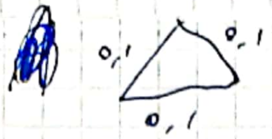
**Ants:**




$\rightarrow$ Probability(All choose 1 or All choose 0)

$$2\left(\frac{1}{2}^3\right) = \frac{1}{4}$$

**Passports:** Starts with 4308, contains 53, 10 digits

- Don't care about first 4 digits, they are set. $\rightarrow$
- Denominator $= 10^{\wedge}10$

| Possibilities | ~~Singles~~ | Doubles |
|---|---|---|
| 53 _ _ _ _ | $\rightarrow 10^4$ | 5353 _ _ |
| _ 53 _ _ _ | $\rightarrow 10^4$ | 53 _ 53 _ |
| _ _ 53 _ _ | $\rightarrow 10^4 - 10^2$ | _ 5353 _ |
| _ _ _ 53 _ | $\rightarrow 10^4 - 10^2 - 10^2$ | 53 _ _ 53 |
| _ _ _ _ 53 | $\rightarrow 10^4 - 10^2 - 10^2 - 10^2$ | _ 53 _ 53 |
| | | _ _ 5353 |

- 535353 appears 3 times, so +1 for over-subtracting

$\rightarrow \dfrac{5(10^4) - 6(10^2) + 1}{10^{10}} = 0.000004$

**Coin:** a) $P(A|B) = \dfrac{P(B|A) \cdot P(A)}{P(B)} = \dfrac{1 \cdot \frac{1}{10}}{\frac{9}{10}\left(\frac{1}{2}\right)^3 + \frac{1}{10}} = 0.4706$

b) ~~$\frac{1}{10}\left(\frac{1}{2}\right) \ldots = 0.55$~~

$$0.4706 + (1 - 0.4706)\frac{1}{2} = 0.735$$

**Family Planning:** Infinite geometric series

$$\sum = \frac{a_1}{1-r} \rightarrow \frac{1/2}{1 - 1/2} = 1, \quad 1:1$$

**Hitchhiker:**

Goal: $1 - P(x=0)$ where $x$ is one period of time

- Periods $= 20/5 = 4$
- 0 cars over all periods $\Rightarrow x^{\wedge}4 = 0.1$, where $x$ = one period
- Probability of 0 cars in one period: $\sqrt[4]{x^4} = \sqrt[4]{0.1} \rightarrow x = 0.1^{1/4}$
- $1 - 0.1^{1/4} = P(\text{At least one car in one period}) = 0.4376$

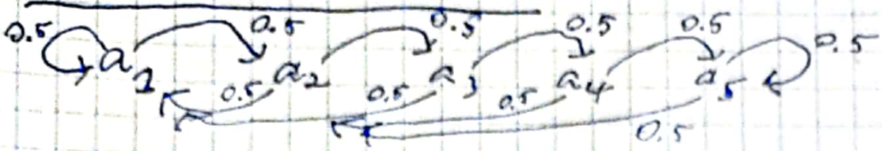
**Sociability:** $P(k) = \dfrac{1}{k^2+k}$, $k \geq 2$ $\rightarrow$ (we know everyone has at least 1 subscriber, so we treat it like zero)

- $P(1) = \frac{1}{1^2+1} = 0.5$, $P(1) - \sum_{k=2}^{\infty} \frac{1}{k_i^2+k_i} = 0.5$
  - If we currently solve for $\sum_{k=2}$, we get 0.5. However, we need $\sum_{k=2}$ to $= 1$
- Therefore, we normalize: $P(1) + \sum_{k=2} \frac{x}{k_i^2+k_i} = 1$, $\sum_{k=2} \frac{x}{k_i^2+k_i} = 0.5 \rightarrow \sum_{k=2}^{\infty} \frac{2}{k^2+k} = 1$

$$\sum_{k=2}^{\infty} \frac{x}{k^2+k} = 1$$

- PMF $= P(k) = \dfrac{2}{k^2+k}$; $k \geq 2$
- CDF $= \sum_{k=2}^{k} \dfrac{2}{k_i^2+k_i}$; $k \geq 2$
  - $\text{CDF}(k) \geq .99 = 199 \rightarrow$ Plug into python or excel
- EV $= \sum_{k=2}^{\infty} k \cdot \text{PMF}(k) = 2 \sum_{k=2}^{\infty} \frac{1}{k+1} = \infty$

# King of the Hill




$a_1 = 0.5(a_1 + a_2 + a_3 + a_4 + a_5) = 0.5$
$a_2 = 0.5a_1 = 0.25$
$a_3 = 0.5a_2 = 0.125$
$a_4 = 0.5a_3 = 0.0625$
$a_5 = 0.5a_4 + 0.5a_5 = 0.0625$

Snail: 3 steps $\{0,1,2,3\}$

- Level 1: Starting corner ($a_1$)
- Level 2: 3 connecting corners to start ($a_2, a_3, a_4$)
- Level 3: 3 connecting corners to level 2 ($a_5, a_6, a_7$), one iteration
- Level 4: End ($a_8$)


$a_1 = 1 + a_2$
$a_2 = 1 + \frac{1}{3}a_1 + \frac{2}{3}a_3$
$a_3 = 1 + \frac{2}{3}a_2$

• Solve for $a_1$:

$a_2 = 1 + \frac{1}{3}a_1 + \frac{2}{3}(1 + \frac{2}{3}a_2)$
$= 1 + \frac{1}{3}a_1 + \frac{2}{3} + \frac{4}{9}a_2$
$a_2 - \frac{4}{9}a_2 = 1\frac{2}{3} + \frac{1}{3}a_1$
$a_2 = 3 + \frac{3}{5}a_1$
$a_1 = 1 + 3 + \frac{3}{5}a_1$
$\frac{2}{5}a_1 = 4$ , $a_1 = 10$ / Expected value of starting from $a_1 = 10$

## Credit Scoring

$P(\text{Bad}/\text{applied}) = \dfrac{P(\text{Applied}/\text{bad}) \cdot P(\text{Bad})}{P(\text{Applied})} = \dfrac{0.8 \cdot 0.05}{(0.8 \cdot 0.05) + (0.95 \cdot 0.3)} = 0.123$

• Updated $P(\text{Bad}) = 0.123$, denoted as $P(\text{Bad}^*)$.

$P(\text{Bad}/\text{repaid}) = \dfrac{P(\text{repaid}/\text{Bad}) \cdot P(\text{Bad}^*)}{P(\text{repaid})} = \dfrac{0.03 \cdot 0.123}{(0.03 \cdot 0.123) + 0.1 \cdot (1 - 0.123)}$

$= 0.0404$