■ Supplementary Materials

Nonlinear PDE Constrained Power and Gas Dispatch: A Koopman-based Global Linearization Approach, Yuan Li et al.

SIMULATION RESULTS

This material, as a supplement to the paper, gives a detailed comparison between the dispatch results and solution results under different models.

Fig. 1-Fig. 4 present the dispatch results for each pipeline obtained by the Koopman-based globally linearized model and the average flow velocity-based locally linearized model, and compares them with the results from the network-level nonlinear PDE simulation. The observations are as follows.

- Overall, the dispatch results of the two models show similarities in values and trends. However, the errors for mass flow rate (MFR) are larger since the MFR is highly sensitive to pressure fluctuations, and small changes in pressure may significantly influence MFR.
- The errors of the globally linearized model are significantly smaller than that of the locally linearized model. The error accumulation phenomenon is pronounced in the locally linearized model. In the network-level simulation, the pressure at the sources and the MFR at the loads are given. As shown in Fig. 3 and Fig. 4, the pressure error shows a gradually increasing trend from the source (i.e. node 8) to the load (i.e. node 16), while the MFR error shows a decreasing trend. In addition, the results also show that most of the errors are unidirectional, and these errors may further accumulate and expand over time.
- In contrast, the globally linearized model exhibits higher accuracy and stability in the prediction process. As shown in Fig. 1 and Fig. 2, the errors are bi-directional (i.e. Fig. 1 (m)) in most pipelines, which means that the errors are distributed in both positive and negative directions. Although the globally linearized model also suffers from the error transfer phenomenon (i.e. Fig. 2 (q)-(s)), the error always stays within a small range. This indicates that the proposed model can effectively control the transfer and accumulation of inter-pipeline errors.
- Fig. 3 and Fig. 4 show the results of the nonlinear PDE model, which reflects the actual operating conditions of the gas network. In the locally linearized model, as shown in Fig. 3, the minimum MFR limits for pipelines 1 and 2 are 30 kg/s. However, after 21:00, the MFR drops below this threshold. Similarly, for pipeline 10, the upper MFR limit is 28 kg/s, but at 04:00, the MFR exceeds this limit. This suggests that although the locally linearized method did not exceed limits during dispatch, there is a risk of violations during actual operation. In contrast, the globally linearized model more accurately reflects real system behavior, preventing limit violations and reducing security risks.

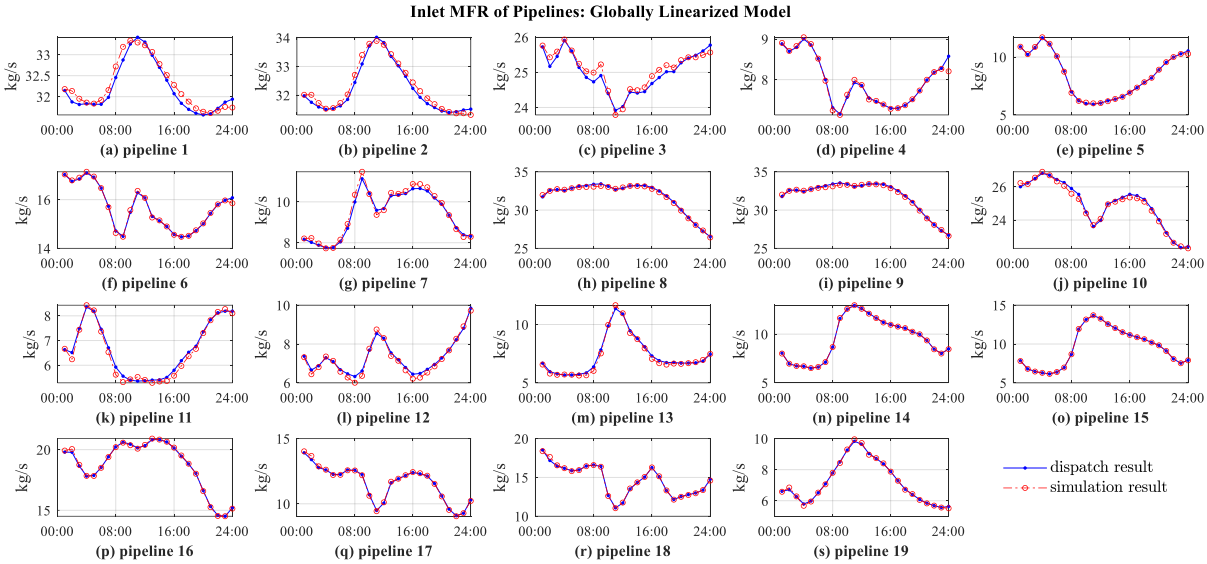

Fig. 1 Inlet MFR of pipelines under the globally linearized model.

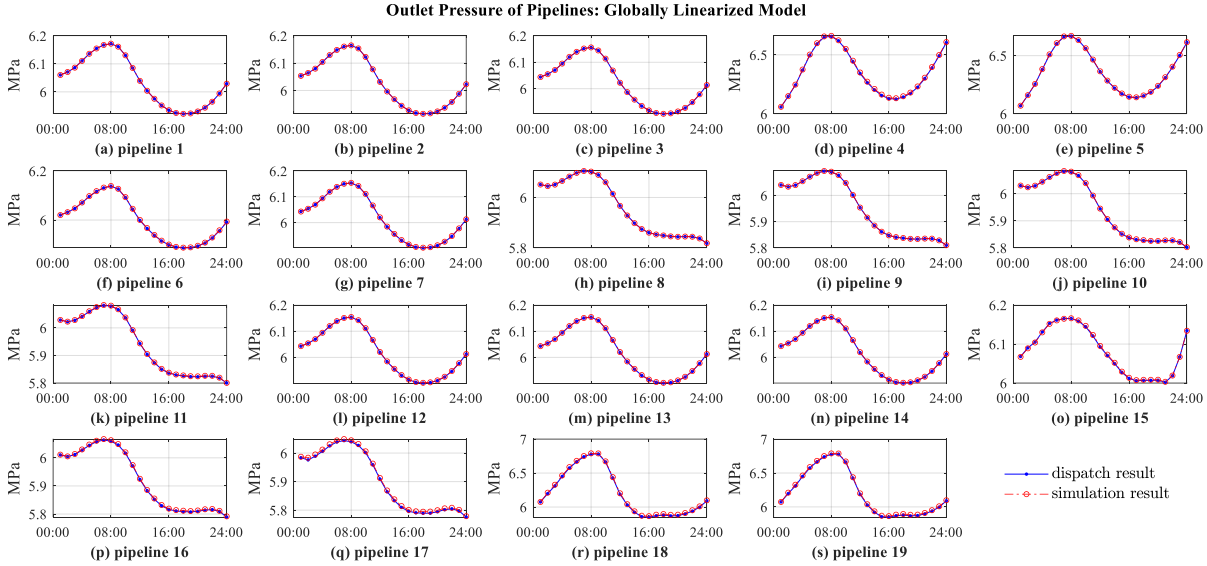

Fig. 2 Outlet pressure of pipelines under the globally linearized model.

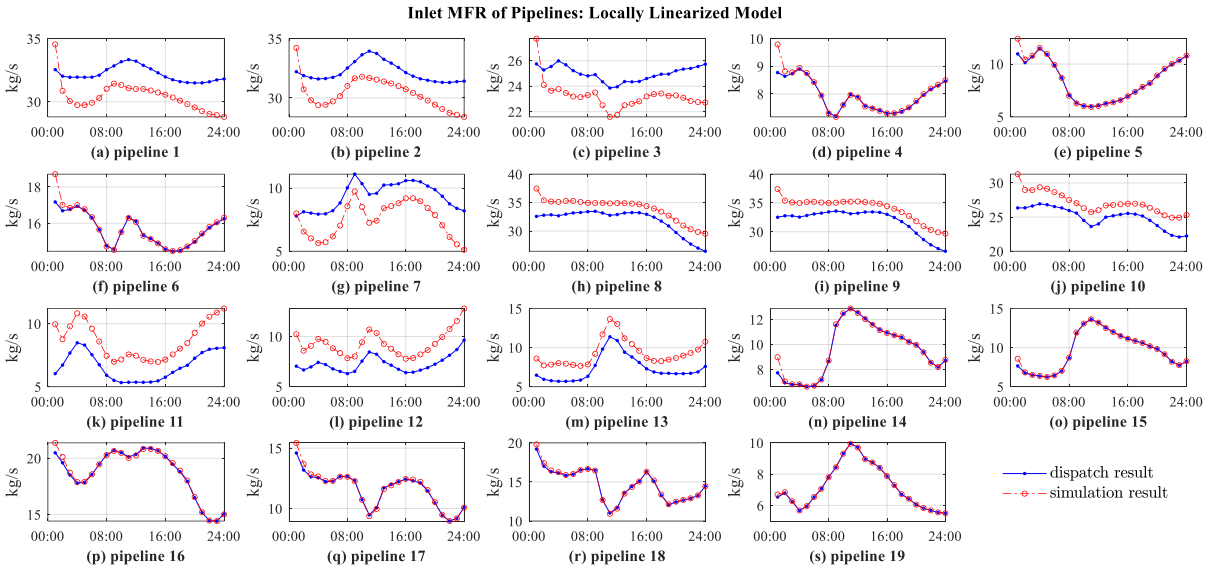

Fig. 3 Inlet MFR of pipelines under the locally linearized model.

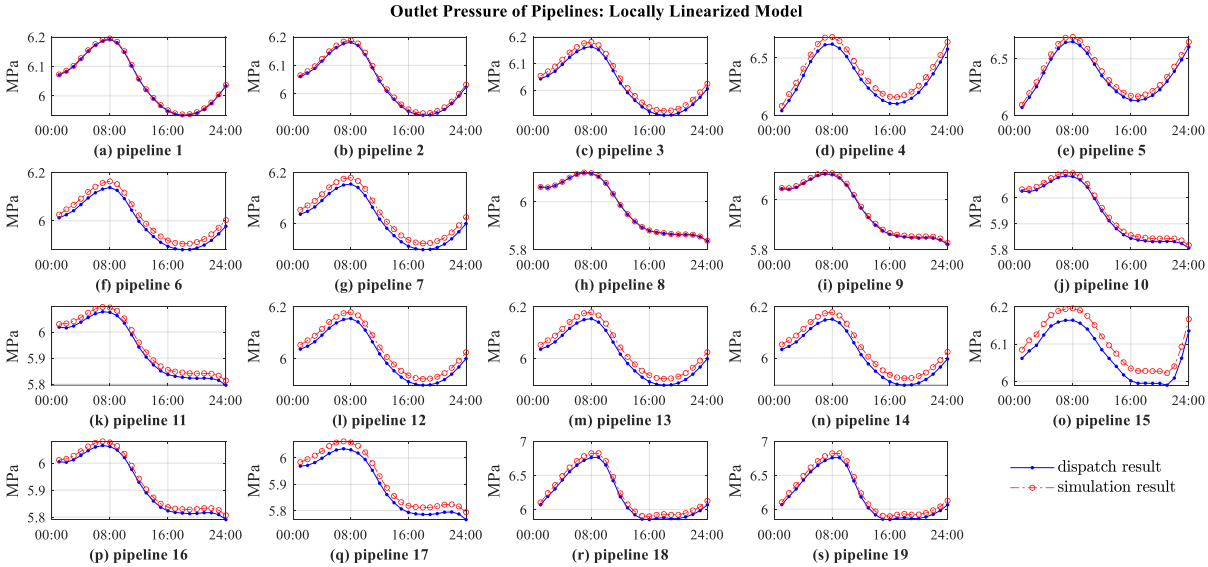

Fig. 4 Outlet pressure of pipelines under the locally linearized model.