Worms. According to the U.S. Department of Agriculture, ten to twenty earthworms per cubic foot is a sign of healthy soil. The soil of a garden is checked by digging 8 holes, each of one-cubic-foot, and counting the earthworms, and the following counts are found: 5, 25, 15, 10, 7, 12, 16, 20.

1. Use the unbiased estimators to estimate the true mean and the true variance of worm density per cubic foot in this garden.
2. What are the mean and variance *of your estimate of the mean*? And how are they estimated?
3. Make a 95% confidence interval for the true mean (the interval between 2.5% and 97.5% quantiles of the estimated mean).

Blood. Three independent blood samples from the same person revealed levels of sugar in blood equal to 70, 70, and 85 mg/dL. If you believe that blood levels are distributed normally, then what is the smallest interval that will cover with 95% probability sugar level in the next blood sample?

Waiting time. Several time intervals spent in a queue are 1, 1, 2, 3, 5, 8, 10, 21, 38, and 81 minutes. Assuming that this time has exponential distribution, estimate its parameter λ using:

- Equation for the mean (sample mean = expected value)
- Equation for the variance (sample variance = expected value)

Is the difference in estimates of λ large? How can it be explained?

City size. In Russia, town (or a city, there is no formal distinction) is defined as a settlement with at least 9000 citizens (although there are many of exceptions). 1000 towns and cities in Russia actually comply with this definition, and their mean size is 100 000 citizens.

1. Assuming that Russian town sizes have Pareto distribution with lower bound 9000, estimate the shape parameter of this distribution (α) using method of moments (with mean).
2. If our assumptions about the distribution are correct, then how many cities with at least one million citizens should we expect?
3. Using your estimate of α, predict 99.9% quantile of this distribution. Because there are 1000 towns in total, we can expect that this quantile is somewhere between the sizes of the first and the second largest cities.

Uniform moments. Using the method of moments, find the estimators for parameters a and b of continuous uniform distribution on $[a, b]$. Because there are two unknown parameters, you need to solve a system of two equations – for mean and for variance.

- Write out the expressions for these estimators.
- Do you think that they are unbiased? Consistent? Give your arguments.
- What do you think may go wrong with this estimates?
- Apply them to estimate a and b on the sample of 5 observations: 0.8, 0.97, 0.31, 0.69, 0.88. Comment on the result.