3D Simulations Support Stalled Convective Mixing of Jupiter's Interior

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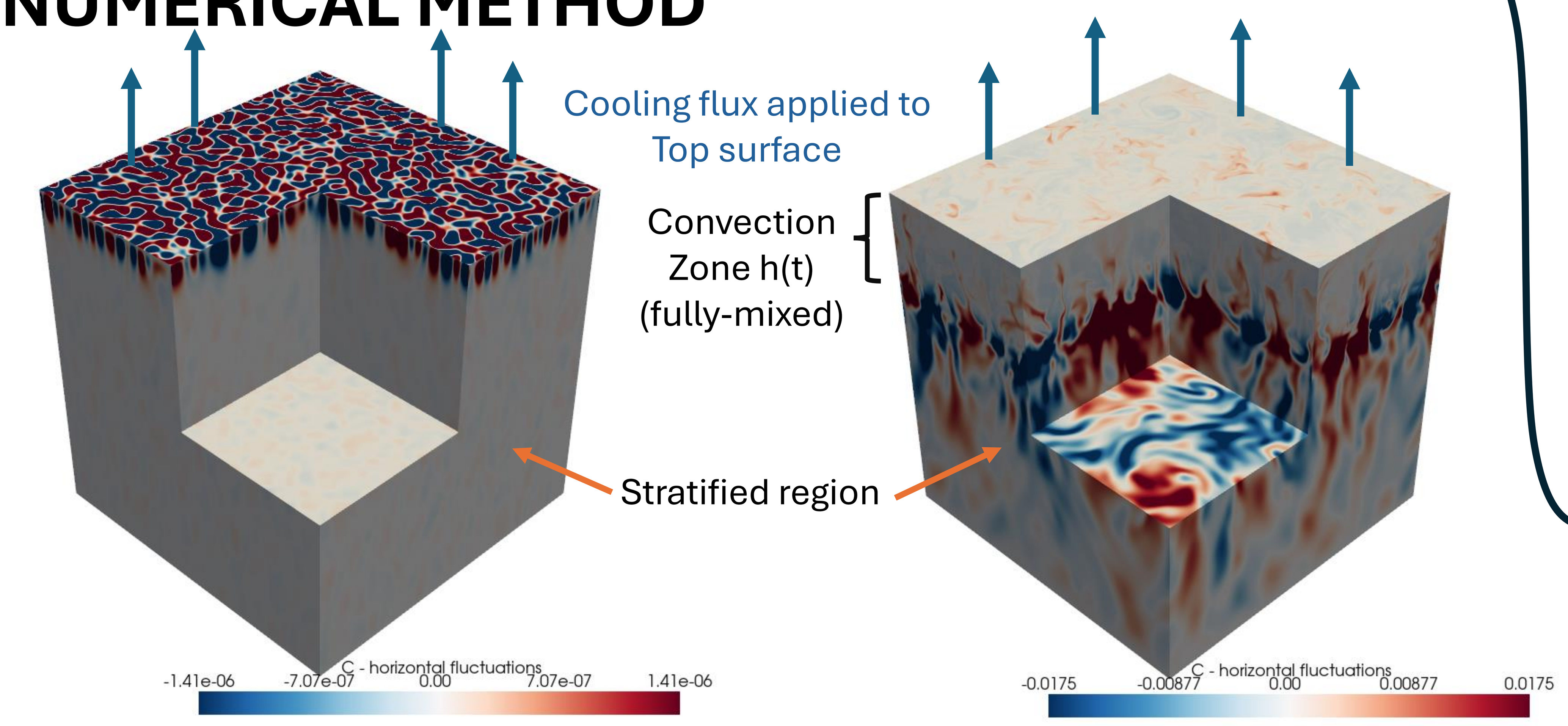
NUMERICAL METHOD



We simulate entrainment and the penetration of a convection zone into a stably-stratified region, using Dedalus [4]. In doing so, we adopt the Boussinesq approximation, where density variations are neglected except in the buoyancy term ($\rho = \rho_0(\beta S - \alpha T)$). The domain is a 3D box with equal dimensions H, periodic boundary conditions in the horizontal directions, and impermeable, stress-free boundaries in the vertical direction. Convection is driven by imposing a cooling flux at the top boundary.

Boussinesq equations in non-dimensionalized form describe the flow:

$$\nabla \cdot \tilde{u} = 0$$

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} + (\tilde{u}\cdot\nabla)\tilde{u} = -\nabla P + \mathrm{Pr}\cdot\mathcal{R}(\tilde{T} - \tilde{S})\hat{z} - \left(\frac{\mathrm{Pr}}{\mathrm{Ek}}\right)\tilde{u}\times\hat{z} + \mathrm{Pr}\nabla^2\tilde{u}$$

$$\frac{\partial \tilde{S}}{\partial \tilde{t}} + (\tilde{u}\cdot\nabla)\tilde{S} = \tau\nabla^2\tilde{S}$$

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} + (\tilde{u}\cdot\nabla)\tilde{T} = \tau\nabla^2\tilde{T}$$

Parameters:

$$\mathcal{R} = \frac{g\beta H^3 S_0}{k_T \nu} = 2\cdot 10^9$$

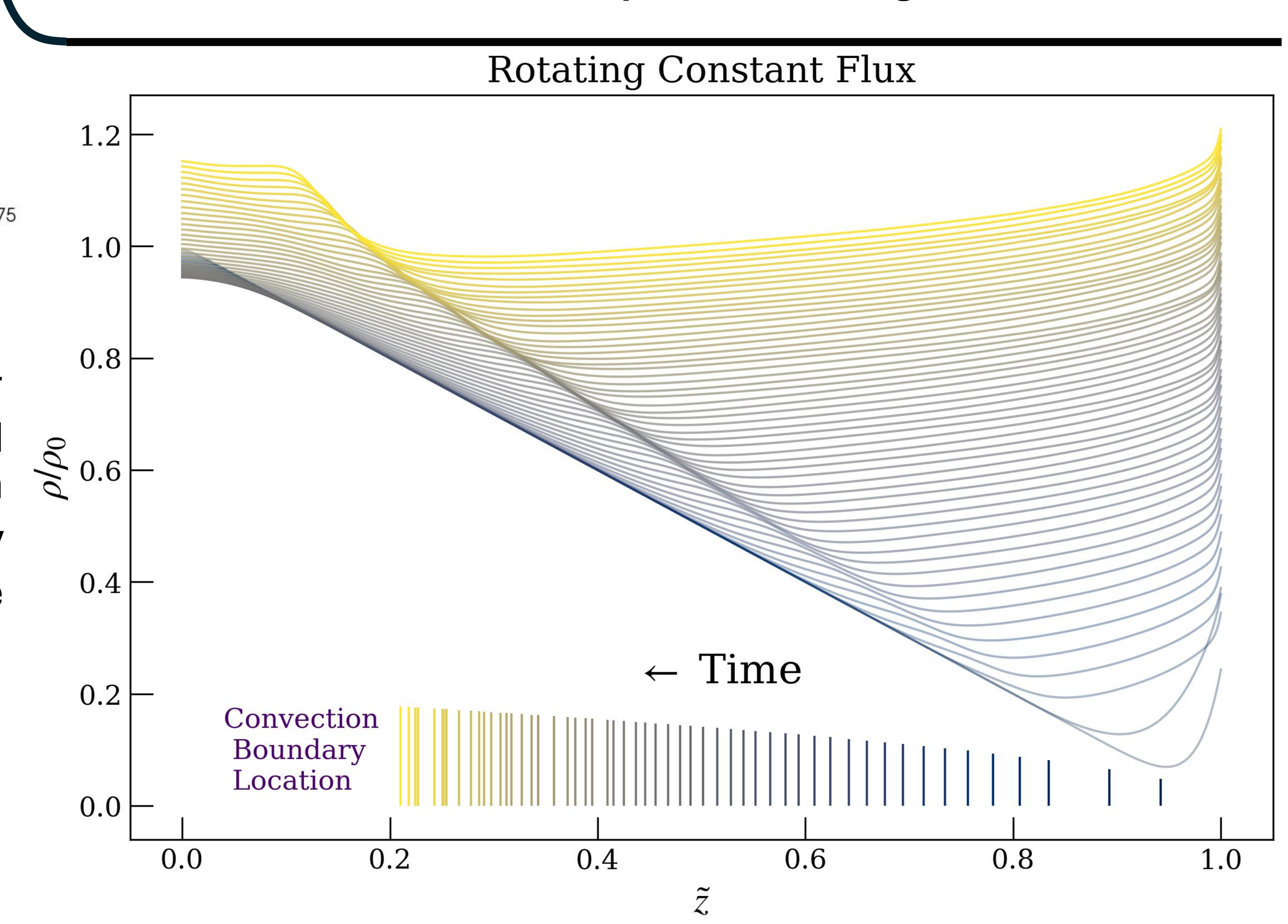
$$\mathrm{Pr} = \frac{\nu}{k_T} = 0.1$$

$$\tau = \frac{k_S}{k_T} = 0.1$$

$$\mathrm{Ek} = \frac{\nu}{2\Omega H^2} = 3\cdot 10^{-6}$$

MOTIVATION

Observations of Jupiter's gravitational moments by Juno imply that, instead of a distinct dense core surrounded by fully mixed light elements, Jupiter still possesses a stably-stratified core [1]. What prevents the primordial concentration gradient from mixing by convection is not understood. Without composition gradients, the convection zone would mix the entire planet in less than a million years [2]. Hindman & Fuentes (2023) used an analytic entrainment model to show that rapid rotation combined with the dwindling surface cooling flux of Jupiter would stall the growth of the convection zone [3]. **Here we test these analytic predictions by carrying out 3D simulations with both a time-dependent cooling flux and rotation.**

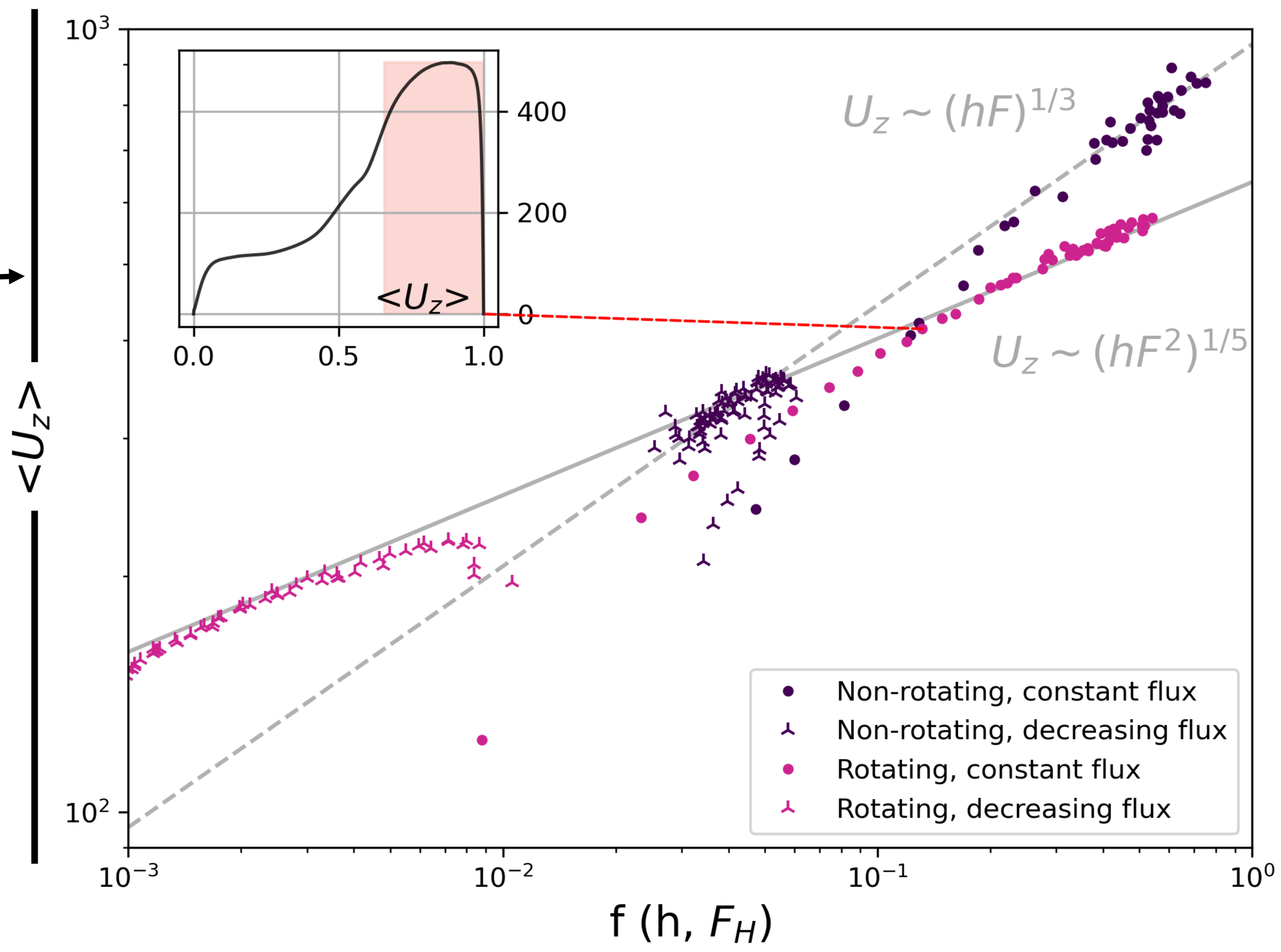


ANALYTIC MODEL

Hindman & Fuentes [1] estimated the convective velocity using mixing-length theory

Non-rotating: $U_{NR} \sim \left(\frac{g\alpha}{\rho_0 c_P}\right)^{1/3}(hF)^{1/3}$

Rotating: $U_R \sim \left(\frac{g\alpha}{\rho_0 c_P}\right)^{2/5}\left(\frac{hF^2}{2\Omega}\right)^{1/5}$



With the assumption that the increase in gravitational potential is equal to the flux of kinetic energy of the flow,

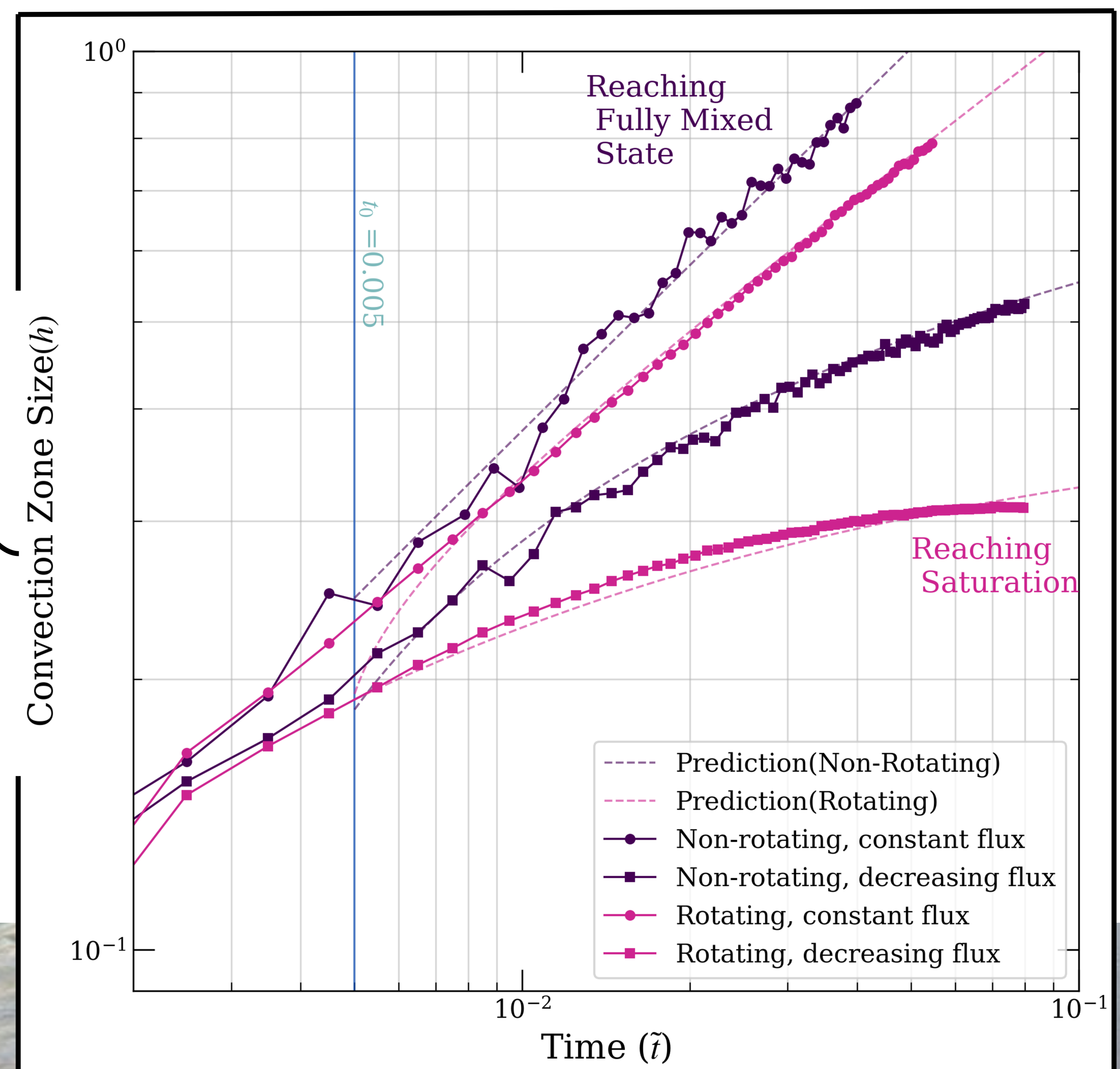
$$g\rho h\frac{dh}{dt} = \gamma U^3,$$

we predict the size of the convection zone $h(t)$ for 4 different configurations:

	Non-rotating	Rotating
Constant Flux $F = F_0$	$\left[(2C)\left(\frac{F_0}{F_c}\right)t\right]^{1/2}$	$\left[(2C)\left(\frac{F_0}{F_c}\right)^{6/5}\left(\frac{\mathcal{R}\cdot\mathrm{Ek}^3}{\mathrm{Pr}^2}\right)^{1/5}t\right]^{5/12}$
Dwindling Flux $F = F_0\left(\frac{t_0}{t_0+t}\right)$	$\left[(2C)\left(\frac{F_0}{F_c}\right)t_0\ln\left\lvert\frac{t+t_0}{t}\right\rvert\right]^{1/2}$	$\left\{(2C)\left(\frac{F_0}{F_c}\right)^{6/5}\left(\frac{\mathcal{R}\cdot\mathrm{Ek}^3}{\mathrm{Pr}^2}\right)^{1/5}t_0\left[1-\left(\frac{t_0}{t_0+t}\right)^{1/5}\right]\right\}^{5/12}$

The flux F_0 is normalized with $F_c = \frac{\rho_0 c_P k_T \beta S_0}{\alpha H}$.

We adopt the efficiency coefficient $C = 1 - \epsilon + 2\gamma$ from the 2D simulations of Fuentes et al. [5]. We take $\boldsymbol{\epsilon = 0.4}$ and $\boldsymbol{\gamma = 0.9}$ based on the values they measured in their simulations. These values give good agreement with the numerical results.



CONCLUSION

Rotation, when combined **with dwindling surface flux**, results in a finite convective mixing depth, potentially explaining Jupiter's diffuse core. **Both** rotation and decreasing flux are essential for the convection zone to stall.

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