

Waste

$$Y = X \cdot p(X < L) + (X - L) \cdot p(X > L)$$

$$Y = X \cdot p(X < L) + (X - L) \cdot (1 - p(X < L))$$

$$Y = \cancel{X p(X < L)} + X - L - \cancel{X p(X < L)} + L \cdot p(X < L)$$

$$Y = X - L + L \cdot p(X < L)$$

$$E(Y) = E(X - L + L \cdot p(X < L))$$

$$E(\text{constant}) = \text{constant}$$

$$E(Y) = E(X) - L + L \cdot p(X < L)$$

$$\boxed{\mu_Y = \mu_X - L + L \cdot p(X < L)} \quad \textcircled{I}$$

$$p(X < L) = \int_{-\infty}^L f_X(x) = F_X(x) = \text{CDF}(X)$$

if  $X$  is normally distributed then:

$$\text{CDF}(X) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{X - \mu_X}{\sigma \sqrt{2}} \right) \right)$$

$$\frac{d}{dx} \mu_Y = \frac{d}{dx} (\mu_X - L + L \cdot \text{CDF}(X)) = 0$$

find ~~maximum~~  
minimum

for  ~~$\mu_X = 2$~~   $L = 2 \text{ [m]}$ ,  $\sigma = 0.02 \text{ [m]}$ : the minimum  
is at  $\mu_X = 2.054 \text{ [m]}$