Model of White Dwarf’s Magnetic Field Evolution

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2024-04-03

Outline

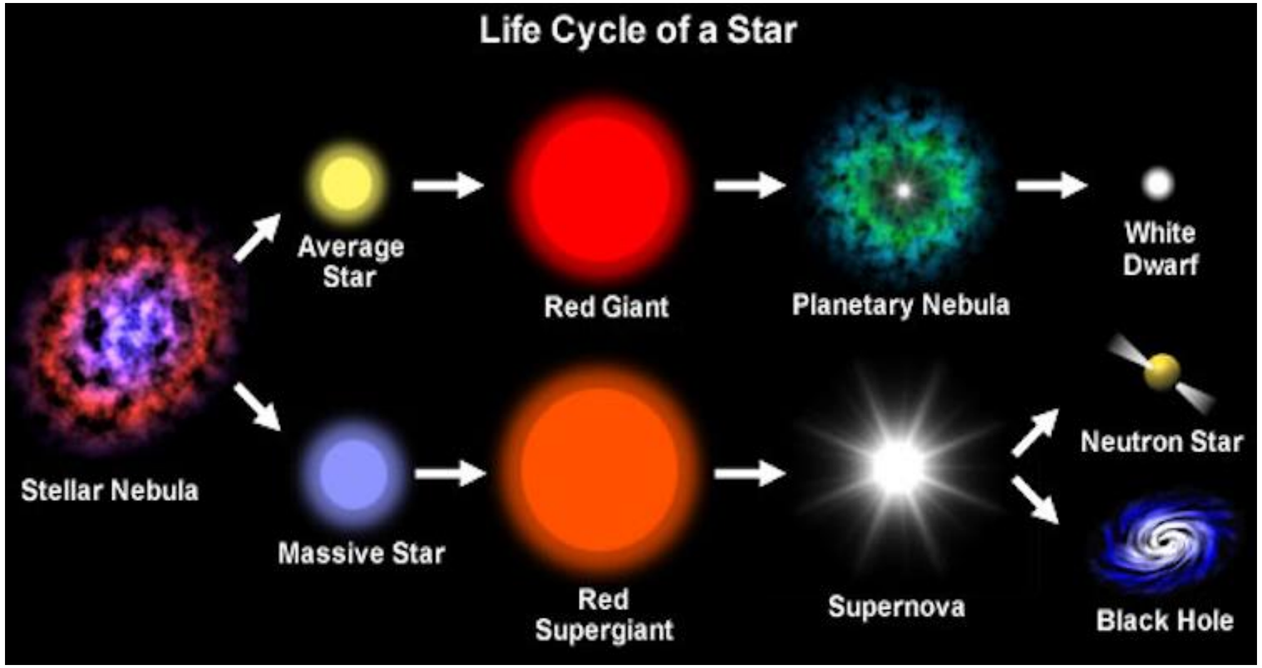
- White Dwarfs
- Observed magnetic field
- Proposed mechanism
- Algorithm

What is a white dwarf?

Remnant of low-mass star
Electron-degenerate matter
Carbon and Oxygen

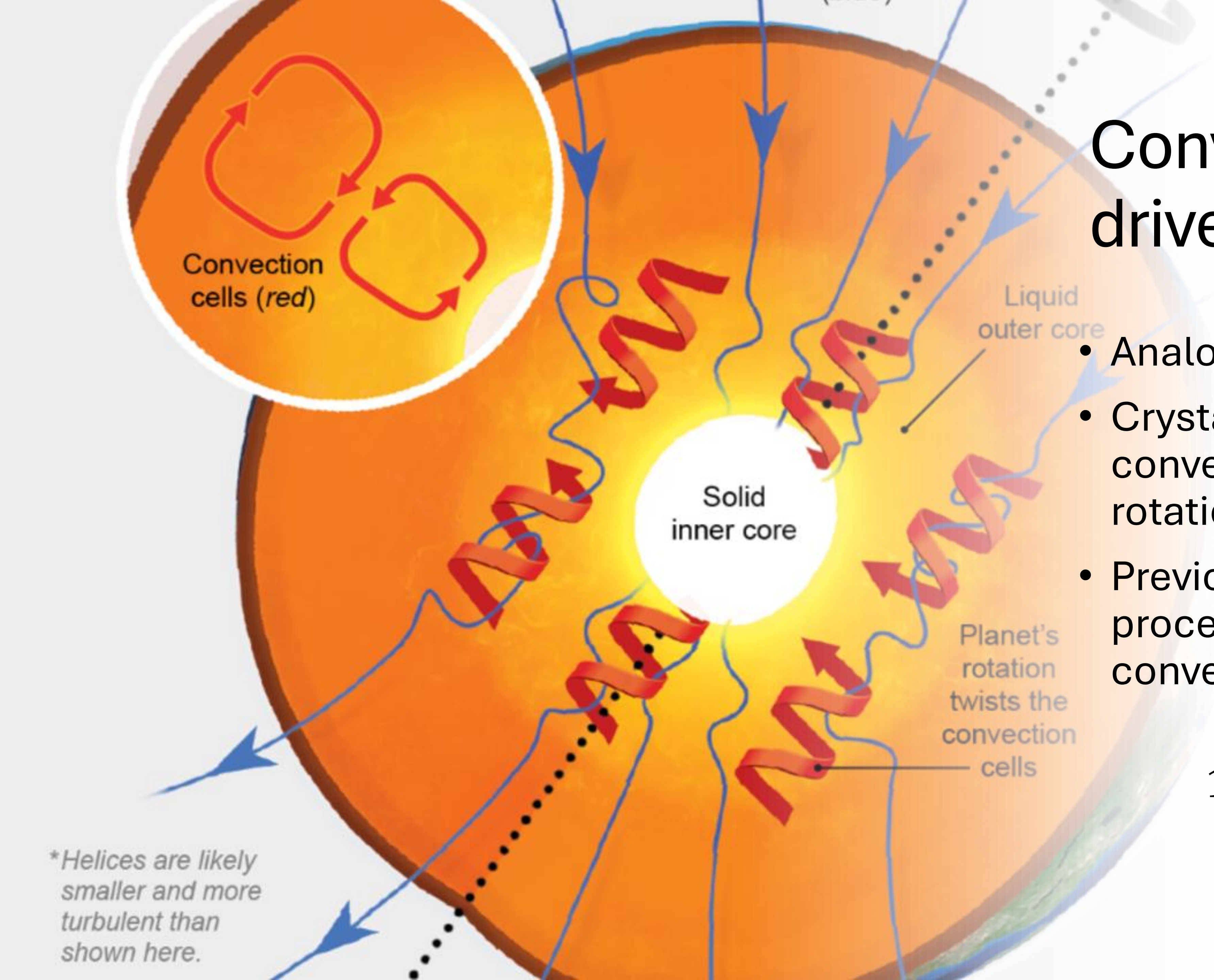
Characteristics

- Thermal emission only
- Mass of sun but size of earth
- Hot but not luminous due to compact size
- Long lifespan $\sim 10^{10}\ yrs$



Mysterious magnetic field

- The split spectral lines indicates white dwarfs' magnetic field
- Observed to have strong magnetic field:
$$10^4 \sim 10^9\ G$$
- The origin of the field is still unclear
- Ideas: convection-driven dynamo?



Convection-driven dynamo

- Analogy to Earth core dynamo
- Crystallization drives convection, combined with rotation creates dynamo
- Previously believed that this process is insufficient with convection velocity:

$$10^{-6} \sim 10^{-5} cm/s$$
$$100\ G$$

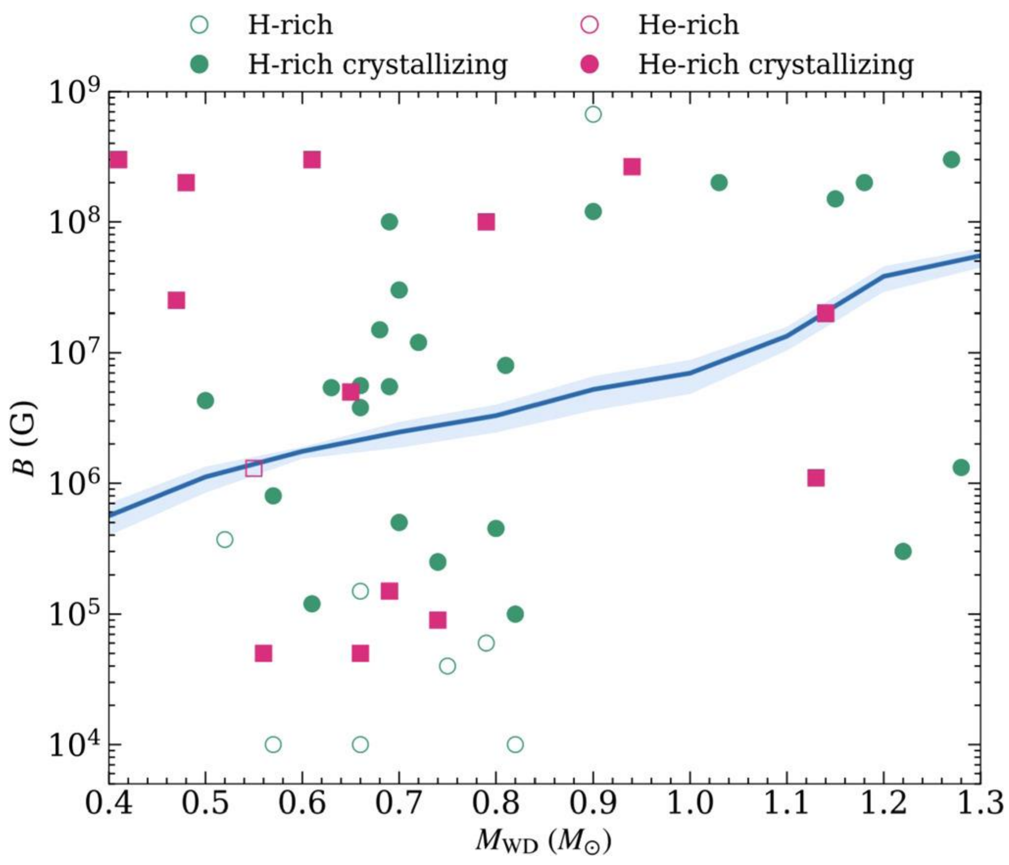
- Strong convection could be possible, but only for a very short time after the onset of crystallization (0.8 to 3 Gyr after birth)

$$10^2 \sim 10^3\ cm/s$$
$$10^6 \sim 10^8\ G$$

- Size of convection zone is still unknown, proposed size [Blatman & Ginzburg (2024)]

$$0.5\ R_{WD} \sim 0.8\ R_{WD}$$

- But initial field doesn't quite match the observation data … …



Fuentes, Castro-Tapia, & Cumming (2024)

Field propagation

With Maxwell's equation

$$\frac{1}{c}\frac{\partial B}{\partial t} = -\nabla \times E$$

$$\frac{4\pi}{c} J = \nabla \times B$$

Ohm's Law

$$J = E\sigma(t)$$

$$\eta = \frac{c^2}{4\pi\sigma}$$

Induction equation

$$\frac{\partial B}{\partial t} = -\nabla \times (\eta(r,t)\nabla \times B)$$

- Assuming an axisymmetric magnetic field
- Translating to vector potential and expanding

$$\vec{B} = \nabla \times \vec{A}$$

$$B_r = \sum_l \frac{l(1+1)}{r^2} R_l(r,t) P_l^1(\cos\theta)$$

- Induction equation reduce to

$$\frac{\partial R_l}{\partial t} = \eta(r,t)\left[\frac{\partial^2 R_l}{\partial r^2} - \frac{l(l+1)R_l}{r^2}\right]$$

Finite differencing

$$\frac{\partial R_l}{\partial t} = \eta(r)\left[\frac{\partial^2 R_l}{\partial r^2} - \frac{l(l+1)R_l}{r^2}\right]$$

$$\frac{\partial R_l}{\partial t} = \frac{R_i^{n+1} - R_i^n}{\Delta t}$$

$$\frac{\partial^2 R_l}{\partial r^2} = \frac{R_{i-1} - 2R_i + R_{i+1}}{(\Delta r)^2} + O(\Delta r)^2$$

Crank-Nicolson method

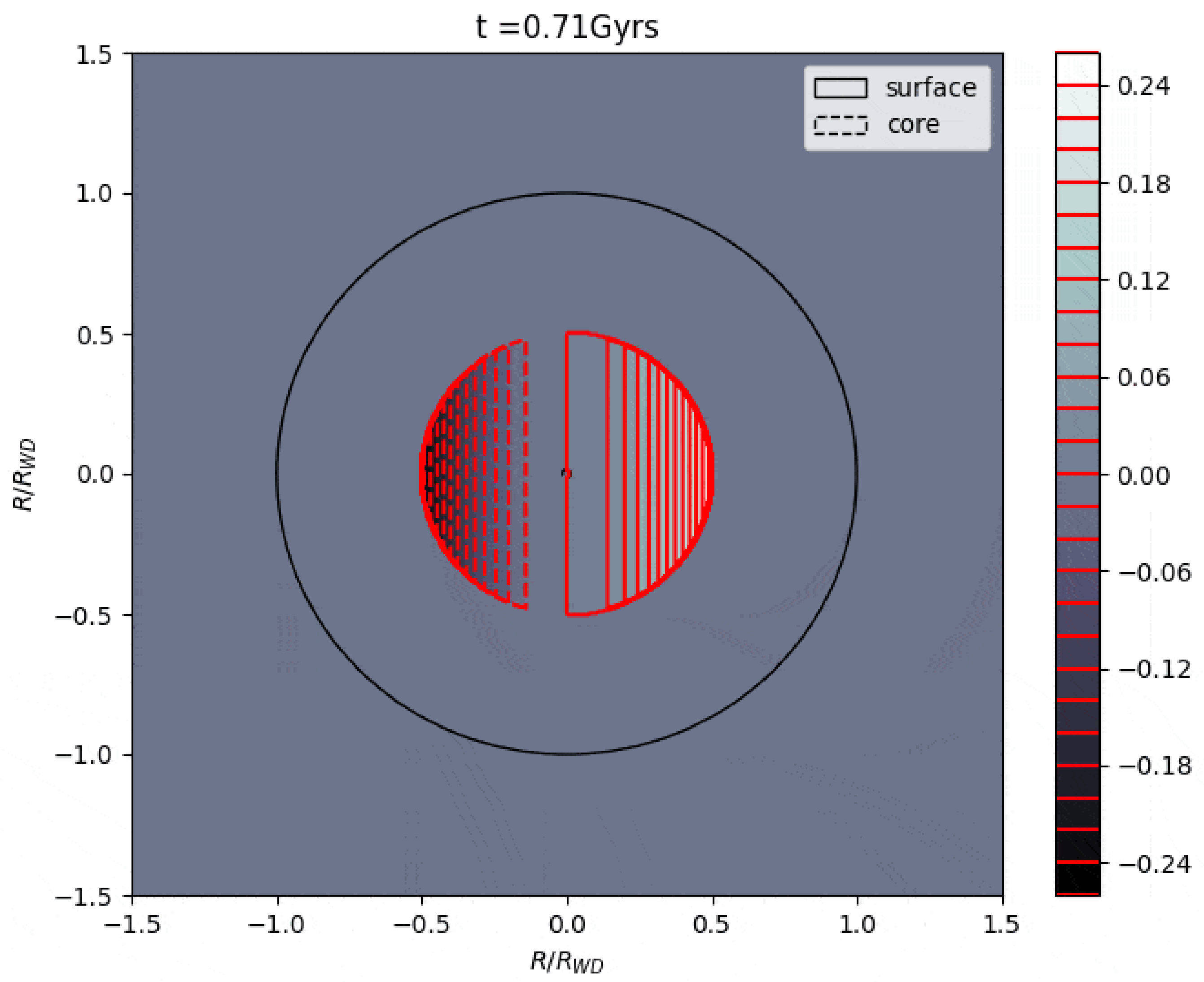
$$\frac{\partial R_l}{\partial t} = \eta(r)\left[\frac{\partial^2 R_l}{\partial r^2} - \frac{l(l+1)R_l}{r^2}\right]$$

$$\frac{R_i^{n+1} - R_i^n}{\Delta t} = \frac{R_{i-1}^n - 2R_i^n + R_{i+1}^n}{(\Delta r)^2}$$

Euler's method

$$\frac{R_i^{n+1} - R_i^n}{\Delta t} = \frac{1}{2}\left[\frac{R_{i-1}^{n+1} - 2R_i^{n+1} + R_{i+1}^{n+1}}{(\Delta r)^2}\right] + \frac{1}{2}\left[\frac{R_{i-1}^n - 2R_i^n + R_{i+1}^n}{(\Delta r)^2}\right]$$

Crank-Nicolson

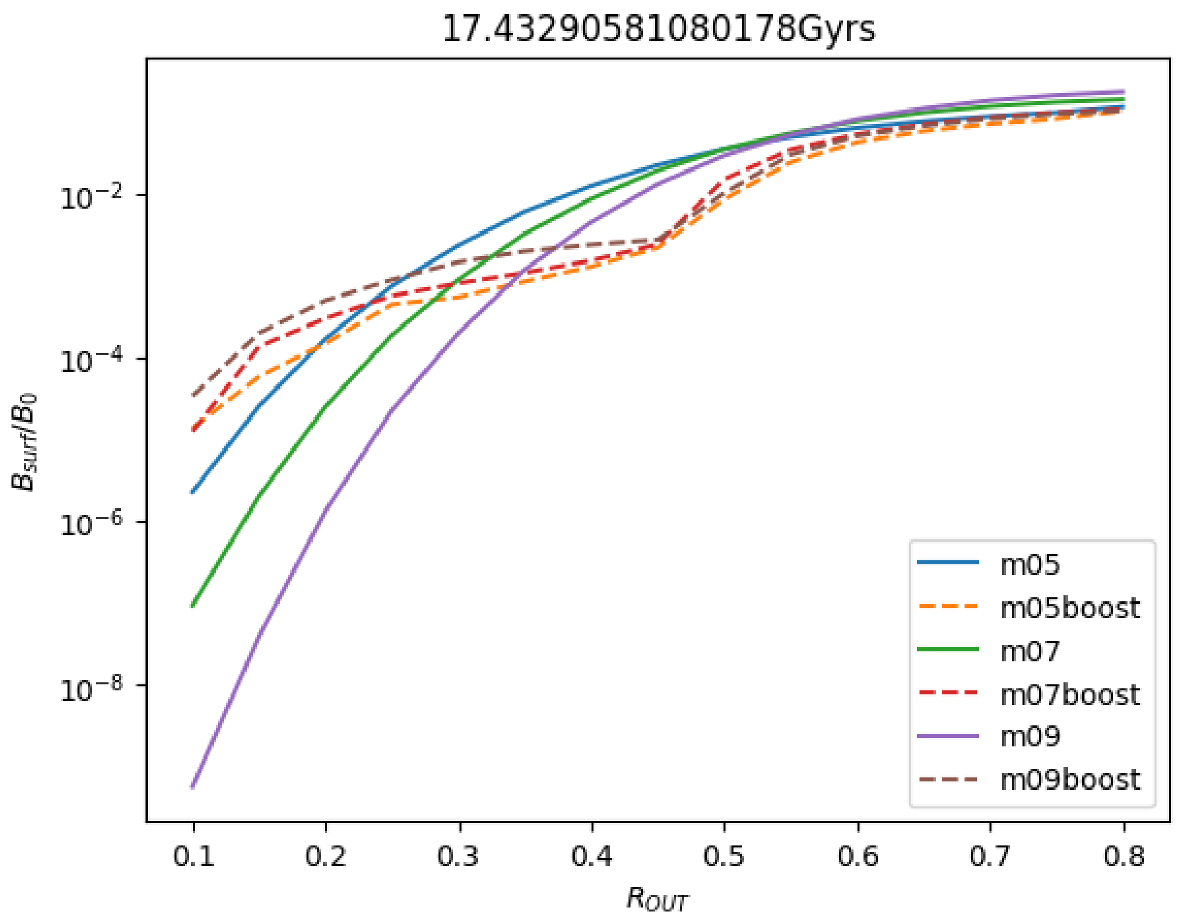

t =0.71Gyrs
surface
core
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
R/R_{WD}
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
R/R_{WD}
0.24
0.18
0.12
0.06
0.00
−0.06
−0.12
−0.18
−0.24

Idea under Development:

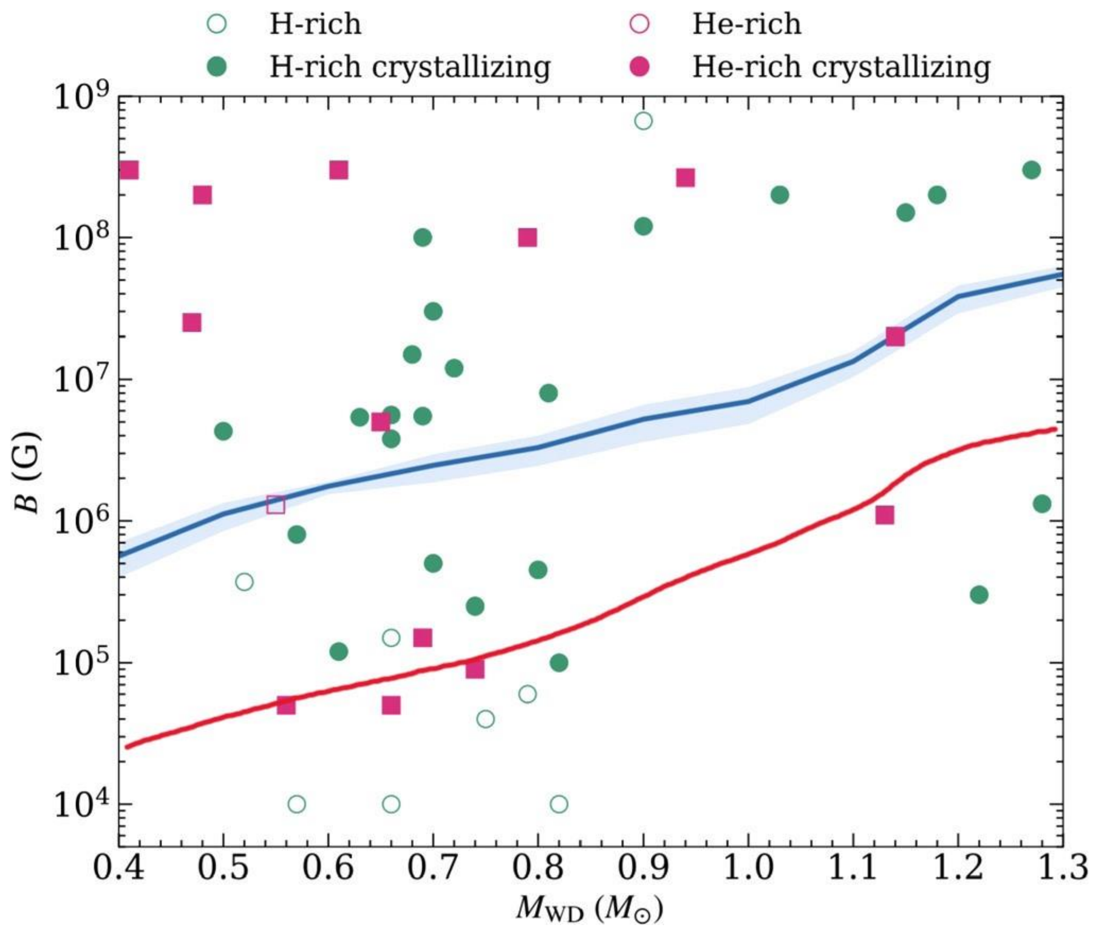
Convection boost:

inefficient convection is still capable of providing some turbulent diffusion, increasing conductivity

- What's the convection velocity …
- How does that change diffusivity…

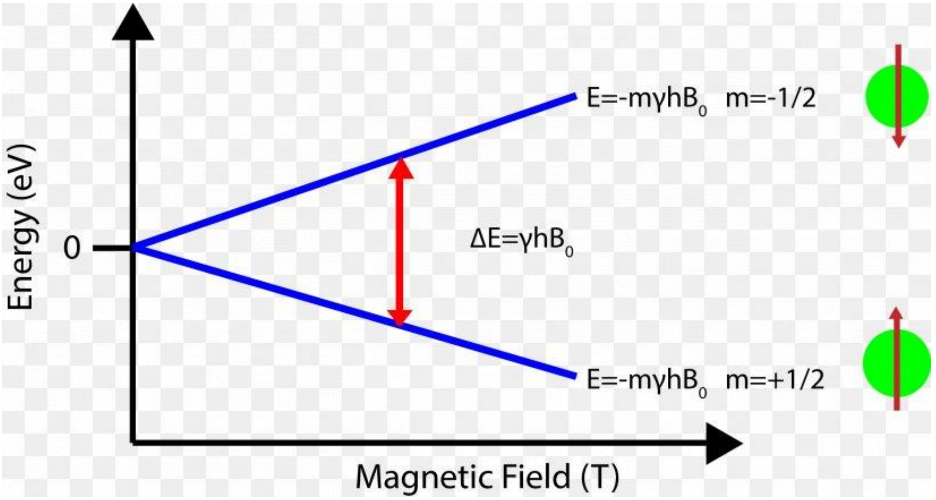


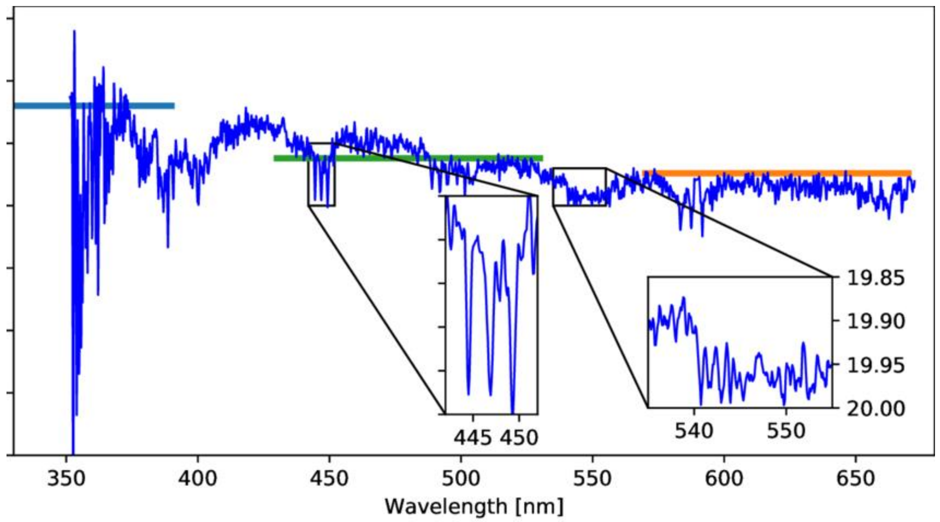
Reduced field strength



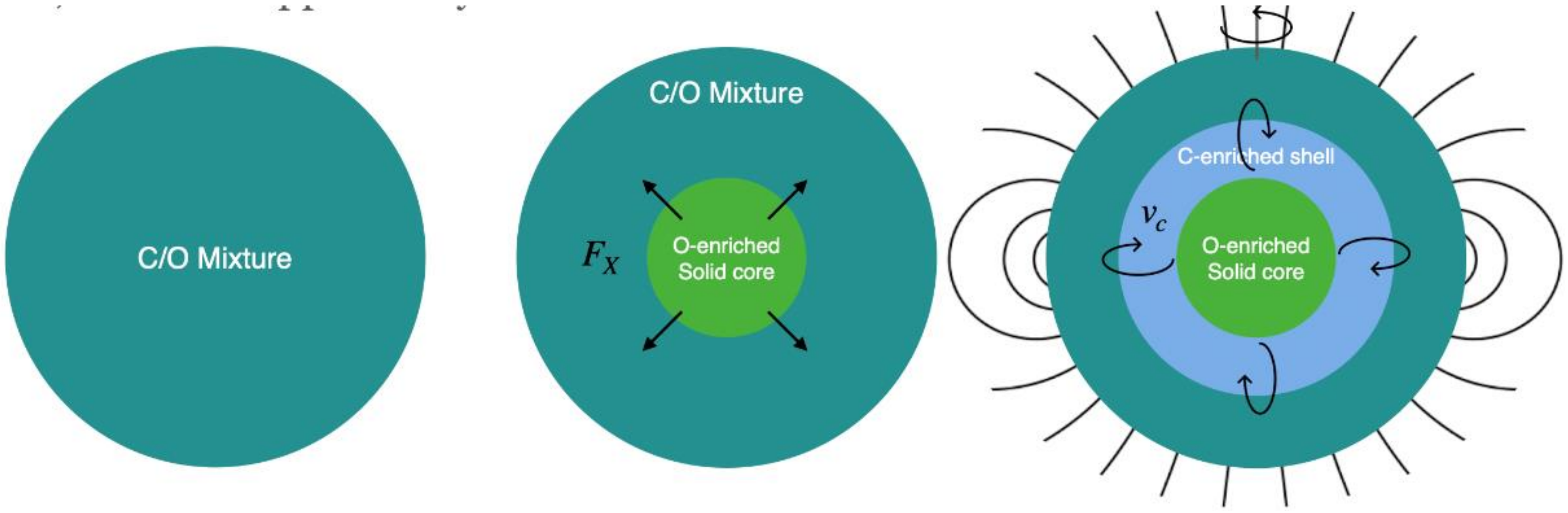
Q&A

- Zeeman effect





- Core convection



Reference

Blatman D., Ginzburg S., 2024, MNRAS, 528, 3153. doi:10.1093/mnras/stae222

Fuentes J. R., Castro-Tapia M., Cumming A., 2024, ApJL, 964, L15. doi:10.3847/2041-8213/ad3100