Department of Physics
Département de Physique

Dark Energy: Fundamental Approach to Challenges in New Cosmology

Zhang, Shu (260944682)

December 6, 2023

Abstract

Starting with very basic modern cosmology: Big Bang theory etc., along with the derivation of a mathematical model of our expanding universe form the very ground up, to trace the existence of a mysterious but important component of the universe: dark energy. And a short discussion of how it would relate to Hubble tension.

1 Introduction

The famous dark energy plays an undoubtfully important role in modern cosmology, and yet we do not have a sufficient explanation either about its origin or its real face. The discovery of it happened not very long ago, and now we have proved it to be the largest component of our universe. If we are fortunate, cosmological measurements and observations over the next decade or more will make sense of it all. Its deep connections to fundamental physics – a new form of energy with repulsive gravity and possible implications for the divergences of quantum theory are definitely something worthy to study. We should start by introducing the modern aspect of cosmology.

2 Model of Universe

2.1 Big Bang model

There is a commonly believed theory of the formation of our universe, the Bing Bang model. Big Bang model describes that our universe originated from a point source of mass and expanded into our present universe, undergoing serval different stages. A simple illustration:Figure 1.

The first stage is inflation, where the universe puffs up faster than the speed of light. The mechanism that drives this expansion is yet unclear but according to causality it must happen: if we look at two opposite directions in the sky we will see their homogenous distribution, but why? If they are so far away from each other, there is no reason that they can communicate with each other. Thus, they must be close together at the very first. The inflation period is such a stage that indicates the universe is primordially a small point mass where anything inside could effectively communicate. Then suddenly, in Planck time scale*, it expands to a considerable size.

What happens next is that energy-produced matter, particularly fundamental particles. A hot, high-density, plasma of matter. As the universe continues to expand, matter cools down, and as they are cooling down, nucleosynthesis begins: protons and neutrons form nuclei. Most of it is Hydrogen nuclei, with a trace of Helium and Lithium. This time the universe is opaque due to free electron scattering†.

The temperature continues to drop, then electrons start to combine with nuclei. This stage is called recombination. When the number density of free electrons drops to a certain point, photons will be able to travel freely, universe is finally transparent to light. At this very moment, we are able to see the ”surface” of the universe, ”inside” the surface is still plasma. Such surface is referred to as the surface of last scattering. In other words, CMB.

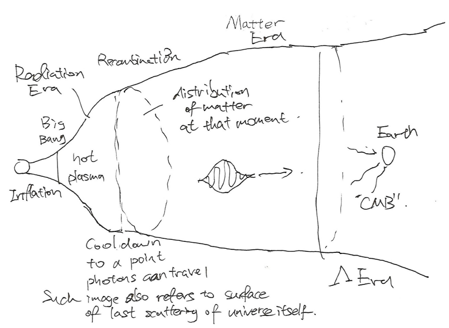

Figure 1: A simple illustration of stages of universe evolution, showing how CMB depicts the distribution of matter.

2.2 CMB

One navie, but intuitive and reasonable, thinking is that if there is a limit to our observable universe, then the background space where there are no stars, or any source

*smallest timescale defined by quantum mechanics

†Particles scatter photon, free electrons contribute to this scattering most (Thomson Scattering) because they have large cross-sectional area

of light, will be completely dark. But sophisticated radio telescope shows conflicting observations, CMB. The cosmic microwave background (CMB) is microwave radiation that we observe at any angle in the sky. It fills all the space of our observable universe. It is a "snapshot" of the universe at the moment it becomes transparent to light. This a great information source of the distribution of matter in the universe. Because there is an anisotropy of the distribution, we could study the frequency associated with it. Thus power spectrum could be plotted, like Figure 2.

2.3 Friedmann Equation

In the previous section I have been indicating that our universe is expanding, this is where a mathematical model, which quantifies the expansion, should be introduced. It is Edwin Hubble first found out that distanced galaxies are moving away from us, and he observed and claimed that the speed of this motion(recession velocity) is linear to the distance to the observer:

$$v_r = H(t)d \tag{1}$$

where the linearity is defined by $H \equiv \frac{1}{R}\frac{dR}{dt}$ the Hubble constant, which, conflicting to what the name suggests, the value is not constant over time as it scales ar the rate of size changing of universe. Further, this could rewritten in scaling factor, where R_0 is the proper coordinates:

$$a(t) = \frac{R(t)}{R_0}$$

Later on, Friedmann contributed to this equation with Einstein's spacetime geometry. If according to general relativity, presence of matter alters the geometry of spacetime in its vicinity, then we could write down the acceleration of space due to Newtonian gravity:

$$\ddot{R} = -\frac{MG}{R^2} = -\frac{(4\pi\rho(t)R^3/3)G}{R^2} = -\frac{4\pi}{3}\rho(t)GR$$

where the density could also be expressed in the scaling relation: $\rho(t) = \frac{1}{R^3}\rho_0$ with that information, rearranging the equation and multiplying both sides by $\dot{R}$:

$$\dot{R}(\ddot{R} + \frac{4\pi}{3}\frac{G\rho_0}{R^2}) = 0$$

Because that $\frac{d(\dot{R}^2)}{dt} = 2\dot{R}\ddot{R}$ and $\frac{d}{dt}(\frac{1}{R}) = \frac{1}{R^2}\dot{R}$. We further rewrite the equation:

$$\frac{1}{2}\frac{d}{dt}(\dot{R}^2) + \frac{4\pi}{3}G\rho_0\frac{\dot{R}}{R^2} = 0,$$

$$\frac{d}{dt}[\dot{R}^2 - \frac{8\pi}{3}\frac{G\rho_0}{R}] = 0.$$

This clearly suggests that if to integrate it over time, we arrive:

$$\dot{R}^2 - \frac{8\pi}{3}\frac{G\rho_0}{R} = K.$$

Where constant K suggests the state of our universe, with $K = 0$ the system being critical, universe is "Flat", energy conserves over time, the expansion will halt at $t = \inf$; $K > 0$ "unbounded" system, free expansion of universe will continue to accelerate; $K < 0$ "bounded" system, the expansion will halt at a finite time, followed with a contraction and collapsing on its own.

Further rewriting the previous equation we reach:

$$(\frac{\dot{R}}{R})^2 - \frac{8\pi G\rho(t)}{3} = -\frac{K}{R^2} \tag{2}$$

Recalling the definition of Hubble constant and using the scaling relation again:

$$H^2(t) \equiv (\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho(t)}{3} - \frac{K}{a^2R_0^2}$$

taking the derivative of time:

$$\frac{d}{dt}(\dot{a}^2 = \frac{8\pi G\rho(t)}{3}a^2 - \frac{K}{R_0^2})$$

$$(\frac{\ddot{a}}{a}) = \frac{4\pi G}{3}(\dot{\rho}\frac{a}{\dot{a}} + 2\rho)$$

Now, at this stage, thermal physics knowledge is needed, recall identity:

$$dE + PdV = \dot{E} + P\dot{V} = 0$$

$$\dot{V} = \frac{d}{dt}(\frac{4\pi}{3}R^3) = \frac{4\pi}{3}R_0^3(3a^2\dot{a}) = 3(\frac{\dot{a}}{a})V$$

and Einstein's famous relation between mass and energy, where ϵ is energy density:

$$E = V\rho c^2 = V\epsilon$$

We can[‡] achieve what Alexander Friedmann derived[1]:

$$(\frac{\ddot{a}}{a}) = -\frac{4\pi G}{3c^2}(\epsilon + 3P) \tag{3}$$

‡the substitution is omitted due length restriction of the page, but the idea behind is simple: take derivative of Einsteins equation, to get $\dot{\rho}$, then replace every ρ in the previous equation

3 Evidence of Dark Energy

With all the information above, we are finally able to peek the face of our universe. By observation, our universe is known to have the following characteristics:

- It is flat(K = 0).(3.4)
- Density distribution suggests an inflation period at the beginning.(2.1)
- Expanding, and the rate of expansion is accelerating(3.3)
- Composition: $\sim 75\%$ dark energy, $\sim 25\%$ matter. Within that 25%, $\sim 20\%$ dark matter, $\sim 5\%$ baryonic matter, $\sim 0.3\%$ neutrino.(3.4)

3.1 "Dark" Particle

In particle physics, a baryon is a type of composite subatomic particle which contains an odd number of valence quarks (at least three). Baryons are also classified as fermions because they have half-integer spin. Thus the proton, neutron and electron are all baryonic matter, while neutrino is not. In our interest, that baryonic matter interacts with light. And non-baryonic matter weakly, or even doesn't interact with photons at all. For example, neutrinos are weakly interacting particles, thus it is very hard for us to detect them. That is the very reason why we grant them the name "dark", if they don't interact with photons, we won't be able to see them.

3.2 Energy or Matter

According to the Friedmann Equation 3 derived previously, and the fact that energy density is given by: $\epsilon = \rho c^2$, the "thing" that drives the expansion of our universe must have negative pressure. And the magnitude of such pressure is in the scale of its energy density. And since this "thing" shows no significant cluster gathering within intergalactic space, it is fair to assume their distribution is relatively uniform. By looking at the ideal gas law

$$PV = nTR$$

normal matter is characterized by $P/c^2 << \rho$. Thus, if the "thing" has pressure which is comparable in magnitude to its energy density, it is more "energy-like" than "matter-like". And the fact that it is not causing matter to gravitationally collapse on it further proves that it is not matter.

3.3 Redshift Measurements

Redshift is the term to describe the situation where an observer receives emission from the source which its wavelength is stretched to be larger. In visual spectrum, a longer wavelength suggests the object is redder[§], hence the name "redshift". The other opposing term will be Blueshift where the wavelength is shorter than at the moment of emission. The principle behind this is the Doppler shift: where the wavelength of emission from a source, which is relatively in motion with the observer, will stretch or compress base on the direction of motion.

In cosmology, there are three main different types of Redshift. The first one is exactly the Doppler Redshift, when objects have a relative motion to us, their wavelength is changed. The next one is gravitational redshift, as general relativity suggests, light path is distorted as the space around massive objects is bent, in this case, the wavelength of light is also changed. The last one is cosmological redshift, which is the one we are particularly interested in to study the expansion of the universe. The fact that space itself also stretches, the light that travels in it will certainly get stretched. We could derive is effect and write it in terms of recessional velocity:

$$z_c = \frac{v_r}{c}$$

Several photometric observations[2] of an apparent Type Ia supernova (SN Ia) at a great distance show a redshift at ~ 1.7[¶]. This supernova, SN 1997ff, was discovered in a repeat observation by the Hubble Space Telescope (HST) of the Hubble Deep Field–North (HDF-N), and serendipitously monitored with NICMOS on HST throughout the Thompson GTO campaign. By monitoring its redshift, we can figure out its change in recessional velocity over time, which can tell us about the rate of expansion of universe. And the results strongly impose that the expansion is **accelerating**.

This discovery is thrilling as firstly we would predict that the expansion will halt for flat universes. But the observation disagrees with that. There is a mysterious

[§] In astrophysics, all observation are things we can see. And light comes in the form of waves. Thus astronomers use red to indicate many scenarios. For stars emitting higher frequency light, they are regarded as "bluer", vice, they are "redder".

[¶] solely looking at the formula I just presented, this number makes no sense as it suggests faster than light expansions. But this is a combination of three different redshifts.

energy that drives our universe to further expand, even to speed up.

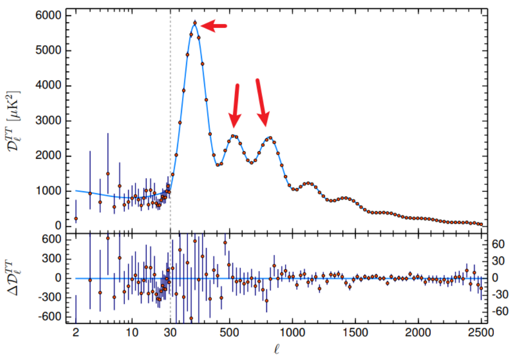

Figure 2: The power Spectrum from Planck's results. Credit to: Planck institution [3]. The peaks in the power spectrum indicate the distribution of the different sources. It could tell us the density of dark energy, baryonic matters and dark matters.

3.4 CMB Surveys

By looking at Equation 2, we could assume that the system is critical (K = 0) for now, then we will have the equation:

$$H^2(t) = \frac{8\pi G\rho(t)}{3}$$

At present time, we will have a critical density $\rho_{c,0}$ that indicates: lower than this number, the universe will collapse on its own:

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} \sim 10^{-29} gcm^{-3}$$

according to the Wilkinson Microwave Anisotropy Probe (WMAP)[4]. Even using a naive way, looking at the local density of a region in the sky, to predict the mass density of the universe, the result would simply disagree with this critical density. Furthermore, we could also derive another relation from Equation 2, if to define a value $\Omega = \frac{\rho}{\rho_c}$, the ratio of density to critical density:

$$1 - \Omega = -\frac{K}{H^2(t)a^2(t)R_0^2} \tag{4}$$

This a better visualization because if $\Omega = 1$, then, we suggest that the right side of the equation is zero, which means that K = 0, we have a flat universe. And the result is indeed that $\Omega = 1.0 \pm 0.04$[5]. Now we have a flat universe, energy and mass are conserved over time, matter density and energy density must sum to the critical density. However, based upon measurements of CMB anisotropy, bulk flows, and the baryonic fraction in clusters(shown in Figure 2), matter only contributes about 1/3 of the critical density $\Omega_M = 0.33 \pm 0.04$[6]. Thus, 2/3 of the critical density is missing. Most of the universe is made of dark energy!

3.5 Einstein' Correction Factor

Mass is the source of the gravitational field and gravity is always attractive in Newtonian physics. However, in general relativity, both energy and pressure source the gravitational field. That the gravity could even be repulsive. This leads to a prediction of the accelerating Universe. But when Einstein derived his theory of general relativity, at that time, the expanding universe was yet an idea unborn. In order to compensate the difference between his results and the flat **but static** universe, he added a "cosmological constant" Λ to what we know as Friedmann equation3:

$$(\frac{\ddot{a}}{a}) = -\frac{4\pi G}{3c^2}(\epsilon + 3P) + \frac{\Lambda}{3}.$$

He quickly discarded the cosmological constant after the discovery of the expansion of the universe‖. This accidentally led to the studies of dark matter, and interestingly became one of the very reasons why our present time is called "Λ Era" instead of the "dark energy Era"1.

3.6 Density Distribution Over Time

In order to escape detection, dark energy must be smoothly distributed; not to interfere with the formation of structure (by inhibiting the growth of density perturbations) the energy density in this component must change more slowly than matter (so that it was subdominant in the past, shown in Figure 1). The pressure associated with dark energy determines how it evolves:

$$\frac{\rho_\Lambda}{\rho_M} \sim (1+z)^{3\omega} \tag{5}$$

where ω is the ratio of the pressure of dark energy to its energy density. But the density of matter is changing faster than this. Thus, our universe used to be dominated by matter instead of dark energy right now. This will be a useful piece of information as it tells us that surveys of the early universe will be a slightly better option for discovering the truth.

‖It is still unclear whether or not Einstein appreciated that his theory predicted the possibility of repulsive gravity

4 Hubble Tension

In the previous section, I showed that there are two approaches to how to derive the Hubble constant from data observations. One this with recessional velocity, then finding out the redshift of the certain light-emitting objects. The other method is looking at the ratio of mass density to the critical density Ω. combining with the scaling ratio to calculate Hubble constant. This leads to the largest, newly raised, challenge in cosmology: Hubble tension. The observations of early universe with redshift and of CMB with mass distribution deviates **a lot** from each other!

4.1 Planck CMB Survey

In Section 3.4 I derived Equation 4, starting from there one could easily find out the relation between total density distribution and the scaling of Hubble constant:

$$H(t) = H_0\sqrt{\Omega_{tot}}$$

And I have proven, or strongly suggested since we haven't actually "seen" any dark energy, that the distribution of universe is roughly 70% of dark energy and 30% of matter, with only a small trace of neutrinos which we could ignore for here because the sim of this paper is not precise derivations. Thus we could break down the total mass density to summation: $\Omega_tot(t) = \Omega_M(t) + \Omega_\Lambda(t)$. Now, it is easy to apply the scaling factor to matter, which is just $\Omega_M(t) = a^{-3}\Omega_{M,0}$, scaling with cubic length. On the other hand, the scaling factor of dark energy is rather known, but from Equation 5, we could get a vague idea of how it evolves with time. According to observation, it is commonly convinced that, $\omega < -0.6$ with 95% certainty. And with that level of confidence in the data, we often take it for $\omega = -1$. In this case, scaling factor is zero order for dark matter, which means dark matter density is **constant** over time. Then, we obtain the equation:

$$H(t) = H_0\sqrt{\Omega_{M,0}a^{-3} + \Omega_{\Lambda,0}}$$

This is neat! which means if we have accurate measurements of present-day density distribution we could calculate H_0. CMB survey from Planck Institution presents a result of[3]:

$$H_0 = 67.4 \pm 0.5 km/s/Mpc.$$

This is great until we look at the early universe behaviour.

4.2 Riess Supernova Redshift

Now it is easy to obtain H_0 for the redshift measurement, as in Equation 1 and the equation of recession velocity derived in Section 3.3:

$$H_0 = \frac{v_r(z)}{d} \simeq \frac{cz}{d}.$$

But this is only **approximation** as the real equations are more sophisticated and complex for reasons I have described also in Section 3.3, the redshift of stars is always a combination of many different factors: gravity, relative motion, etc. However, it is sufficient enough for this paper.

The logic behind this is extremely simple, we only have to measure the distance and redshift of stars, but practically, it is a lot harder for stars with extreme distance, especially when measuring their distance. Because physics distance is always changing due to the expanding universe. We need many calibrations to be able to measure their absolute magnitude**. Astronomers use stars called "standard candles" to calibrate magnitude measurements. They are often variables††. And they are extremely bright. Then we could use them as the "standard" to get the relative magnitude of interested objects. This is not only **a** tool but a sequential process with repetitive operations. Thus, this is the "Cosmic Distance Ladder", we climb up step by step to the desired distance.

With this technique, Riess' team presents their result[7]:

$$H_0 = 73.04 \pm 1.04 km/s/Mpc.$$

which again looks promising on its own. But it is terrifying when align this with Planck's result. They **5 standard deviations** away, making it implausible to reconcile the two by any chance!

4.3 Why?

The simple answer will be nobody knows yet, it is ongoing tension with many different proposals aiming to solve it. But looking back to the derivation I made all along, one could notice that the **biggest assumption** I made, or rather say cosmologists made, is $\omega = -1$! This is an extremely risky assumption. Hence the most popular argument is that this not right: dark energy density is **NOT** constant over time at all.

**Distance measurements are always carried out by comparing the luminosity of stars, or the absolute magnitude. As the only thing we could do is look at the stars and "see" them.

††stars with periodically varying flux due to different reasons

5 Conclusion

Dark energy is a mysterious invisible stuff in our universe. But through observation, we discover that not only it is not trivial but rather important to the present universe, as it contributes to $\sim 75\%$ of critical density. And yet we are not able to detect any of them, how sad! And there is also the famous ongoing conflict in different observations, the Hubble Tension. In a word, dark matter leaves us with a lot of questions that one day we might be able to figure out. Most of them seem to suggest the need for a brand-new concept/cosmological model to revisit the mysterious dark energy. Maybe we are on the edge of a revolutionary change since the last time it happened was more than 30 years ago.

References

[1]A. Friedman, "On the curvature of space", General Relativity and Gravitation **31**, 1991–2000 (1999).

[2]A. G. Riess and P. E. Nugent, "The farthest known supernova: support for an accelerating universe and a glimpse of the epoch of deceleration", The Astrophysical Journal **560**, 49–71 (2001).

[3]N. Aghanim, Y. Akrami, and etl, "Planck2018 results: vi. cosmological parameters", Astronomy amp; Astrophysics **641**, A6 (2020).

[4]C. L. Bennett, M. Bay, and Halpern, "Themicrowave anisotropy probemission", The Astrophysical Journal **583**, 1–23 (2003).

[5]C. Pryke, N. W. Halverson, E. M. Leitch, J. Kovac, J. E. Carlstrom, W. L. Holzapfel, and M. Dragovan, "Cosmological parameter extraction from the first season of observations with the degree angular scale interferometer", The Astrophysical Journal **568**, 46–51 (2002).

[6]M. S. Turner, "The case for [tinf][ital]m[/ital][/tinf] = 0.33 ± 0.035", The Astrophysical Journal **576**, L101–L104 (2002).

[7]A. G. Riess, S. Casertano, W. Yuan, L. M. Macri, and D. Scolnic, "Large magellanic cloud cepheid standards provide a 1 foundation for the determination of the hubble constant and stronger evidence for physics beyond cdm", The Astrophysical Journal **876**, 85 (2019).