

# Basis functions for Legendre polynomials

Shuai Wu

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## General Overview

This document describes a function for computing Legendre polynomials and their derivatives, as well as Legendre-Gauss quadrature nodes and weights.

## Function Descriptions

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### 0.1 `lp.m`

**Purpose** This function is for calculating the value of the  $n$ -th Legendre polynomial and its derivative at the point  $x$ .

**Syntax** `[dy, y] = lp(n, x)`

#### Input Parameters

- **n**: Non-negative integer, the degree of the Legendre polynomial;
- **x**: Scalar or vector, the point(s) at which to evaluate the polynomial.

#### Output

- **y**: Value of the  $n$ th Legendre polynomial at point  $x$ , matching the size of  $x$ ;
- **dy**: Derivative of the  $n$ th Legendre polynomial at point  $x$ , matching the size of  $x$ .

#### Example Usage

```
Input :   n = 5; x = [-0.5; 0; 0.5];
Output :  y = [-0.0898; 0; 0.0898];
          dy = [-2.2265; 1.8750; -2.2265].
```

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### 0.2 `lpgns.m`

**Purpose** Computes Gauss-Legendre quadrature nodes and weights for  $n$  points.

**Syntax** `[x, w] = lpgns(n)`

## Input Parameters

- **n**: Positive integer, the number of quadrature nodes.

## Output

- **x**: Vector of length **n**, containing the Gauss-Legendre quadrature nodes in  $[-1, 1]$ ;
- **w**: Vector of length **n**, containing the corresponding quadrature weights.

## Example Usage

```
Input :    n = 5;  
Output :  x = [-0.9062; -0.5385; 0; 0.5385; 0.9062];  
           w = [0.2369; 0.4786; 0.5689; 0.4786; 0.2369].
```

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## Related functions

[lpm.m](#): Computes all Legendre polynomials up to degree **n** and their derivatives at given points.

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## Visual Examples

**Legendre polynomials:** One can see the Legendre polynomials in Figure 1.

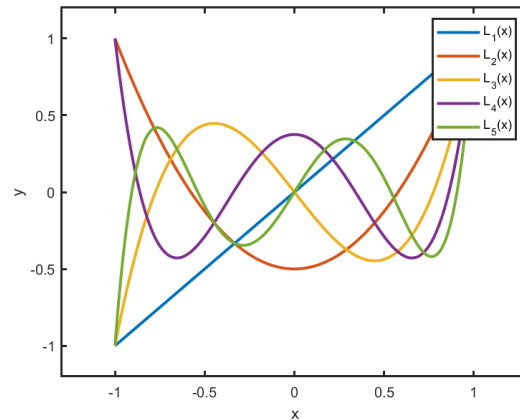


Figure 1: Visual Legendre polynomials.

**Orthogonality of Legendre polynomials:** Legendre polynomials perform inner products on each other, the spy of the matrix in Figure 2. (The value smaller than  $1e-15$  is be ignored)

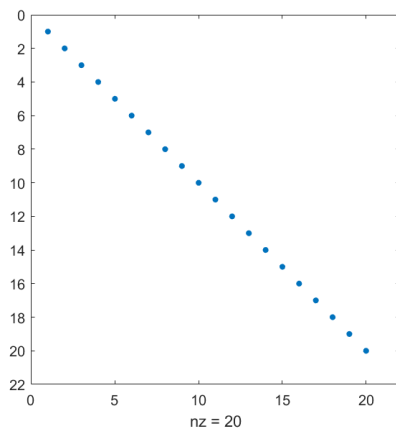


Figure 2: Visual Legendre polynomials.