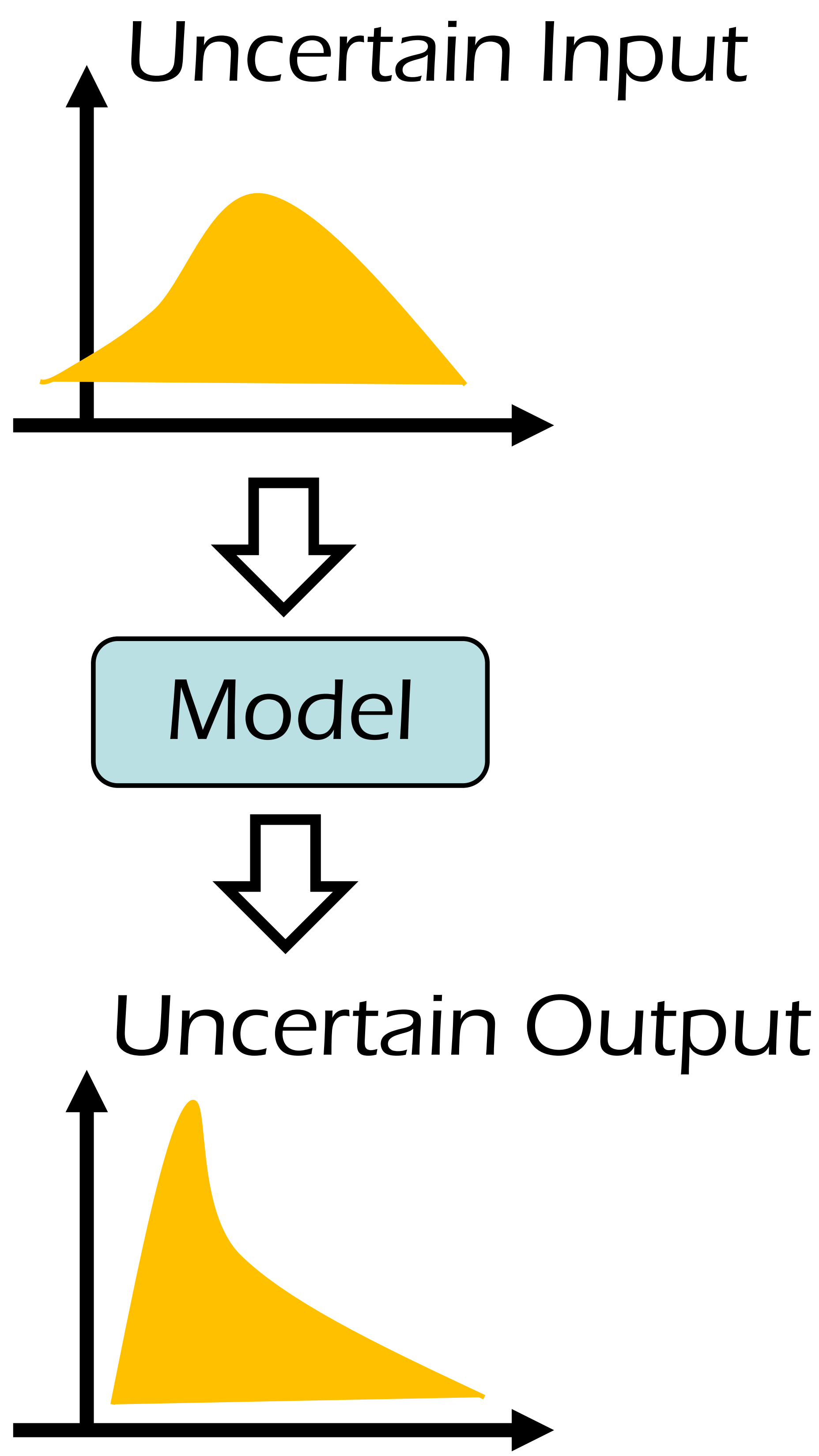
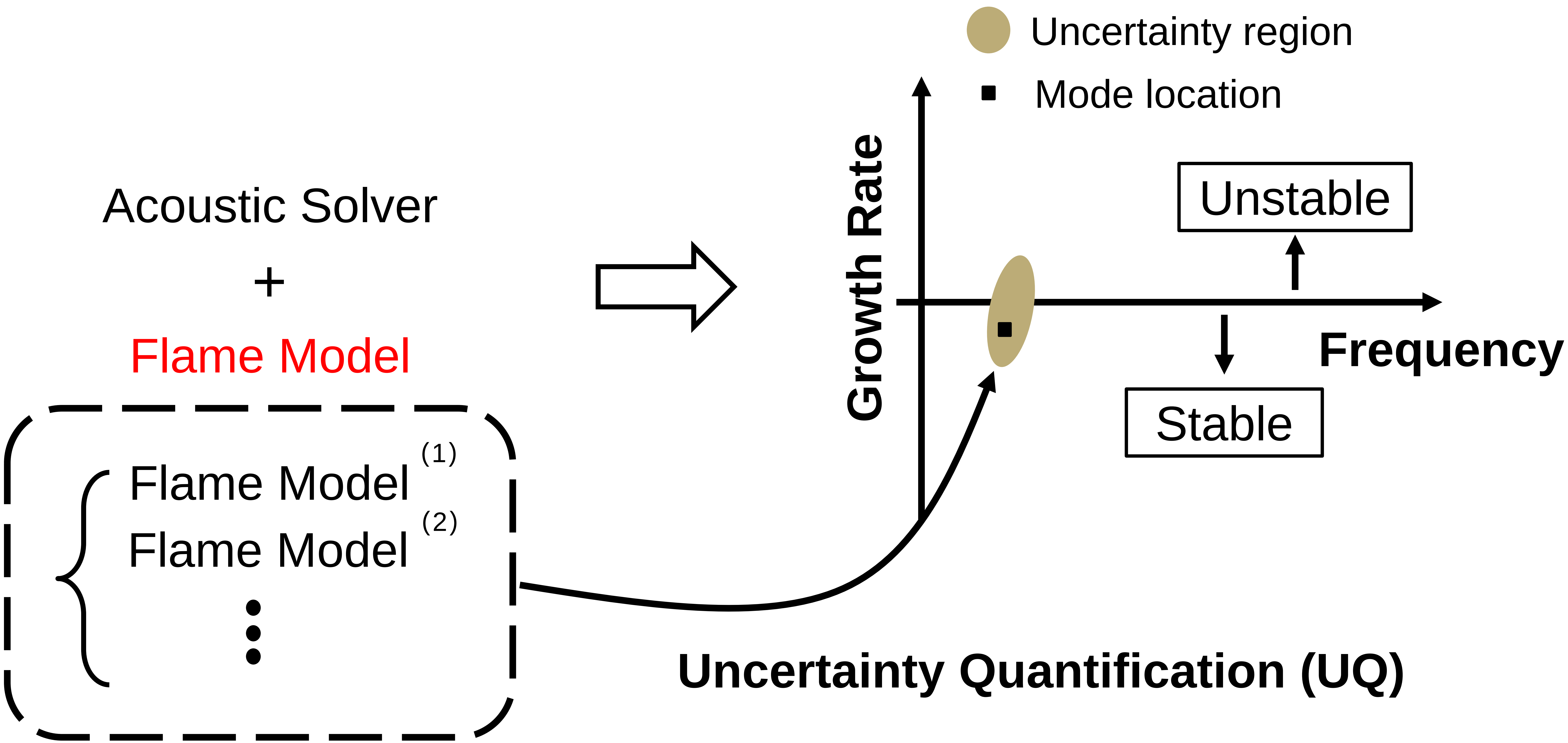
Quantification and Propagation of Uncertainties in Identification of Flame Impulse Response for Thermoacoustic Stability Analysis



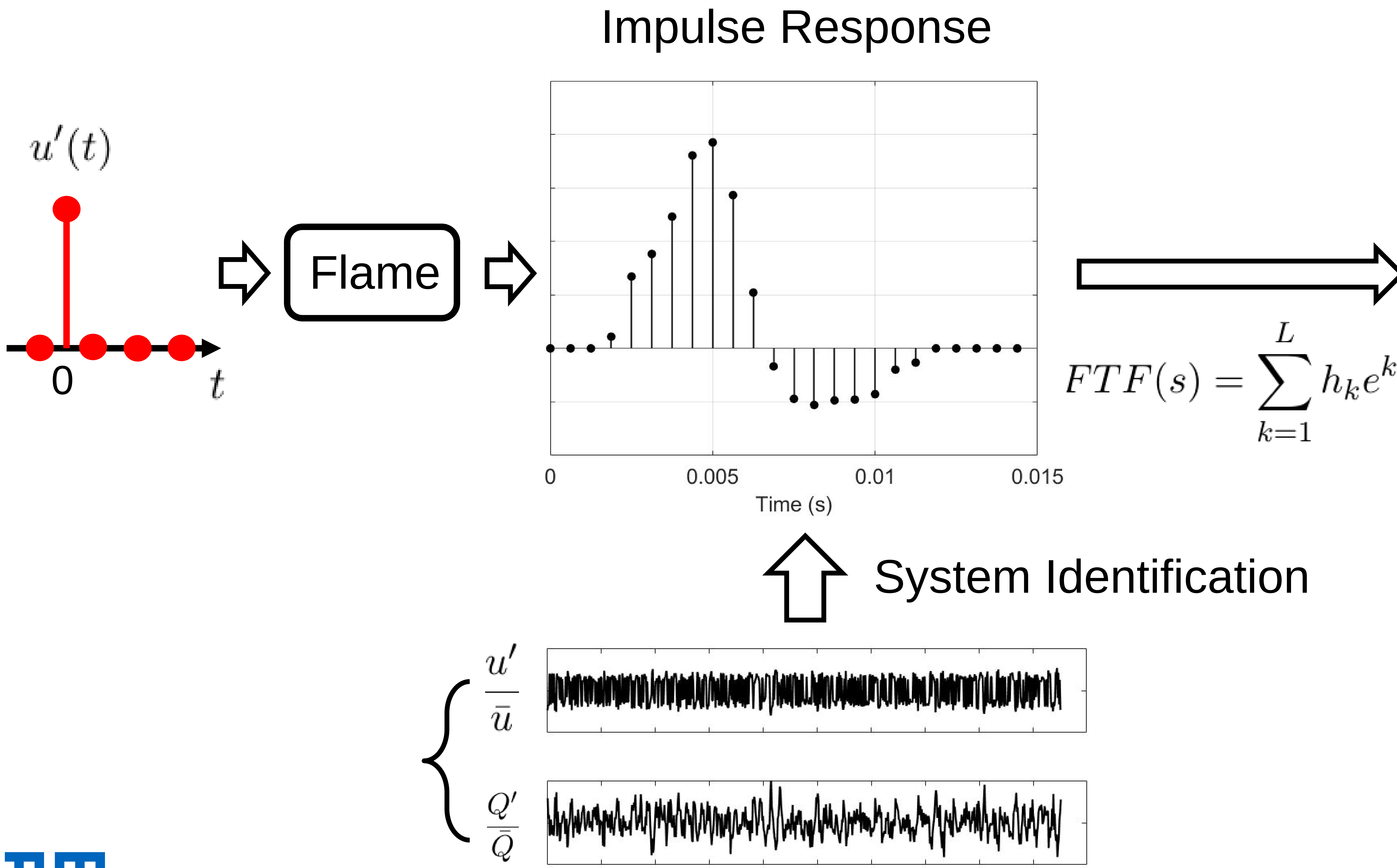
S. Guo, C. Silva, A. Ghani, W. Polifke

ASME Turbo Expo 2018
GT2018-75644

Uncertainty quantification is required to achieve robust thermoacoustic instability prediction



We focus on flame impulse response model in the current study



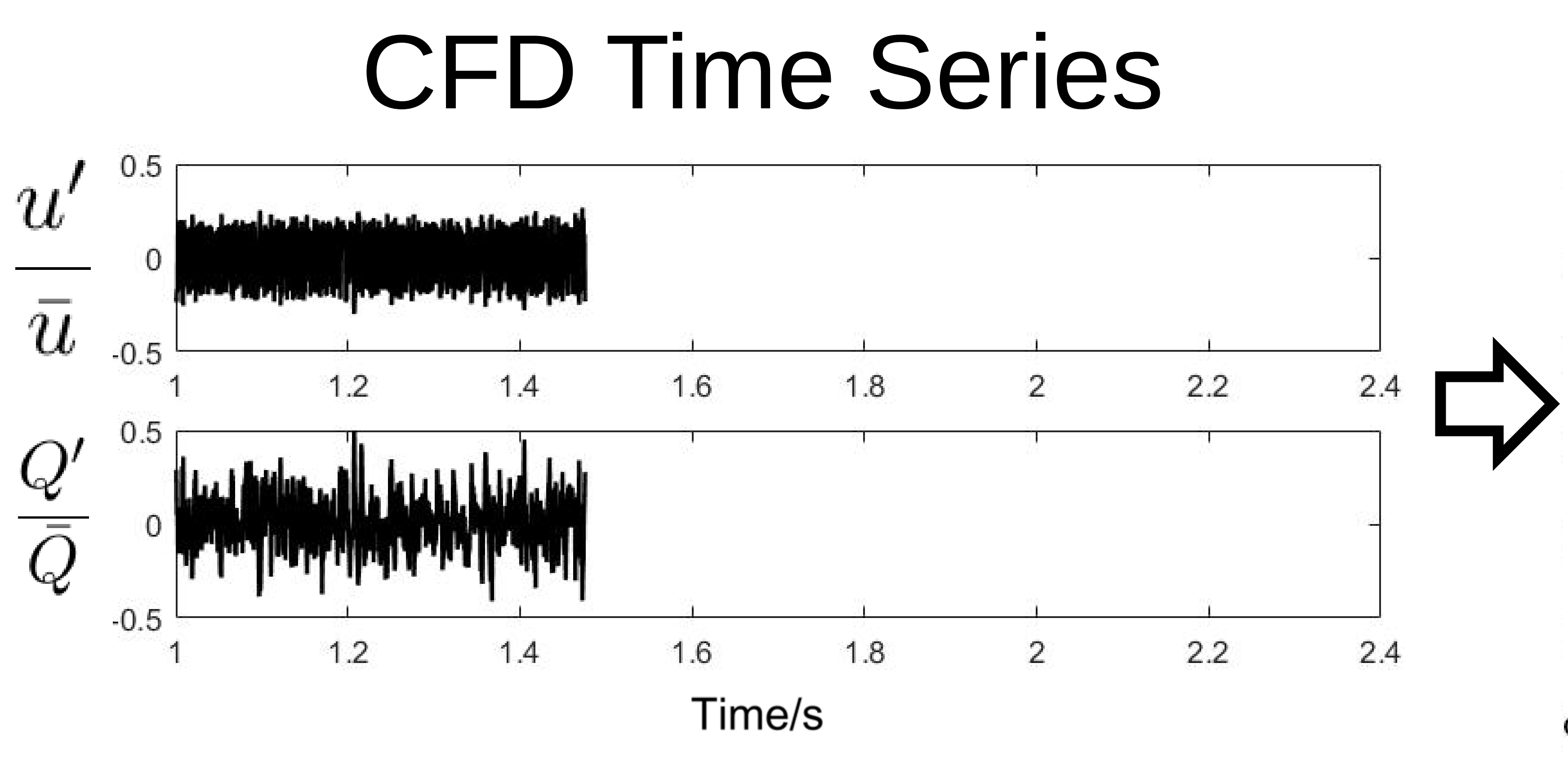
Acoustic Solver

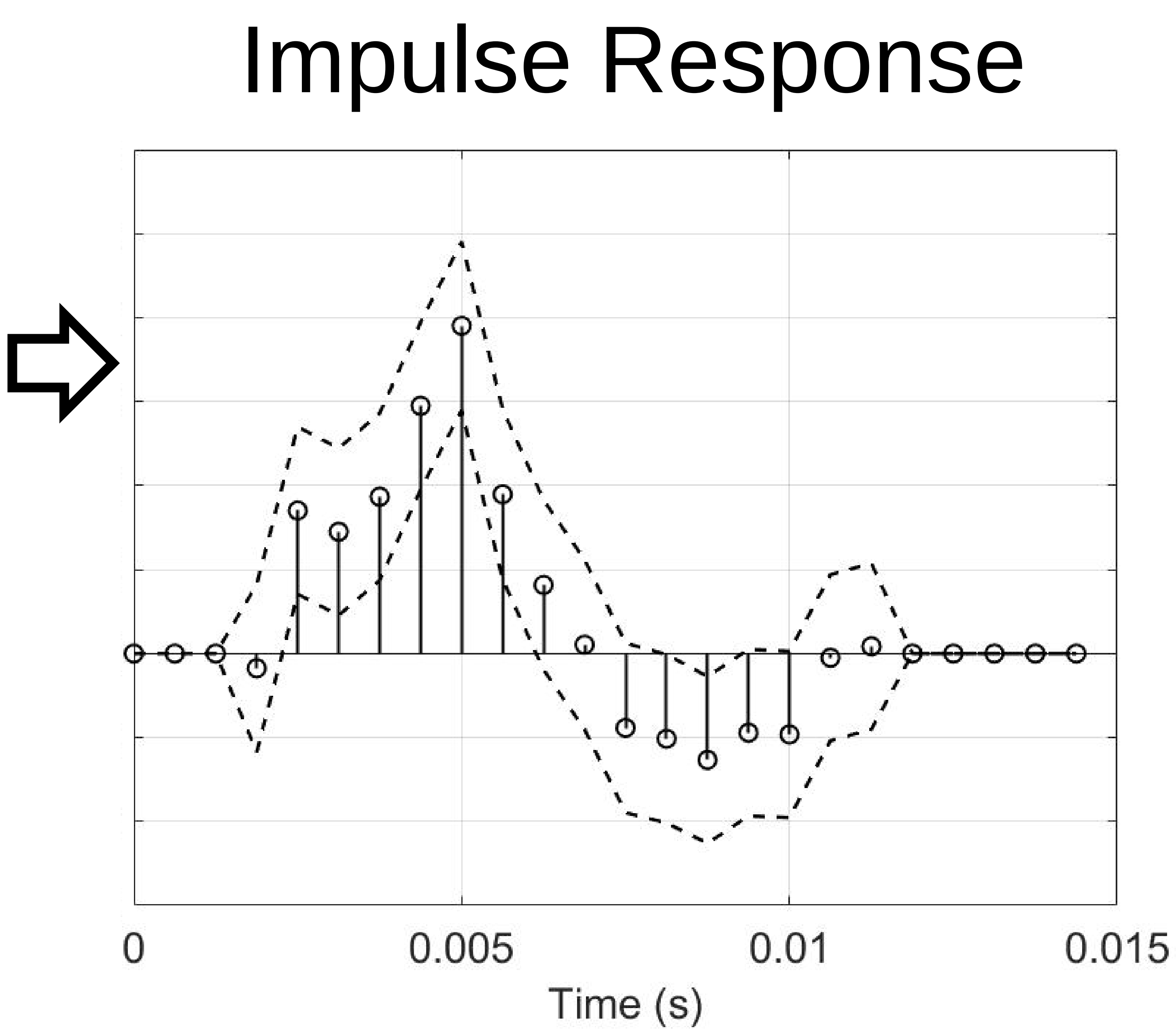
+

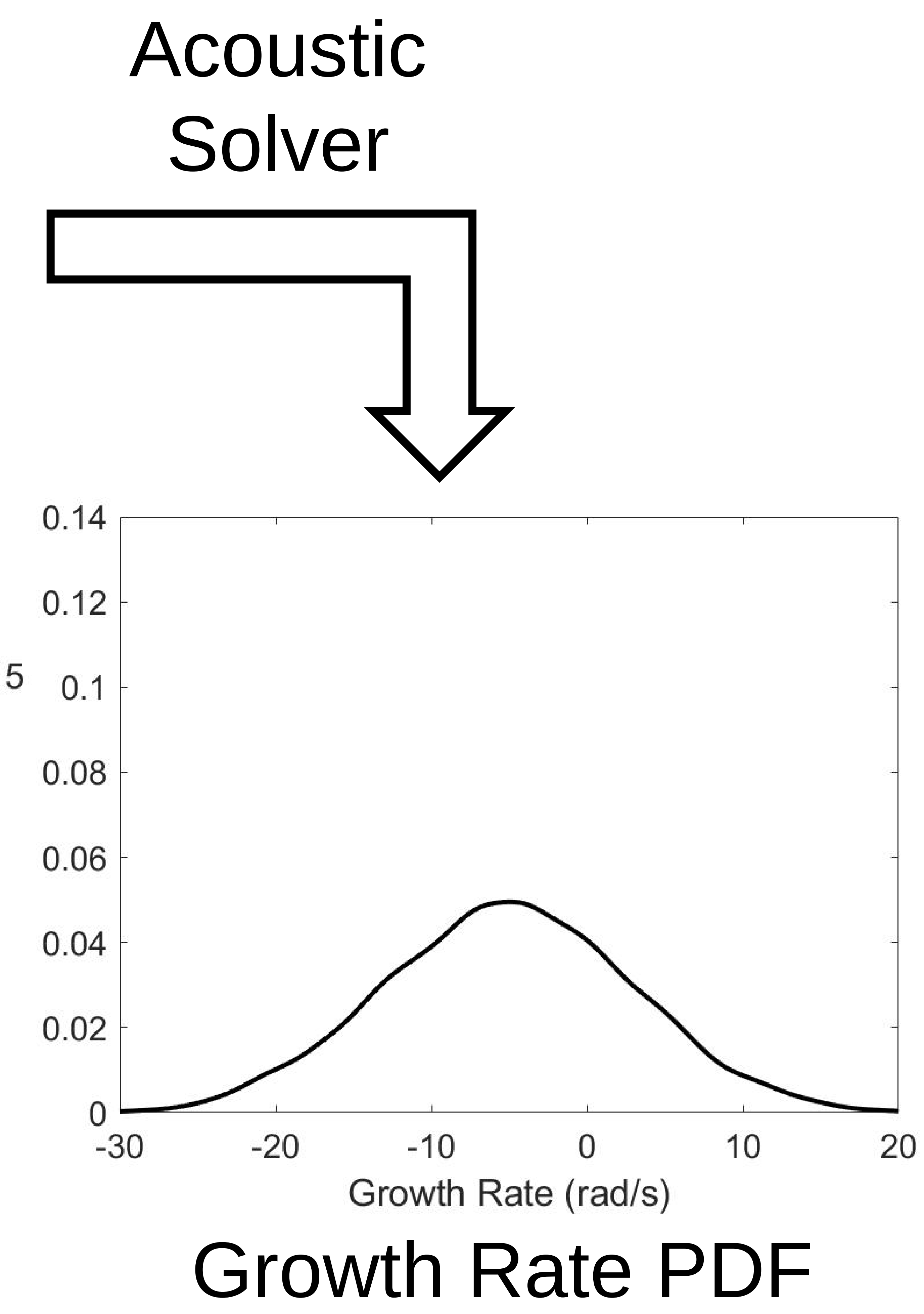
Flame Model

$$FTF(s) = \sum_{k=1}^{L} h_k e^{k\Delta t s}$$

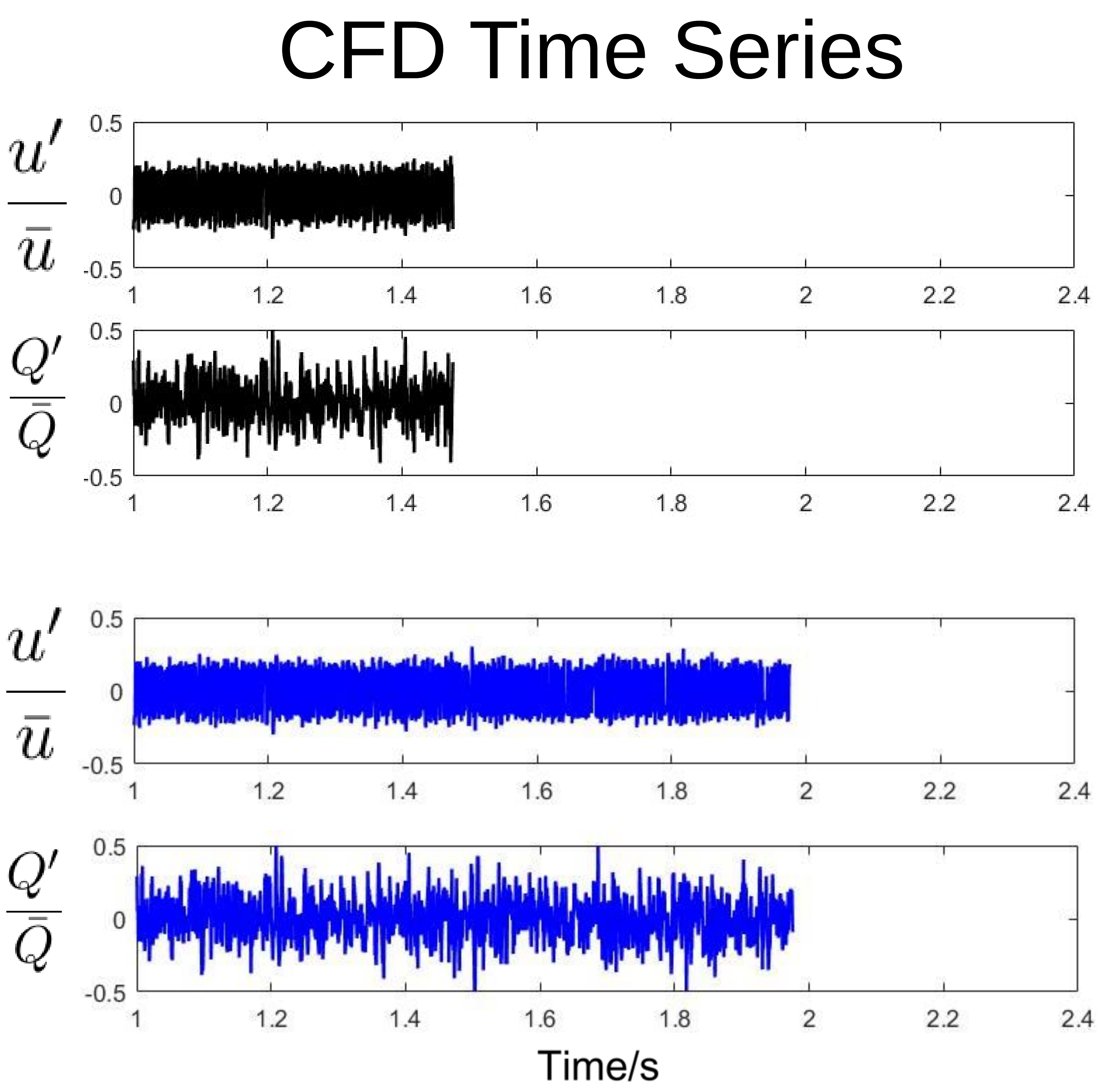
We are interested in the impact of flame impulse response model on thermoacoustic instability prediction

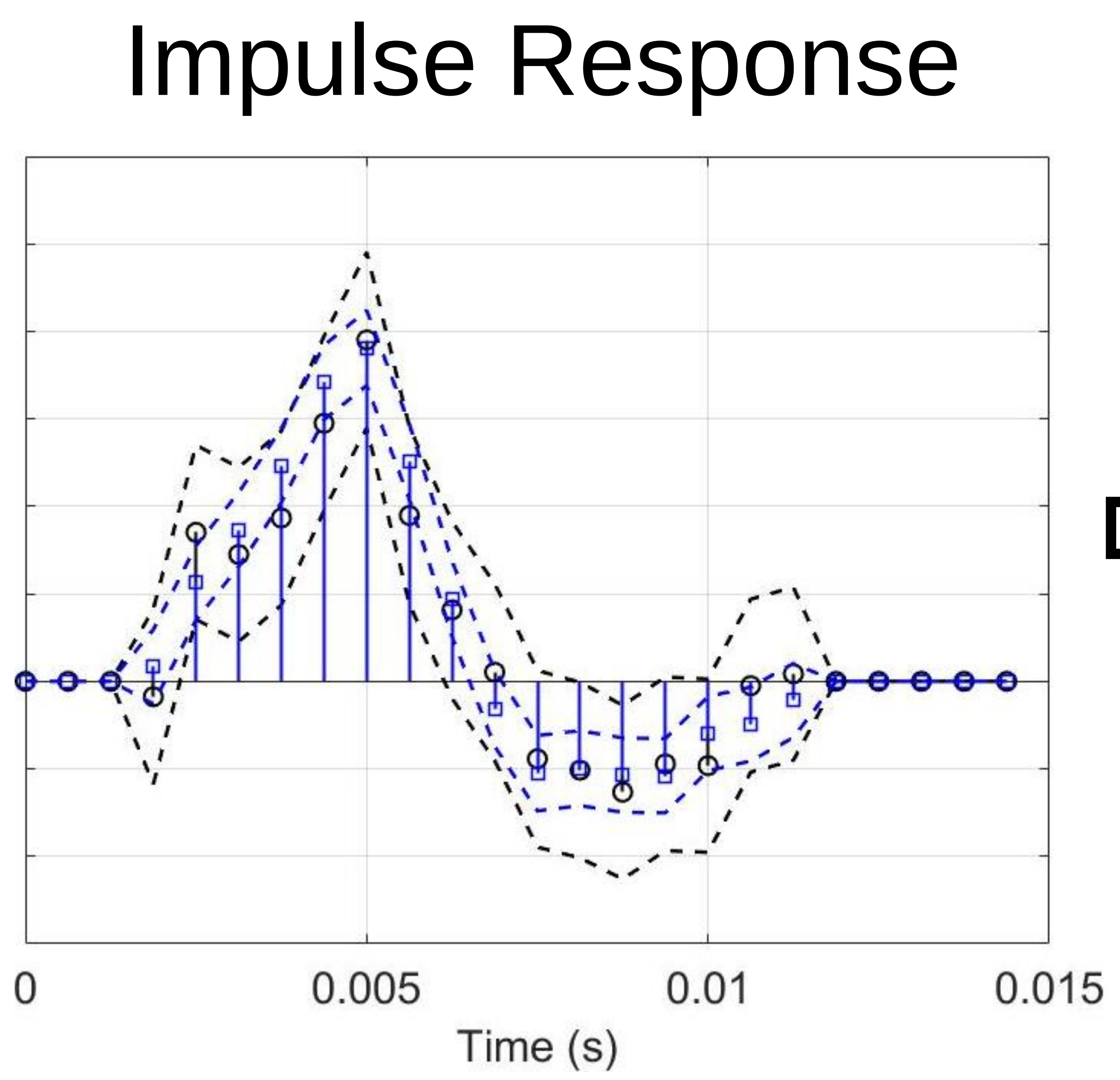


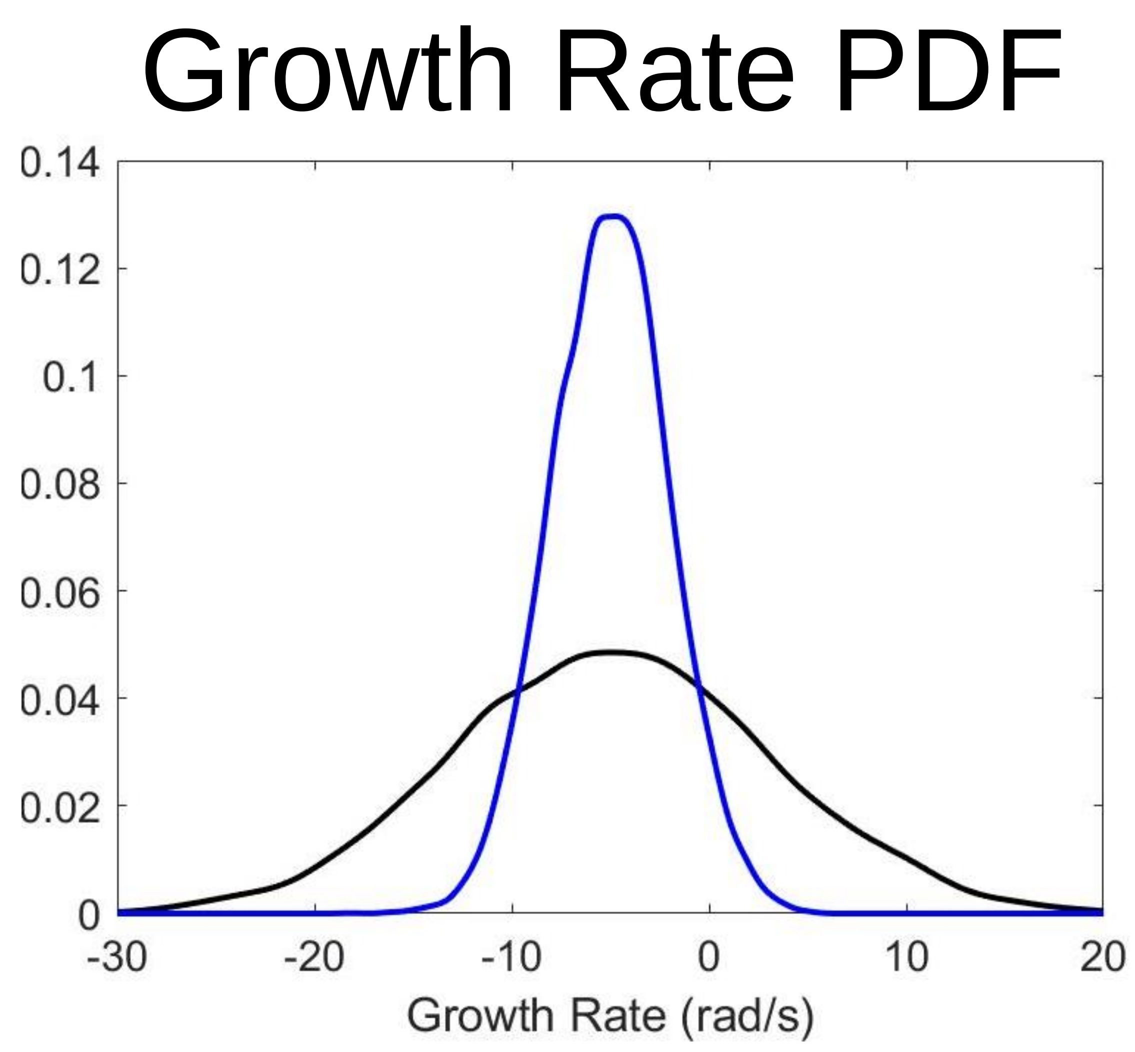




Longer CFD time series yields less uncertain identification of impulse response model as well as growth rate prediction

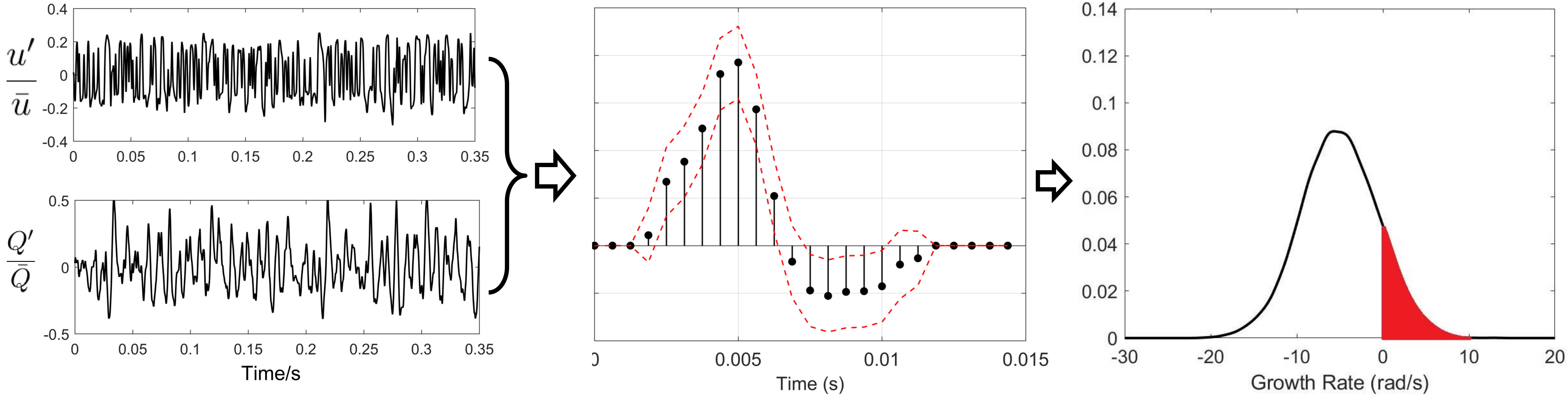






Motivation of our current study

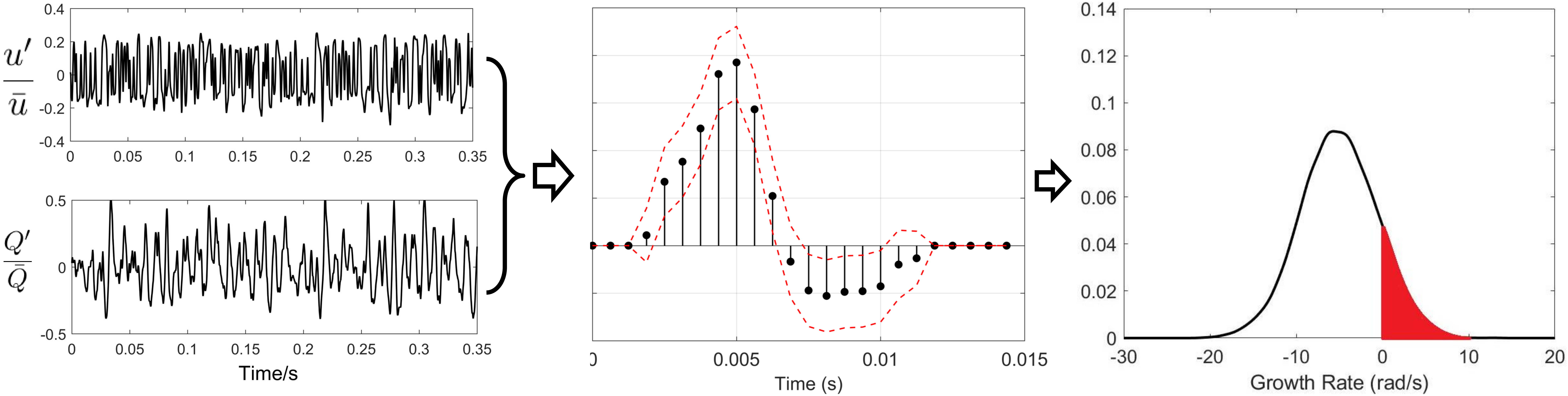
Problem: How to determine the adequate length of CFD time series to achieve a satisfactory uncertainty reduction in growth rate prediction?



Presentation overview

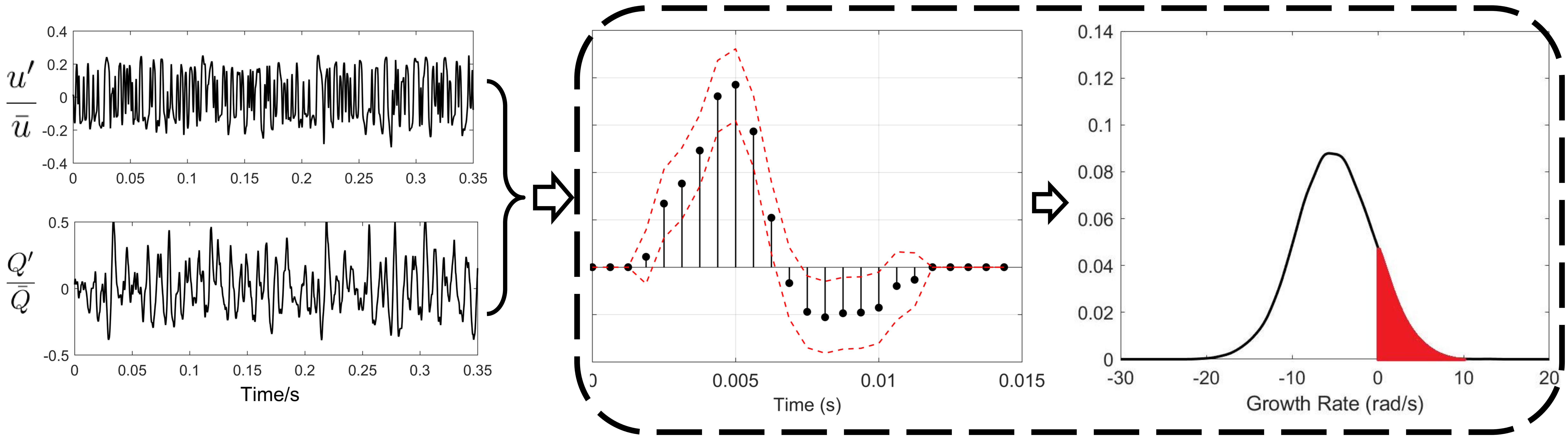
- Motivation
- Methodology
- Thermoacoustic case studies
- Results
- Conclusions

Methodology – divide and conquer



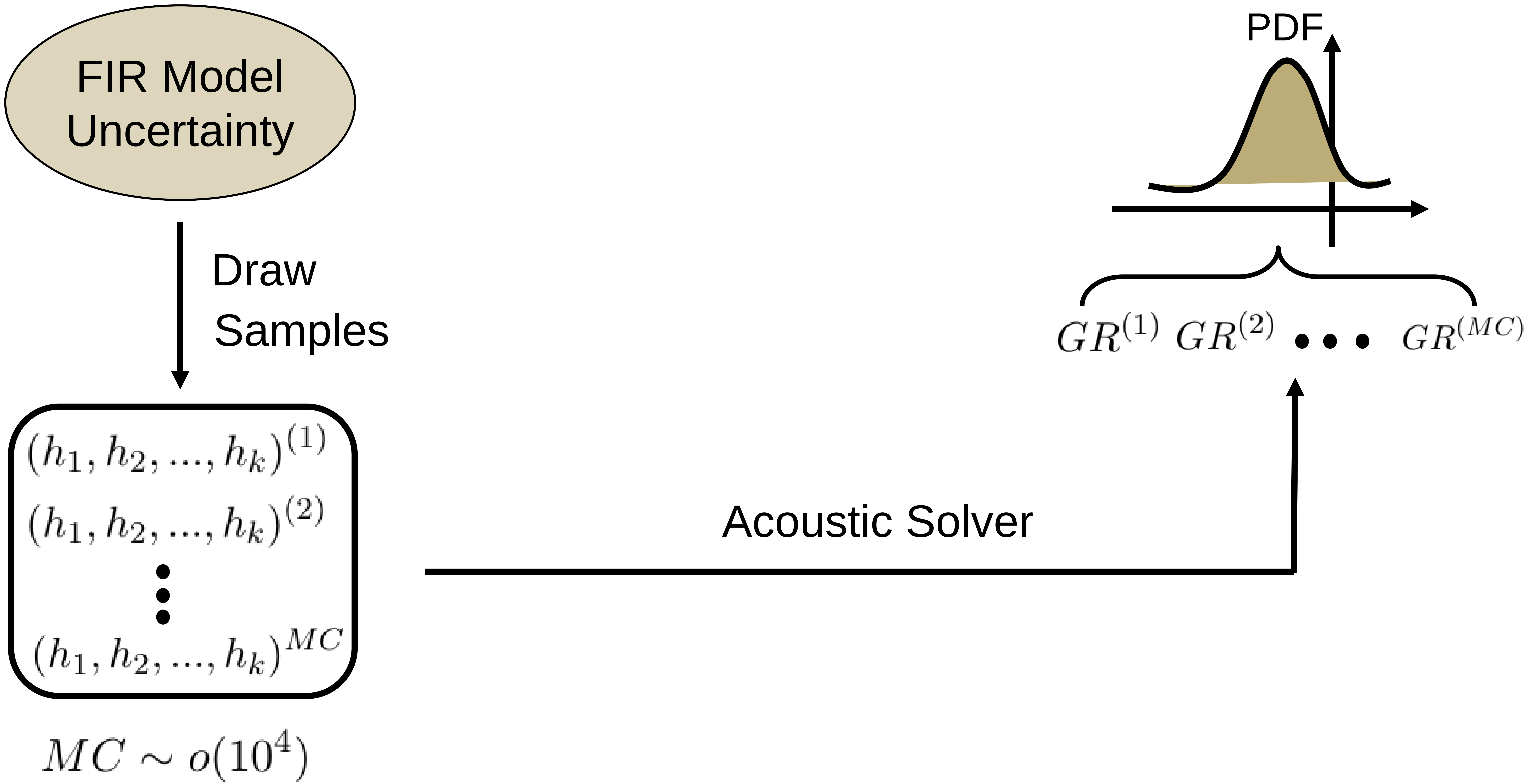
First we need to solve the problem of efficient forward uncertainty propagation

Problem: How to efficiently propagate uncertainties in flame impulse response model to the growth rate predictions?



Forward Uncertainty Propagation

Monte Carlo is an accurate yet very expensive method to achieve forward uncertainty propagation



Response Surface Modeling is an alternative for reducing the sample numbers, but it suffers from "Curse of Dimensionality"

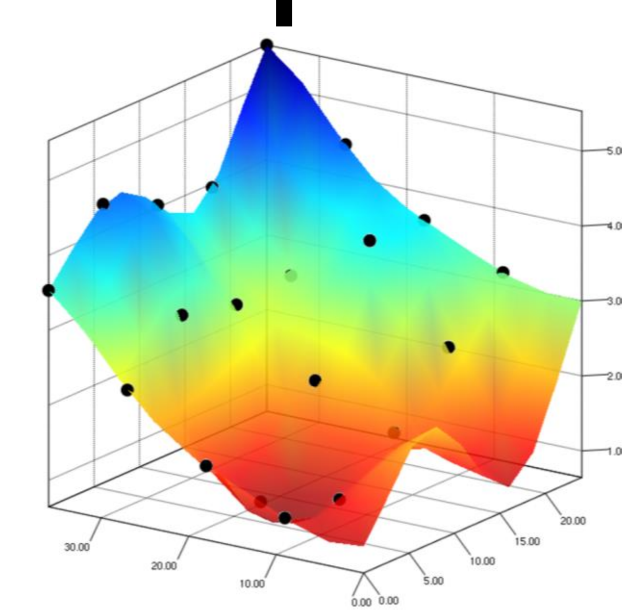
FIR Model Uncertainty

Draw Samples

$$(h_1, h_2, ..., h_k)^{(1)}$$
$$(h_1, h_2, ..., h_k)^{(2)}$$
$$\vdots$$
$$(h_1, h_2, ..., h_k)^{RS}$$

Acoustic Solver

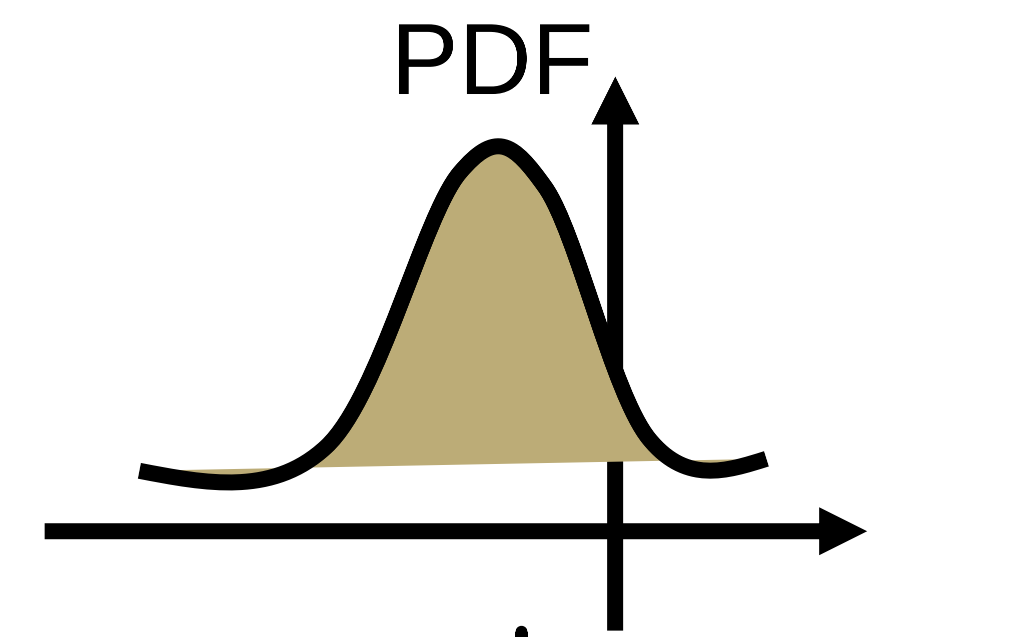
$$GR^{(1)}$$
$$GR^{(2)}$$
$$\vdots$$
$$GR^{(RS)}$$



$$GR \approx Poly(h_1, h_2, ..., h_k)$$

Monte Carlo

$GR_1 \quad GR_2 \quad \cdots$



$RS < MC$ $\quad \boxed{RS \sim e^k}$

"Curse of Dimensionality"

Active Subspace identifies and exploits important directions in input space: Example

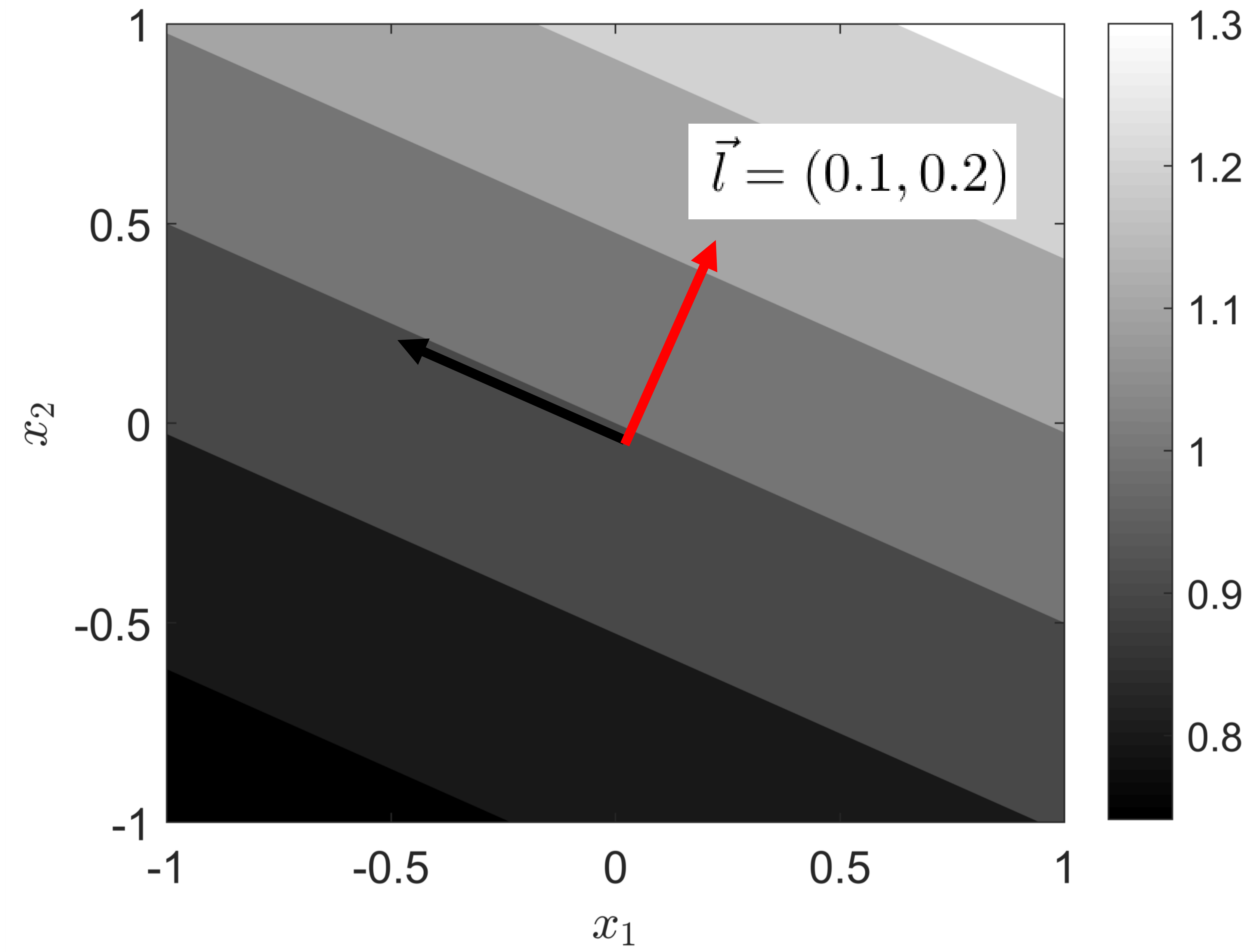
Example:

$$y = e^{(0.1x_1+0.2x_2)}$$

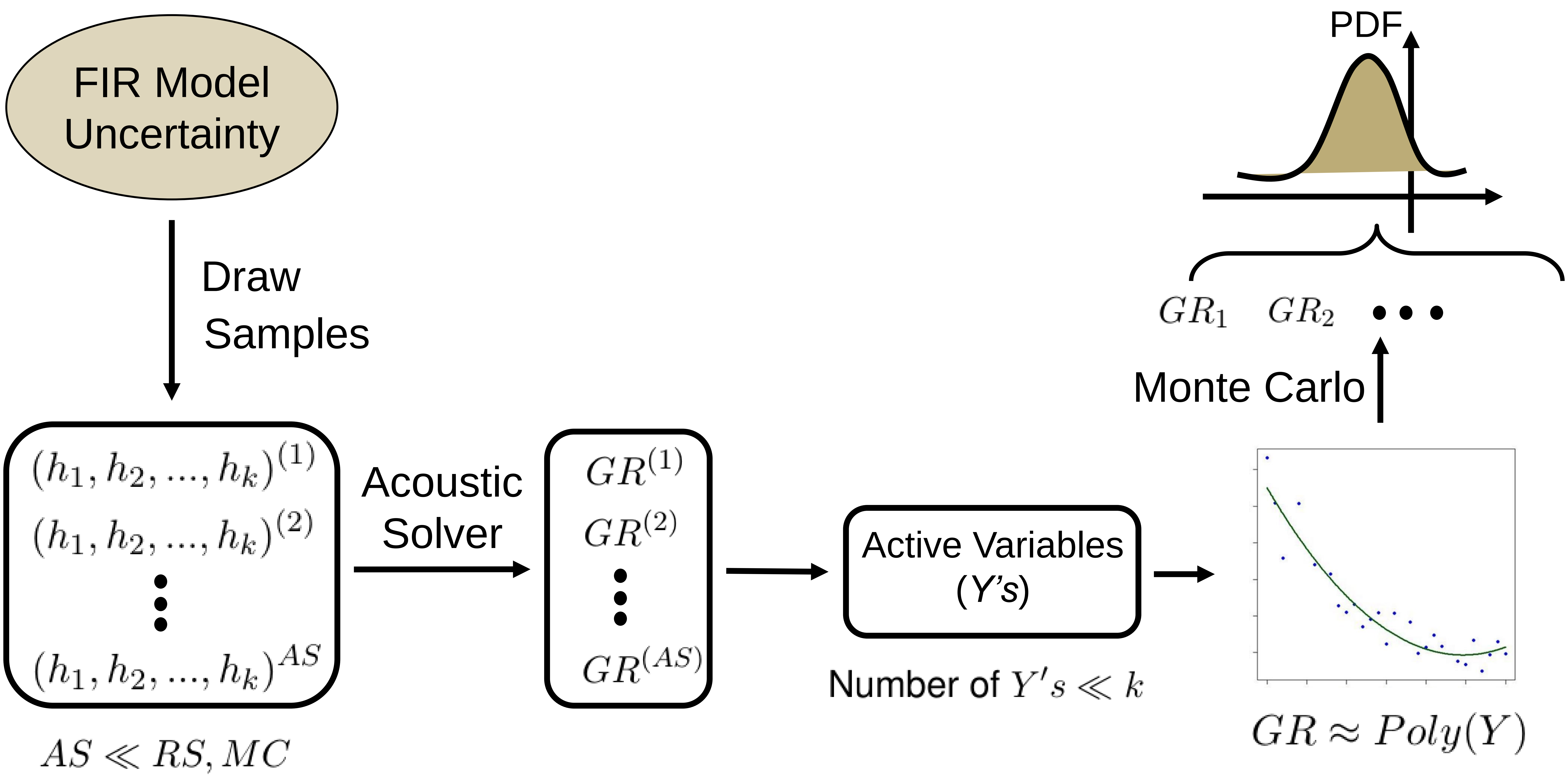
Approximate

$$AV = 0.1x_1 + 0.2x_2$$

$$y \approx f(AV)$$



Active Subspace approach offers much more efficient uncertainty propagation by reducing the problem dimensionality



Presentation overview

- Motivation
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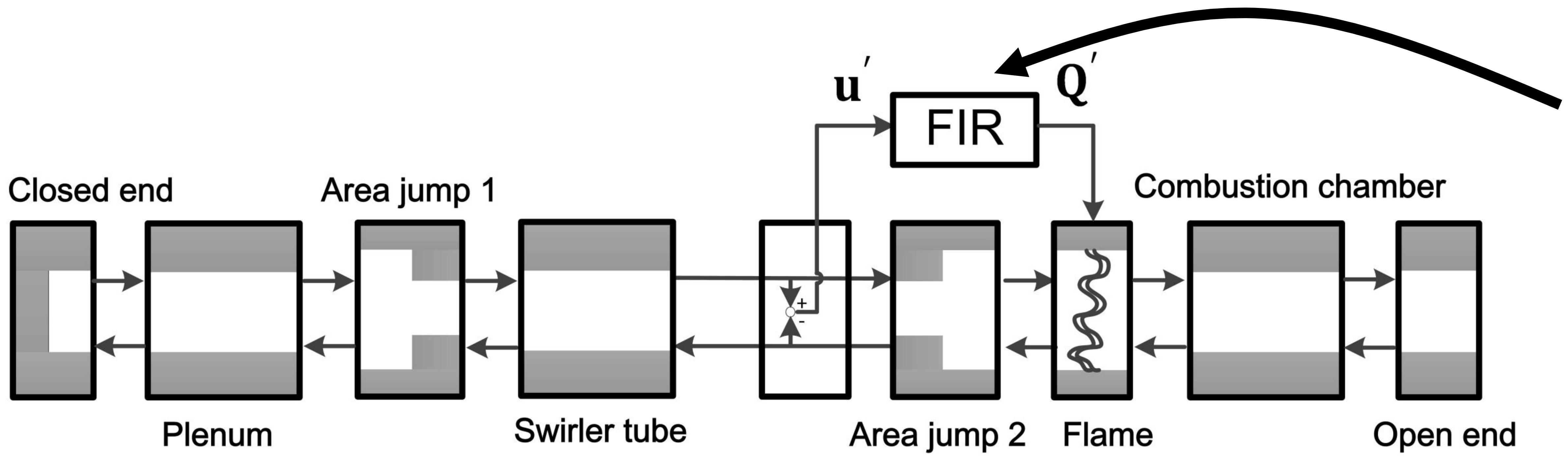
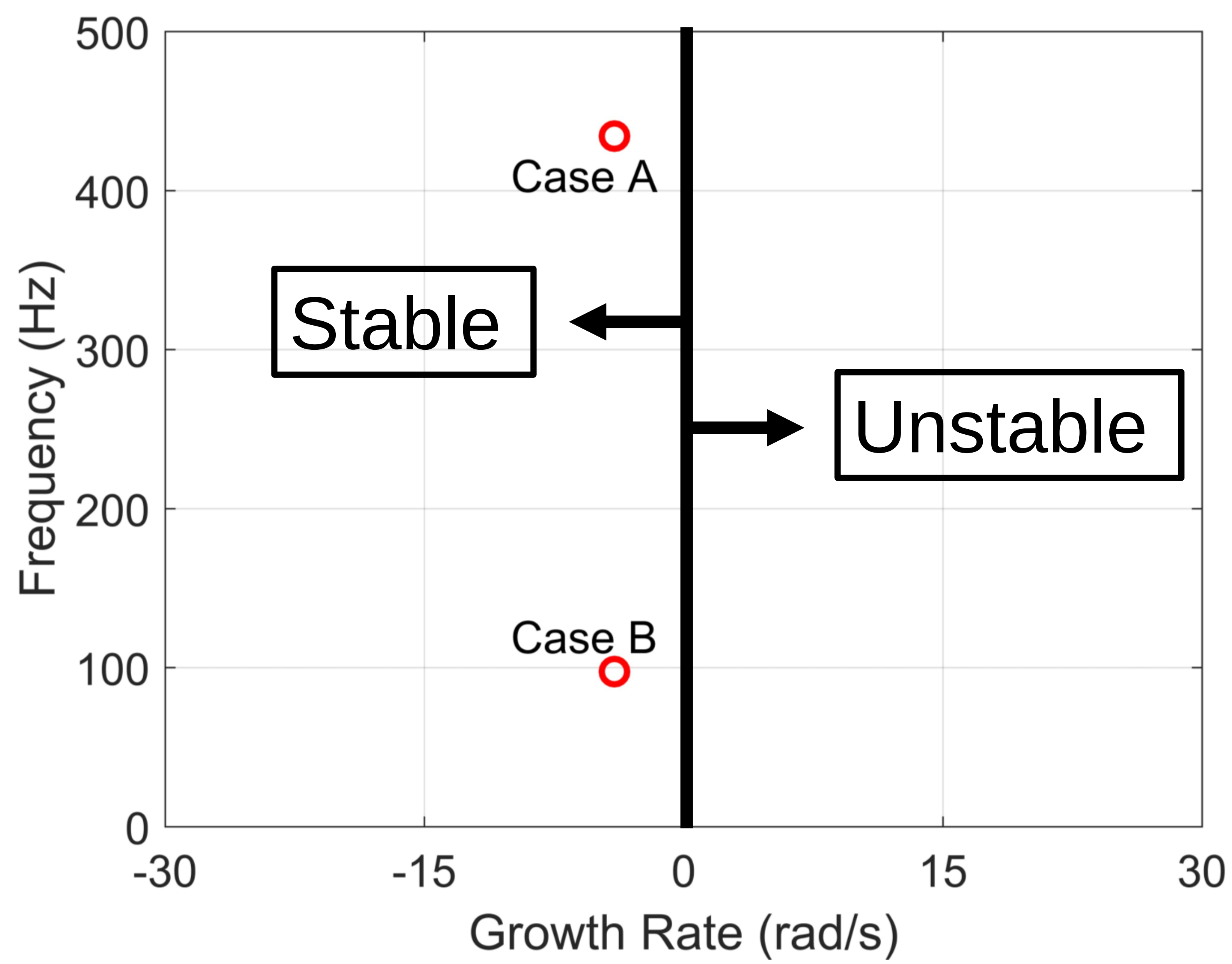
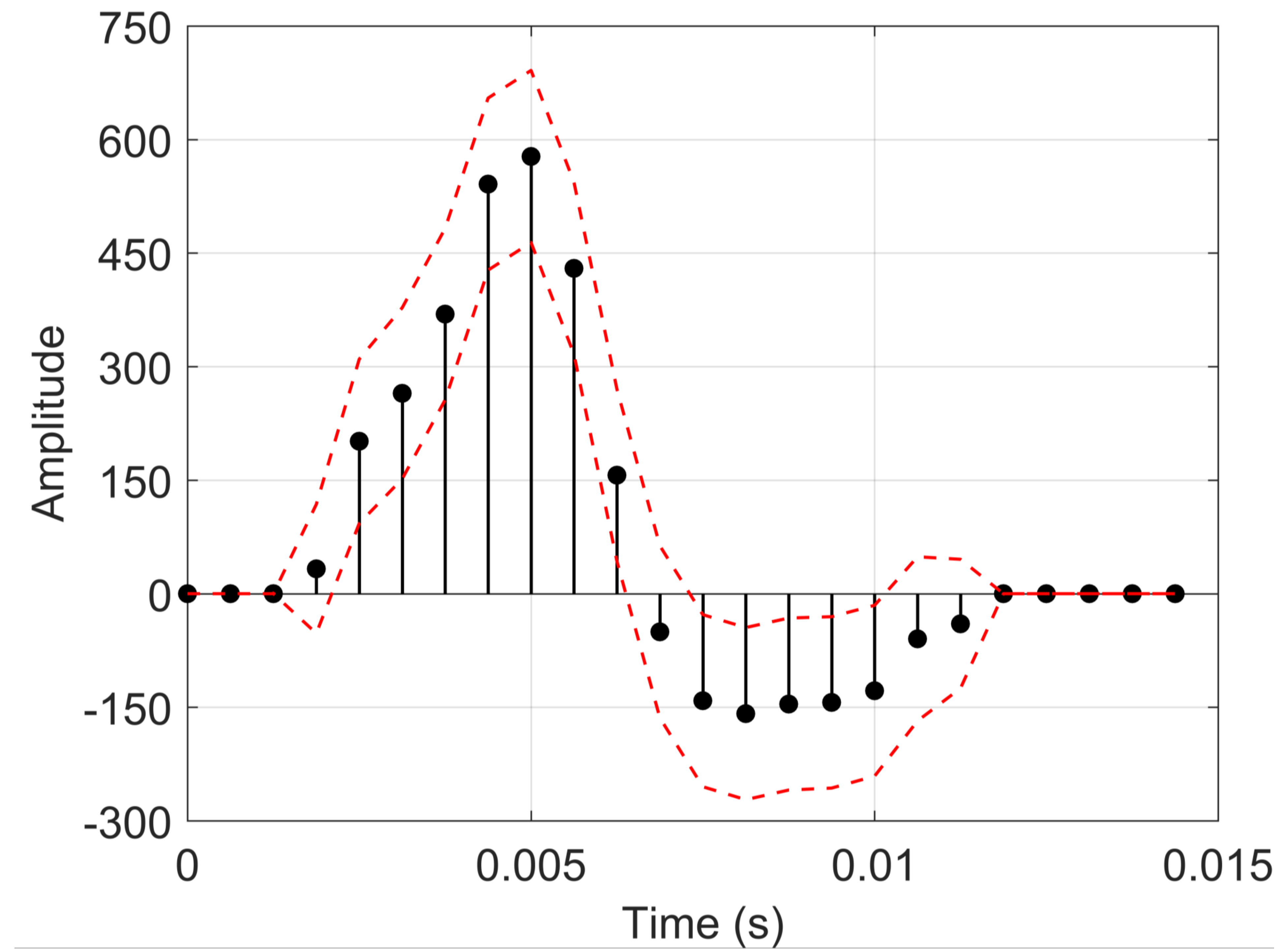
- Thermoacoustic case studies
- Results discussion
- Conclusions

Case studies: acoustic network model, impulse response identification and thermoacoustic mode specification







$$(h_1, h_2, \ldots, h_{16})$$

Multivariate Normal Distribution

Active Subspace approach identified a single active variable in each case

$$Y^A = a_1^A \cdot h_1 + a_2^A \cdot h_2 + ... + a_{16}^A \cdot h_{16}$$

$$Y^B = a_1^B \cdot h_1 + a_2^B \cdot h_2 + ... + a_{16}^B \cdot h_{16}$$

Dimensionality Reduction: 16D ⇨ 1D

1

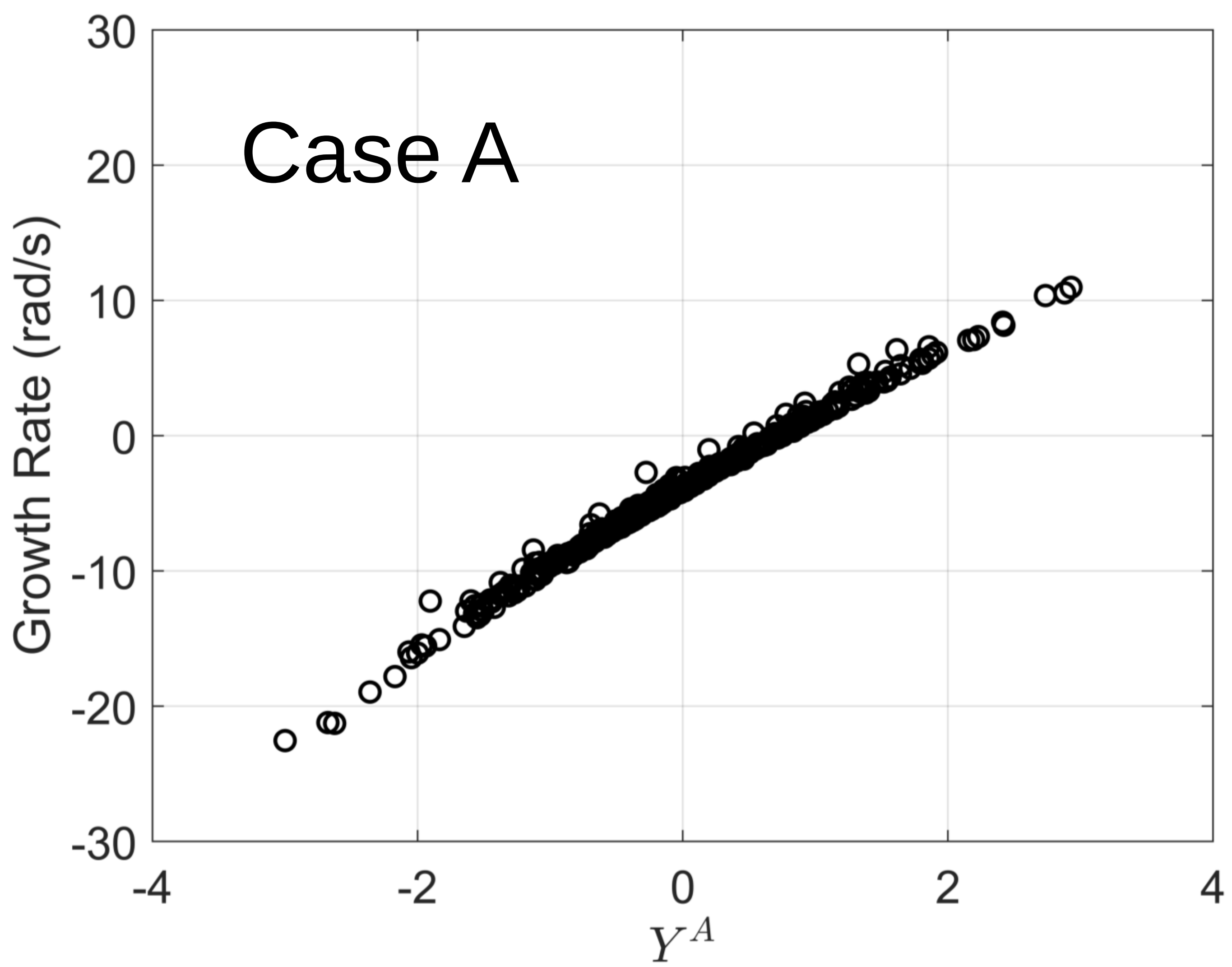
Subspace Identification

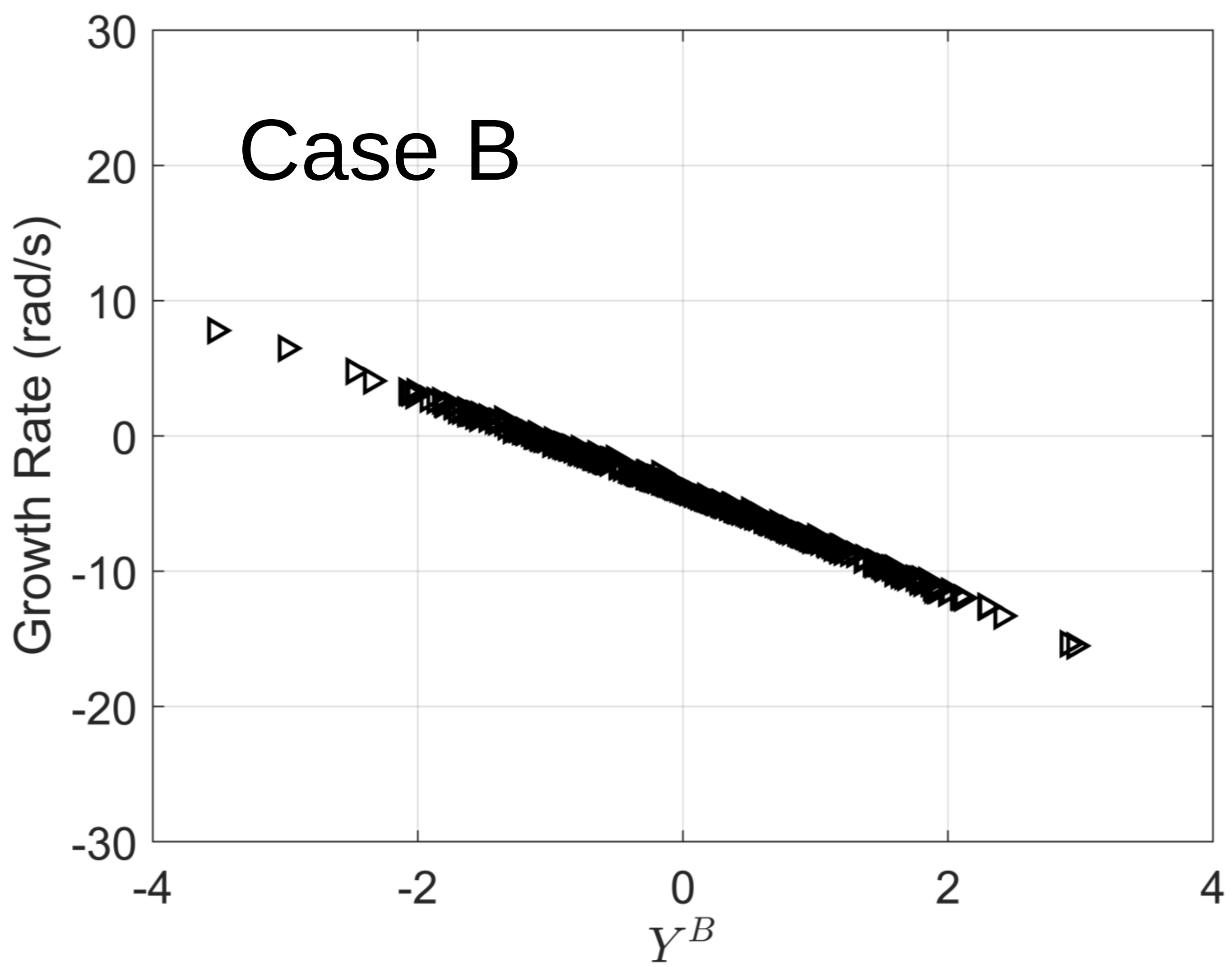
A surrogate model is trained to map from active variable value to growth rate value for each case

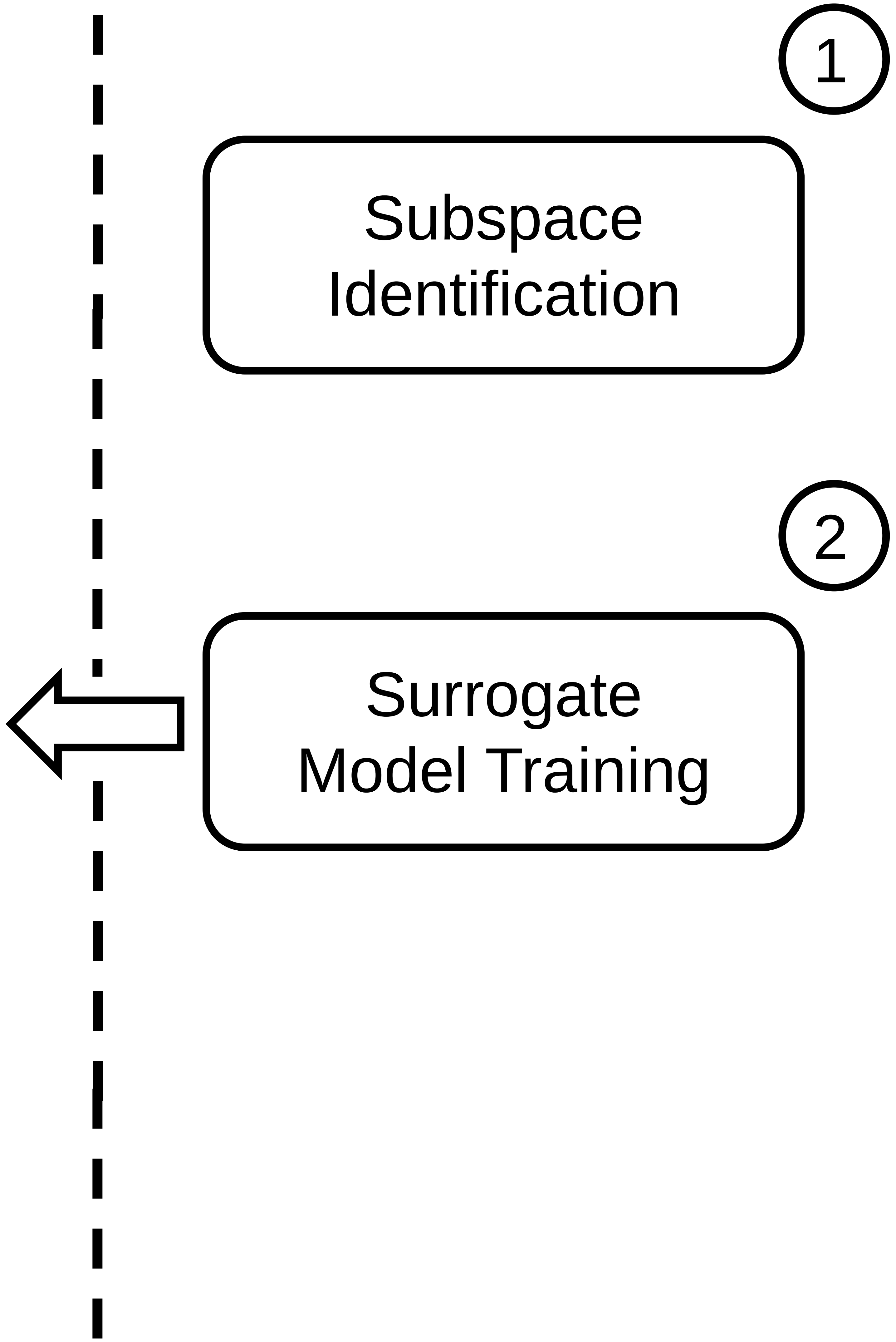
$$Y^A = a_1^A \cdot h_1 + a_2^A \cdot h_2 + ... + a_{16}^A \cdot h_{16}$$

$$Y^B = a_1^B \cdot h_1 + a_2^B \cdot h_2 + ... + a_{16}^B \cdot h_{16}$$

Dimensionality Reduction: 16D ⇨ 1D



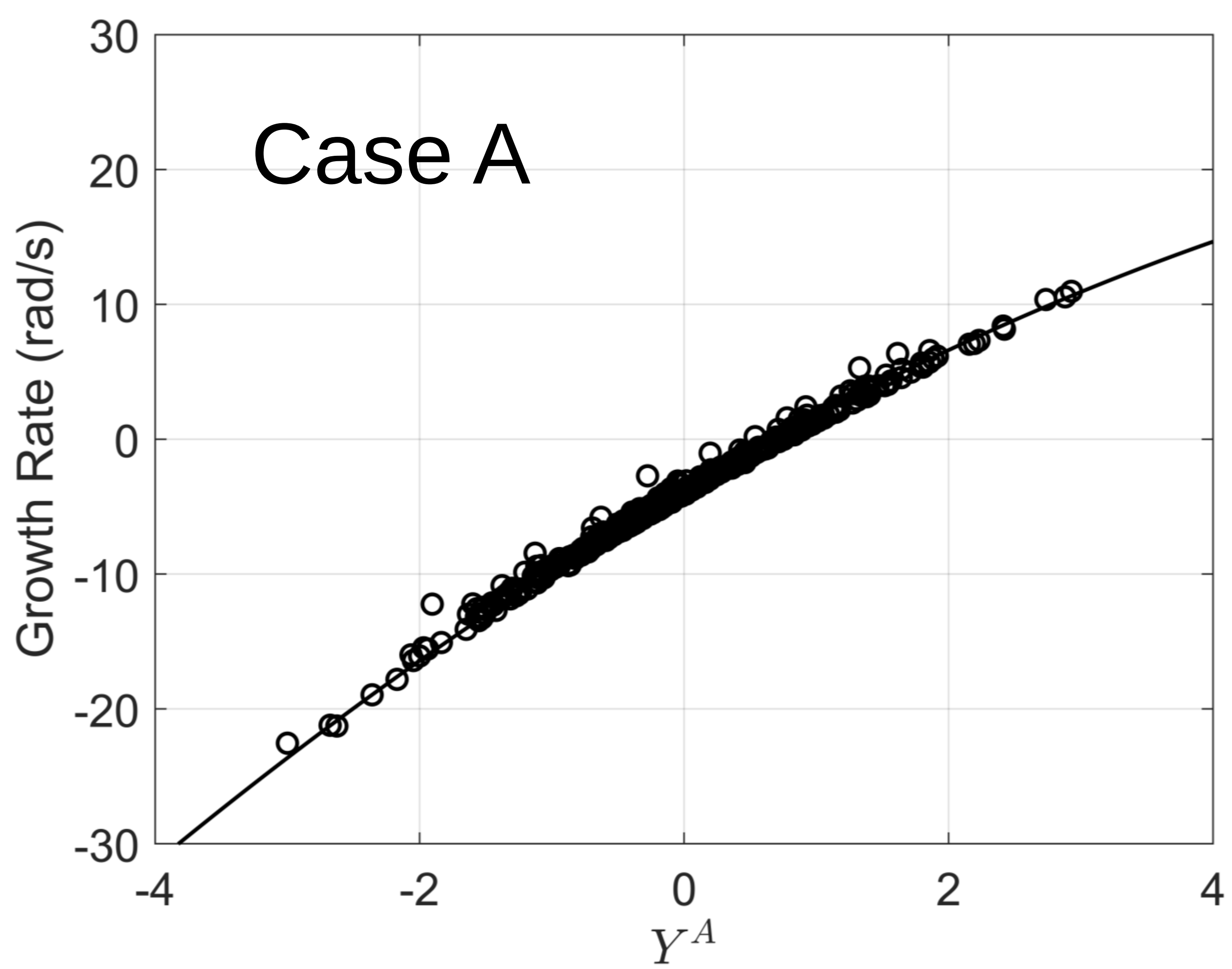


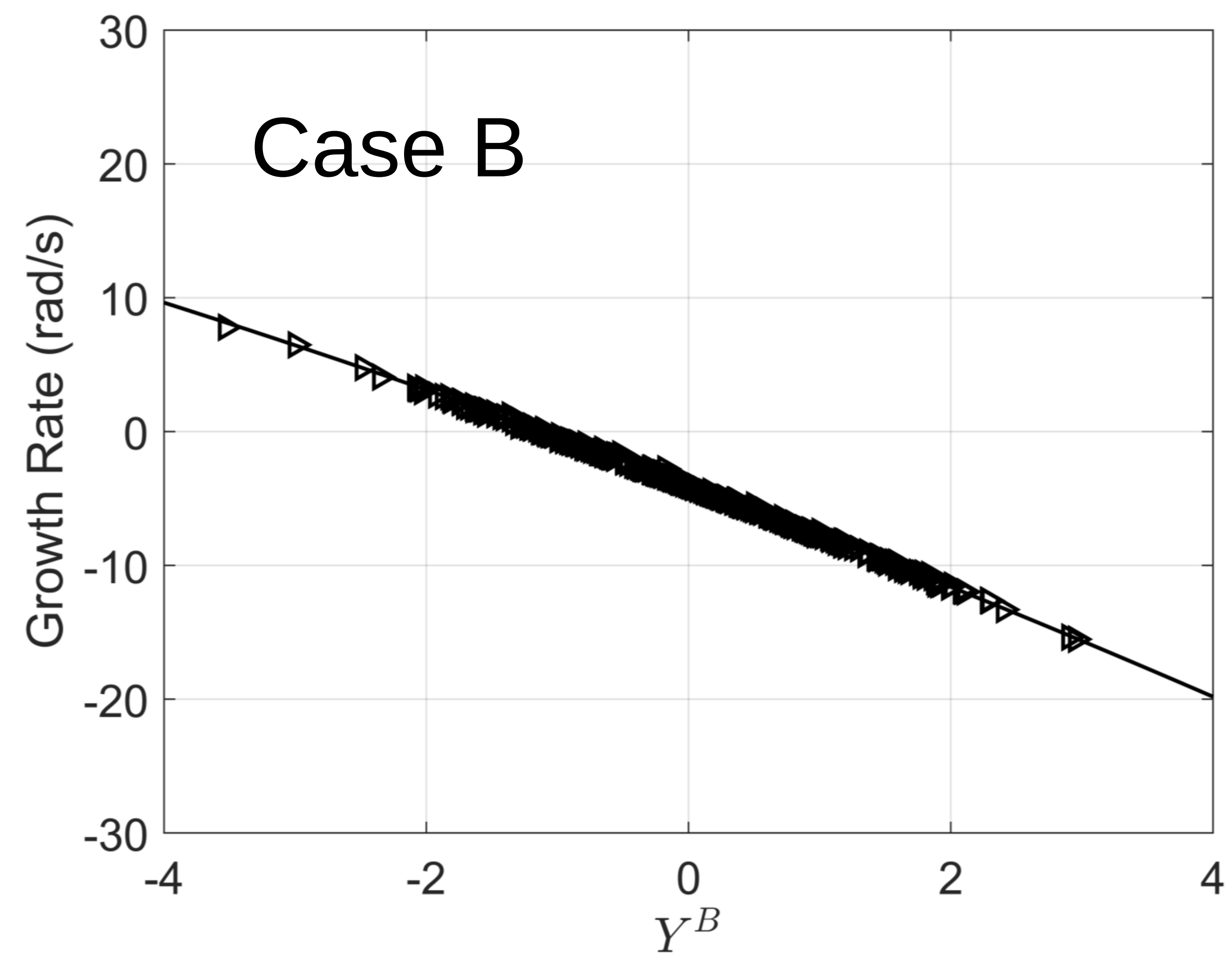


A quadratic function is trained to map from active variable value to growth rate value for each case

$$Y^A = a_1^A \cdot h_1 + a_2^A \cdot h_2 + \ldots + a_{16}^A \cdot h_{16}$$

$$Y^B = a_1^B \cdot h_1 + a_2^B \cdot h_2 + \ldots + a_{16}^B \cdot h_{16}$$





$$GR^A = \beta_0^B + \beta_1^B(Y^A) + \beta_2^B(Y^A)^2$$

$$GR^B = \beta_0^B + \beta_1^B(Y^B) + \beta_2^B(Y^B)^2$$

1

Subspace Identification

2

Surrogate Model Training

Monte Carlo simulation can be directly applied on the surrogate model to accelerate the UQ analysis

$$\boxed{{\color{red}Y^{A/B}} = a_1^{A/B} \cdot h_1 + a_2^{A/B} \cdot h_2 + \dots + a_{16}^{A/B} \cdot h_{16}}$$

$$\boxed{GR^{A/B} = \beta_0^{A/B} + \beta_1^{A/B} \cdot {\color{red}Y^{A/B}} + \beta_2^{A/B} \cdot ({\color{red}Y^{A/B}})^2}$$

$$(h_1, h_2, \dots, h_{16})^{(1)}$$

$$\vdots$$

$$(h_1, h_2, \dots, h_{16})^{(20000)}$$

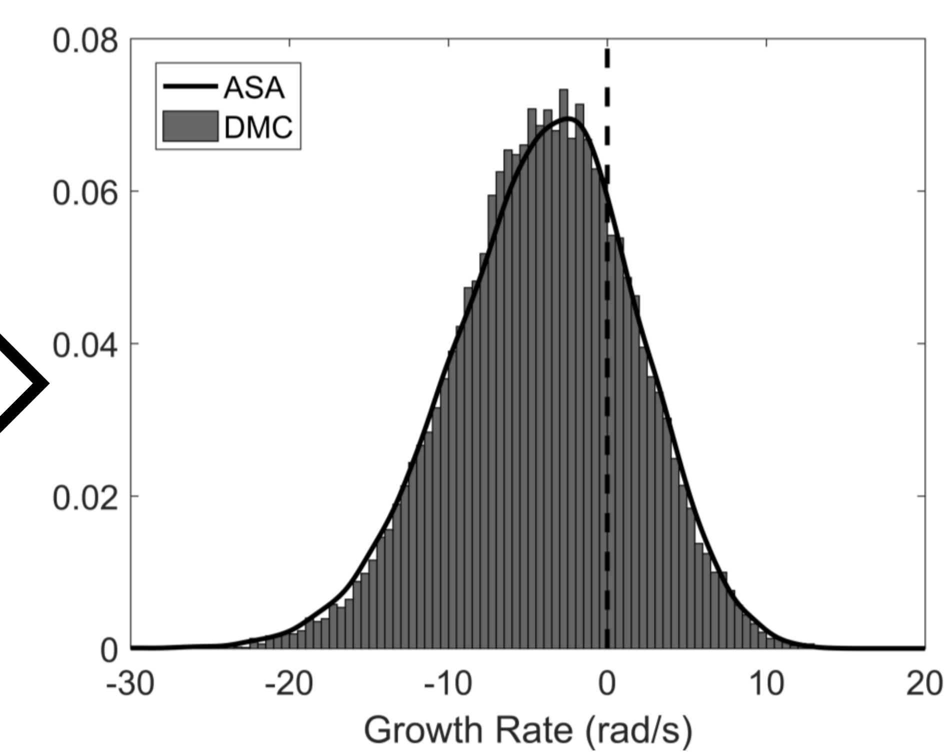
Samples

$$GR^{(1)}$$

$$\vdots$$

$$GR^{(20000)}$$

PDF

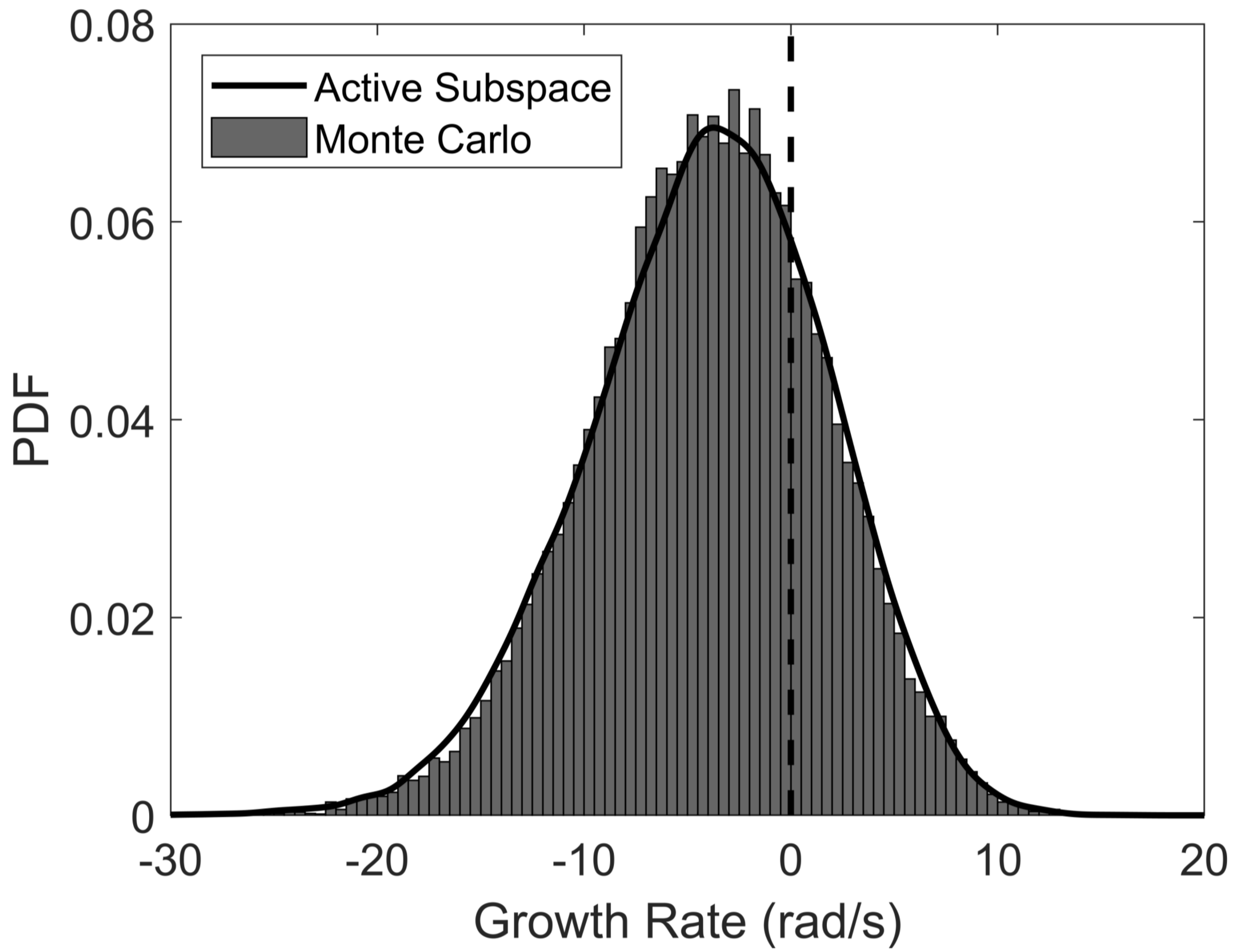


1 Subspace Identification

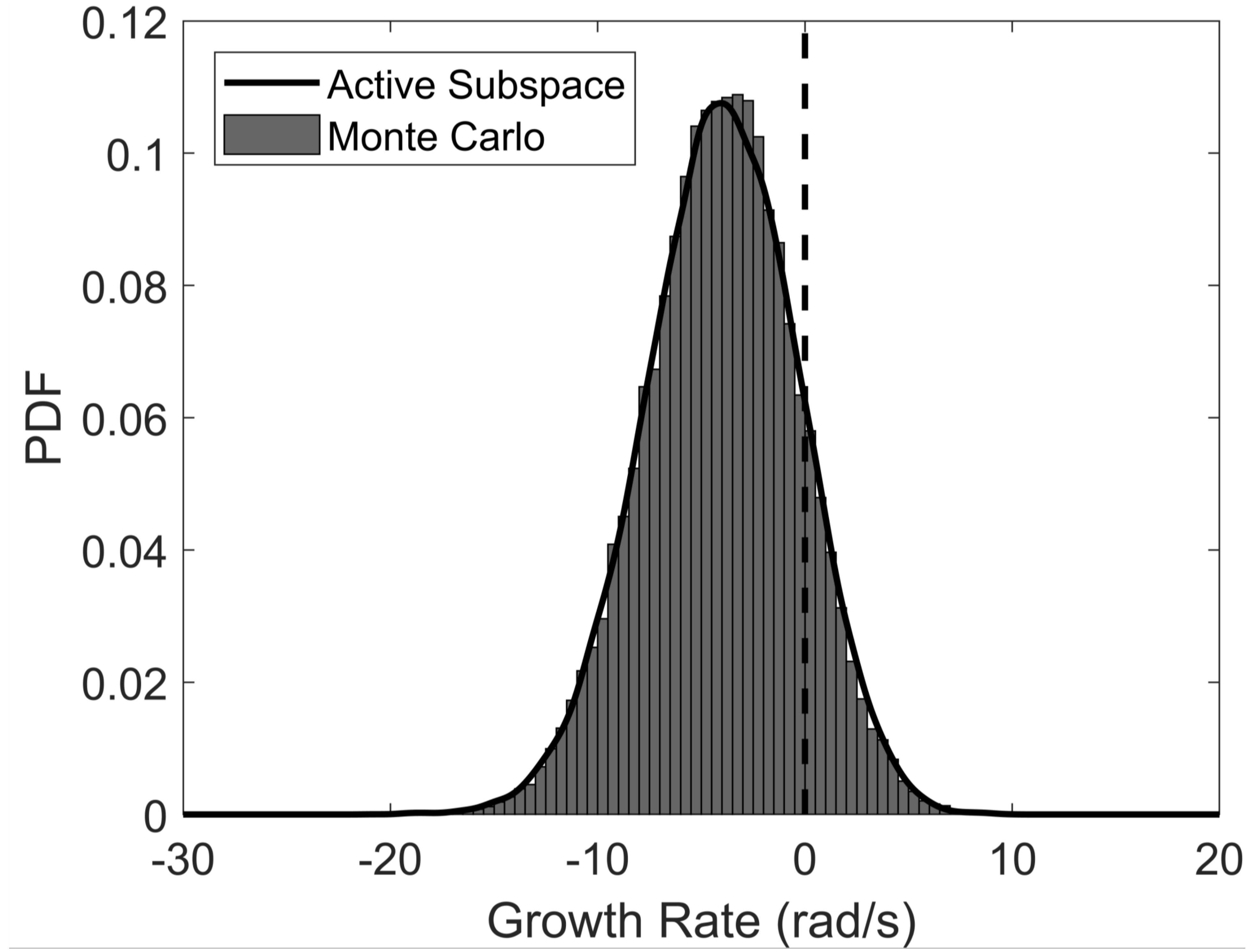
2 Surrogate Model Training

3 UQ Analysis

Active Subspace replicated the reference Monte Carlo results with 50 times less computational cost



Case A



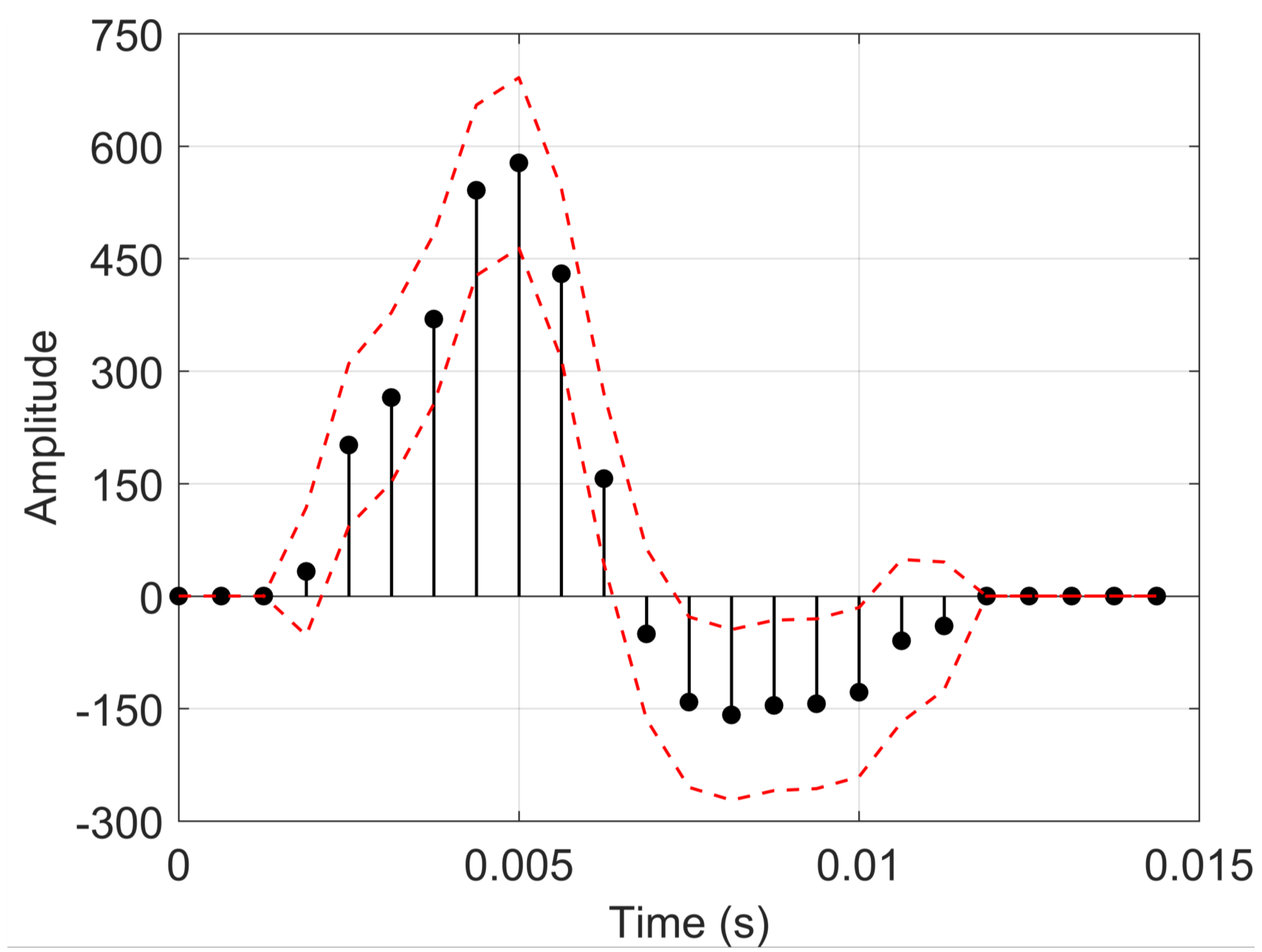
Case B

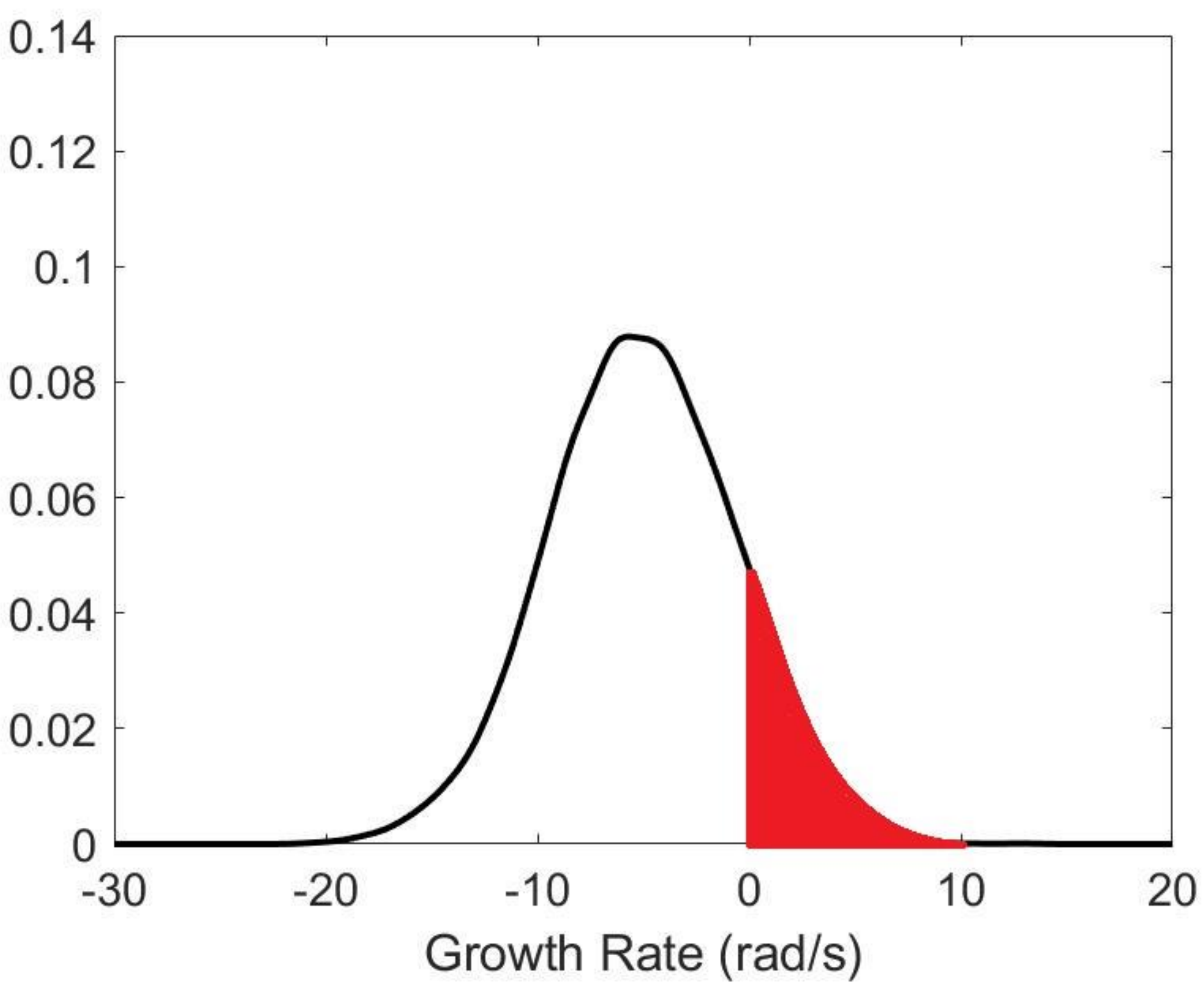
Computational Cost:
- Monte Carlo: 20000 acoustic network calculations
- Active Subspace: 400 acoustic network calculations

Active Subspace is very efficient in quantifying the impact of FIR model uncertainty on thermoacoustic instability prediction

Problem Solved:

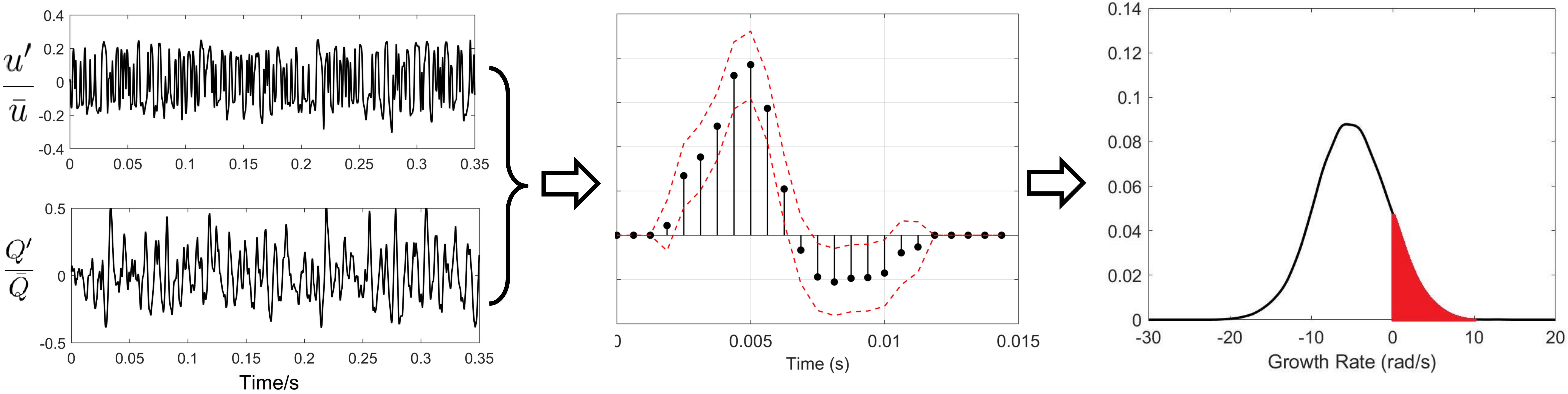
How to efficiently propagate uncertainties in flame impulse response model to the growth rate predictions?



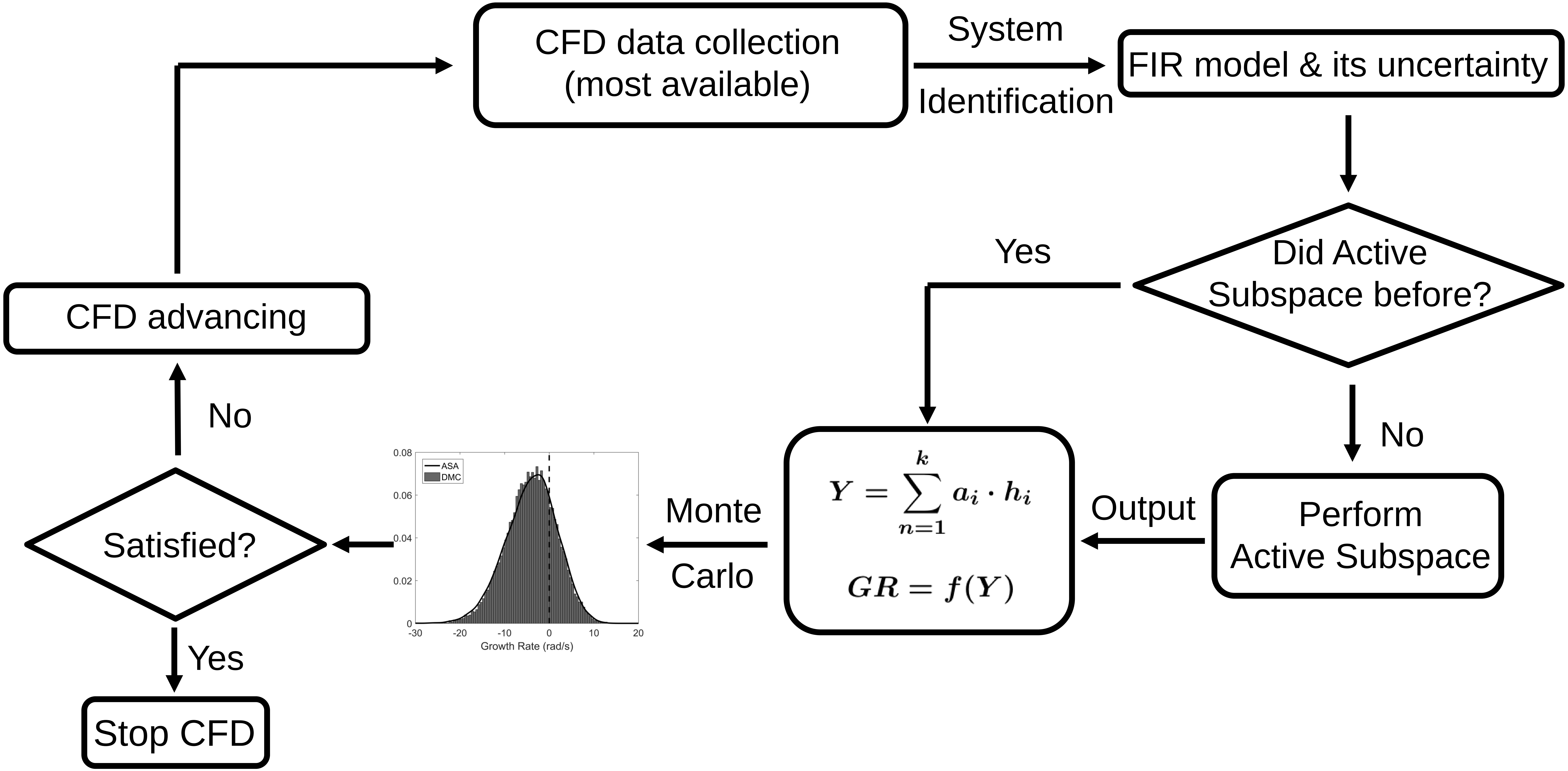


The main question we want to address

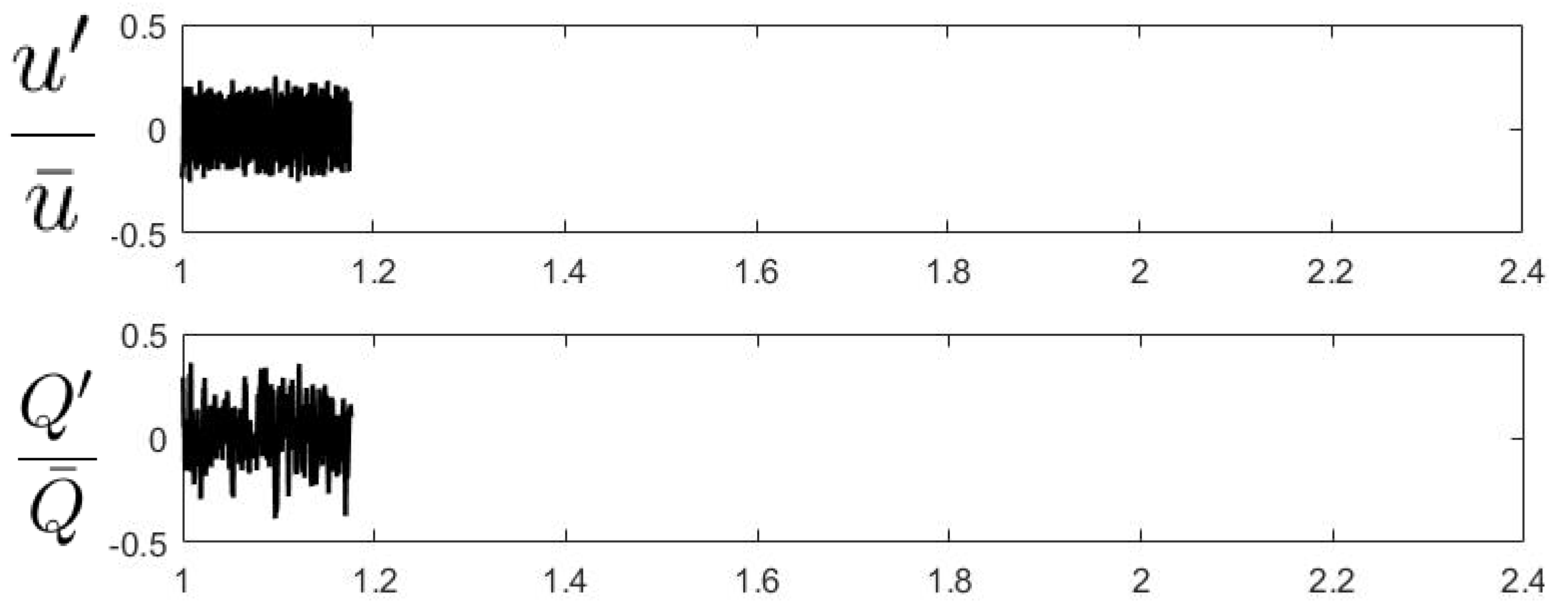
Problem: How to determine the adequate length of CFD time series to achieve a satisfactory uncertainty reduction in growth rate prediction?

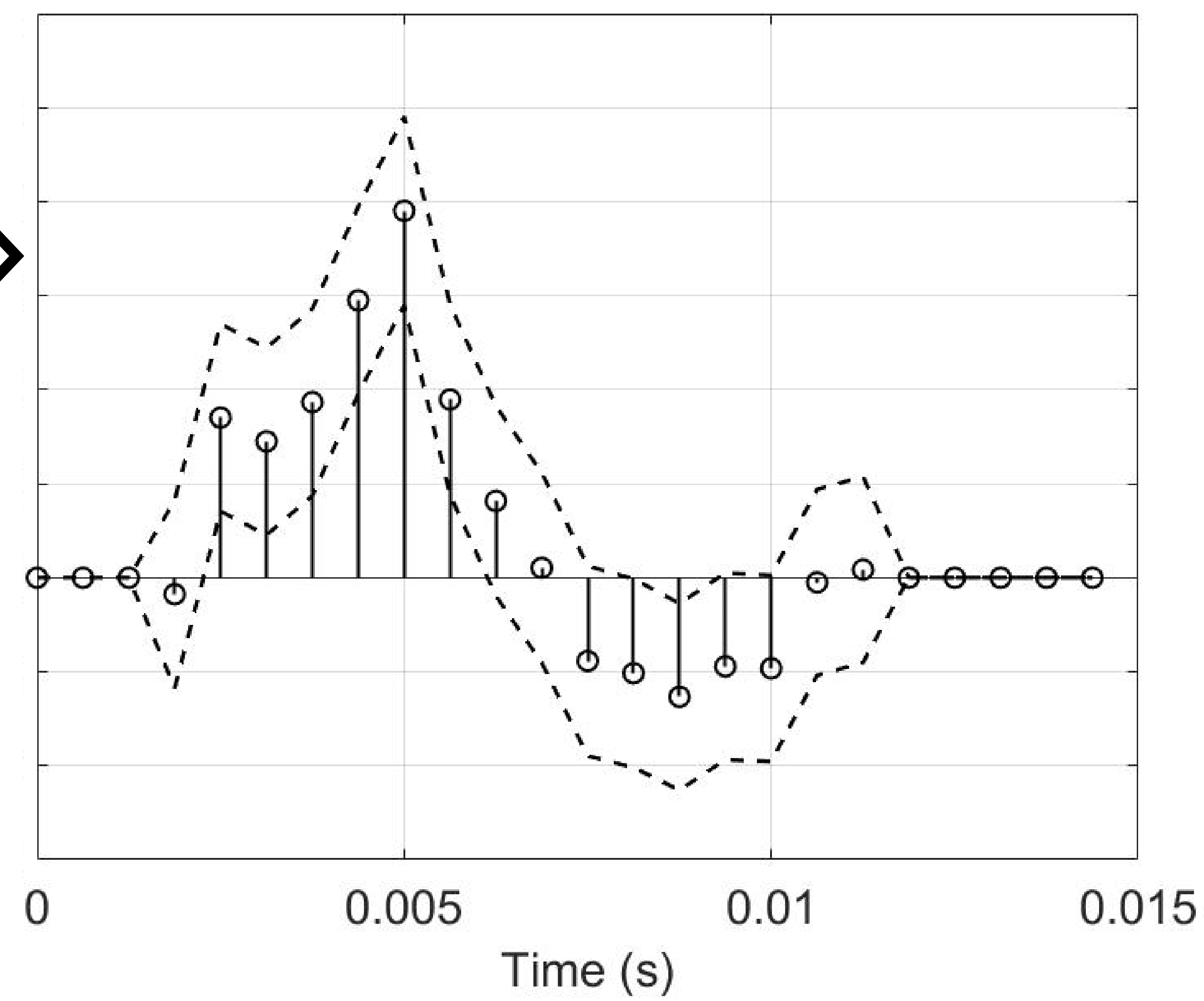


Procedure for estimating the CFD time series length based on Active Subspace approach



Procedure demonstration

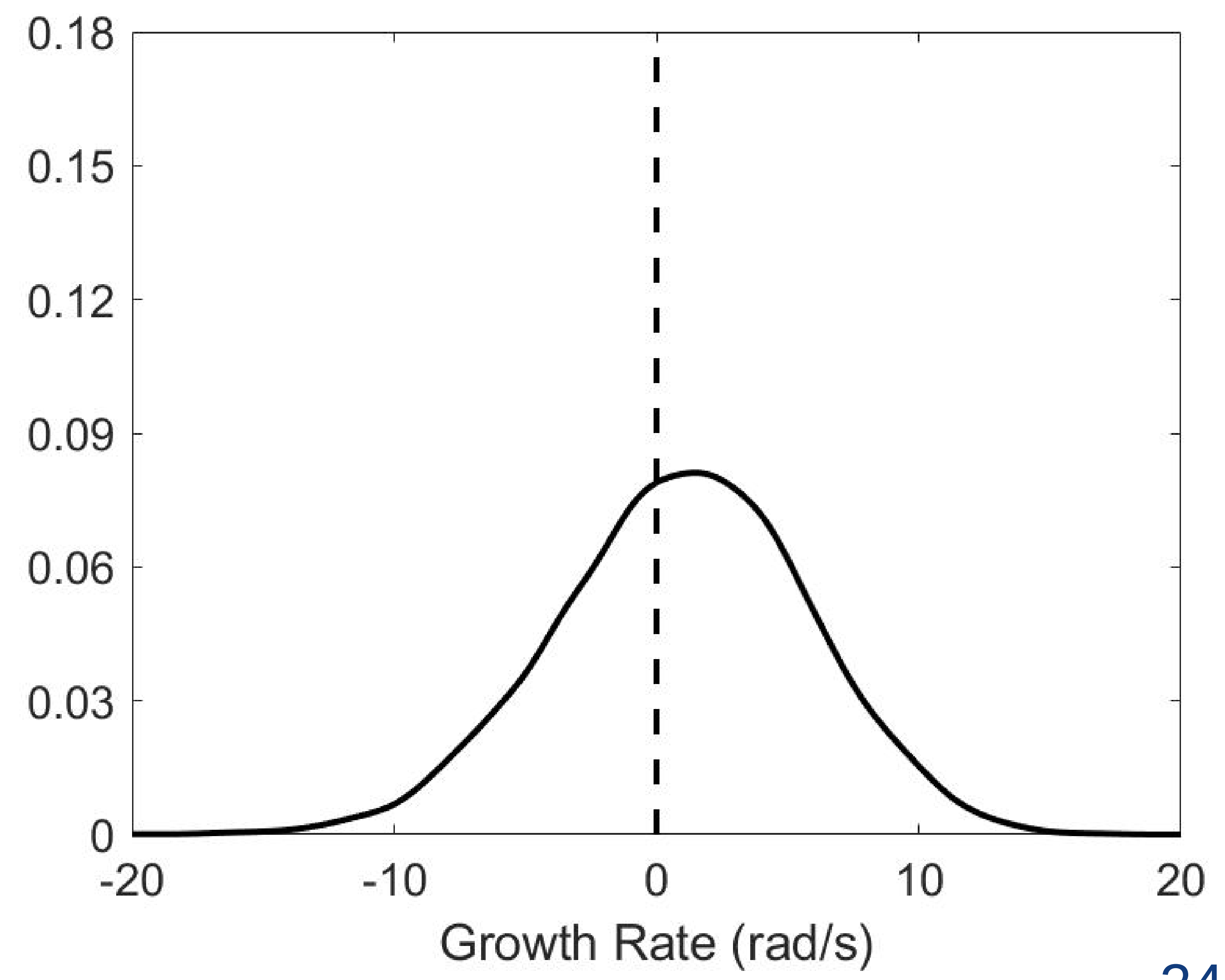




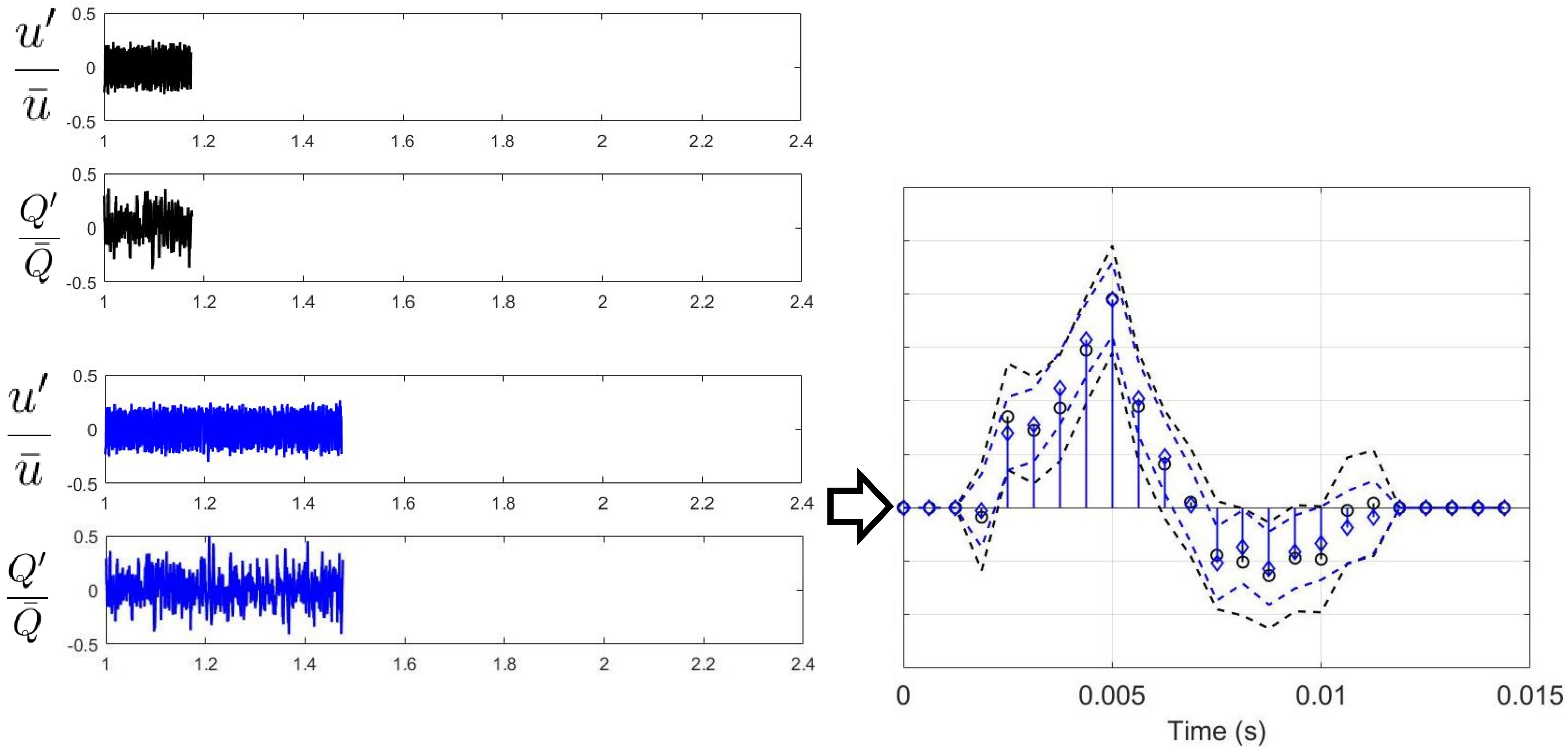
$$Y = \sum_{n=1}^{k} a_i \cdot h_i$$

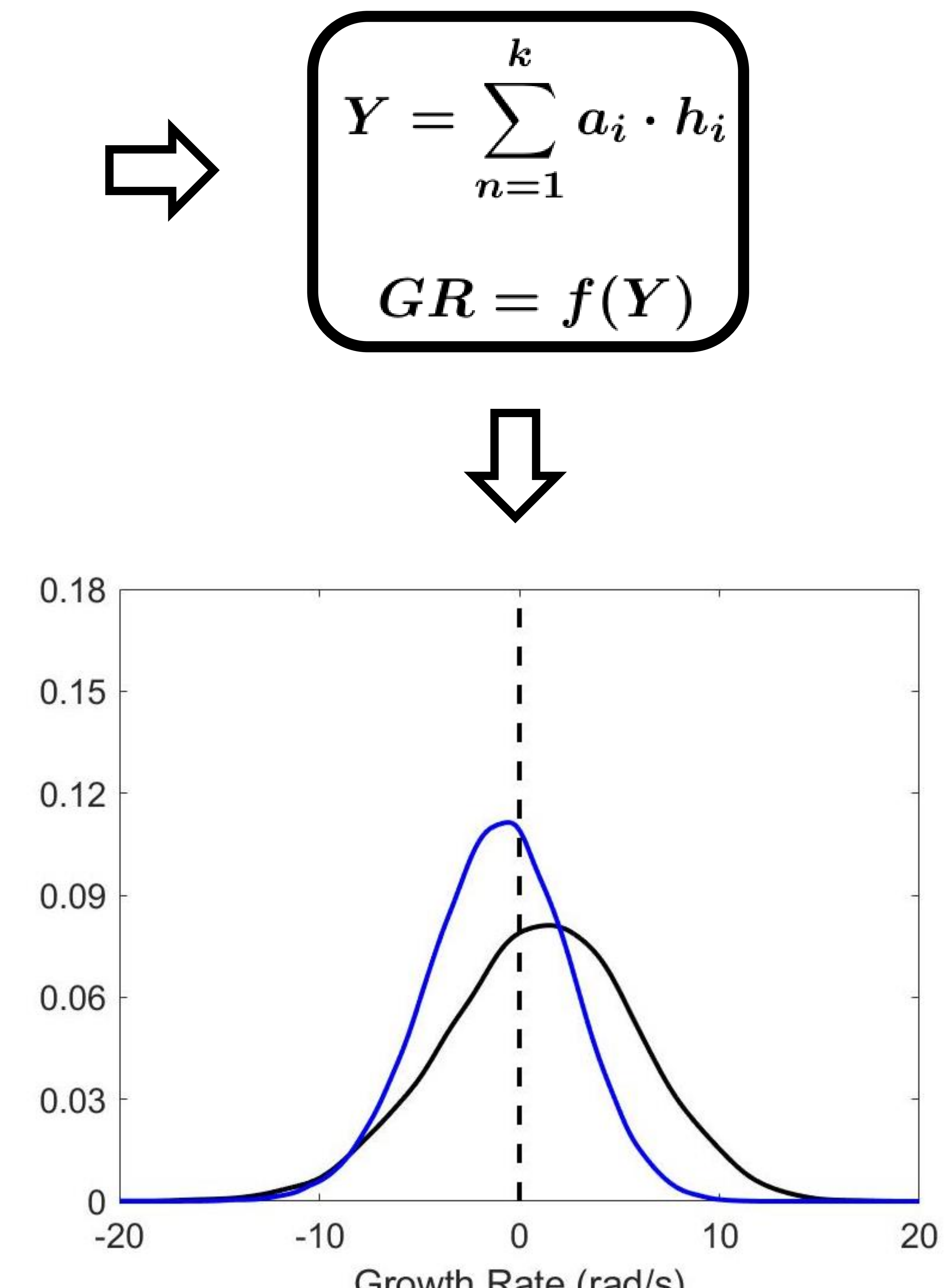
$$GR = f(Y)$$

Active Subspace

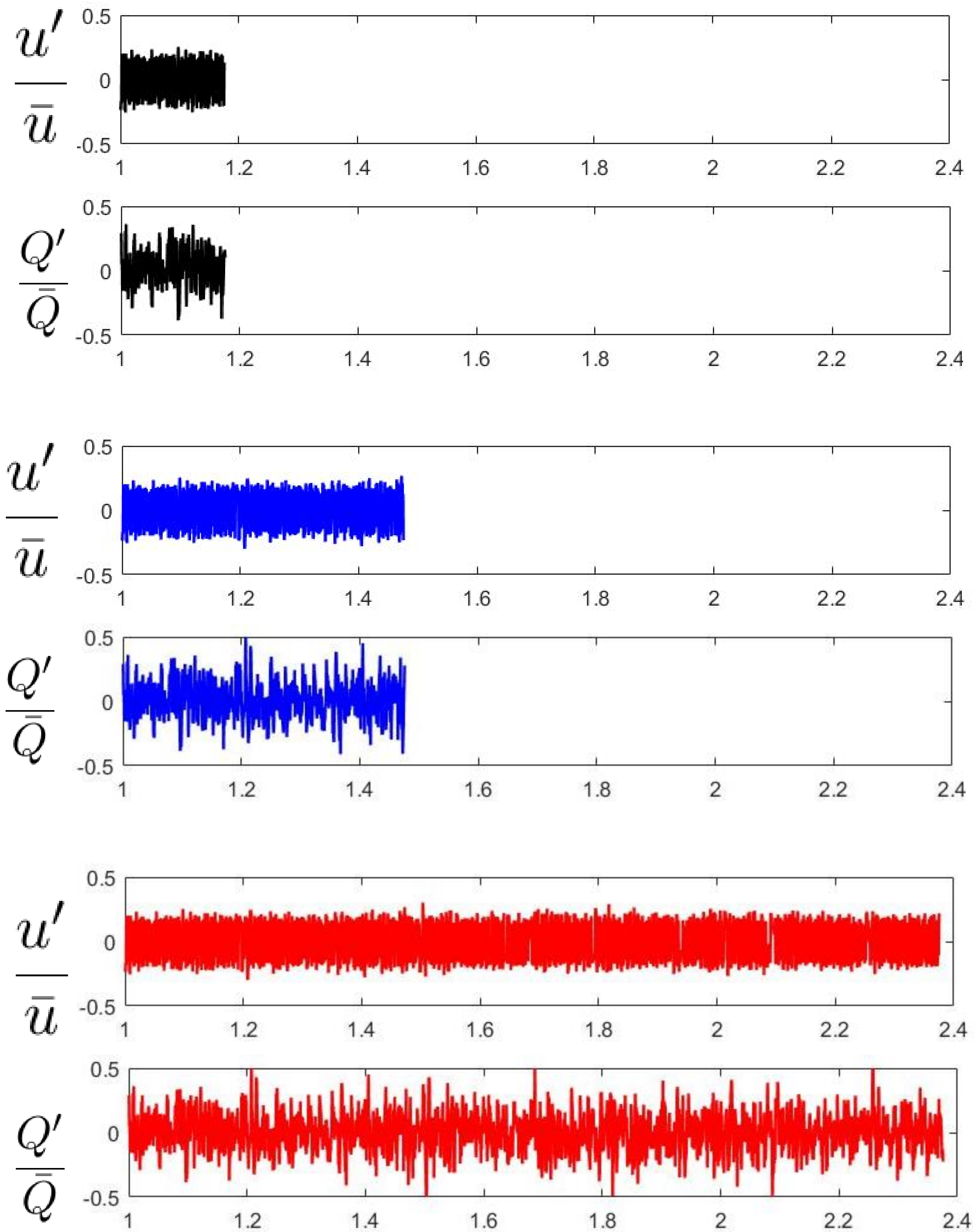


Procedure demonstration



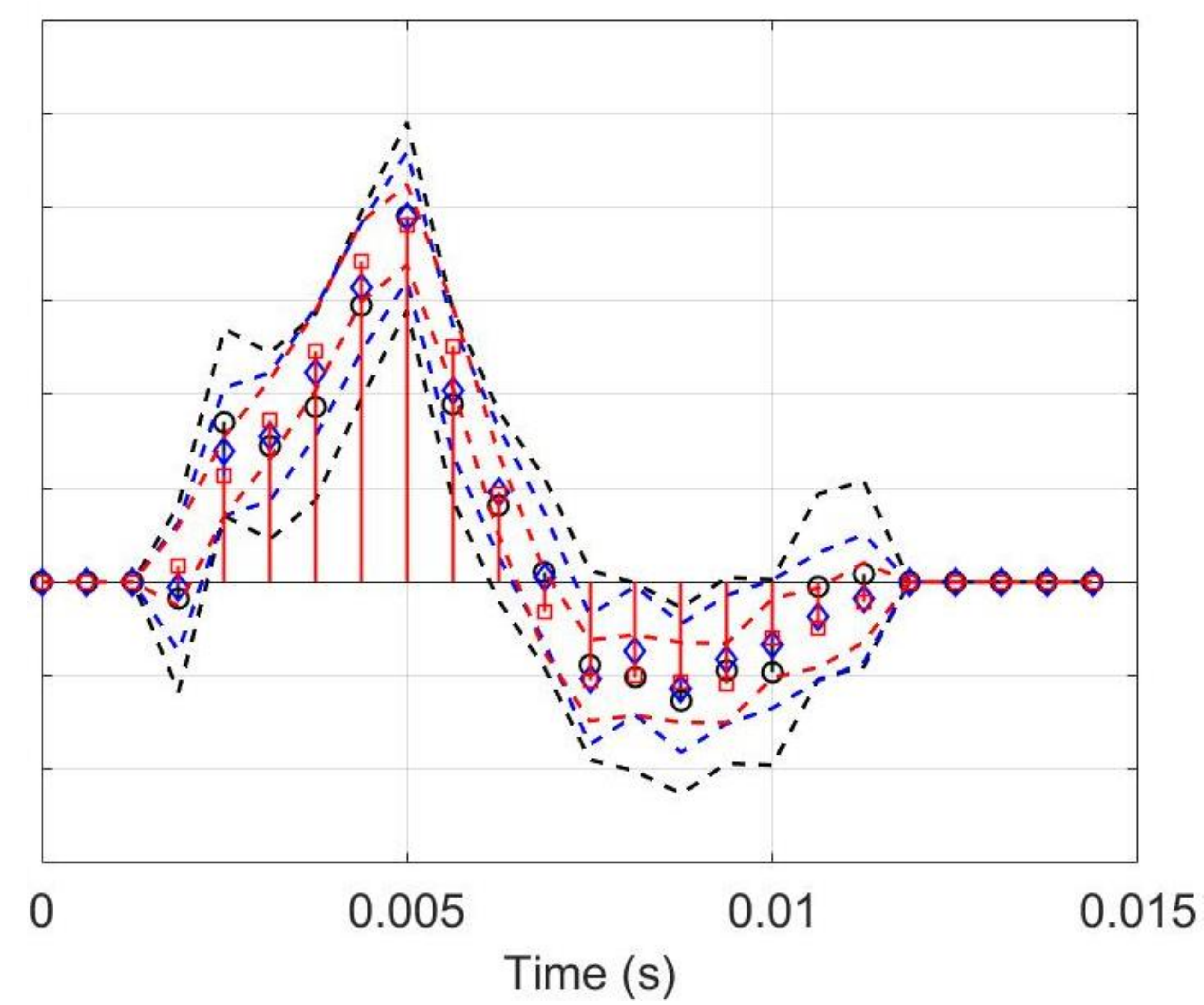


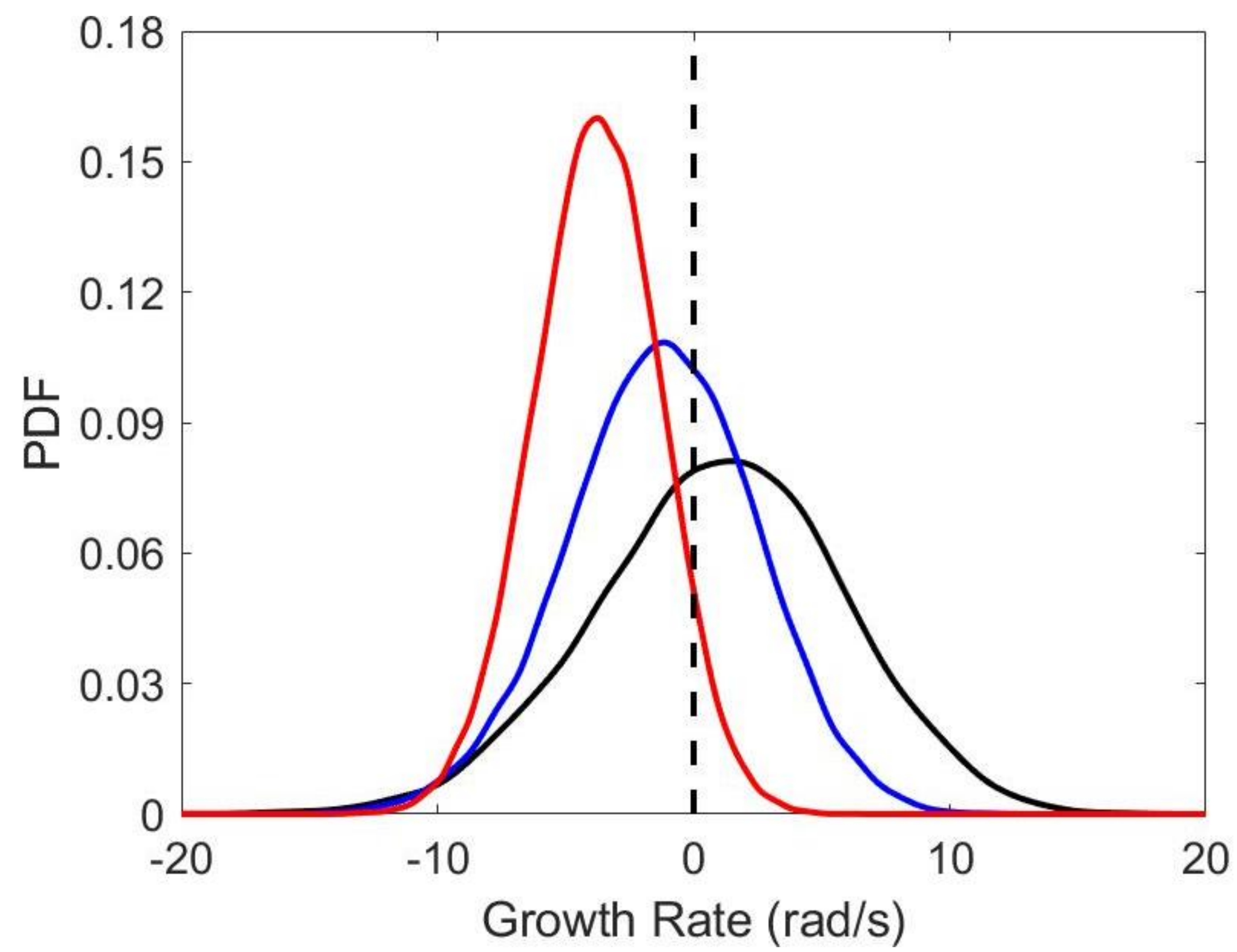
Procedure demonstration



$$Y = \sum_{n=1}^{k} a_i \cdot h_i$$

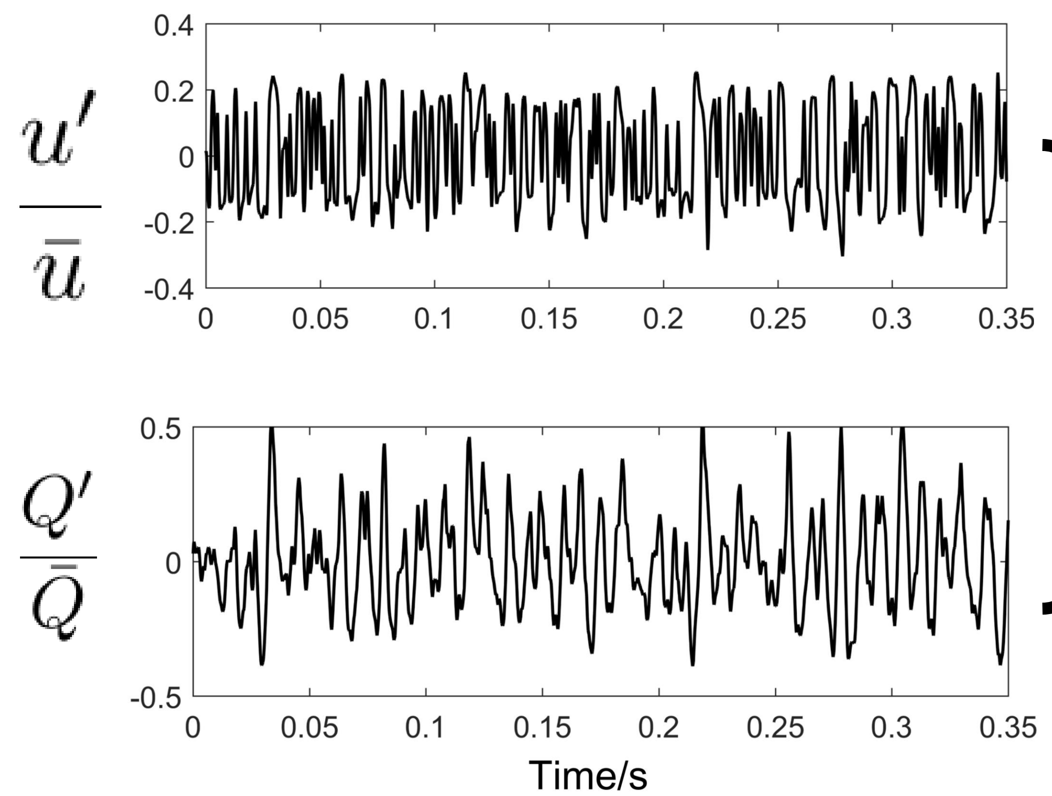
$$GR = f(Y)$$

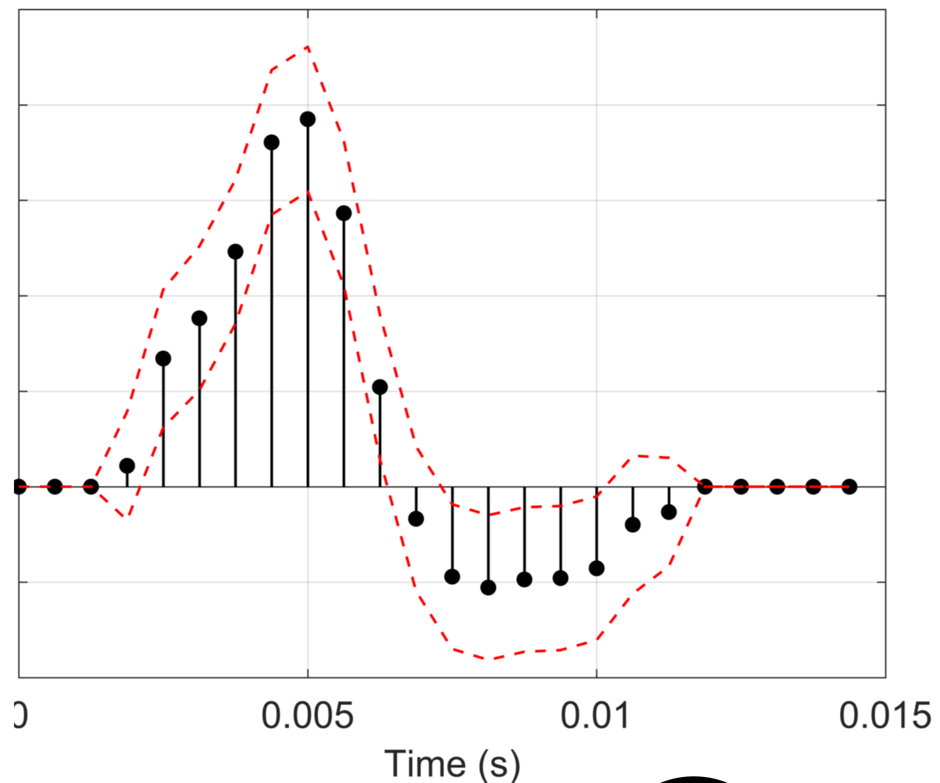


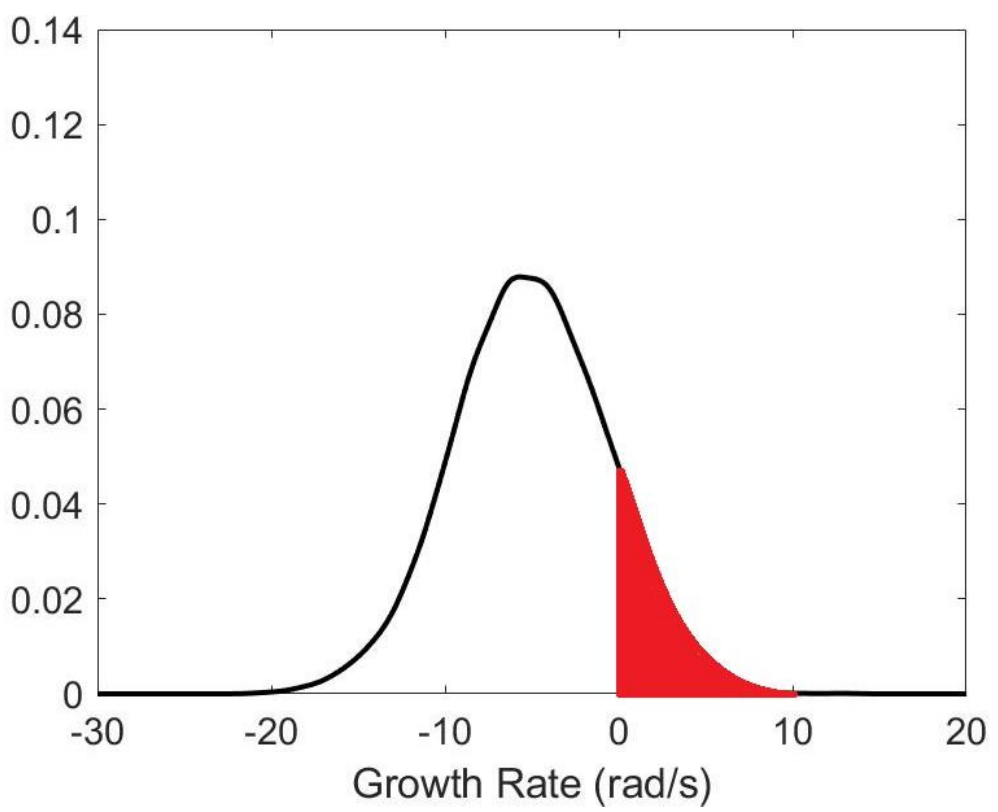


Conclusions

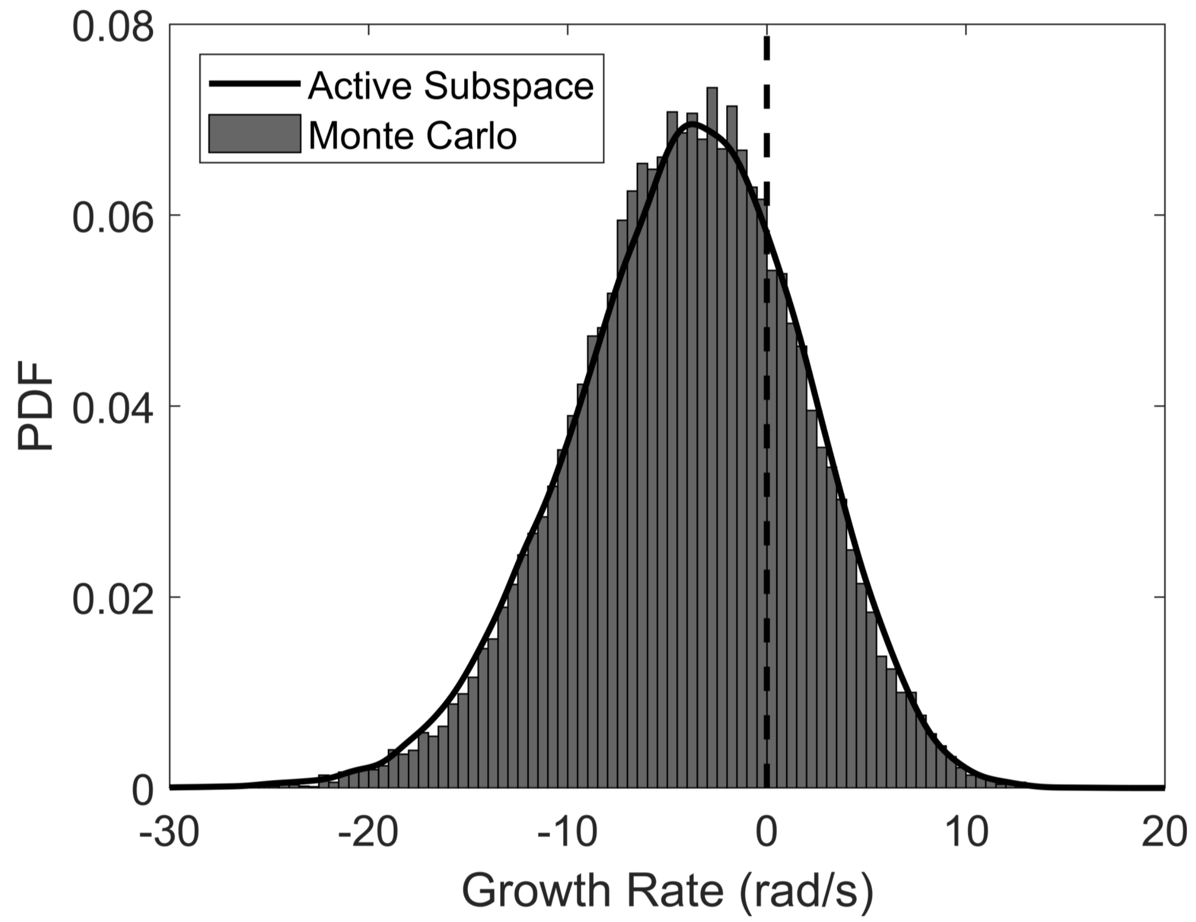
1 A procedure to estimate adequate length of CFD time series



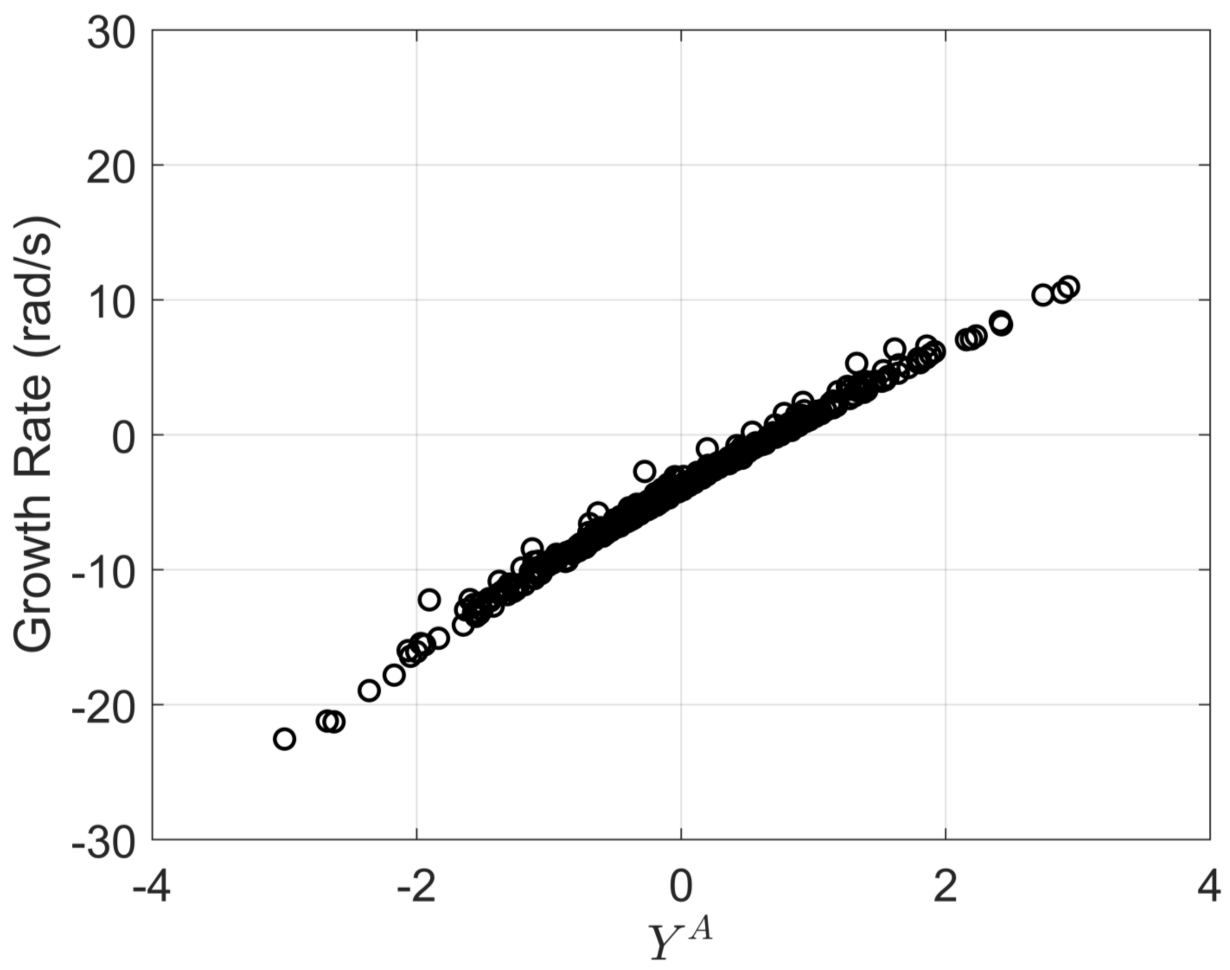




2 Active Subspace offers more efficient UQ analysis



3 One Active Variable is sufficient



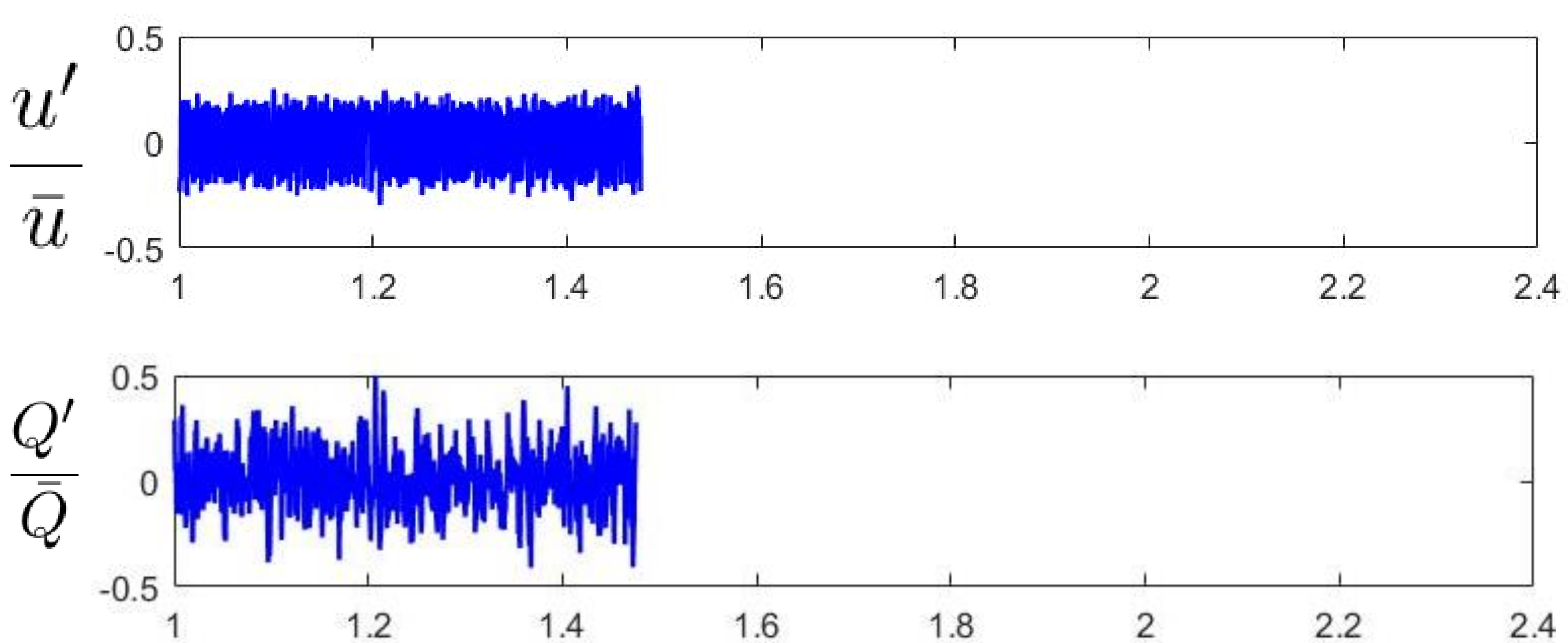
Back-up Slides

Impulse Response Identification

$$\underbrace{\frac{\dot{Q}'_n}{\bar{\dot{Q}}} = \frac{1}{\bar{u}_u}\sum_{k=0}^{L-1} h_k u'_{u,n-k}}_{\text{Convolution}}$$

u'_u Velocity fluctuation upstream of the flame

$\dot{Q}'$ Global heat release rate fluctuation

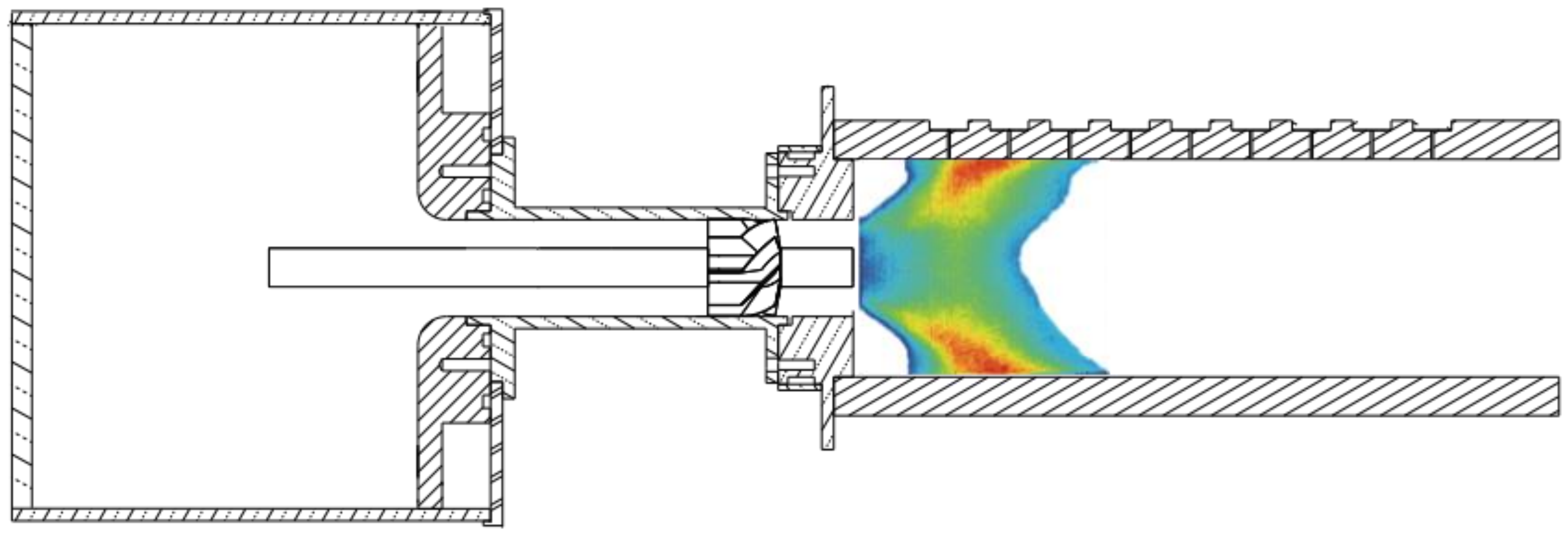


Least Square $\Rightarrow$

Estimations

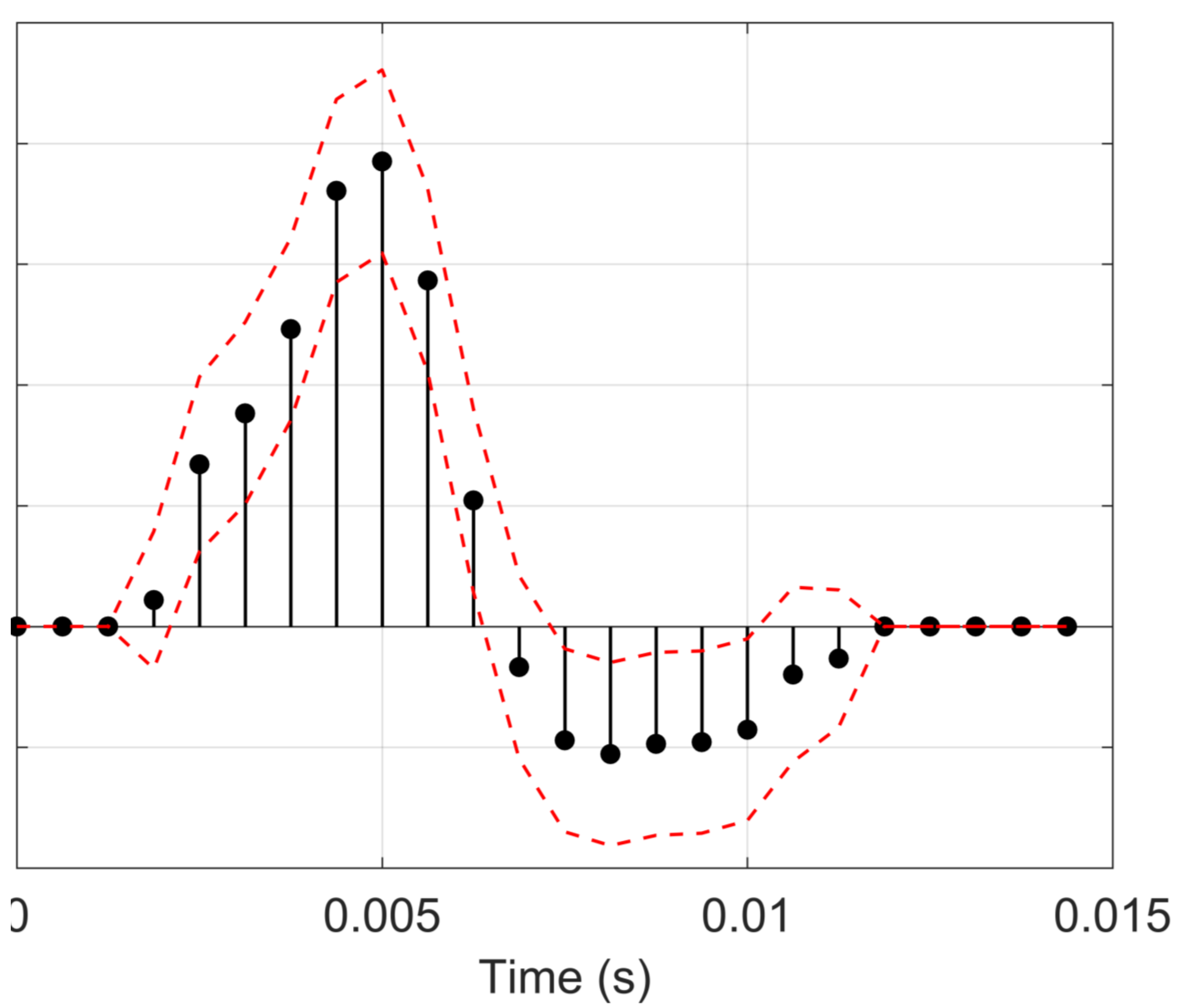
Estimation Error
(Covariance matrix)

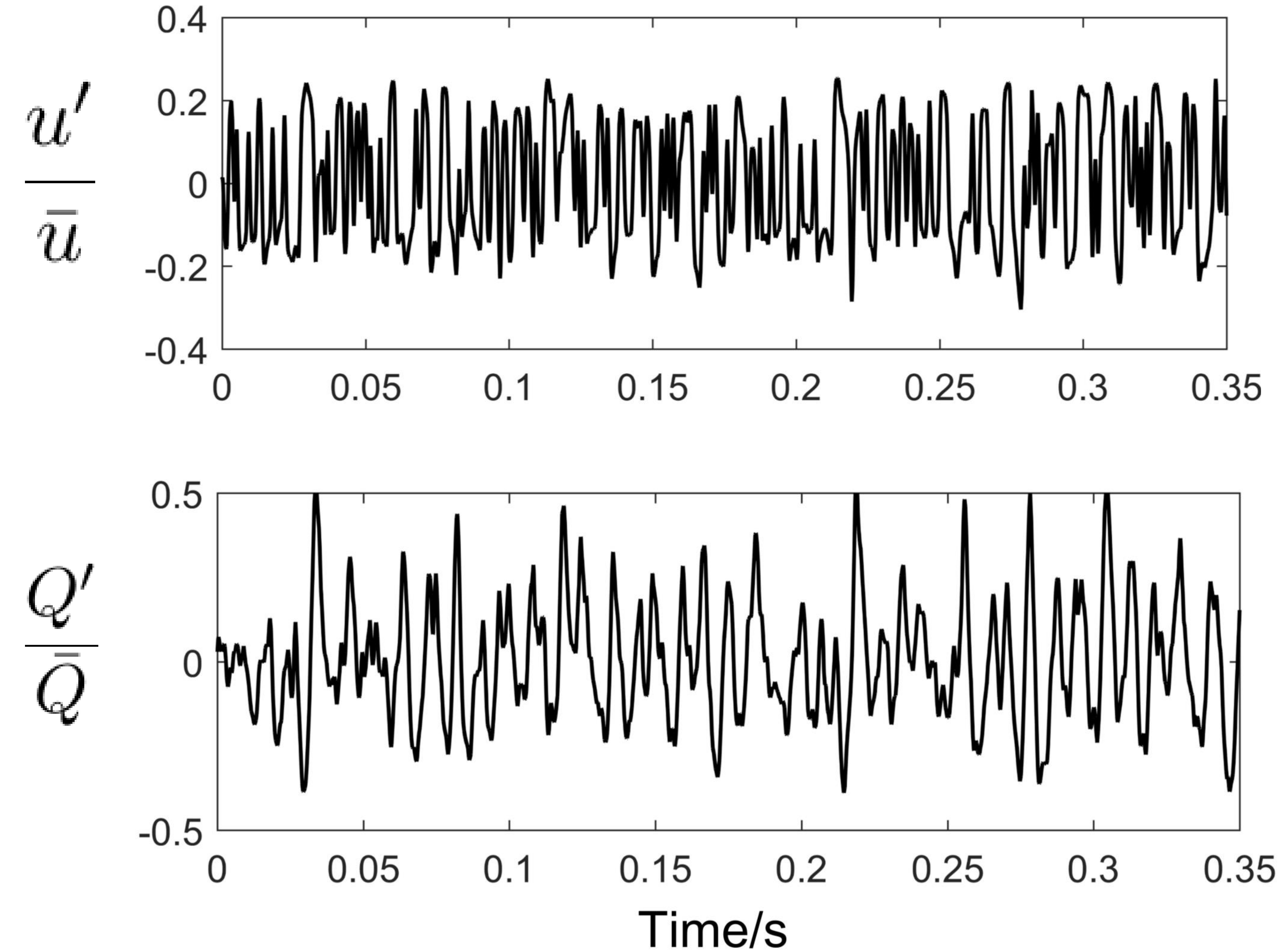
Current Impulse Response Model is identified through LES simulations



Test Rig: A turbulent, premixed Swirl Burner

95% CI

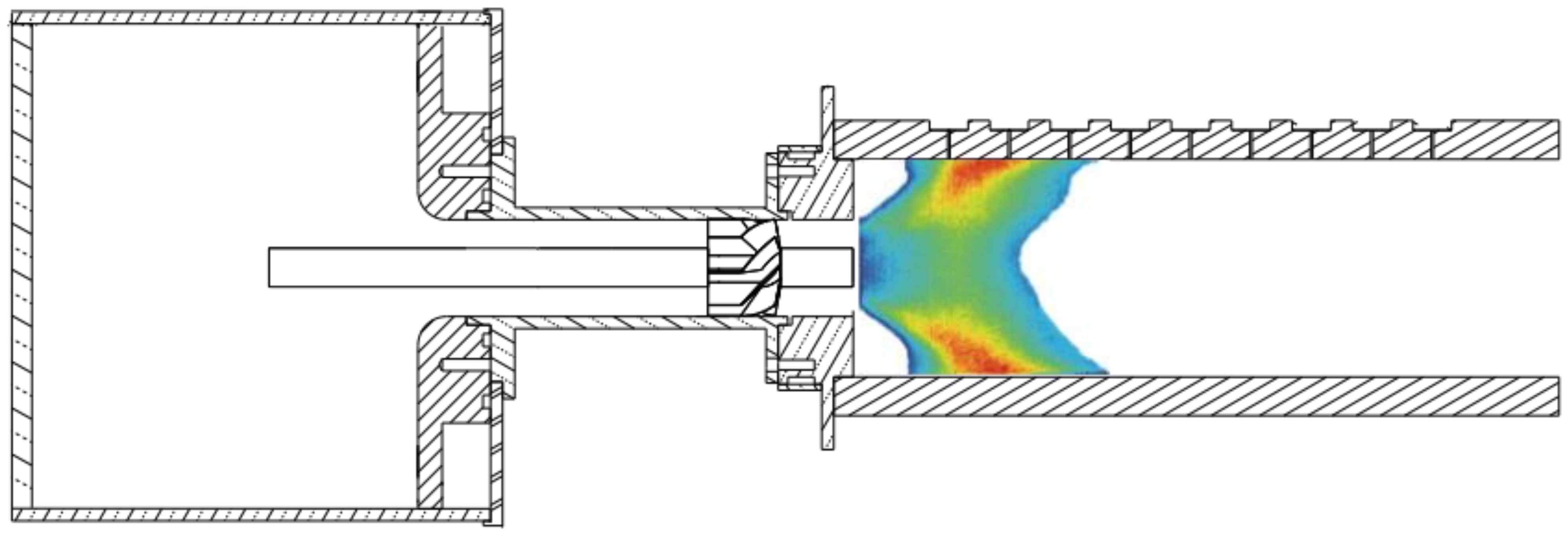


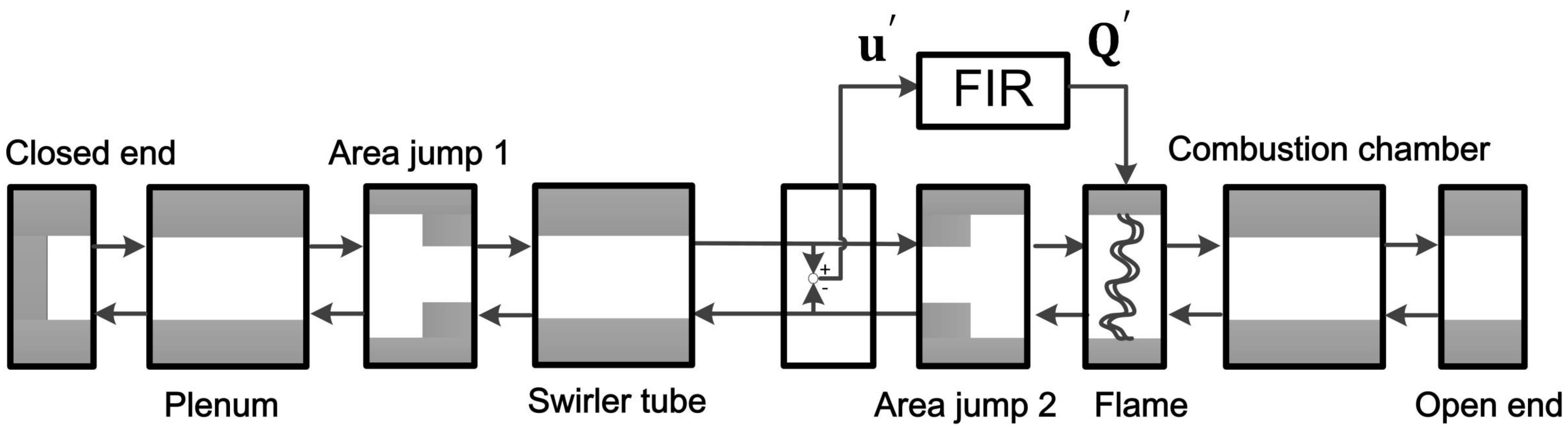


LES time series[1]

[1] Luis Tay-Wo-Chong et al., 2010 in ASME

Current Acoustic Network Model and its model parameters



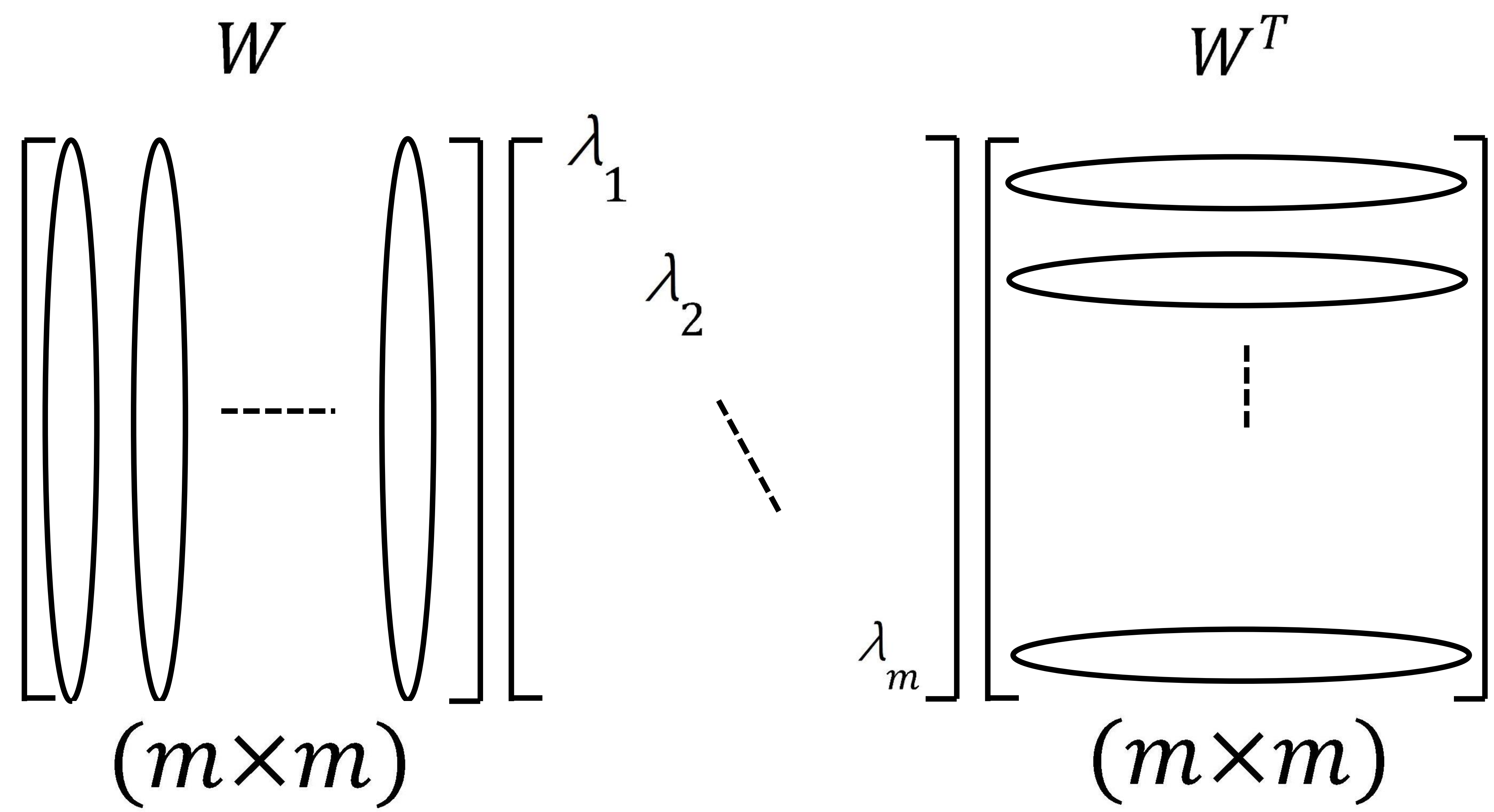


Acoustic element	Parameters
Closed end	Reflection coefficient = 1
Plenum	Length = 0.17m
	Sound speed = 343 m/s
Area jump 1	Area ratio = 29.76
Swirler tube	Length = 0.18m
	Sound speed = 343 m/s
Area jump 2	Area ratio = 0.13
Flame	Relative temperature jump = 5.59
	Ratio of specific impedances = 2.57
FIR	Impulse response model
Combustion chamber	$Length_A$ = 0.51m; $Length_B$ = 0.6m
	Sound speed = 880 m/s
Combustor exit	Reflection coefficient$_A$= -0.9883
	Reflection coefficient$_B$= -0.6351

Active Subspace Identification

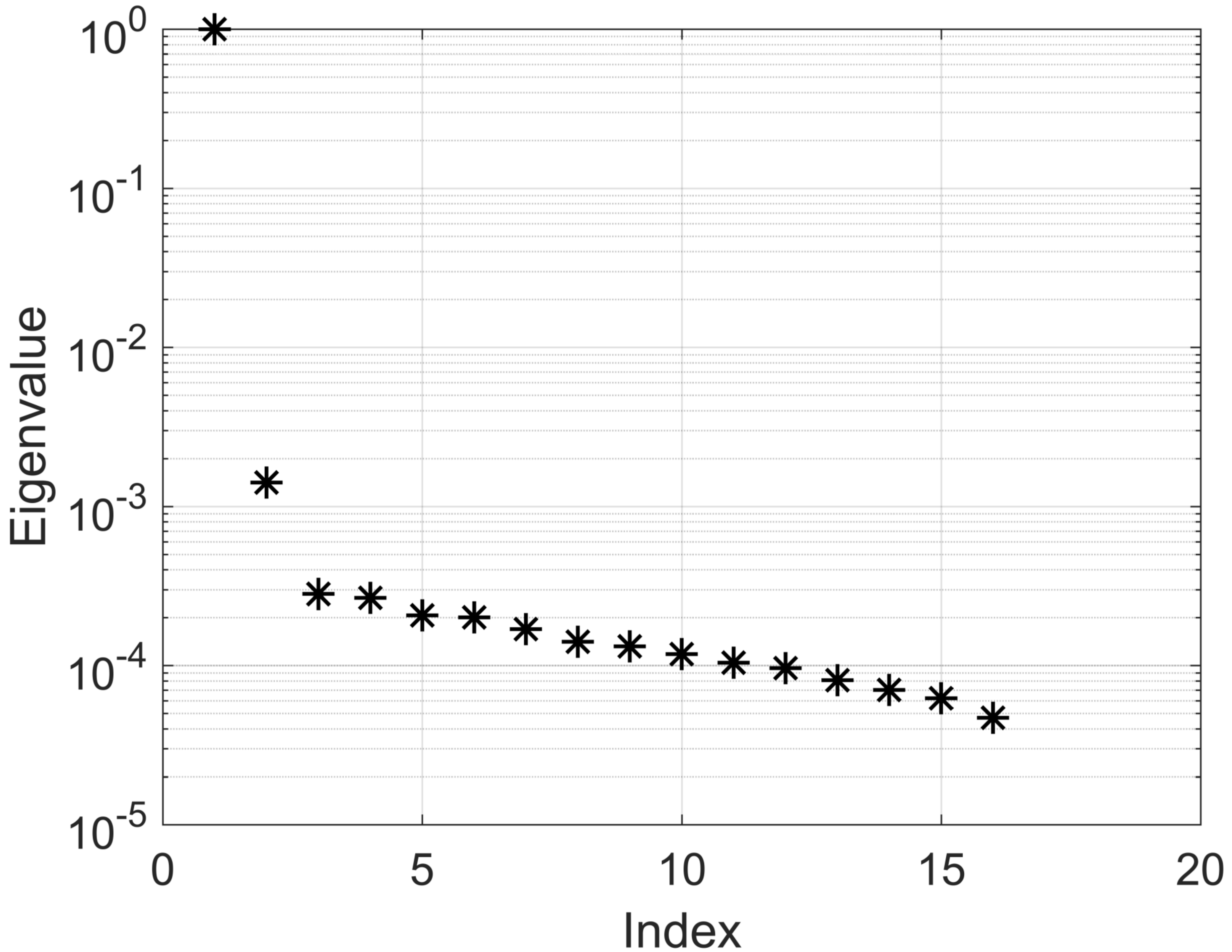
$$f = f(\vec{x}) \qquad \vec{x} \in R^m$$

$$\underset{(m\times m)}{C} = E[\underbrace{(\nabla_x f)}_{(m\times 1)}\underbrace{(\nabla_x f)^T}_{(1\times m)}] =$$

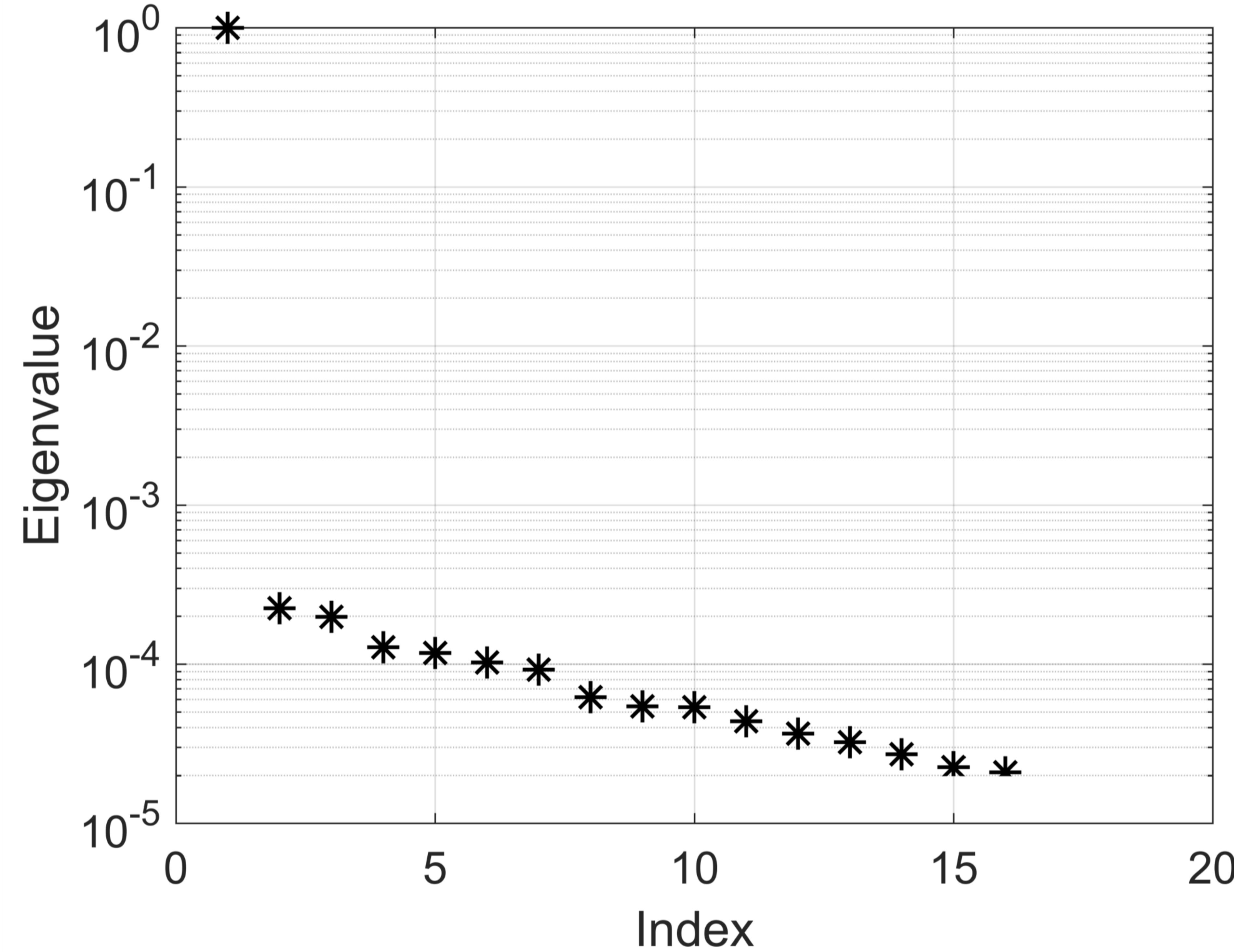


Covariance matrix of gradient vector

Active Subspace Identification

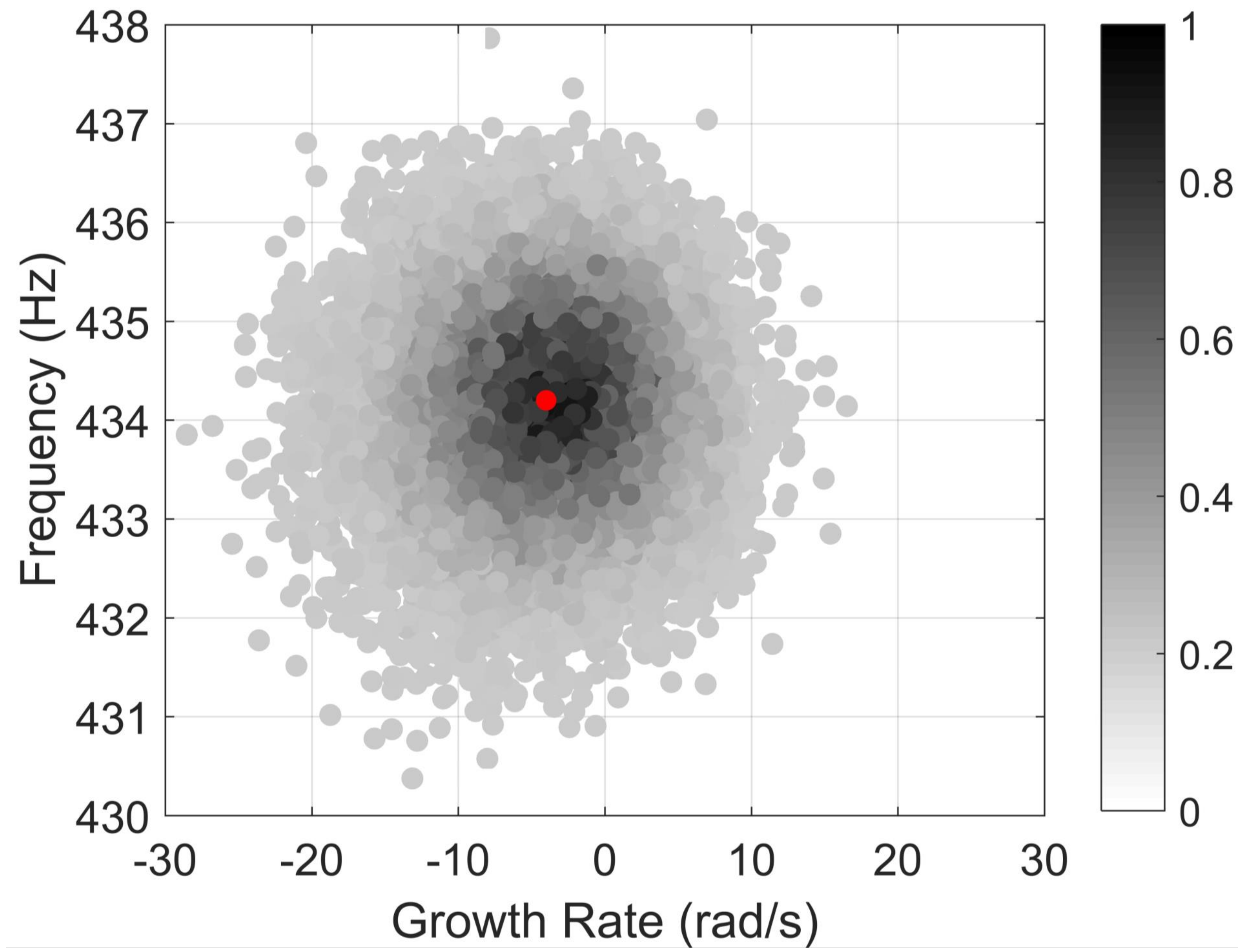


Case A

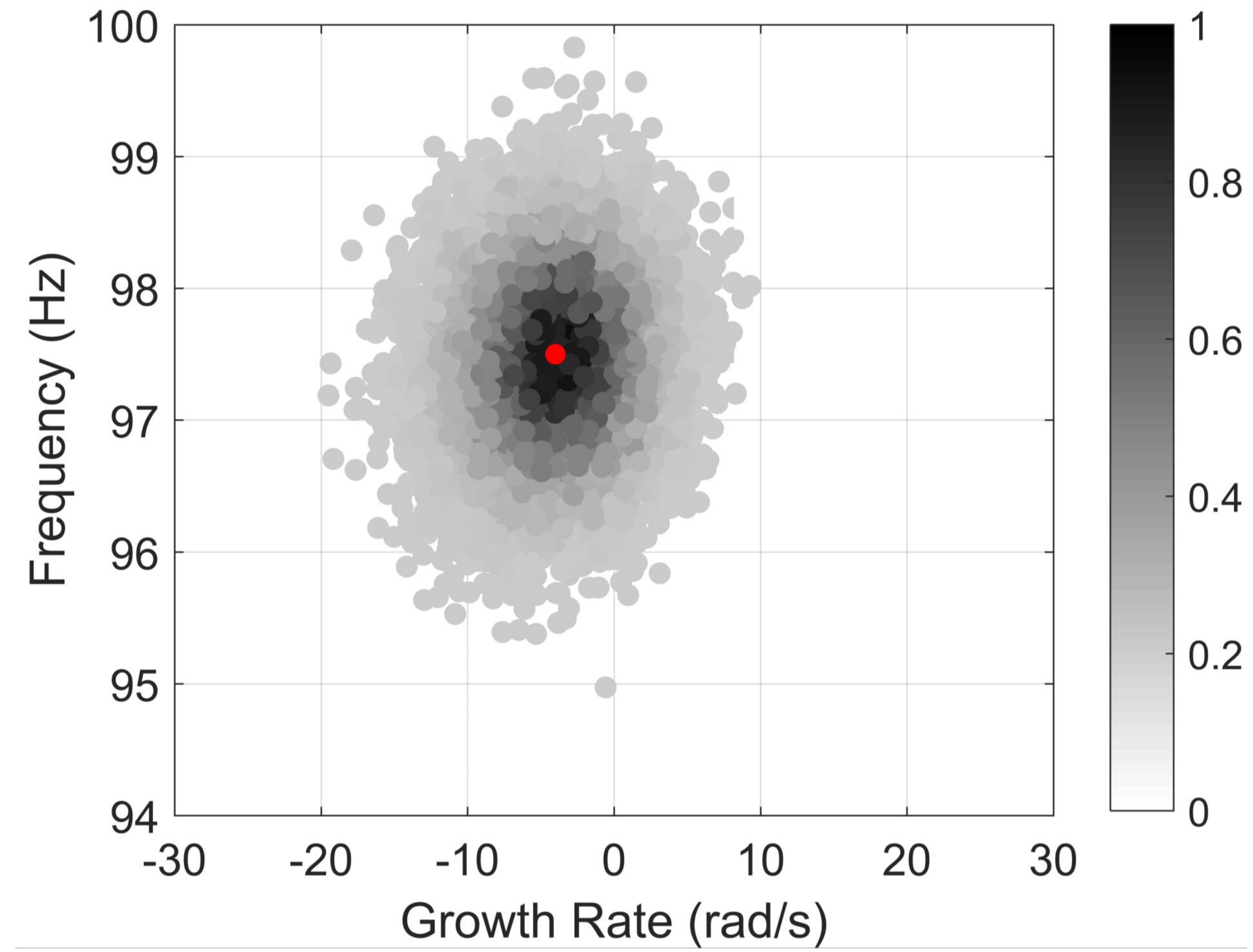


Case B

Monte Carlo results for case A & B

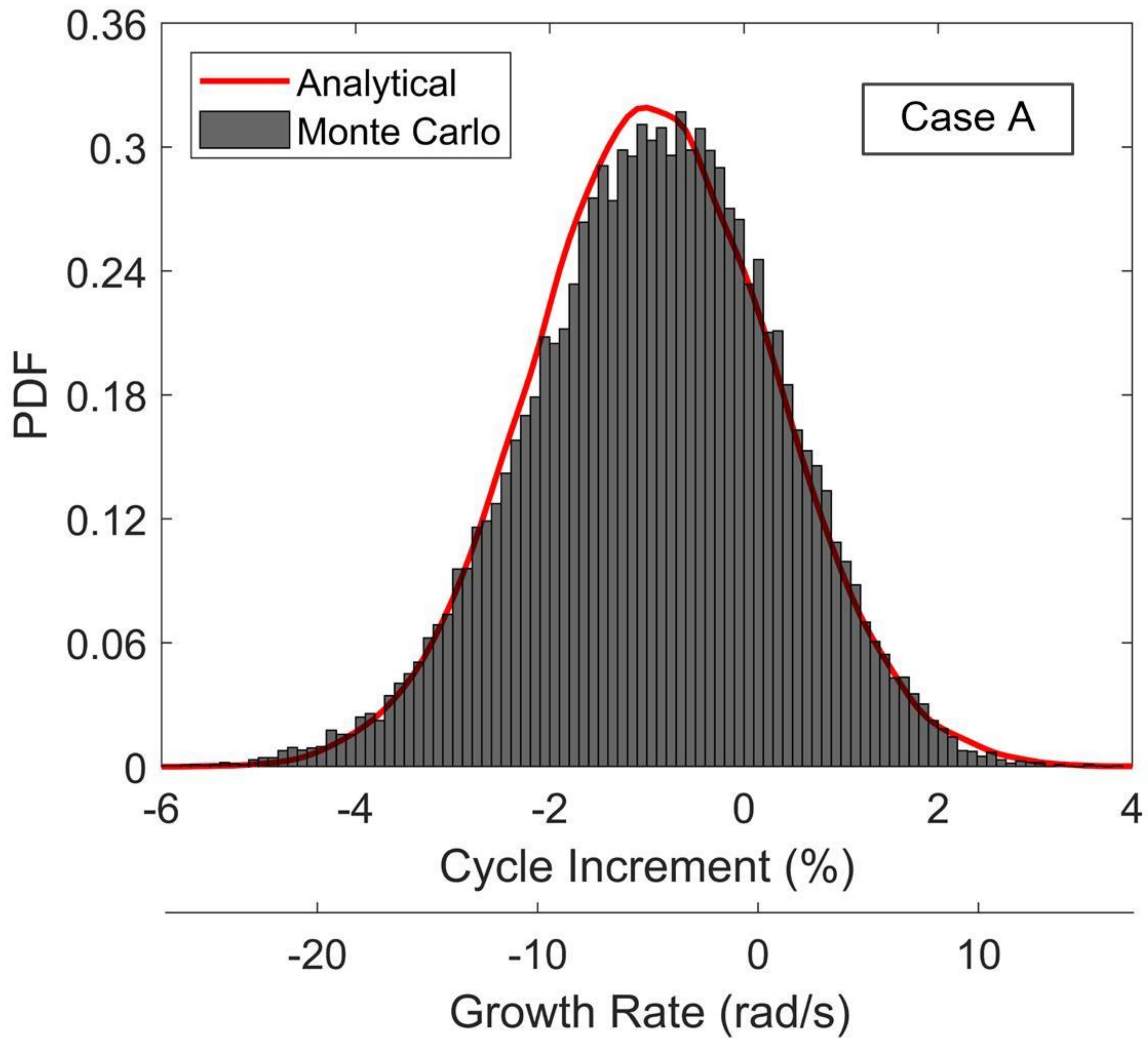


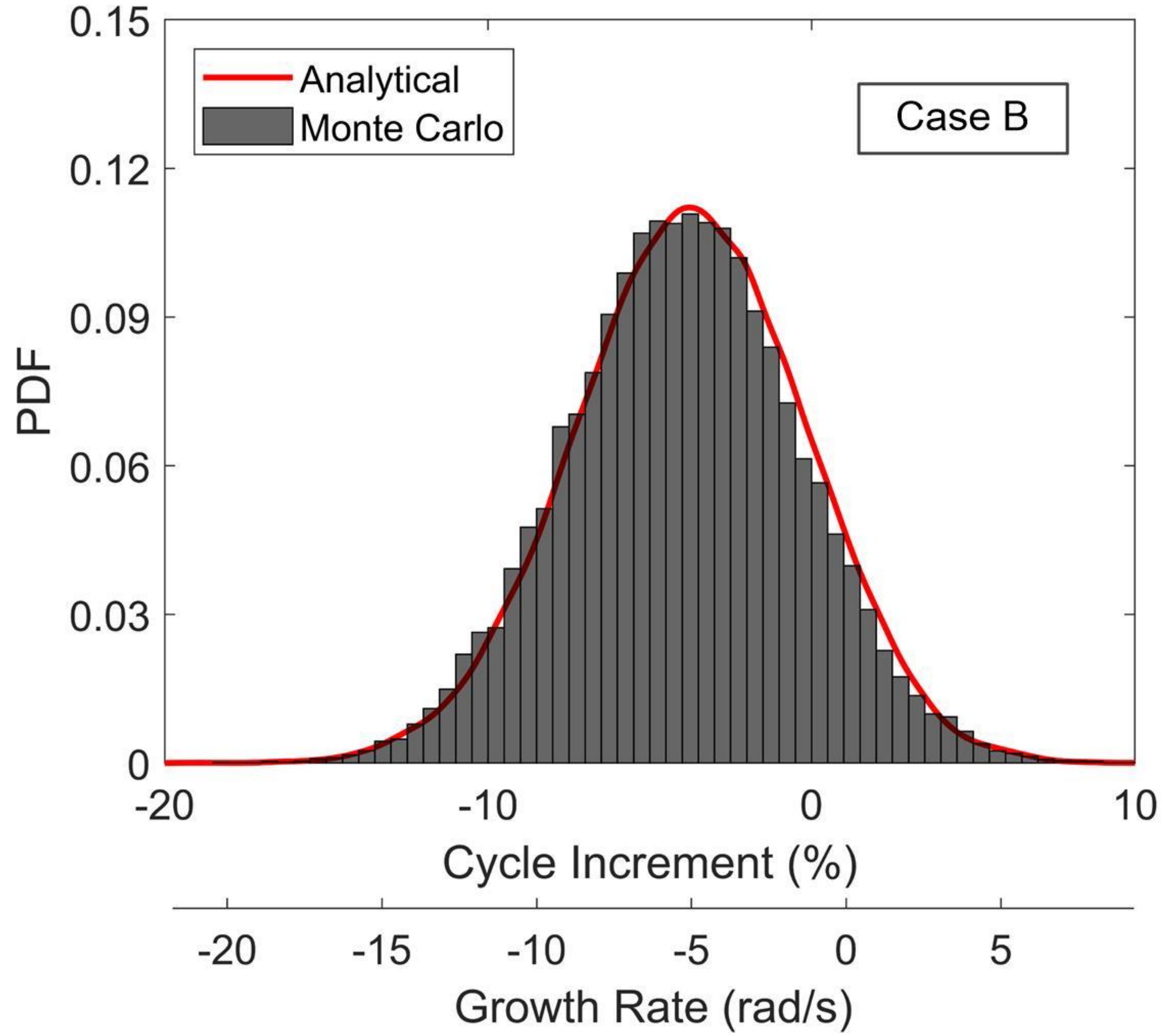
Case A



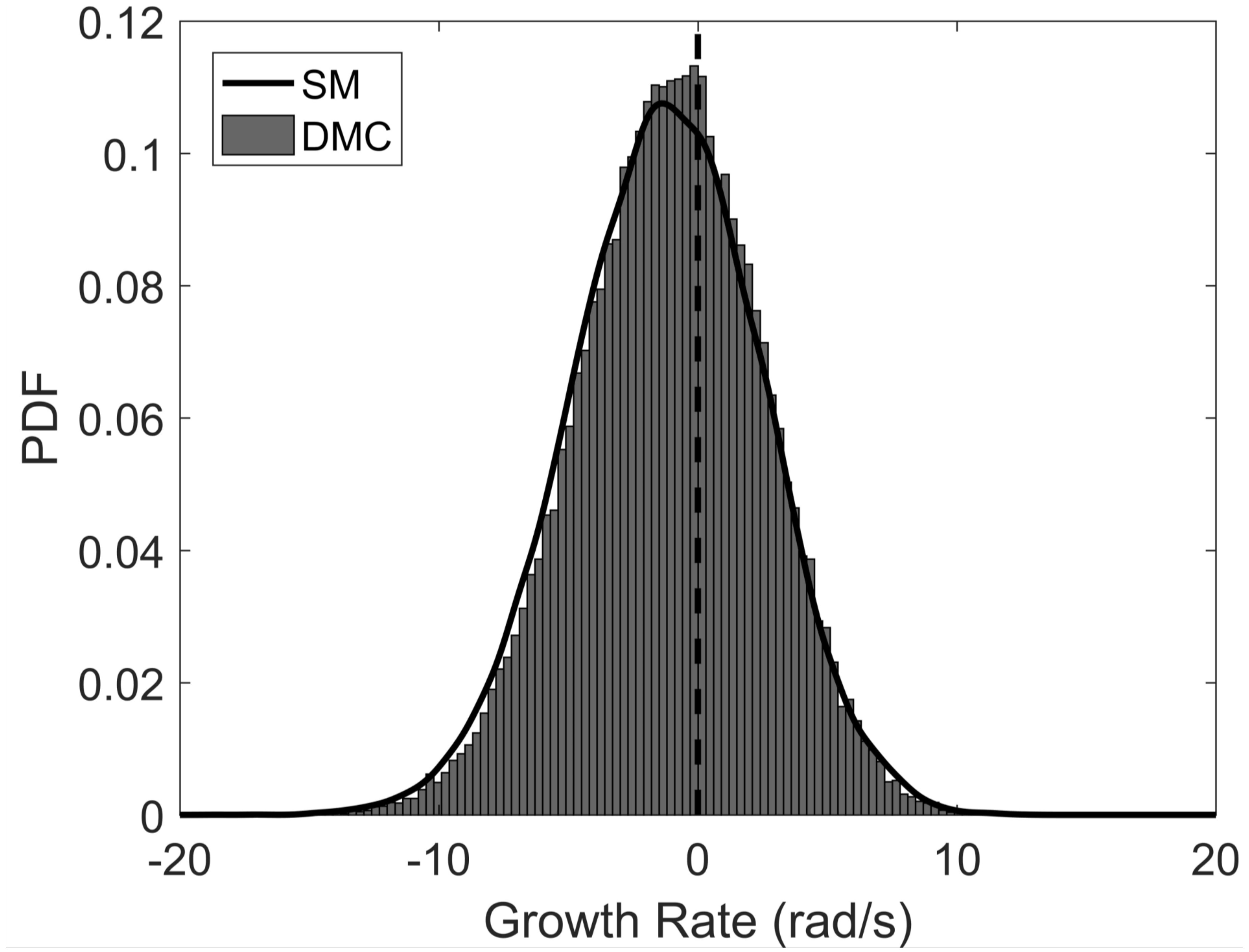
Case B

PDF of modal growth rate in terms of cycle increment

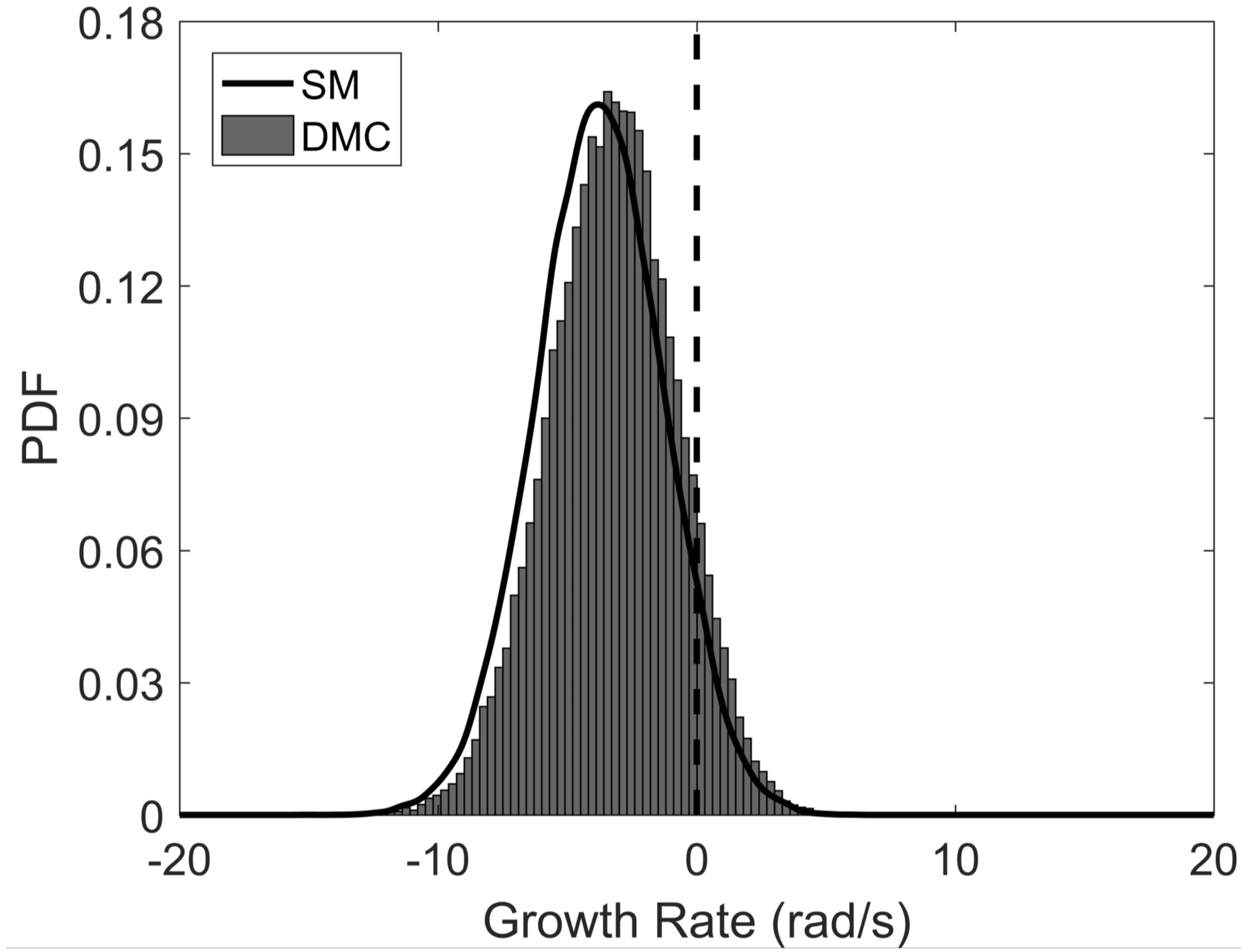




Procedure validation



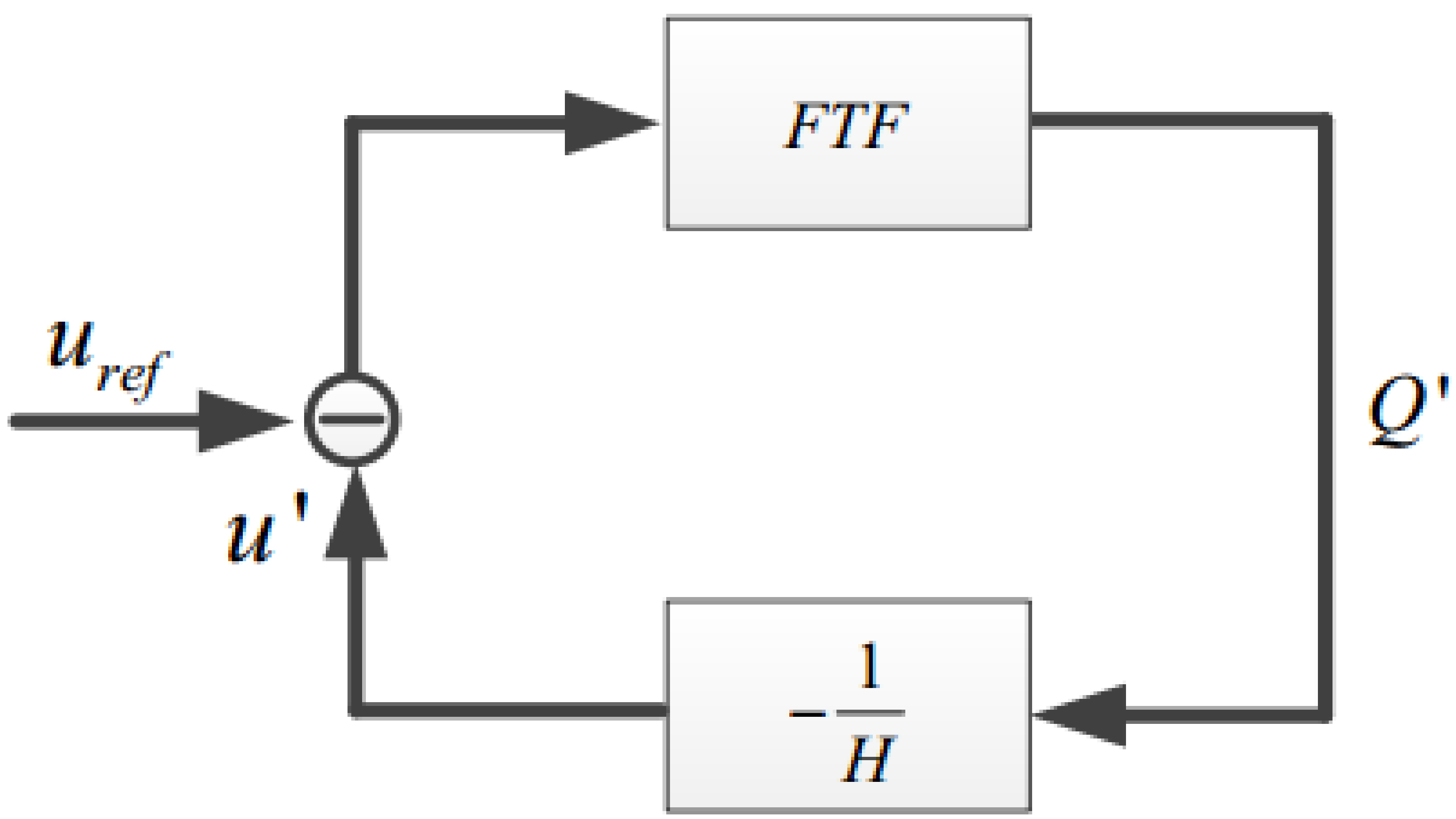
500 ms



1400 ms

Discussion on the single active variable

1. This active variable approximates to first order the casual relationship between variations of the FIR model coefficients and variations of the model growth rate



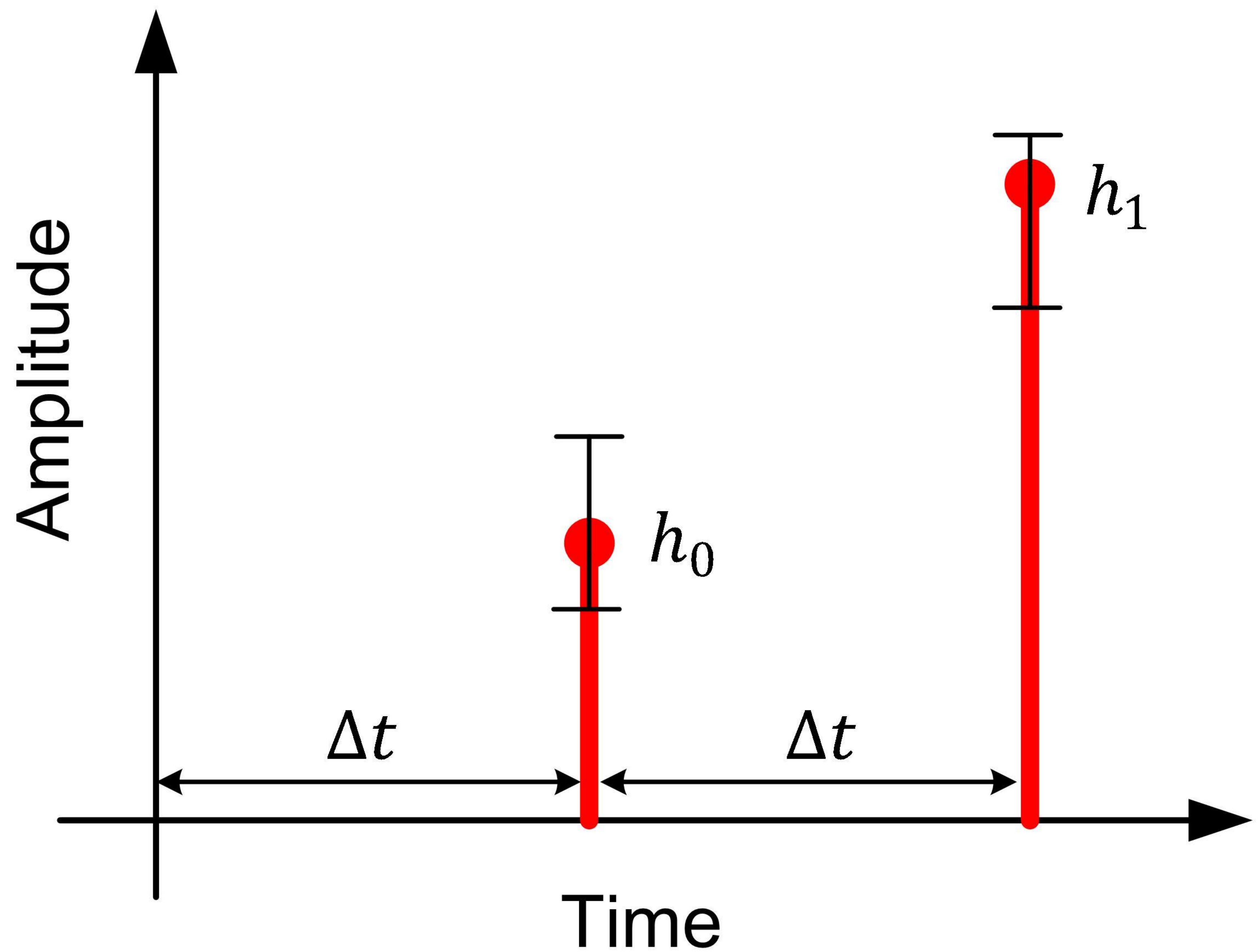
$$FTF(\omega - \boldsymbol{i}\sigma) = H(\omega - \boldsymbol{i}\sigma) \quad \omega, \sigma \in \mathbb{R}$$

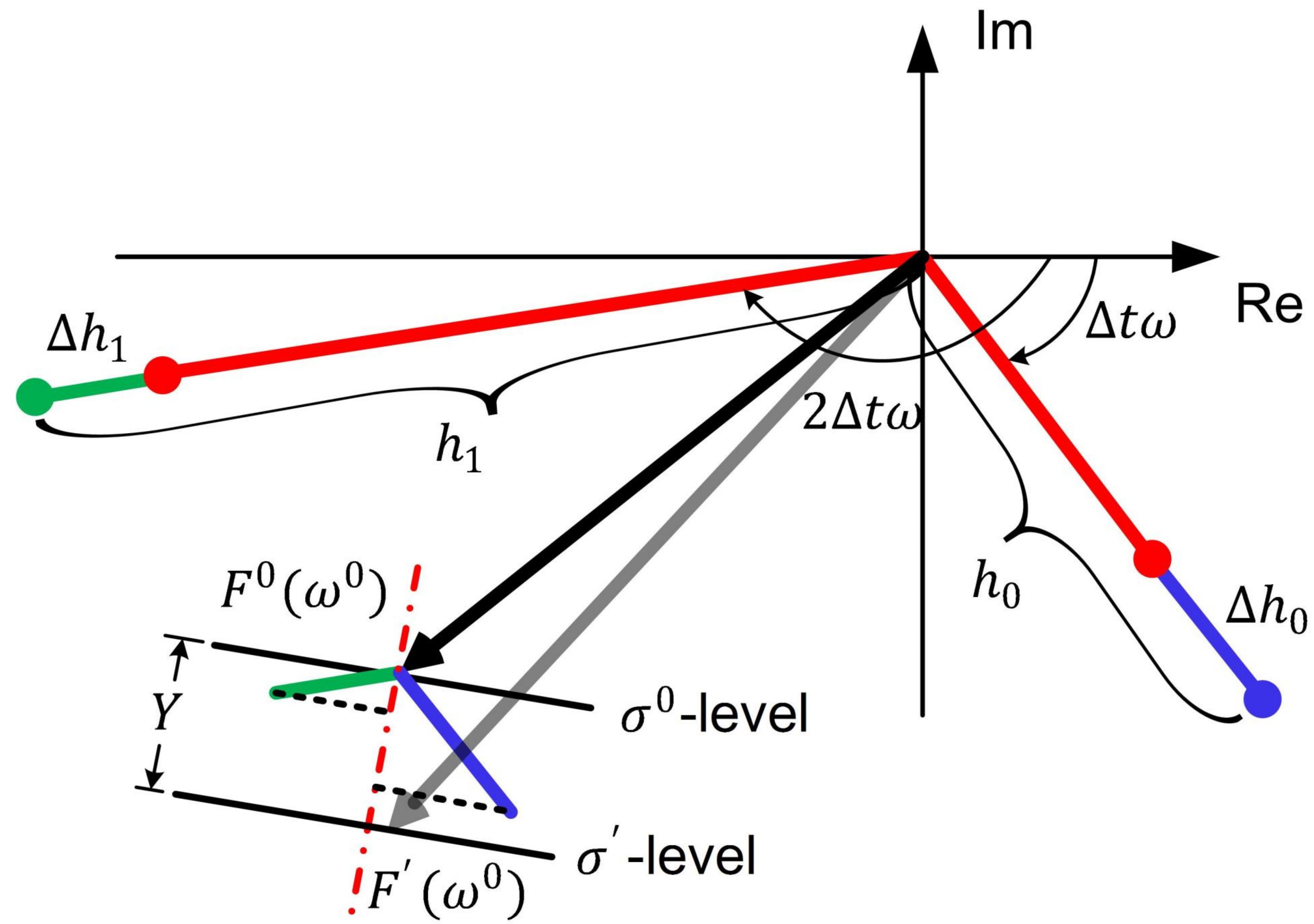
2. It is possible to analytically derive the linear combination coefficients in active variable expression

Discussion on the single active variable

3. This first-order approximation fails when uncertainties in impulse response model is to large, which is not allowed in real application of flame impulse response model

Discussion on the single active variable (phasor plot demo)





$$\mathrm{F}(\omega) = \sum_{k=0}^{L-1} h_k e^{-i(k+1)\Delta t\omega}, \quad \omega \in \mathbb{R} \Rightarrow \mathrm{F}(\omega^0) = h_0 e^{-i\Delta t\omega^0} + h_1 e^{-i(2\Delta t)\omega^0}$$