■ Supplementary Information

Paper title: Physics-Informed Data-Driven Aggregate Model of District Heating Network for Integrated Energy Optimization

This document, as a supplement to the paper, consists of three sections. The first section gives the derivations of several equations; the remaining two sections separately give the simulation results of Case I and II in detail.

I. DERIVATIONS

A. Derivation of the Aggregate Model (AGM) of Single-Source District Heating Network (DHN)

Assume that the node numbers on the shortest path of the source node k to the load node v are $k, k+1, k+2, \ldots, v-1, v$. The pipe numbers on the path are $k+1, k+2, \ldots, v-1, v$.

According to the node method, the temperature of node $k+1$ can be expressed as:

$$\tau_{s,n}^{k+1,t} = \left(1-\eta_p^{k+1}\right)\alpha_p^{k+1}\tau_{s,n}^{k,t-\gamma_p^{k+1}} + \left(1-\eta_p^{k+1}\right)\left(1-\alpha_p^{k+1}\right)\tau_{s,n}^{k,t-\gamma_p^{k+1}-1} + \eta_p^{k+1}\tau_{amb} \tag{1-1}$$

Assume that

$$\begin{aligned} a_s^{i-1,i,0} &= \left(1-\eta_p^i\right)\alpha_p^i \\ a_s^{i-1,i,1} &= \left(1-\eta_p^i\right)\left(1-\alpha_p^i\right) \end{aligned} \tag{1-2}$$

we have

$$\begin{aligned} \tau_{s,n}^{k+2,t} &= \left(1-\eta_p^{k+2}\right)\alpha_p^{k+2}\tau_{s,n}^{k+1,t-\gamma_p^{k+2}} + \left(1-\eta_p^{k+2}\right)\left(1-\alpha_p^{k+2}\right)\tau_{s,n}^{k+1,t-\gamma_p^{k+2}-1} + \eta_p^{k+2}\tau_{amb} \\ &= a_s^{k+1,k+2,0}\tau_{s,n}^{k+1,t-\gamma_p^{k+2}} + a_s^{k+1,k+2,1}\tau_{s,n}^{k+1,t-\gamma_p^{k+2}-1} + \eta_p^{k+2}\tau_{amb} \\ &= a_s^{k+1,k+2,0}\left(a_s^{k,k+1,0}\tau_{s,n}^{k,t-\gamma_p^{k+1}-\gamma_p^{k+2}} + a_s^{k,k+1,1}\tau_{s,n}^{k,t-\gamma_p^{k+1}-1-\gamma_p^{k+2}} + \eta_p^{k+1}\tau_{amb}\right) \\ &+ a_s^{k+1,k+2,1}\left(a_s^{k,k+1,0}\tau_{s,n}^{k,t-\gamma_p^{k+1}-\gamma_p^{k+2}-1} + a_s^{k,k+1,1}\tau_{s,n}^{k,t-\gamma_p^{k+1}-1-\gamma_p^{k+2}-1} + \eta_p^{k+1}\tau_{amb}\right) \\ &+ \eta_p^{k+2}\tau_{amb} \\ &= a_s^{k+1,k+2,0}a_s^{k,k+1,0}\tau_{s,n}^{k,t-\gamma_p^{k+1}-\gamma_p^{k+2}} \\ &+ \left(a_s^{k+1,k+2,0}a_s^{k,k+1,1} + a_s^{k+1,k+2,1}a_s^{k,k+1,0}\right)\tau_{s,n}^{k,t-\gamma_p^{k+1}-\gamma_p^{k+2}-1} \\ &+ a_s^{k+1,k+2,1}a_s^{k,k+1,1}\tau_{s,n}^{k,t-\gamma_p^{k+1}-\gamma_p^{k+2}-2} \\ &+ \left(1-\left(1-\eta_p^{k+1}\right)\left(1-\eta_p^{k+2}\right)\right)\tau_{amb} \end{aligned} \tag{1-3}$$

Assume that

$$\tau_{s,n}^{k+2,t} = a_s^{k,k+2,0}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+2}} + a_s^{k,k+2,1}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+2}-1} + a_s^{k,k+2,2}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+2}-2} + b_s^{k,k+2}\tau_{amb} \tag{1-4}$$

wherein

$$\begin{aligned} a_s^{k,k+2,0} &= a_s^{k+1,k+2,0}a_s^{k,k+1,0} \\ a_s^{k,k+2,1} &= a_s^{k+1,k+2,0}a_s^{k,k+1,1} + a_s^{k+1,k+2,1}a_s^{k,k+1,0} \\ a_s^{k,k+2,2} &= {}_s^{k+1,k+2,1}\ a_s^{k,k+1,1} \\ b_s^{k,k+2} &= 1-\left(1-\eta_p^{k+1}\right)\left(1-\eta_p^{k+2}\right) \\ \gamma_{agg}^{k,k+2} &= \gamma_p^{k+1} + \gamma_p^{k+2} \end{aligned} \tag{1-5}$$

we have

$$
\begin{aligned}
\tau_{s,n}^{k+3,t} &= a_s^{k+2,k+3,0}\tau_{s,n}^{k+2,t-\gamma_p^{k+3}} + a_s^{k+2,k+3,1}\tau_{s,n}^{k+2,t-\gamma_p^{k+3}-1} + \eta_p^{k+3}\tau_{amb} \\
&= a_s^{k+2,k+3,0}\left(a_s^{k,k+2,0}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+3}} + a_s^{k,k+2,1}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+3}-1} + a_s^{k,k+2,2}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+3}-2} + b_s^{k,k+2}\tau_{amb}\right) \\
&+ a_s^{k+2,k+3,1}\left(a_s^{k,k+2,0}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+3}-1} + a_s^{k,k+2,1}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+3}-2} + a_s^{k,k+2,2}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+3}-3} + b_s^{k,k+2}\tau_{amb}\right) \\
&+ \eta_p^{k+3}\tau_{amb} \\
&= a_s^{k+2,k+3,0}a_s^{k,k+2,0}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+3}} \\
&+ \left(a_s^{k+2,k+3,0}a_s^{k,k+2,1} + a_s^{k+2,k+3,1}a_s^{k,k+2,0}\right)\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+3}-1} \\
&+ \left(a_s^{k+2,k+3,0}a_s^{k,k+2,2} + a_s^{k+2,k+3,1}a_s^{k,k+2,1}\right)\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+3}-2} \\
&+ a_s^{k+2,k+3,1}a_s^{k,k+2,2}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+3}-3} \\
&+ \left(1-\left(1-\eta_p^{k+1}\right)\left(1-\eta_p^{k+2}\right)\left(1-\eta_p^{k+3}\right)\right)\tau_{amb}
\end{aligned} \tag{1-6}
$$

Based on the equations (1-1)-(1-6), we assume that the temperature equation for N_{k+n} is as follows:

$$
\tau_{s,n}^{k+n,t} = \sum_{i=0}^{n}\left(a_s^{k,k+n,i}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+n}-i}\right) + b_s^{k,k+n}\tau_{amb} \tag{1-7}
$$

wherein,

$$
a_s^{k,k+n,i} = \begin{cases} a_s^{k+n-1,k+n,0}\cdot a_s^{k,k+n-1,0} & i=0 \\ a_s^{k+n-1,k+n,0}\cdot a_s^{k,k+n-1,i} & \\ \quad + a_s^{k+n-1,k+n,1}\cdot a_s^{k,k+n-1,i-1} & i=1,\ldots,n-1 \\ a_s^{k+n-1,k+n,1}\cdot a_s^{k,k+n-1,n-1} & i=n \end{cases} \tag{1-8}
$$

$$
b_s^{k,k+n} = 1-\prod_{j\in[1:n]}\left(1-\eta_p^j\right) \tag{1-9}
$$

$$
\gamma_{agg}^{k,k+n} = \sum_{j\in[1:n]}\gamma_p^j \tag{1-10}
$$

When n=1,2,3, equation(1-7)-(1-10) is valid, assuming that the equation is valid when $n = n_p$ ($1 \le n_p \le v-k-1$), then the temperature equation of node $n_p + 1$ can be derived by the node method:

$$
\begin{aligned}
\tau_{s,n}^{k+n_p+1,t} &= a_s^{k+n_p,k+n_p+1,0}\tau_{s,n}^{k+n_p,t-\gamma_p^{k+n_p+1}} + a_s^{k+n_p,k+n_p+1,1}\tau_{s,n}^{k+n_p,t-\gamma_p^{k+n_p+1}-1} + \eta_p^{k+n_p+1}\tau_{amb} \\
&= a_s^{k+n_p,k+n_p+1,0}\left(\sum_{i=0}^{n_p}\left(a_s^{k,k+n_p,i}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+n_p}-\gamma_p^{k+n_p+1}-i}\right)+b_s^{k,k+n_p}\tau_{amb}\right) \\
&+ a_s^{k+n_p,k+n_p+1,1}\left(\sum_{i=0}^{n_p}\left(a_s^{k,k+n_p,i}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+n_p}-\gamma_p^{k+n_p+1}-i-1}\right)+b_s^{k,k+n_p}\tau_{amb}\right) \\
&+ \eta_p^{k+n_p+1}\tau_{amb} \\
&= \sum_{i=0}^{n_p}\left(a_s^{k+n_p,k+n_p+1,0}a_s^{k,k+n_p,i}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+n_p}-\gamma_p^{k+n_p+1}-i}\right)+a_s^{k+n_p,k+n_p+1,0}b_s^{k,k+n_p}\tau_{amb} \\
&+ \sum_{i=0}^{n_p}\left(a_s^{k+n_p,k+n_p+1,1}a_s^{k,k+n_p,i}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+n_p}-\gamma_p^{k+n_p+1}-i-1}\right)+a_s^{k+n_p,k+n_p+1,1}b_s^{k,k+n_p}\tau_{amb} \\
&+ \eta_p^{k+n_p+1}\tau_{amb} \\
&= \sum_{i=0}^{n_p+1}\left(a_s^{k,k+n_p+1,i}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+n_p+1}-i}\right)+\left(1-\eta_p^{k+n_p+1}\right)b_s^{k,k+n_p}\tau_{amb}+\eta_p^{k+n_p+1}\tau_{amb} \\
&= \sum_{i=0}^{n_p+1}\left(a_s^{k,k+n_p+1,i}\tau_{s,n}^{k,t-\gamma_{agg}^{k,k+n_p+1}-i}\right)+b_s^{k,k+n_p+1}\tau_{amb}
\end{aligned} \tag{1-11}
$$

wherein

$$
a_s^{k,k+n_p+1,i} = \begin{cases} a_s^{k+n_p,k+n_p+1,0}\cdot a_s^{k,k+n_p,0} & i=0 \\ a_s^{k+n_p,k+n_p+1,0}\cdot a_s^{k,k+n_p,i} & \\ \quad + a_s^{k+n_p,k+n_p+1,1}\cdot a_s^{k,k+n_p,i-1} & i=1,\ldots,n_p \\ a_s^{k+n_p,k+n_p+1,1}\cdot a_s^{k,k+n_p,n_p} & i=n_p+1 \end{cases} \tag{1-12}
$$

$$
b_s^{k,k+n_p+1} = 1-\prod_{j\in[1:n_p+1]}\left(1-\eta_p^j\right) \tag{1-13}
$$

$$
\gamma_{agg}^{k,k+n_p+1} = \sum_{j\in[1:n_p+1]}\gamma_p^j \tag{1-14}
$$

Obviously, the equation (1-11)-(1-14) still satisfied (1-7)-(1-10). Therefore, on the shortest path of source node k to load node v, equation (1-7)-(1-10) always stand up. Hence, we can get the general expression of AGM of supply network of single source heat supply network.

$$
\tau_{s,l}^{v,t} = \sum_{i=0}^{N_p^{k,v}}\left(a_s^{k,v,i}\tau_{s,src}^{k,t-\gamma_{agg}^{k,v}-i}\right)+b_s^{k,v}\tau_{amb} \quad \forall v\in\Phi_{ln} \tag{1-15}
$$

wherein

$$
a_s^{k,v,i} = \begin{cases} a_s^{v-1,v,0}a_s^{k,v-1,0}, & i=0 \\ \sum_{n=0}^{1}a_s^{v-1,v,n}a_s^{k,v-1,i-n}, & 1\le i\le v-k-1 \\ a_s^{v-1,v,1}a_s^{k,v-1,v-k-1}, & i=v-k \end{cases} \tag{1-16}
$$

$$
b_s^{k,v} = 1-\prod_{j\in\Phi_p^{k,v}}\left(1-\eta_p^j\right) \tag{1-17}
$$

$$
\gamma_{agg}^{k,v} = \sum_{j\in\Phi_p^{k,v}}\gamma_p^j \tag{1-18}
$$

$$
\begin{cases} a_s^{i-1,i,0} = \left(1-\eta_p^i\right)\alpha_p^i \\ a_s^{i-1,i,1} = \left(1-\eta_p^i\right)\left(1-\alpha_p^i\right) \end{cases} \tag{1-19}
$$

B. Derivation of the Constraint of Aggregate Parameters

The AGM for the multi-source DHN is:

$$\tau_{s,l}^{v,t}=\sum_{k\in\Phi_{sn}}\xi_{src}^{k,v}\left(\sum_{i=0}^{N_p^{k,v}}a_s^{k,v,i}\tau_{s,src}^{k,t-\gamma_{agg}^{k,v}-i}+b_s^{k,v}\tau_{amb}\right)\ \forall v\in\Phi_{ln} \tag{2-1}$$

$$\tau_{r,src}^{k,t}=\sum_{v\in\Phi_{ln}}\xi_{l}^{k,v}\left(\sum_{i=0}^{N_p^{k,v}}a_r^{k,v,i}\tau_{r,l}^{v,t-\gamma_{agg}^{k,v}-i}+b_r^{k,v}\tau_{amb}\right)\ \forall k\in\Phi_{sn} \tag{2-2}$$

wherein,

$$\xi_{src}^{k,v}=\frac{m_l^{k,v}}{\sum_{k\in\Phi_{sn}}m_l^{k,v}},\xi_l^{k,v}=\frac{m_{src}^{k,v}}{\sum_{k\in\Phi_{ln}}m_{src}^{k,v}} \tag{2-3}$$

We have:

$$\begin{gathered}\tilde{a}_s^{k,v,i}=\begin{cases}\xi_{src}^{k,v}a_s^{k,v,i-\gamma_{agg}^{k,v}} & 0\le i-\gamma_{agg}^{k,v}\le N_p^{k,v}\\ 0 & others\end{cases}\\ \tilde{a}_r^{k,v,i}=\begin{cases}\xi_{l}^{k,v}a_r^{k,v,i-\gamma_{agg}^{k,v}} & 0\le i-\gamma_{agg}^{k,v}\le N_p^{k,v}\\ 0 & others\end{cases}\end{gathered} \tag{2-4}$$

$$\tilde{b}_s^{v}=\sum_{k\in\Phi_{sn}}\xi_{src}^{k,v}b_s^{k,v},\tilde{b}_r^{k}=\sum_{v\in\Phi_{ln}}\xi_{l}^{k,v}b_r^{k,v} \tag{2-5}$$

$$\tilde{\boldsymbol{a}}_s^{v}=\begin{bmatrix}\tilde{a}_s^{1,v,\Gamma} & \cdots & \tilde{a}_s^{N_s,v,\Gamma}\\ \vdots & \ddots & \vdots\\ \tilde{a}_s^{1,v,0} & \cdots & \tilde{a}_s^{N_s,v,0}\end{bmatrix},\tilde{\boldsymbol{a}}_r^{k}=\begin{bmatrix}\tilde{a}_r^{k,1,\Gamma} & \cdots & \tilde{a}_r^{k,1,\Gamma}\\ \vdots & \ddots & \vdots\\ \tilde{a}_r^{k,1,0} & \cdots & \tilde{a}_r^{k,N_l,0}\end{bmatrix} \tag{2-6}$$

Note that:

$$a_s^{i-1,i,0}+a_s^{i-1,i,1}=1-\eta_p^i \tag{2-7}$$

First of all, we use mathematical induction to prove the following equation in single source DHN:

$$1^{\mathrm{T}}\boldsymbol{a}_s^{k+n}1+b_s^{k+n}=1,\quad n\in[1,v-k] \tag{2-8}$$

When $n=1$, we have:

$$1^{\mathrm{T}}\boldsymbol{a}_s^{k+1}1+b_s^{k+1}=\left(1-\eta_p^{k+1}\right)\alpha_p^{k+1}+\left(1-\eta_p^{k+1}\right)\left(1-\alpha_p^{k+1}\right)+\eta_p^{k+1}=1 \tag{2-9}$$

Assuming when $n=n_p, 1\le n_p\le v-k-1$, the equation (2-8) holds, when $n=n_p+1$,we have:

$$\begin{aligned}1^{\mathrm{T}}\boldsymbol{a}_s^{k+n_p}1+b_s^{k+n_p}&=a_s^{k+n_p,k+n_p+1,0}\left(\sum_{i=0}^{n_p}a_s^{k,k+n_p,i}+b_s^{k,k+n_p}\right)\\ &+a_s^{k+n_p,k+n_p+1,1}\left(\sum_{i=0}^{n_p}a_s^{k,k+n_p,i}+b_s^{k,k+n_p}\right)\\ &+\eta_p^{k+n_p+1}\\ &=a_s^{k+n_p,k+n_p+1,0}+a_s^{k+n_p,k+n_p+1,1}+\eta_p^{k+n_p+1}\\ &=1\end{aligned} \tag{2-10}$$

The proof is complete.

Obviously, when $n=k-v$, the equation still holds, i.e.

$$1^{\mathrm{T}}\boldsymbol{a}_s^{v}1+b_s^{v}=1,\quad \forall v\in\Phi_{sn} \tag{2-11}$$

Then, we promote the equation (2-11) to multi-source DHN below:

$$
\begin{aligned}
1^{\mathrm{T}}\tilde{\boldsymbol{a}}_s^v 1+\tilde{b}_s^v &= \sum_{k\in\Phi_{sn}}\sum_{i=0}^{N_p^{k,v}}\xi_{src}^{k,v}a_s^{k,v,i}+\sum_{k\in\Phi_{sn}}\xi_{src}^{k,v}b_s^{k,v} \\
&= \sum_{k\in\Phi_{sn}}\xi_{src}^{k,v}\left(\sum_{i=0}^{N_p^{k,v}}a_s^{k,v,i}+b_s^{k,v}\right) \\
&= \sum_{k\in\Phi_{sn}}\frac{m_l^{k,v}}{\sum_{k\in\Phi_{sn}}m_l^{k,v}} \\
&= 1
\end{aligned} \tag{2-12}
$$

The proof for the return network is consistent with the above process, so it is omitted here. In summary, we can obtain the following equation:

$$
1^{\mathrm{T}}\tilde{\boldsymbol{a}}_s^v 1+\tilde{b}_s^v=1,\quad \forall v\in\Phi_{sn},\quad 1^{\mathrm{T}}\tilde{\boldsymbol{a}}_r^k 1+\tilde{b}_r^k=1,\quad \forall k\in\Phi_{ln} \tag{2-13}
$$

II. Results of Case I

This section briefly introduces Case I and shows the test results of Case I with analysis. Part A gives the case description. Part B gives the estimation results of aggregate parameters in 6 tests. Part C presents the goodness-of-fit metrics of the aggregate model (AGM) to the original data in 6 tests and makes a brief analysis. Part D gives the comparison between the original data and the model-fitting data.

A. Case Description

The district heating network (DHN) in Case 1 is shown in Fig. 1. The DHN consists of 2 heat sources and 3 heat loads. The training and test data include 400 and 100 samples, respectively, with a resolution of 60 minutes. We perform several tests to verify the proposed method, the settings of which are given in Table I.

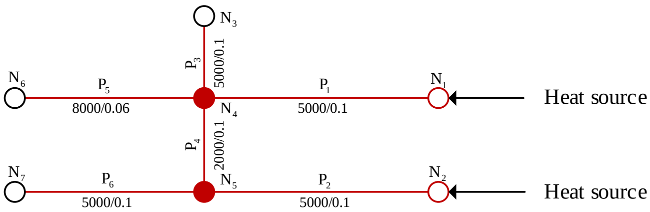


Fig. 1 The DHN structure of case 1.

TABLE I
SETTINGS OF TESTS

Test No.	1	2	3	4	5[a]	6[b]
Standard deviation σ	0	1.0%	1.0%	1.0%	1.0%	1.0%
Proportion of outliers	0	0	10%	20%	10%	10%

Note: [a] excluding normalization constraints; [b] excluding sparsity constraints.

B. Estimation Results of Aggregate Parameters

The aggregate parameters of 3 load nodes and 2 source nodes estimated by the least square estimator (LSE) and Huber M-estimator (HME) are given below. The following conclusions can be drawn from the following experimental results:

1) Compared with LSE, the aggregate parameters estimated by HME are more stable;

2) It can be seen from Test 5 that the removal of normalization could negatively impact aggregate parameters' stability.

TABLE II
AGGREGATE PARAMETERS OF N_3 SOLVED BY HME IN CASE I

Coefficients of $\tau_{s,src}^{1,t-2-i}$

Test No.	$\gamma_{agg}^{1,3}$	i=-2	i=-1	i=0	i=1	i=2	i=3
0	1	/	/	0.017	0.45	0.20	/
1	1	/	0*	0.017	0.45	0.20	/
2	1	/	2.7e-3	0.018	0.44	0.20	/
3	1	/	0.017	0.023	0.46	0.21	/
4	0	0.038	0.030	0.050	0.46	/	/
5	1	/	0.012	0.020	0.45	0.21	/

Test No.	$\gamma_{agg}^{2,3}$	Coefficients of $\tau_{s,src}^{2,t-2-i}$						b_s^k
		i=-1	i=0	i=1	i=2	i=3	i=4	
0	2	/	0*	4.9e-3	0.093	0.11	/	0.12
1	2	/	0*	4.9e-3	0.093	0.11	/	0.12
2	2	/	4.5e-3	0.0067	0.090	0.12	/	0.12
3	1	0.019	0.021	0.016	0.12	/	/	0.12
4	2	/	0.019	0.031	0.11	0.14	/	0.12
5	1	0.015	0.016	0.013	0.11	/	/	0*

Test No.	$\gamma_{agg}^{1,3}$	Coefficients of $\tau_{s,src}^{1,t-3-i}$					
		i=0	i=1	i=2	i=3	i=4	i=5
6	3	0.44	0.20	0*	0*	8.3e-3	1.3e-3

Test No.	$\gamma_{agg}^{2,3}$	Coefficients of $\tau_{s,src}^{2,t-2-i}$						b_s^k
		i=0	i=1	i=2	i=3	i=4	i=5	
6	2	5.2e-3	5.5e-3	0.094	0.11	0*	0.012	0.12

Note: The parameter below 1e-4 is denoted as 0*; "/" denotes the model does not include this parameter, i.e., the default value is 0.

TABLE III

AGGREGATE PARAMETERS OF N_6 SOLVED BY HME IN CASE I

Test No.	$\gamma_{agg}^{1,6}$	Coefficients of $\tau_{s,src}^{1,t-2-i}$					
		i=-2	i=-1	i=0	i=1	i=2	i=3
0	2	/	/	0.38	0.23	0.026	0
1	2	/	/	0.38	0.23	0.026	0*
2	1	/	3.4e-4	0.38	0.22	0.023	/
3	1	/	3.7e-3	0.38	0.22	0.025	/
4	0	6.5e-3	3.7e-3	0.38	0.22	/	/
5	1	/	6.4e-3	0.37	0.22	0.022	/

Test No.	$\gamma_{agg}^{2,6}$	Coefficients of $\tau_{s,src}^{2,t-2-i}$						b_s^k
		i=-1	i=0	i=1	i=2	i=3	i=4	
0	2	/	1.4e-3	0.075	0.11	0.015	/	0.17
1	2	/	1.4e-3	0.075	0.11	0.015	/	0.17
2	2	/	6.9e-3	0.079	0.10	0.016	/	0.17
3	1	6.2e-3	8.2e-3	0.080	0.11	/	/	0.17
4	2	/	5.8e-3	0.079	0.11	0.019	/	0.17
5	1	3.4e-3	5.3e-3	0.078	0.11	/	/	0.094

Test No.	$\gamma_{agg}^{1,6}$	Coefficients of $\tau_{s,src}^{1,t-1-i}$					
		i=0	i=1	i=2	i=3	i=4	i=5
6	1	6.9e-4	0.38	0.22	0.023	5.0e-4	0*

Test No.	$\gamma_{agg}^{2,6}$	Coefficients of $\tau_{s,src}^{2,t-2-i}$						b_s^k
		i=0	i=1	i=2	i=3	i=4	i=5	
6	2	5.5e-3	0.078	0.11	0.012	0*	7.2e-3	0.17

Note: The parameter below 1e-4 is denoted as 0*; "/" denotes the model does not include this parameter, i.e., the default value is 0.

TABLE IV

AGGREGATE PARAMETERS OF N_7 SOLVED BY HME IN CASE I

Test No.	$\gamma_{agg}^{1,7}$	Coefficients of $\tau_{s,src}^{1,t-2-i}$					
		i=-2	i=-1	i=0	i=1	i=2	i=3
0	/	/	/	/	/	/	/
1	1	/	0*	0*	0*	0*	/
2	0	0*	2.9e-3	2.7e-3	0*	/	/
3	0	0*	4.1e-3	1.9e-3	0*	/	/
4	2	/	/	9.5e-3	0*	0*	1.2e-3
5	0	0*	2.4e-3	3.7e-4	0*	/	/

Test No.	$\gamma_{agg}^{2,7}$	Coefficients of $\tau_{s,src}^{2,t-1-i}$						b_s^k
		i=-1	i=0	i=1	i=2	i=3	i=4	
0	1	/	0.012	0.63	0.28	/	/	0.086
1	0	0*	0.012	0.63	0.28	/	/	0.085
2	0	2.8e-3	7.2e-3	0.63	0.27	/	/	0.086
3	1	/	0.010	0.63	0.27	0*	/	0.085
4	2	/	/	0.63	0.28	0*	0*	0.086
5	1	/	8.4e-3	0.62	0.27	0*	/	0.056

Test No.	$\gamma_{agg}^{1,7}$	Coefficients of $\tau_{s,src}^{1,t-0-i}$						
		i=0	i=1	i=2	i=3	i=4	i=5	i=6
6	0	0*	3.5e-3	1.6e-3	0*	0*	1.6e-3	/

Test No.	$\gamma_{agg}^{2,7}$	Coefficients of $\tau_{s,src}^{2,t-1-i}$						b_s^k
		i=0	i=1	i=2	i=3	i=4	i=5	
6	1	9.7e-3	0.62	0.27	0*	0*	0*	0.085

Note: The parameter below 1e-4 is denoted as 0*; "/" denotes the model does not include this parameter, i.e., the default value is 0.

TABLE V
AGGREGATE PARAMETERS OF N_1 SOLVED BY HME IN CASE I

Test No.	$\gamma_{agg}^{1,3}$	Coefficients of $\tau_{r,l}^{3,t-1-i}$					
		i=-1	i=0	i=1	i=2	i=3	i=4
0	2	/	/	0.011	0.30	0.13	/
1	1	/	0*	0.011	0.30	0.13	/
2	1	/	0.030	0.029	0.28	0.14	/
3	1	/	0.028	0.028	0.28	0.14	/
4	0	2.0e-3	0.050	0.032	0.38	/	/
5	1	/	0.028	0.028	0.28	0.14	/

Test No.	$\gamma_{agg}^{1,6}$	Coefficients of $\tau_{r,l}^{6,t-3-i}$					
		i=-1	i=0	i=1	i=2	i=3	i=4
0	2	/	/	0.25	0.15	0.017	/
1	2	/	/	0.25	0.15	0.017	0*
2	1	/	0*	0.21	0.14	0.029	/
3	1	/	0*	0.21	0.15	0.034	/
4	0	0.032	3.2e-3	0.21	0.12	/	/
5	1	/	0*	0.21	0.14	0.035	/

Test No.	$\gamma_{agg}^{1,7}$	Coefficients of $\tau_{r,l}^{7,t-1-i}$						b_r^k
		i=-1	i=0	i=1	i=2	i=3	i=4	
0	1	/	/	/	/	/	/	0.11
1	1	/	0*	0*	0*	0*	/	0.11
2	0	1.9e-3	4.5e-4	0*	0*	/	/	0.11
3	0	7.2e-4	1.2e-3	0*	0*	/	/	0.11
4	2	/	/	0.010	7.8e-3	3.2e-3	0.014	0.11
5	0	1.1e-3	1.5e-3	0*	0*	/	/	0.14

Test No.	$\gamma_{agg}^{1,3}$	Coefficients of $\tau_{r,l}^{3,t-3-i}$					
		i=0	i=1	i=2	i=3	i=4	i=5
6	3	0.28	0.14	0*	0*	0*	5.9e-3

Test No.	$\gamma_{agg}^{1,6}$	Coefficients of $\tau_{r,l}^{6,t-1-i}$					
		i=0	i=1	i=2	i=3	i=4	i=5
6	1	0*	0.24	0.16	0.031	0*	0*

Test No.	$\gamma_{agg}^{1,7}$	Coefficients of $\tau_{r,l}^{7,t-0-i}$							b_r^k
		i=0	i=1	i=2	i=3	i=4	i=5	i=6	
6	0	8.0e-4	5.6e-4	0*	0*	0*	7.2e-3	/	0.13

Note: The parameter below 1e-4 is denoted as 0*; "/" denotes the model does not include this parameter, i.e., the default value is 0.

TABLE VI
AGGREGATE PARAMETERS OF N_2 SOLVED BY HME IN CASE I

Test No.	$\gamma_{agg}^{2,3}$	Coefficients of $\tau_{r,l}^{3,t-2-i}$					
		i=-1	i=0	i=1	i=2	i=3	i=4
0	2	/	0*	2.5e-3	0.046	0.057	/
1	2	/	0*	2.5e-3	0.046	0.057	/
2	2	/	0.012	0*	0.065	0.054	/
3	1	7.9e-3	0.017	0*	0.098	/	/
4	2	/	0.016	0*	0.071	0.054	/
5	1	1.6e-3	0.012	0*	0.090	/	/

Test No.	$\gamma_{agg}^{2,6}$	Coefficients of $\tau_{r,l}^{6,t-2-i}$					
		i=-1	i=0	i=1	i=2	i=3	i=4
0	2	/	6.8e-4	0.038	0.054	7.4e-3	/
1	2	/	6.8e-4	0.038	0.054	7.4e-3	/
2	2	/	0*	0.038	0.051	0*	/
3	1	0*	0*	0.040	0.050	/	/
4	2	/	0*	0.033	0.048	0*	/
5	1	0*	0*	0.041	0.045	/	/

Test No.	$\gamma_{agg}^{2,7}$	Coefficients of $\tau_{r,l}^{7,t-1-i}$					b_r^k
		i=-1	i=0	i=1	i=2	i=3	
0	1	/	8.6e-3	0.47	0.21	/	0.11
1	0	0*	8.6e-3	0.47	0.21	/	0.11

2	0	2.8e-3	9.8e-3	0.45	0.21	/	0.11
3	1	/	0.014	0.46	0.21	0*	0.11
4	2	/	/	0.46	0.21	0*	0.11
5	1	/	8.6e-3	0.45	0.21	0*	0.021

Test No.	$\gamma_{agg}^{2,3}$	Coefficients of $\tau_{r,l}^{3,t-2-i}$					
		i=0	i=1	i=2	i=3	i=4	i=5
6	2	5.2e-3	0*	0.068	0.051	0*	0*

Test No.	$\gamma_{agg}^{2,6}$	Coefficients of $\tau_{r,l}^{6,t-2-i}$					
		i=0	i=1	i=2	i=3	i=4	i=5
6	2	0*	0.042	0.048	1.8e-3	0*	0*

Test No.	$\gamma_{agg}^{2,7}$	Coefficients of $\tau_{r,l}^{7,t-2-i}$						b_r^k
		i=0	i=1	i=2	i=3	i=4	i=5	
6	2	0.012	0.45	0.20	0*	4.8e-3	1.8e-3	0.11

Note: The parameter below 1e-4 is denoted as 0*; "/" denotes the model does not include this parameter, i.e., the default value is 0.

TABLE VII
AGGREGATE PARAMETERS OF N_3 SOLVED BY LSE IN CASE I

Test No.	$\gamma_{agg}^{1,3}$	Coefficients of $\tau_{s,src}^{1,t-2-i}$					
		i=-2	i=-1	i=0	i=1	i=2	i=3
0	1	/	/	0.017	0.45	0.20	/
1	2	/	/	0.017	0.45	0.20	0*
2	1	/	3.4e-3	0.020	0.44	0.20	/
3	1	/	0.090	0*	0.37	0.18	/
4	0	0*	0*	0.098	0.45	/	/

Test No.	$\gamma_{agg}^{2,3}$	Coefficients of $\tau_{s,src}^{2,t-2-i}$						b_s^k
		i=-1	i=0	i=1	i=2	i=3	i=4	
0	2	/	0*	4.9e-3	0.093	0.11	/	0.12
1	2	/	0*	4.9e-3	0.093	0.11	/	0.12
2	2	/	5.8e-3	4.6e-3	0.092	0.11	/	0.12
3	1	0.072	3.8e-3	0	0.16	/	/	0.12
4	2	/	0.061	0.038	0.11	0.12	/	0.12

Note: The parameter below 1e-4 is denoted as 0*; "/" denotes the model does not include this parameter, i.e., the default value is 0.

TABLE VIII
AGGREGATE PARAMETERS OF N_6 SOLVED BY LSE IN CASE I

Test No.	$\gamma_{agg}^{1,6}$	Coefficients of $\tau_{s,src}^{1,t-2-i}$					
		i=-2	i=-1	i=0	i=1	i=2	i=3
0	2	/	/	0.38	0.23	0.026	0
1	2	/	/	0.38	0.23	0.026	0*
2	1	/	1.8e-3	0.38	0.22	0.025	/
3	1	/	0.092	0.29	0.16	0.014	/
4	2	/	/	0.35	0.24	0*	0*

Test No.	$\gamma_{agg}^{2,6}$	Coefficients of $\tau_{s,src}^{2,t-2-i}$						b_s^k
		i=-1	i=0	i=1	i=2	i=3	i=4	
0	2	/	1.4e-3	0.075	0.11	0.015	/	0.17
1	2	/	1.4e-3	0.075	0.11	0.015	/	0.17
2	2	/	7.2e-3	0.077	0.11	0.012	/	0.17
3	1	0.078	3.9e-3	0.046	0.15	/	/	0.16
4	2	/	0.052	0.059	0.10	0.030	/	0.16

Note: The parameter below 1e-4 is denoted as 0*; "/" denotes the model does not include this parameter, i.e., the default value is 0.

TABLE IX
AGGREGATE PARAMETERS OF N_7 SOLVED BY LSE IN CASE I

Test No.	$\gamma_{agg}^{1,7}$	Coefficients of $\tau_{s,src}^{1,t-2-i}$						
		i=-2	i=-1	i=0	i=1	i=2	i=3	i=3
0	/	/	/	/	/	/	/	/
1	0*	0*	0*	0*	0*	/	/	/
2	1	/	2.6e-3	2.9e-3	0*	0*	/	/
3	0	0*	0.068	0*	0*	/	/	/
4	1	/	0*	0.064	0.041	0*	/	/

Test No.	$\gamma_{agg}^{2,7}$	Coefficients of $\tau_{s,src}^{2,t-1-i}$						b_s^k
		i=-1	i=0	i=1	i=2	i=3	i=4	
0	1	/	0.012	0.63	0.28	/	/	0.086
1	1	/	0.012	0.63	0.28	0*	/	0.086
2	0	1.8e-3	9.5e-3	0.63	0.27	/	/	0.085
3	1	/	0.069	0.54	0.20	0.037	/	0.085
4	2	/	/	0.56	0.24	0	0	0.091

Note: The parameter below 1e-4 is denoted as 0*; "/" denotes the model does not include this parameter, i.e., the default value is 0.

TABLE X
AGGREGATE PARAMETERS OF N_1 SOLVED BY LSE IN CASE I

Test No.	$\gamma_{agg}^{1,3}$	Coefficients of $\tau_{r,l}^{3,t-1-i}$					
		i=-1	i=0	i=1	i=2	i=3	i=4
0	2	/	/	0.011	0.30	0.13	/
1	2	/	/	0.011	0.30	0.13	0
2	1	/	0.025	0.028	0.28	0.14	/
3	1	/	0*	0.080	0.17	0.12	/
4	0	0.16	0*	0.022	0*	/	/

Test No.	$\gamma_{agg}^{1,6}$	Coefficients of $\tau_{r,l}^{6,t-3-i}$					
		i=-1	i=0	i=1	i=2	i=3	i=4
0	2	/	/	0.25	0.15	0.017	/
1	2	/	/	0.25	0.15	0.017	0*
2	1	/	0*	0.22	0.14	0.034	/
3	1	/	0*	0.28	0.11	0.080	/
4	2	/	/	0*	0*	0.50	0*

Test No.	$\gamma_{agg}^{1,7}$	Coefficients of $\tau_{r,l}^{7,t-1-i}$							b_r^k
		i=-1	i=0	i=1	i=2	i=3	i=4	i=5	
0	1	/	/	/	/	/	/	/	0.11
1	0	0*	0*	0*	0*	/	/	/	0.13
2	1	/	1.6e-3	0*	0*	0*	/	/	0.14
3	0	0*	0.042	0.023	0*	/	/	/	0.11
4	1	/	0.074	7.6e-3	0.14	0*	/	/	0.11

Note: The parameter below 1e-4 is denoted as 0*; "/" denotes the model does not include this parameter, i.e., the default value is 0.

TABLE XI
AGGREGATE PARAMETERS OF N_2 SOLVED BY LSE IN CASE I

Test No.	$\gamma_{agg}^{2,3}$	Coefficients of $\tau_{r,l}^{3,t-2-i}$					
		i=-1	i=0	i=1	i=2	i=3	i=4
0	2	/	0*	2.5e-3	0.046	0.057	/
1	2	/	0*	2.5e-3	0.046	0.057	/
2	2	/	5.8e-3	0*	0.062	0.052	/
3	1	0*	0.041	0*	0.042	/	/
4	2	/	0*	0*	0*	0*	/

Test No.	$\gamma_{agg}^{2,6}$	Coefficients of $\tau_{r,l}^{6,t-2-i}$					
		i=-1	i=0	i=1	i=2	i=3	i=4
0	2	/	6.8e-4	0.038	0.054	7.4e-3	/
1	2	/	6.8e-4	0.038	0.054	7.4e-3	/
2	2	/	0*	0.040	0.053	3.8e-3	/
3	1	0*	0.094	0*	0*	/	/
4	2	/	0*	0*	0.11	0*	/

Test No.	$\gamma_{agg}^{2,7}$	Coefficients of $\tau_{r,l}^{7,t-1-i}$				b_r^k
		i=-1	i=0	i=1	i=2	i=3	
0	1	/	8.6e-3	0.47	0.21	0*	0.11
1	1	/	8.6e-3	0.47	0.21	0*	0.11
2	0	3.8e-3	9.8e-3	0.45	0.21	/	0.11
3	1	/	0.022	0.57	0.087	0.068	0.074
4	2	/	/	0.52	0.31	0*	0.060

Note: The parameter below 1e-4 is denoted as 0*; "/" denotes the model does not include this parameter, i.e., the default value is 0.

C. Goodness-of-Fit Metrics of the AGM

The goodness-of-fit metrics of the AGM estimated by HME and LSE for the original data are shown as follows. In general, models estimated by LSE could be unreliable when there are outliers in the data. Especially the AGM of the return network because there are more load nodes than source nodes in this DH. Therefore, there are more aggregate parameters in the AGM of the return network, which is more likely to be affected by bad data. Meanwhile, the model estimated by HME has good goodness-of-fit metrics for data with Gaussian errors or outliers and can be regarded as a more suitable estimation algorithm for AGM.

TABLE XII
GOODNESS-OF-FIT METRICS OF THE ARM IN CASE 1 (TRAINING DATA)

Test No.	Estimator	$\boldsymbol{N_3}$

		RMSE (℃)	MAPE (℃)	R^2
2	LSE	0.41	0.0051	0.99
	HME	0.40	0.0051	0.99
3	LSE	1.8	0.023	0.84
	HME	1.1	0.015	0.94
4	LSE	2.1	0.027	0.79
	HME	1.9	0.025	0.82
5	HME	1.1	0.015	0.94
6	HME	0.44	0.0055	0.99

Test No.	Estimator	N_6		
		RMSE (℃)	MAPE (℃)	R^2
2	LSE	0.37	0.0050	0.99
	HME	0.36	0.0049	0.99
3	LSE	1.7	0.024	0.82
	HME	0.39	0.0053	0.99
4	LSE	1.1	0.016	0.93
	HME	0.43	0.0058	0.99
5	HME	0.39	0.0053	0.99
6	HME	0.37	0.0050	0.99

Test No.	Estimator	N_7		
		RMSE (℃)	MAPE (℃)	R^2
2	LSE	0.53	0.0061	0.99
	HME	0.53	0.0061	0.99
3	LSE	1.4	0.017	0.95
	HME	0.53	0.0061	0.99
4	LSE	1.2	0.014	0.96
	HME	0.54	0.0062	0.99
5	HME	0.53	0.0061	0.99
6	HME	0.53	0.0061	0.99

Test No.	Estimator	N_1		
		RMSE (℃)	MAPE (℃)	R^2
2	LSE	0.21	0.0044	0.99
	HME	0.21	0.0045	0.99
3	LSE	2.4	0.063	0.19
	HME	0.21	0.0045	0.99
4	LSE	4.0	0.097	-1.30
	HME	0.67	0.015	0.94
5	HME	0.21	0.0045	0.99
6	HME	0.23	0.0048	0.99

Test No.	Estimator	N_2		
		RMSE (℃)	MAPE (℃)	R^2
2	LSE	0.29	0.0051	0.99
	HME	0.30	0.0051	0.99
3	LSE	3.1	0.063	0.28
	HME	0.40	0.0067	0.99
4	LSE	4.6	0.097	-0.65
	HME	0.31	0.0052	0.99
5	HME	0.38	0.0065	0.99
6	HME	0.30	0.0052	0.99

TABLE XIII
GOODNESS-OF-FIT METRICS OF THE ARM IN CASE 1 (TEST DATA)

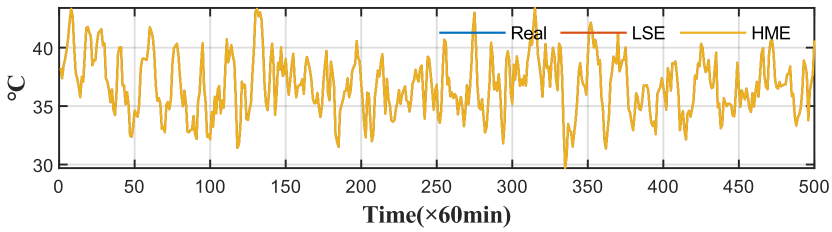
Test No.	Estimator	N_3		
		RMSE (℃)	MAPE (℃)	R^2
2	LSE	0.41	0.0054	0.99
	HME	0.42	0.0054	0.99
3	LSE	1.5	0.019	0.87
	HME	1.0	0.013	0.94
4	LSE	2.0	0.027	0.75
	HME	1.8	0.025	0.80
5	HME	0.99	0.013	0.94
6	HME	0.44	0.0054	0.99

Test No.	Estimator	N_6		
		RMSE (℃)	MAPE (℃)	R^2
2	LSE	0.37	0.0052	0.99
	HME	0.37	0.0052	0.99
3	LSE	1.8	0.025	0.76
	HME	0.39	0.0053	0.99
4	LSE	1.2	0.017	0.90
	HME	0.42	0.0057	0.99
5	HME	0.37	0.0050	0.99

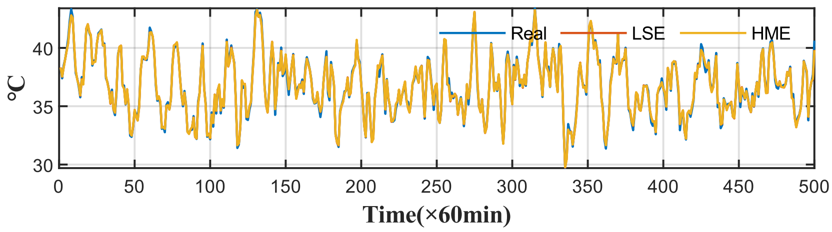
6	HME	0.36	0.0050	0.99
Test No.	Estimator	N_7		
		RMSE (℃)	MAPE (℃)	R^2
2	LSE	0.56	0.0063	0.99
	HME	0.56	0.0063	0.99
3	LSE	1.4	0.016	0.94
	HME	0.56	0.0063	0.99
4	LSE	1.3	0.015	0.94
	HME	0.57	0.0066	0.99
5	HME	0.56	0.0062	0.99
6	HME	0.56	0.0063	0.99
Test No.	Estimator	N_1		
		RMSE (℃)	MAPE (℃)	R^2
2	LSE	0.23	0.0050	0.99
	HME	0.24	0.0052	0.98
3	LSE	2.5	0.066	-0.63
	HME	0.24	0.0051	0.98
4	LSE	4.3	0.11	-3.94
	HME	0.63	0.014	0.90
5	HME	0.24	0.0051	0.98
6	HME	0.22	0.0048	0.99
Test No.	Estimator	N_2		
		RMSE (℃)	MAPE (℃)	R^2
2	LSE	0.34	0.0056	0.99
	HME	0.34	0.0057	0.99
3	LSE	3.1	0.063	0.22
	HME	0.42	0.0069	0.99
4	LSE	4.8	0.099	-0.90
	HME	0.35	0.0058	0.99
5	HME	0.41	0.0068	0.99
6	HME	0.35	0.0058	0.99

D. Result Comparison

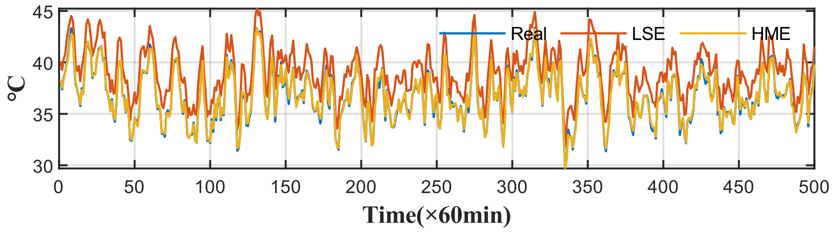
In this part, we compare the original data and the model-fitting data. The results show that when there are outliers in the original data, the AGM estimated based on HME has a better fitting effect on the original data, especially in the return network. (Note: In the line chart below, the first 400 samples are training samples, and the last 100 are test samples)



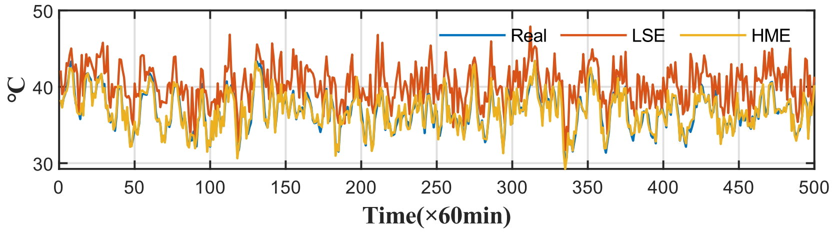
(a-1) N_1-Test 1



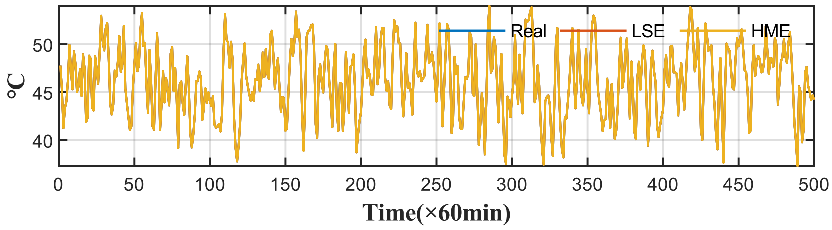
(a-2) N_1-Test 2



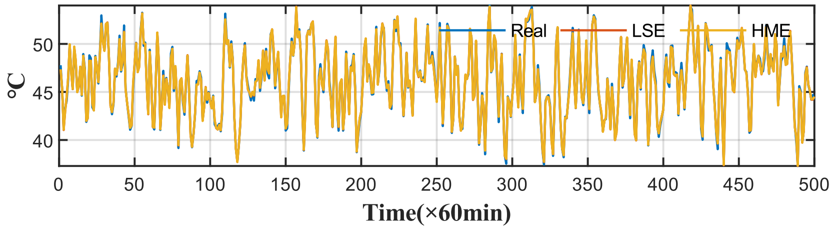
(a-3) N_1-Test 3



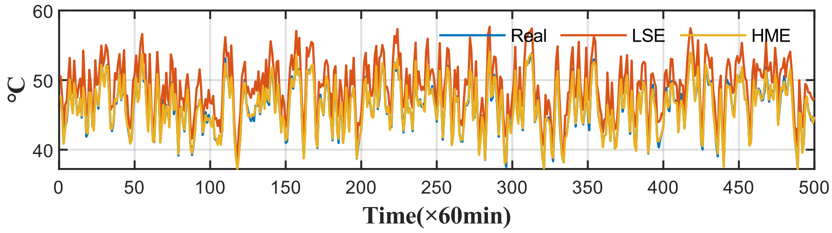
(a-4) N_1-Test 4



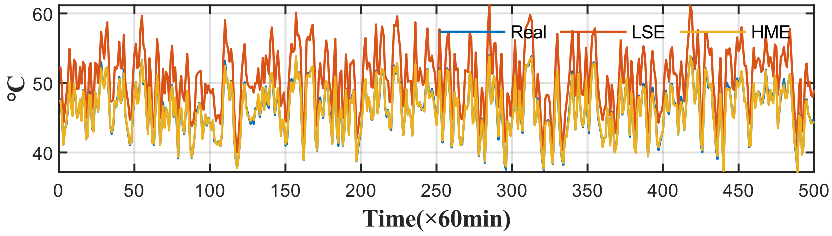
(b-1) N_2-Test 1



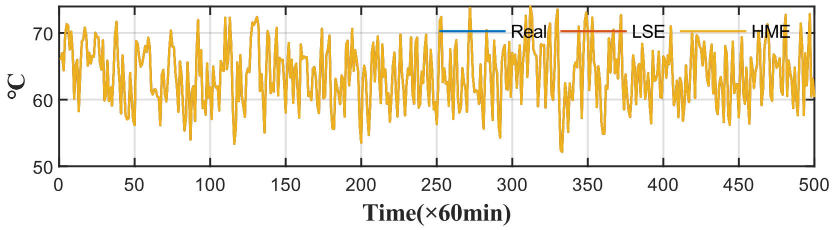
(b-2) N_2-Test 2



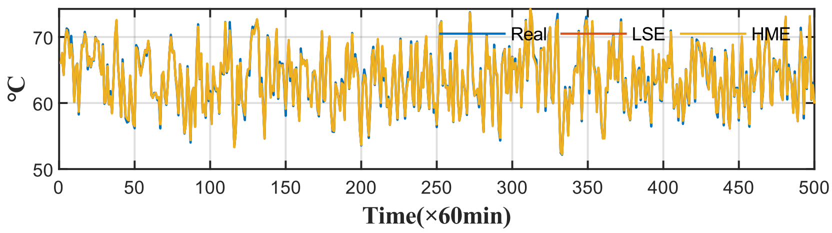
(b-3) N_2-Test 3



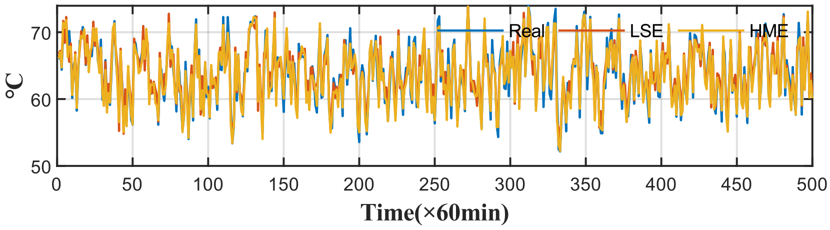
(b-4) N_2-Test 4



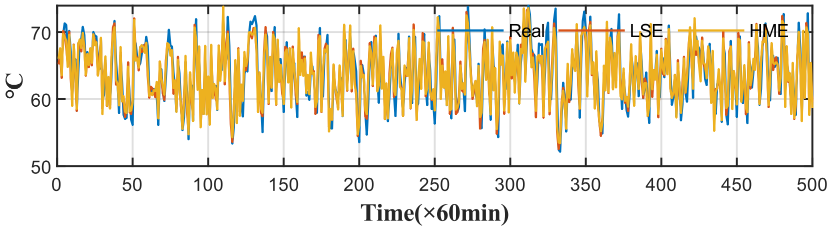
(c-1) N_3-Test 1



(c-2) N_3-Test 2



(c-3) N_3-Test 3



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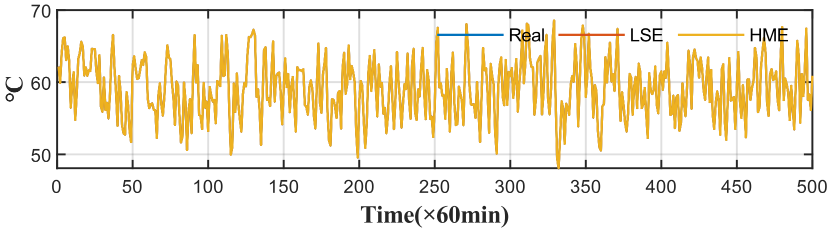
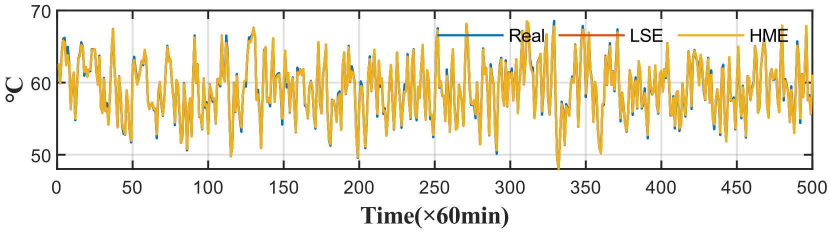
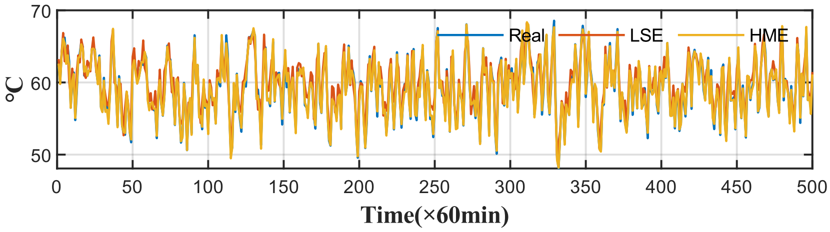


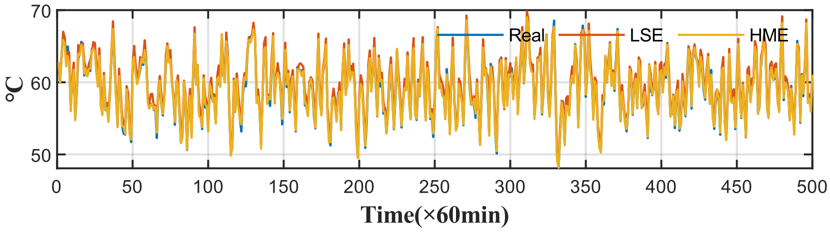
(d-1) N_6-Test 1



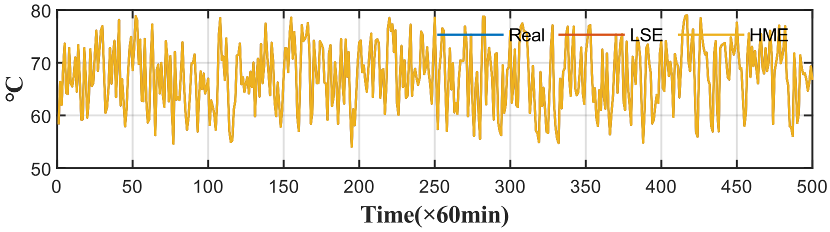
(d-2) N_6-Test 2



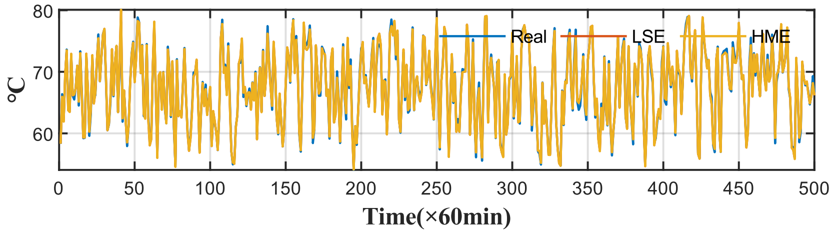
(d-3) N_6-Test 3



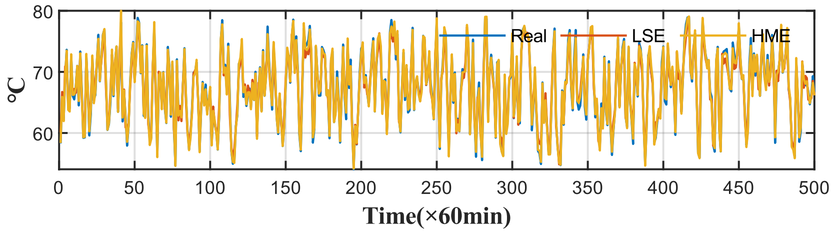
(d-4) N_6-Test 4



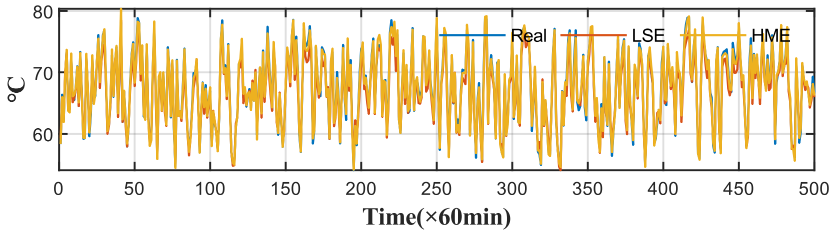
(e-1) N_7-Test 1



(e-2) N_7-Test 2



(e-3) N_7-Test 3



(e-4) N_7-Test 4

Fig. 2 Temperature comparison in Case I.

III. RESULTS OF CASE II

A. *Case Description*

The structure of the 42-node DHN in case II is based on the DHN located in Beijing, China, the topology of which is given in Fig. 3. The DHN in this case consists of 1 heat source and 21 heat loads. In Fig. 3, an unmarked load node indicates that no real data is available for that node. The training and test data include 200 and 100 samples, respectively, with a resolution of 30 minutes. Due to incomplete real-world operation data, only the AGMs of partial supply network are tested in this section, which is sufficient to prove the effectiveness of the AGM. Considering the computational cost and accuracy, we use the 3-horizon AGM in case II.

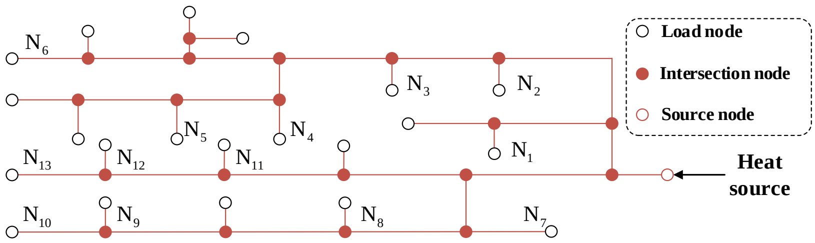


Fig. 3 Topology of DHN in case II.

B. *Goodness-of-Fit Metrics of the AGM*

The goodness-of-fit metrics for the AGM of the load nodes with available data are shown below. Note that the goodness-of-fit metrics of some nodes are slightly worse because there are outliers in this part of real data (e.g., test data of $N_6 \sim N_{11}$). Although the outliers have been filtered by HME, the outliers in real data will still have a negative impact on the goodness-of-fit metrics. In general, the AGMs obtained by HME are superior.

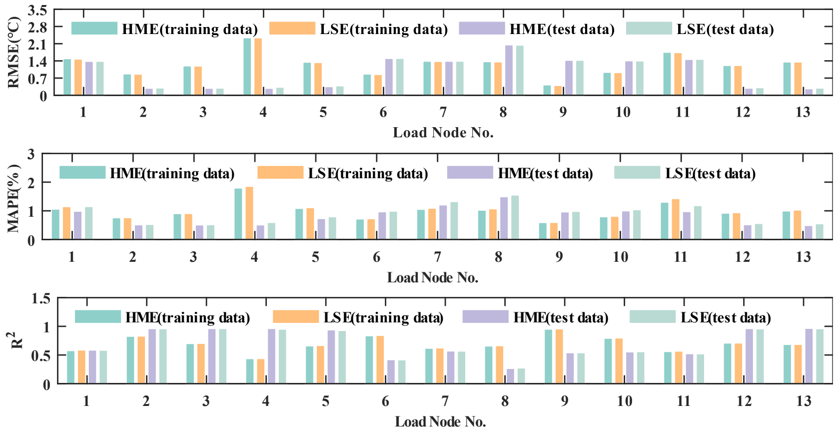
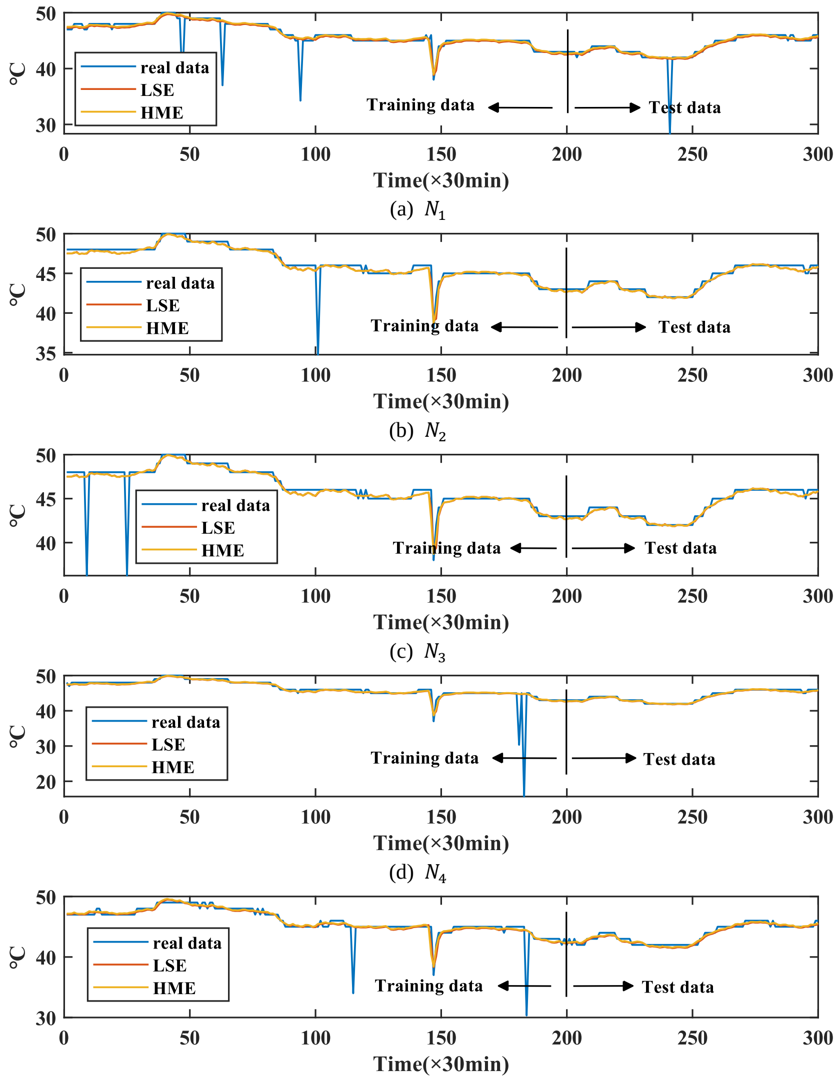


Fig. 4 Goodness-of-fit metrics of the AGM in case II.

C. Data Comparison

The comparison between the real temperature and the AGM fitted temperature of each node is shown below. Obviously, the AGM can effectively filter outliers in real data and accurately display the trend of temperature changes.



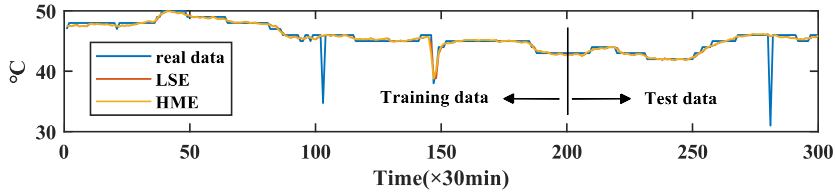
(a) N_1

(b) N_2

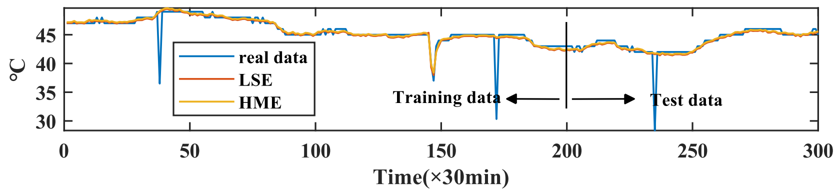
(c) N_3

(d) N_4

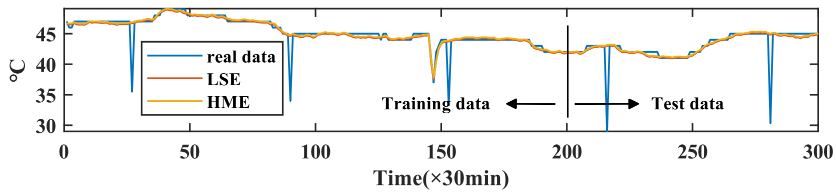
(e) N_5



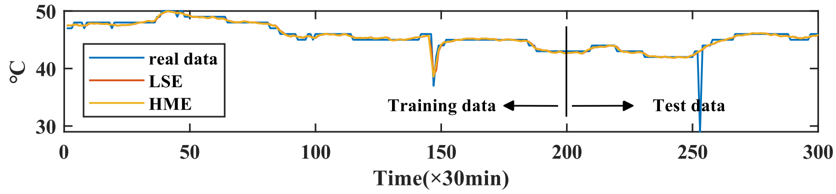
(f) N_6



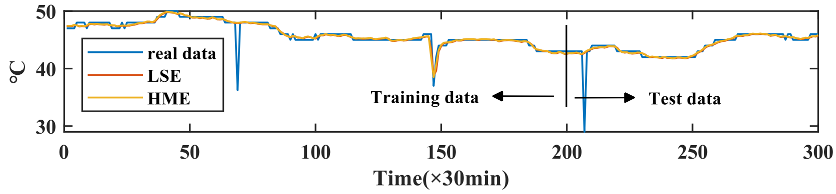
(g) N_7



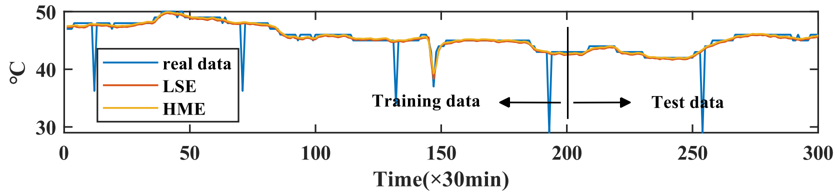
(h) N_8



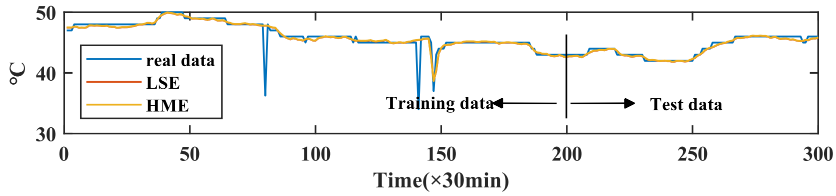
(i) N_9



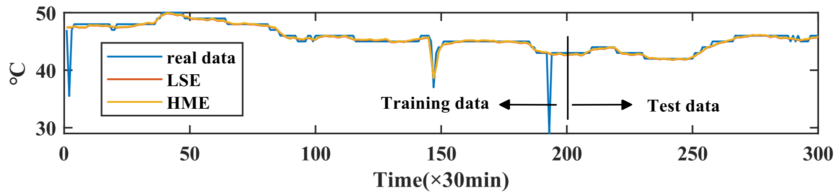
(j) N_{10}



(k) N_{11}



(l) N_{12}



(m) N_{13}

Fig. 5 Temperature comparison in Case II.