 10.5 | 14.3 | 12.8 | 12.5 |
| | Heavy tailed | 0.05 | 16.5 | 15.3 | 11.8 | 31.8 | 27.7 | 23.4 |
| | | 0.10 | 24.4 | 22.3 | 19.7 | 40.8 | 36.0 | 30.9 |
| | Skewed | 0.05 | 10.5 | 9.1 | 8.2 | 26.0 | 19.5 | 14.2 |
| | | 0.10 | 18.5 | 16.2 | 15.7 | 36.3 | 30.2 | 24.1 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 5.2 | 5.1 | 4.9 | 5.3 | 4.8 | 5.0 |
| | | 0.10 | 9.9 | 10.4 | 10.3 | 10.5 | 10.3 | 10.2 |
| | Heavy tailed | 0.05 | 9.9 | 9.3 | 8.2 | 13.1 | 11.7 | 9.0 |
| | | 0.10 | 15.1 | 14.4 | 13.8 | 19.4 | 17.1 | 15.6 |
| | Skewed | 0.05 | 6.1 | 5.7 | 5.5 | 8.1 | 7.1 | 6.2 |
| | | 0.10 | 11.8 | 12.0 | 11.3 | 14.5 | 13.3 | 12.8 |
| Heavy tailed | Normal | 0.05 | 12.0 | 10.5 | 9.1 | 22.4 | 20.6 | 17.4 |
| | | 0.10 | 18.4 | 16.0 | 14.7 | 31.8 | 27.6 | 24.6 |
| | Heavy tailed | 0.05 | 22.9 | 20.2 | 16.9 | 39.9 | 38.0 | 31.0 |
| | | 0.10 | 32.4 | 29.4 | 25.3 | 50.7 | 47.2 | 40.4 |
| | Skewed | 0.05 | 16.2 | 14.3 | 12.2 | 33.0 | 30.0 | 23.2 |
| | | 0.10 | 23.0 | 20.9 | 18.3 | 42.6 | 37.9 | 31.9 |
| Skewed | Normal | 0.05 | 10.6 | 9.2 | 8.5 | 20.7 | 18.1 | 14.1 |
| | | 0.10 | 18.1 | 16.0 | 15.2 | 28.9 | 24.2 | 21.1 |
| | Heavy tailed | 0.05 | 17.4 | 15.0 | 11.6 | 33.2 | 29.2 | 21.4 |
| | | 0.10 | 24.4 | 21.9 | 18.8 | 43.1 | 36.8 | 30.8 |
| | Skewed | 0.05 | 13.9 | 12.9 | 8.6 | 26.4 | 20.0 | 15.4 |
| | | 0.10 | 21.2 | 19.9 | 17.3 | 35.0 | 28.7 | 23.4 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 5.1 | 5.1 | 4.3 | 6.0 | 5.4 | 5.9 |
| | | 0.10 | 8.9 | 8.9 | 8.9 | 10.7 | 10.8 | 9.8 |
| | Heavy tailed | 0.05 | 6.1 | 5.3 | 5.0 | 5.0 | 5.5 | 5.3 |
| | | 0.10 | 11.3 | 10.5 | 9.7 | 9.9 | 10.8 | 10.9 |
| | Skewed | 0.05 | 4.8 | 4.4 | 4.9 | 4.7 | 5.8 | 4.8 |
| | | 0.10 | 9.7 | 9.2 | 9.6 | 9.5 | 9.5 | 8.8 |
| Heavy tailed | Normal | 0.05 | 23.9 | 21.4 | 16.9 | 41.0 | 36.7 | 29.7 |
| | | 0.10 | 30.1 | 27.9 | 24.9 | 49.9 | 46.3 | 40.0 |
| | Heavy tailed | 0.05 | 32.0 | 29.6 | 22.7 | 54.2 | 49.8 | 40.5 |
| | | 0.10 | 39.2 | 36.7 | 31.4 | 62.4 | 58.6 | 50.9 |
| | Skewed | 0.05 | 27.9 | 24.1 | 19.5 | 50.1 | 46.1 | 37.1 |
| | | 0.10 | 39.0 | 34.0 | 28.5 | 59.2 | 56.1 | 48.7 |
| Skewed | Normal | 0.05 | 20.1 | 18.4 | 13.3 | 36.7 | 30.8 | 20.8 |
| | | 0.10 | 28.7 | 25.9 | 20.1 | 46.9 | 39.6 | 32.9 |
| | Heavy tailed | 0.05 | 23.2 | 20.4 | 16.6 | 44.2 | 33.8 | 26.3 |
| | | 0.10 | 32.7 | 28.8 | 24.7 | 55.8 | 45.3 | 37.2 |
| | Skewed | 0.05 | 20.0 | 17.1 | 13.9 | 41.5 | 31.0 | 22.7 |
| | | 0.10 | 29.6 | 27.0 | 21.1 | 52.9 | 41.0 | 34.1 |

Table 32: Percentage of tests rejecting normality of the random slope using two rotations and $s = 55$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 6.2 | 6.2 | 4.6 | 5.5 | 5.1 | 5.2 |
| | | 0.10 | 10.5 | 10.6 | 10.1 | 11.2 | 11.4 | 9.9 |
| | Heavy tailed | 0.05 | 13.5 | 11.7 | 9.9 | 21.4 | 19.9 | 14.6 |
| | | 0.10 | 20.1 | 19.5 | 16.0 | 30.2 | 26.8 | 23.2 |
| | Skewed | 0.05 | 7.9 | 7.8 | 7.9 | 15.1 | 11.7 | 10.1 |
| | | 0.10 | 14.7 | 13.5 | 14.4 | 23.2 | 20.3 | 16.3 |
| Heavy tailed | Normal | 0.05 | 9.6 | 9.0 | 7.8 | 11.2 | 9.9 | 8.9 |
| | | 0.10 | 14.3 | 13.9 | 14.3 | 17.8 | 15.9 | 15.1 |
| | Heavy tailed | 0.05 | 18.4 | 16.8 | 12.8 | 37.3 | 33.5 | 27.5 |
| | | 0.10 | 28.0 | 24.5 | 20.8 | 46.1 | 42.8 | 37.1 |
| | Skewed | 0.05 | 13.8 | 12.0 | 9.6 | 28.4 | 24.6 | 21.0 |
| | | 0.10 | 20.5 | 19.0 | 16.0 | 38.6 | 34.3 | 30.2 |
| Skewed | Normal | 0.05 | 5.6 | 5.8 | 6.0 | 8.1 | 6.6 | 5.7 |
| | | 0.10 | 11.6 | 11.9 | 11.1 | 13.6 | 12.6 | 11.4 |
| | Heavy tailed | 0.05 | 16.3 | 14.9 | 12.0 | 28.7 | 25.2 | 20.2 |
| | | 0.10 | 22.9 | 21.8 | 19.0 | 37.7 | 33.8 | 28.1 |
| | Skewed | 0.05 | 10.6 | 8.7 | 7.8 | 24.6 | 19.2 | 13.7 |
| | | 0.10 | 16.8 | 15.9 | 13.5 | 35.0 | 27.9 | 23.1 |
| | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 4.9 | 5.1 | 5.3 | 4.7 | 4.6 | 5.3 |
| | | 0.10 | 10.3 | 10.6 | 10.6 | 9.6 | 10.4 | 9.5 |
| | Heavy tailed | 0.05 | 8.9 | 8.4 | 8.2 | 11.5 | 10.2 | 7.5 |
| | | 0.10 | 14.8 | 14.7 | 13.4 | 17.6 | 16.8 | 14.2 |
| | Skewed | 0.05 | 5.5 | 5.2 | 4.9 | 6.7 | 6.2 | 6.2 |
| | | 0.10 | 9.8 | 10.3 | 9.8 | 12.8 | 12.4 | 12.4 |
| Heavy tailed | Normal | 0.05 | 13.6 | 11.2 | 8.7 | 24.3 | 20.8 | 17.0 |
| | | 0.10 | 18.5 | 17.7 | 15.5 | 32.1 | 29.2 | 24.7 |
| | Heavy tailed | 0.05 | 21.8 | 20.7 | 16.7 | 39.1 | 37.3 | 30.2 |
| | | 0.10 | 32.3 | 28.7 | 25.6 | 48.7 | 45.6 | 39.5 |
| | Skewed | 0.05 | 15.3 | 13.8 | 11.7 | 32.2 | 27.9 | 22.2 |
| | | 0.10 | 22.3 | 20.8 | 19.4 | 41.0 | 37.5 | 32.1 |
| Skewed | Normal | 0.05 | 10.9 | 9.1 | 7.8 | 19.5 | 15.8 | 12.1 |
| | | 0.10 | 18.1 | 16.1 | 15.3 | 28.1 | 24.1 | 20.5 |
| | Heavy tailed | 0.05 | 15.7 | 14.2 | 11.7 | 30.9 | 26.1 | 18.7 |
| | | 0.10 | 23.7 | 21.1 | 18.7 | 39.9 | 35.8 | 28.1 |
| | Skewed | 0.05 | 12.9 | 11.7 | 8.9 | 22.8 | 17.7 | 14.0 |
| | | 0.10 | 20.4 | 19.2 | 16.3 | 30.6 | 24.7 | 22.1 |
| | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 4.7 | 5.4 | 4.3 | 5.7 | 5.7 | 4.8 |
| | | 0.10 | 8.9 | 9.6 | 9.9 | 10.5 | 9.2 | 10.0 |
| | Heavy tailed | 0.05 | 4.4 | 4.7 | 4.4 | 4.7 | 4.8 | 5.6 |
| | | 0.10 | 10.5 | 10.5 | 9.5 | 10.6 | 11.2 | 11.0 |
| | Skewed | 0.05 | 4.9 | 4.6 | 4.7 | 4.4 | 4.7 | 4.7 |
| | | 0.10 | 9.4 | 8.7 | 8.7 | 8.8 | 9.3 | 9.0 |
| Heavy tailed | Normal | 0.05 | 22.4 | 20.7 | 17.7 | 39.9 | 35.3 | 29.6 |
| | | 0.10 | 30.2 | 28.0 | 25.2 | 48.4 | 44.6 | 38.2 |
| | Heavy tailed | 0.05 | 30.1 | 27.3 | 21.7 | 53.0 | 48.1 | 38.6 |
| | | 0.10 | 36.7 | 33.5 | 29.3 | 60.4 | 57.3 | 49.9 |
| | Skewed | 0.05 | 27.7 | 24.5 | 18.6 | 47.5 | 45.1 | 37.7 |
| | | 0.10 | 37.9 | 34.0 | 28.6 | 55.6 | 52.7 | 46.3 |
| Skewed | Normal | 0.05 | 18.9 | 16.1 | 12.4 | 34.4 | 27.6 | 20.7 |
| | | 0.10 | 28.4 | 25.3 | 21.4 | 45.1 | 36.6 | 31.3 |
| | Heavy tailed | 0.05 | 21.9 | 19.7 | 14.6 | 41.7 | 31.3 | 24.3 |
| | | 0.10 | 30.7 | 28.2 | 23.4 | 53.1 | 43.0 | 35.7 |
| | Skewed | 0.05 | 19.5 | 16.4 | 13.0 | 40.8 | 29.1 | 22.8 |
| | | 0.10 | 29.2 | 25.6 | 21.5 | 51.0 | 41.0 | 34.6 |

Table 33: Percentage of tests rejecting normality of the random slope using two rotations and $s = 50$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 5.3 | 4.6 | 4.7 | 5.7 | 5.8 | 4.4 |
| | | 0.10 | 10.5 | 10.5 | 9.4 | 9.5 | 10.4 | 10.2 |
| | Heavy tailed | 0.05 | 12.5 | 10.9 | 8.4 | 16.7 | 15.5 | 13.1 |
| | | 0.10 | 17.3 | 17.1 | 15.6 | 25.7 | 22.8 | 19.6 |
| | Skewed | 0.05 | 6.9 | 6.9 | 6.8 | 12.6 | 11.8 | 8.8 |
| | | 0.10 | 13.9 | 13.9 | 12.8 | 19.9 | 18.0 | 15.9 |
| Heavy tailed | Normal | 0.05 | 9.1 | 8.8 | 9.1 | 12.0 | 10.6 | 8.8 |
| | | 0.10 | 15.1 | 14.9 | 14.8 | 19.0 | 16.9 | 14.3 |
| | Heavy tailed | 0.05 | 17.6 | 15.6 | 12.1 | 34.3 | 30.6 | 24.9 |
| | | 0.10 | 26.0 | 23.5 | 19.8 | 44.2 | 40.0 | 34.9 |
| | Skewed | 0.05 | 11.8 | 10.5 | 8.4 | 23.8 | 20.8 | 16.8 |
| | | 0.10 | 19.5 | 17.5 | 13.9 | 32.8 | 29.5 | 26.5 |
| Skewed | Normal | 0.05 | 6.7 | 6.5 | 5.8 | 6.8 | 5.3 | 4.5 |
| | | 0.10 | 12.0 | 12.0 | 11.3 | 13.7 | 11.3 | 10.6 |
| | Heavy tailed | 0.05 | 15.6 | 13.6 | 10.7 | 25.5 | 22.7 | 18.4 |
| | | 0.10 | 22.3 | 21.6 | 17.6 | 34.2 | 29.4 | 25.8 |
| | Skewed | 0.05 | 9.7 | 8.3 | 6.8 | 18.3 | 13.4 | 10.9 |
| | | 0.10 | 16.5 | 16.3 | 12.7 | 27.2 | 22.4 | 19.2 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | | |
| Normal | Normal | 0.05 | 5.4 | 5.0 | 4.9 | 4.8 | 5.0 | 4.3 |
| | | 0.10 | 10.3 | 9.8 | 10.4 | 9.6 | 9.3 | 9.6 |
| | Heavy tailed | 0.05 | 8.1 | 8.0 | 7.1 | 10.4 | 8.6 | 7.9 |
| | | 0.10 | 13.8 | 13.6 | 12.8 | 17.2 | 15.7 | 12.8 |
| | Skewed | 0.05 | 5.0 | 4.7 | 5.3 | 6.6 | 6.1 | 5.4 |
| | | 0.10 | 10.8 | 10.1 | 9.3 | 12.8 | 11.4 | 10.5 |
| Heavy tailed | Normal | 0.05 | 13.6 | 11.6 | 9.7 | 23.7 | 20.3 | 16.3 |
| | | 0.10 | 19.8 | 18.5 | 15.5 | 31.7 | 28.4 | 24.8 |
| | Heavy tailed | 0.05 | 21.8 | 19.0 | 15.9 | 36.2 | 33.9 | 28.7 |
| | | 0.10 | 30.3 | 27.9 | 23.4 | 45.2 | 42.2 | 37.8 |
| | Skewed | 0.05 | 14.7 | 13.5 | 12.1 | 30.6 | 26.2 | 22.1 |
| | | 0.10 | 21.5 | 20.3 | 18.5 | 40.8 | 36.0 | 30.8 |
| Skewed | Normal | 0.05 | 10.7 | 9.7 | 7.8 | 18.9 | 15.7 | 12.9 |
| | | 0.10 | 18.5 | 15.8 | 15.9 | 29.1 | 24.4 | 20.3 |
| | Heavy tailed | 0.05 | 16.1 | 14.4 | 11.0 | 28.8 | 23.8 | 18.4 |
| | | 0.10 | 23.7 | 21.1 | 17.8 | 38.3 | 32.9 | 27.4 |
| | Skewed | 0.05 | 13.1 | 10.5 | 8.7 | 22.7 | 15.9 | 12.2 |
| | | 0.10 | 19.4 | 17.6 | 14.2 | 31.3 | 25.5 | 20.4 |
| | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | | |
| Normal | Normal | 0.05 | 5.0 | 4.9 | 4.8 | 5.6 | 5.7 | 5.4 |
| | | 0.10 | 9.0 | 10.1 | 10.1 | 11.4 | 10.8 | 9.2 |
| | Heavy tailed | 0.05 | 5.2 | 4.5 | 4.3 | 5.2 | 5.7 | 5.0 |
| | | 0.10 | 10.0 | 9.6 | 8.8 | 11.0 | 10.9 | 11.8 |
| | Skewed | 0.05 | 4.1 | 4.3 | 3.9 | 4.8 | 5.2 | 4.1 |
| | | 0.10 | 8.2 | 8.4 | 8.5 | 8.8 | 8.9 | 9.1 |
| Heavy tailed | Normal | 0.05 | 22.1 | 19.8 | 16.2 | 37.8 | 34.4 | 28.0 |
| | | 0.10 | 27.9 | 26.2 | 22.8 | 46.8 | 44.2 | 38.3 |
| | Heavy tailed | 0.05 | 27.5 | 24.9 | 18.8 | 49.5 | 45.1 | 37.3 |
| | | 0.10 | 35.5 | 32.2 | 29.3 | 56.9 | 54.3 | 46.5 |
| | Skewed | 0.05 | 27.5 | 24.2 | 19.2 | 46.5 | 42.3 | 31.3 |
| | | 0.10 | 36.2 | 32.3 | 28.1 | 55.2 | 51.7 | 44.2 |
| Skewed | Normal | 0.05 | 18.1 | 16.4 | 12.0 | 34.0 | 26.5 | 20.2 |
| | | 0.10 | 27.2 | 24.0 | 20.4 | 43.7 | 36.1 | 29.1 |
| | Heavy tailed | 0.05 | 20.7 | 18.4 | 15.2 | 38.0 | 30.0 | 21.9 |
| | | 0.10 | 29.4 | 26.5 | 23.2 | 51.0 | 40.2 | 31.7 |
| | Skewed | 0.05 | 19.2 | 16.7 | 12.3 | 37.4 | 28.7 | 22.1 |
| | | 0.10 | 28.6 | 25.2 | 21.1 | 49.1 | 38.9 | 31.8 |

Table 34: Percentage of tests rejecting normality of the random slope using two rotations and $s = 45$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | $\sigma^2_\varepsilon = 4,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | |
| Normal | Normal | 0.05 | 5.1 | 5.2 | 5.7 | 4.4 | 4.1 | 5.2 |
| | | 0.10 | 9.8 | 10.3 | 9.5 | 8.9 | 9.1 | 9.9 |
| | Heavy tailed | 0.05 | 10.0 | 9.7 | 8.1 | 15.3 | 14.6 | 11.6 |
| | | 0.10 | 16.9 | 15.9 | 13.1 | 23.0 | 21.1 | 18.7 |
| | Skewed | 0.05 | 6.8 | 6.4 | 6.2 | 12.3 | 11.1 | 8.2 |
| | | 0.10 | 12.3 | 11.5 | 12.5 | 18.6 | 16.2 | 15.0 |
| Heavy tailed | Normal | 0.05 | 9.6 | 9.5 | 9.0 | 12.0 | 10.0 | 9.3 |
| | | 0.10 | 16.6 | 15.1 | 14.5 | 18.2 | 16.8 | 15.6 |
| | Heavy tailed | 0.05 | 16.7 | 14.9 | 11.8 | 30.5 | 27.3 | 22.2 |
| | | 0.10 | 23.4 | 21.8 | 18.4 | 39.6 | 36.3 | 32.0 |
| | Skewed | 0.05 | 11.4 | 10.9 | 7.9 | 21.2 | 17.7 | 13.6 |
| | | 0.10 | 17.9 | 16.5 | 14.8 | 30.2 | 26.8 | 21.3 |
| Skewed | Normal | 0.05 | 6.2 | 6.1 | 6.2 | 8.4 | 7.6 | 5.5 |
| | | 0.10 | 13.4 | 12.7 | 11.6 | 14.0 | 12.3 | 11.2 |
| | Heavy tailed | 0.05 | 14.2 | 13.4 | 8.3 | 23.1 | 20.9 | 16.6 |
| | | 0.10 | 21.7 | 18.9 | 16.4 | 31.0 | 27.4 | 23.8 |
| | Skewed | 0.05 | 9.4 | 8.0 | 7.6 | 17.6 | 13.3 | 11.5 |
| | | 0.10 | 15.8 | 15.0 | 13.9 | 26.0 | 22.1 | 19.4 |
| | | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | |
| Normal | Normal | 0.05 | 4.7 | 4.6 | 6.0 | 4.6 | 4.6 | 4.7 |
| | | 0.10 | 9.6 | 10.0 | 9.8 | 10.0 | 9.7 | 9.8 |
| | Heavy tailed | 0.05 | 8.0 | 7.8 | 6.7 | 9.3 | 8.4 | 8.1 |
| | | 0.10 | 12.8 | 12.4 | 11.3 | 15.5 | 14.3 | 13.8 |
| | Skewed | 0.05 | 6.1 | 5.2 | 5.3 | 6.4 | 5.9 | 6.1 |
| | | 0.10 | 11.0 | 10.9 | 10.6 | 12.1 | 11.4 | 11.5 |
| Heavy tailed | Normal | 0.05 | 12.8 | 11.7 | 9.7 | 20.7 | 18.6 | 14.9 |
| | | 0.10 | 19.7 | 18.3 | 15.1 | 29.4 | 26.4 | 24.8 |
| | Heavy tailed | 0.05 | 19.8 | 17.7 | 16.1 | 34.0 | 30.8 | 25.7 |
| | | 0.10 | 29.3 | 26.6 | 21.6 | 42.5 | 39.9 | 34.2 |
| | Skewed | 0.05 | 15.2 | 14.4 | 11.2 | 29.1 | 26.0 | 21.1 |
| | | 0.10 | 22.1 | 20.4 | 18.6 | 37.9 | 33.8 | 29.7 |
| Skewed | Normal | 0.05 | 11.3 | 9.3 | 8.2 | 20.0 | 17.0 | 12.3 |
| | | 0.10 | 18.9 | 16.1 | 14.5 | 29.6 | 24.9 | 20.2 |
| | Heavy tailed | 0.05 | 14.4 | 12.9 | 10.4 | 27.3 | 23.7 | 18.9 |
| | | 0.10 | 22.0 | 19.8 | 17.1 | 36.6 | 30.6 | 26.1 |
| | Skewed | 0.05 | 12.3 | 10.3 | 8.4 | 22.0 | 16.6 | 12.3 |
| | | 0.10 | 20.7 | 17.7 | 15.6 | 30.9 | 26.2 | 19.8 |
| | | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | | |
| Normal | Normal | 0.05 | 4.9 | 5.2 | 6.0 | 5.0 | 4.9 | 5.3 |
| | | 0.10 | 10.2 | 9.7 | 10.5 | 10.6 | 10.9 | 10.2 |
| | Heavy tailed | 0.05 | 5.2 | 4.6 | 4.2 | 5.0 | 5.0 | 5.2 |
| | | 0.10 | 9.8 | 9.1 | 9.7 | 10.2 | 10.4 | 10.5 |
| | Skewed | 0.05 | 4.1 | 4.3 | 4.4 | 4.8 | 5.5 | 5.4 |
| | | 0.10 | 8.1 | 8.6 | 8.8 | 9.5 | 9.8 | 9.5 |
| Heavy tailed | Normal | 0.05 | 20.9 | 18.3 | 14.4 | 35.8 | 32.4 | 26.0 |
| | | 0.10 | 27.7 | 25.5 | 22.5 | 44.6 | 41.3 | 35.9 |
| | Heavy tailed | 0.05 | 25.4 | 23.3 | 17.9 | 43.6 | 40.3 | 32.7 |
| | | 0.10 | 32.5 | 30.9 | 27.1 | 53.2 | 49.5 | 43.2 |
| | Skewed | 0.05 | 26.6 | 23.6 | 18.7 | 44.6 | 40.9 | 32.3 |
| | | 0.10 | 34.3 | 31.1 | 28.1 | 53.4 | 51.1 | 43.6 |
| Skewed | Normal | 0.05 | 19.1 | 16.5 | 13.2 | 32.8 | 27.2 | 19.8 |
| | | 0.10 | 26.3 | 22.5 | 21.0 | 44.2 | 36.3 | 29.9 |
| | Heavy tailed | 0.05 | 19.4 | 17.8 | 14.5 | 35.6 | 27.8 | 21.5 |
| | | 0.10 | 27.6 | 25.5 | 22.0 | 47.3 | 38.4 | 31.3 |
| | Skewed | 0.05 | 17.7 | 14.9 | 11.4 | 36.3 | 27.3 | 20.3 |
| | | 0.10 | 27.6 | 24.3 | 20.3 | 45.6 | 37.4 | 30.7 |

Table 35: Percentage of tests rejecting normality of the random slope using two rotations and $s = 40$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | $\sigma^2_\varepsilon = 4,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 5.7 | 5.5 | 5.4 | 4.5 | 4.4 | 4.3 |
| | | 0.10 | 10.3 | 10.2 | 10.0 | 9.3 | 10.5 | 9.0 |
| | Heavy tailed | 0.05 | 8.4 | 8.9 | 7.7 | 13.6 | 12.1 | 9.6 |
| | | 0.10 | 15.7 | 15.1 | 12.9 | 21.6 | 19.9 | 17.1 |
| | Skewed | 0.05 | 6.3 | 6.5 | 5.7 | 9.5 | 8.4 | 7.4 |
| | | 0.10 | 12.1 | 12.3 | 11.4 | 16.1 | 15.4 | 13.2 |
| Heavy tailed | Normal | 0.05 | 9.8 | 9.2 | 7.6 | 13.6 | 12.6 | 10.5 |
| | | 0.10 | 15.0 | 14.4 | 14.7 | 19.5 | 17.8 | 16.4 |
| | Heavy tailed | 0.05 | 14.5 | 12.8 | 9.8 | 27.6 | 25.0 | 20.0 |
| | | 0.10 | 22.0 | 20.2 | 15.8 | 36.5 | 33.6 | 29.4 |
| | Skewed | 0.05 | 11.2 | 10.0 | 8.4 | 19.1 | 17.1 | 14.2 |
| | | 0.10 | 17.0 | 16.0 | 15.1 | 28.1 | 25.1 | 22.3 |
| Skewed | Normal | 0.05 | 7.3 | 6.0 | 5.3 | 8.9 | 8.2 | 6.9 |
| | | 0.10 | 12.9 | 12.6 | 11.4 | 15.6 | 14.2 | 12.7 |
| | Heavy tailed | 0.05 | 11.7 | 10.1 | 8.7 | 19.5 | 16.9 | 13.7 |
| | | 0.10 | 19.0 | 18.7 | 14.8 | 27.9 | 25.1 | 20.6 |
| | Skewed | 0.05 | 8.4 | 8.4 | 6.7 | 15.3 | 12.2 | 8.1 |
| | | 0.10 | 15.3 | 14.5 | 12.0 | 23.0 | 20.1 | 15.7 |
| | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 1$ | | | | | |
| Normal | Normal | 0.05 | 4.6 | 4.7 | 4.8 | 3.1 | 3.7 | 4.2 |
| | | 0.10 | 8.9 | 9.7 | 9.0 | 9.2 | 8.4 | 8.3 |
| | Heavy tailed | 0.05 | 6.9 | 7.1 | 6.7 | 7.8 | 7.2 | 6.8 |
| | | 0.10 | 13.2 | 12.3 | 11.0 | 15.1 | 13.0 | 12.0 |
| | Skewed | 0.05 | 6.2 | 5.8 | 5.3 | 5.5 | 4.8 | 5.2 |
| | | 0.10 | 10.4 | 10.9 | 10.4 | 11.2 | 11.4 | 9.9 |
| Heavy tailed | Normal | 0.05 | 12.4 | 10.6 | 8.5 | 22.0 | 19.2 | 15.9 |
| | | 0.10 | 18.8 | 16.4 | 14.9 | 30.3 | 28.6 | 25.4 |
| | Heavy tailed | 0.05 | 17.8 | 16.3 | 13.0 | 31.9 | 30.0 | 23.6 |
| | | 0.10 | 25.8 | 24.1 | 20.6 | 40.1 | 37.3 | 32.5 |
| | Skewed | 0.05 | 13.5 | 12.1 | 10.2 | 28.4 | 24.5 | 20.0 |
| | | 0.10 | 21.3 | 18.8 | 17.5 | 36.0 | 33.3 | 28.9 |
| Skewed | Normal | 0.05 | 9.9 | 8.9 | 7.8 | 19.6 | 16.1 | 13.1 |
| | | 0.10 | 18.4 | 16.2 | 14.5 | 27.3 | 22.4 | 20.6 |
| | Heavy tailed | 0.05 | 12.0 | 10.1 | 8.5 | 24.8 | 19.3 | 15.6 |
| | | 0.10 | 20.4 | 18.0 | 14.8 | 34.9 | 30.5 | 23.9 |
| | Skewed | 0.05 | 12.0 | 9.9 | 8.2 | 20.6 | 15.1 | 11.8 |
| | | 0.10 | 18.0 | 15.9 | 13.7 | 27.8 | 23.0 | 19.6 |
| | | | $\sigma^2_\varepsilon = 1,\quad \sigma^2_{b_0} = \sigma^2_{b_1} = 4$ | | | | | |
| Normal | Normal | 0.05 | 5.1 | 5.8 | 5.5 | 5.0 | 5.4 | 5.1 |
| | | 0.10 | 10.3 | 9.5 | 9.9 | 9.8 | 10.0 | 9.7 |
| | Heavy tailed | 0.05 | 5.5 | 4.9 | 5.0 | 5.5 | 5.1 | 4.3 |
| | | 0.10 | 9.8 | 9.5 | 9.3 | 10.0 | 10.3 | 9.2 |
| | Skewed | 0.05 | 4.0 | 3.8 | 3.7 | 5.0 | 4.5 | 6.0 |
| | | 0.10 | 7.8 | 7.3 | 9.0 | 10.2 | 10.6 | 9.4 |
| Heavy tailed | Normal | 0.05 | 20.2 | 18.5 | 14.8 | 34.6 | 30.7 | 24.5 |
| | | 0.10 | 26.6 | 25.0 | 21.3 | 42.8 | 40.1 | 34.1 |
| | Heavy tailed | 0.05 | 22.6 | 21.0 | 16.3 | 39.7 | 36.6 | 30.7 |
| | | 0.10 | 31.7 | 29.0 | 23.8 | 50.0 | 46.6 | 40.2 |
| | Skewed | 0.05 | 24.9 | 21.1 | 17.1 | 40.7 | 38.8 | 31.7 |
| | | 0.10 | 32.6 | 30.4 | 26.1 | 49.0 | 46.9 | 42.0 |
| Skewed | Normal | 0.05 | 17.2 | 15.7 | 12.3 | 29.4 | 24.7 | 18.5 |
| | | 0.10 | 25.3 | 22.7 | 20.1 | 40.8 | 33.6 | 27.4 |
| | Heavy tailed | 0.05 | 17.7 | 15.9 | 12.0 | 37.0 | 29.2 | 21.0 |
| | | 0.10 | 25.5 | 23.7 | 19.7 | 47.1 | 38.4 | 30.8 |
| | Skewed | 0.05 | 16.3 | 14.8 | 11.8 | 33.3 | 26.4 | 19.3 |
| | | 0.10 | 25.4 | 23.1 | 18.8 | 45.3 | 37.0 | 29.2 |

Table 36: Percentage of tests rejecting normality of the random slope using two rotations and $s = 35$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | $\sigma_\varepsilon^2 = 4,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | |
| Normal | Normal | 0.05 | 5.5 | 5.1 | 5.0 | 4.9 | 5.0 | 4.0 |
| | | 0.10 | 9.7 | 9.6 | 8.7 | 10.6 | 10.6 | 10.0 |
| | Heavy tailed | 0.05 | 9.0 | 8.4 | 6.7 | 12.2 | 11.9 | 7.8 |
| | | 0.10 | 14.4 | 13.5 | 12.2 | 19.4 | 17.1 | 14.8 |
| | Skewed | 0.05 | 5.4 | 5.4 | 4.1 | 7.4 | 6.8 | 5.7 |
| | | 0.10 | 10.0 | 9.6 | 9.1 | 15.0 | 14.3 | 11.9 |
| Heavy tailed | Normal | 0.05 | 9.7 | 9.1 | 7.3 | 12.1 | 10.8 | 8.4 |
| | | 0.10 | 15.6 | 14.2 | 13.4 | 17.6 | 16.7 | 14.8 |
| | Heavy tailed | 0.05 | 12.6 | 11.2 | 8.6 | 25.7 | 22.1 | 18.1 |
| | | 0.10 | 20.1 | 18.0 | 15.0 | 32.9 | 31.2 | 27.8 |
| | Skewed | 0.05 | 10.8 | 9.2 | 7.3 | 16.1 | 14.1 | 11.4 |
| | | 0.10 | 15.0 | 14.2 | 13.1 | 24.2 | 21.4 | 18.4 |
| Skewed | Normal | 0.05 | 7.1 | 6.5 | 5.5 | 9.8 | 8.8 | 7.4 |
| | | 0.10 | 12.3 | 11.3 | 11.3 | 15.5 | 14.2 | 14.1 |
| | Heavy tailed | 0.05 | 11.3 | 9.2 | 7.1 | 17.5 | 14.7 | 11.5 |
| | | 0.10 | 17.1 | 15.8 | 13.9 | 25.6 | 23.2 | 19.6 |
| | Skewed | 0.05 | 7.6 | 7.2 | 6.7 | 13.5 | 11.0 | 9.1 |
| | | 0.10 | 15.1 | 13.7 | 12.2 | 22.7 | 18.4 | 14.2 |
| | | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | | |
| Normal | Normal | 0.05 | 4.2 | 4.2 | 5.1 | 4.1 | 4.3 | 5.5 |
| | | 0.10 | 9.1 | 9.4 | 10.1 | 10.0 | 10.4 | 11.2 |
| | Heavy tailed | 0.05 | 6.6 | 6.5 | 5.8 | 8.2 | 7.9 | 6.6 |
| | | 0.10 | 11.4 | 11.1 | 11.1 | 14.7 | 13.5 | 11.0 |
| | Skewed | 0.05 | 5.3 | 5.6 | 5.0 | 5.7 | 5.1 | 5.7 |
| | | 0.10 | 10.9 | 11.5 | 10.7 | 10.5 | 10.3 | 11.3 |
| Heavy tailed | Normal | 0.05 | 11.4 | 10.8 | 9.7 | 21.8 | 20.3 | 17.4 |
| | | 0.10 | 18.7 | 17.4 | 14.4 | 30.8 | 27.4 | 26.0 |
| | Heavy tailed | 0.05 | 16.1 | 14.7 | 11.9 | 31.5 | 28.4 | 22.9 |
| | | 0.10 | 23.1 | 21.3 | 19.0 | 39.8 | 37.2 | 31.1 |
| | Skewed | 0.05 | 12.1 | 11.4 | 9.3 | 24.8 | 22.3 | 18.8 |
| | | 0.10 | 18.7 | 16.9 | 17.4 | 33.4 | 30.6 | 26.6 |
| Skewed | Normal | 0.05 | 9.9 | 8.7 | 8.0 | 20.8 | 16.5 | 13.2 |
| | | 0.10 | 18.5 | 16.0 | 13.1 | 27.8 | 24.2 | 20.9 |
| | Heavy tailed | 0.05 | 11.2 | 10.1 | 8.3 | 20.7 | 18.5 | 14.2 |
| | | 0.10 | 18.0 | 15.9 | 14.5 | 29.9 | 25.2 | 20.8 |
| | Skewed | 0.05 | 10.2 | 9.9 | 8.2 | 18.7 | 14.4 | 11.2 |
| | | 0.10 | 18.4 | 16.4 | 14.2 | 27.0 | 22.1 | 18.5 |
| | | | | $\sigma_\varepsilon^2 = 1,\quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | | |
| Normal | Normal | 0.05 | 5.1 | 5.1 | 4.6 | 4.0 | 4.4 | 3.9 |
| | | 0.10 | 9.1 | 9.0 | 9.0 | 8.2 | 7.9 | 8.4 |
| | Heavy tailed | 0.05 | 5.7 | 5.0 | 4.7 | 5.2 | 5.5 | 4.3 |
| | | 0.10 | 10.0 | 9.0 | 10.2 | 10.8 | 10.9 | 9.0 |
| | Skewed | 0.05 | 4.6 | 4.4 | 3.9 | 4.8 | 5.2 | 5.5 |
| | | 0.10 | 8.3 | 8.3 | 9.1 | 10.0 | 9.7 | 10.9 |
| Heavy tailed | Normal | 0.05 | 18.9 | 18.0 | 13.9 | 33.7 | 30.2 | 25.3 |
| | | 0.10 | 25.8 | 24.7 | 20.6 | 42.2 | 39.7 | 35.4 |
| | Heavy tailed | 0.05 | 21.2 | 18.9 | 15.1 | 36.7 | 33.4 | 27.7 |
| | | 0.10 | 28.1 | 25.7 | 23.9 | 45.9 | 43.0 | 36.3 |
| | Skewed | 0.05 | 21.6 | 19.5 | 15.3 | 38.7 | 36.0 | 29.3 |
| | | 0.10 | 30.9 | 29.1 | 23.4 | 47.2 | 44.4 | 39.7 |
| Skewed | Normal | 0.05 | 15.5 | 14.8 | 12.1 | 29.3 | 23.3 | 17.8 |
| | | 0.10 | 25.3 | 22.5 | 19.0 | 38.8 | 31.8 | 27.0 |
| | Heavy tailed | 0.05 | 17.0 | 15.1 | 11.6 | 33.0 | 26.0 | 19.6 |
| | | 0.10 | 26.1 | 23.5 | 20.1 | 43.2 | 36.6 | 28.7 |
| | Skewed | 0.05 | 16.2 | 13.2 | 11.7 | 31.6 | 25.8 | 18.2 |
| | | 0.10 | 23.3 | 20.6 | 17.6 | 43.1 | 36.0 | 28.2 |

Table 37: Percentage of tests rejecting normality of the random slope using two rotations and $s = 30$.

| Distributions | | Nominal | Rotation | | | Varimax rotation | | |
|---|---|---|---|---|---|---|---|---|
| Random effects | Errors | $\alpha$ | AD | CVM | KS | AD | CVM | KS |
| | | | | | $\sigma_\varepsilon^2 = 4, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | |
| Normal | Normal | 0.05 | 4.7 | 5.1 | 4.1 | 5.3 | 4.7 | 4.5 |
| | | 0.10 | 9.6 | 10.9 | 9.2 | 11.5 | 11.2 | 10.2 |
| | Heavy tailed | 0.05 | 8.3 | 7.2 | 5.7 | 10.8 | 8.7 | 8.9 |
| | | 0.10 | 12.2 | 11.8 | 10.6 | 17.4 | 15.3 | 14.9 |
| | Skewed | 0.05 | 5.6 | 5.1 | 5.3 | 7.3 | 6.8 | 5.7 |
| | | 0.10 | 10.5 | 10.1 | 9.9 | 14.2 | 12.9 | 11.6 |
| Heavy tailed | Normal | 0.05 | 9.4 | 9.0 | 7.8 | 11.7 | 10.6 | 8.3 |
| | | 0.10 | 14.9 | 13.9 | 14.6 | 17.5 | 17.0 | 15.0 |
| | Heavy tailed | 0.05 | 13.6 | 10.9 | 8.9 | 21.7 | 20.2 | 15.9 |
| | | 0.10 | 19.9 | 18.9 | 17.1 | 28.5 | 27.8 | 23.2 |
| | Skewed | 0.05 | 9.1 | 8.7 | 7.2 | 14.1 | 12.8 | 10.1 |
| | | 0.10 | 16.3 | 14.9 | 13.2 | 22.3 | 19.7 | 17.2 |
| Skewed | Normal | 0.05 | 6.6 | 5.6 | 5.6 | 9.7 | 7.9 | 7.0 |
| | | 0.10 | 13.0 | 11.5 | 10.2 | 15.0 | 14.3 | 12.9 |
| | Heavy tailed | 0.05 | 10.7 | 9.6 | 7.7 | 14.7 | 13.6 | 10.3 |
| | | 0.10 | 16.9 | 16.9 | 14.4 | 23.1 | 20.8 | 17.6 |
| | Skewed | 0.05 | 9.0 | 7.9 | 7.2 | 10.9 | 9.6 | 8.7 |
| | | 0.10 | 15.4 | 14.2 | 12.7 | 19.3 | 16.7 | 15.3 |
| | | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 1$ | | | |
| Normal | Normal | 0.05 | 3.8 | 4.3 | 5.1 | 5.5 | 5.6 | 4.8 |
| | | 0.10 | 9.3 | 10.1 | 11.7 | 10.7 | 11.3 | 10.5 |
| | Heavy tailed | 0.05 | 6.3 | 6.2 | 5.5 | 7.3 | 6.6 | 5.7 |
| | | 0.10 | 12.1 | 12.1 | 11.2 | 13.2 | 11.8 | 11.5 |
| | Skewed | 0.05 | 5.0 | 4.7 | 4.9 | 5.3 | 5.7 | 5.2 |
| | | 0.10 | 10.4 | 9.9 | 10.4 | 11.6 | 11.6 | 9.9 |
| Heavy tailed | Normal | 0.05 | 11.4 | 10.6 | 9.7 | 20.4 | 19.0 | 15.1 |
| | | 0.10 | 18.7 | 18.2 | 15.2 | 27.6 | 24.9 | 23.2 |
| | Heavy tailed | 0.05 | 15.7 | 14.9 | 11.5 | 27.2 | 25.6 | 19.8 |
| | | 0.10 | 22.8 | 22.2 | 18.9 | 34.9 | 33.3 | 27.1 |
| | Skewed | 0.05 | 11.2 | 11.1 | 8.5 | 22.9 | 20.3 | 16.9 |
| | | 0.10 | 18.6 | 17.3 | 15.4 | 31.0 | 28.3 | 25.7 |
| Skewed | Normal | 0.05 | 10.0 | 9.6 | 8.0 | 19.1 | 16.7 | 13.0 |
| | | 0.10 | 17.9 | 15.1 | 14.4 | 26.2 | 24.0 | 21.2 |
| | Heavy tailed | 0.05 | 10.4 | 9.0 | 8.8 | 20.4 | 16.6 | 12.8 |
| | | 0.10 | 18.0 | 16.3 | 13.7 | 27.7 | 22.9 | 19.3 |
| | Skewed | 0.05 | 10.7 | 10.0 | 7.2 | 18.5 | 15.1 | 11.8 |
| | | 0.10 | 18.2 | 15.4 | 13.8 | 27.5 | 23.2 | 20.9 |
| | | | | | $\sigma_\varepsilon^2 = 1, \quad \sigma_{b_0}^2 = \sigma_{b_1}^2 = 4$ | | | |
| Normal | Normal | 0.05 | 3.7 | 4.0 | 3.4 | 3.9 | 3.8 | 3.6 |
| | | 0.10 | 8.2 | 8.9 | 8.2 | 8.3 | 8.6 | 8.4 |
| | Heavy tailed | 0.05 | 5.3 | 5.9 | 5.3 | 5.6 | 6.0 | 4.8 |
| | | 0.10 | 11.6 | 11.3 | 11.2 | 11.8 | 11.9 | 10.6 |
| | Skewed | 0.05 | 5.6 | 5.3 | 4.8 | 5.9 | 5.8 | 5.2 |
| | | 0.10 | 9.2 | 9.0 | 10.1 | 10.8 | 11.5 | 10.8 |
| Heavy tailed | Normal | 0.05 | 16.6 | 15.6 | 12.8 | 30.4 | 27.6 | 22.0 |
| | | 0.10 | 24.8 | 21.5 | 19.6 | 38.7 | 35.3 | 31.0 |
| | Heavy tailed | 0.05 | 17.1 | 15.7 | 12.7 | 32.9 | 31.1 | 24.8 |
| | | 0.10 | 25.8 | 22.9 | 20.4 | 40.8 | 38.5 | 34.8 |
| | Skewed | 0.05 | 18.8 | 17.2 | 13.9 | 34.0 | 31.5 | 25.5 |
| | | 0.10 | 28.1 | 26.6 | 20.8 | 41.4 | 40.2 | 34.5 |
| Skewed | Normal | 0.05 | 14.0 | 12.9 | 11.0 | 26.1 | 22.3 | 17.3 |
| | | 0.10 | 22.7 | 20.2 | 17.6 | 37.0 | 30.0 | 25.3 |
| | Heavy tailed | 0.05 | 15.2 | 13.6 | 10.0 | 28.7 | 23.3 | 18.3 |
| | | 0.10 | 23.8 | 20.7 | 17.0 | 38.8 | 31.4 | 26.8 |
| | Skewed | 0.05 | 13.9 | 11.6 | 9.9 | 28.8 | 23.1 | 18.6 |
| | | 0.10 | 21.9 | 19.9 | 17.8 | 38.8 | 33.0 | 28.4 |

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