Evaluating the impact of uncertainty in flame impulse response model on thermoacoustic instability prediction: A dimensionality reduction approach

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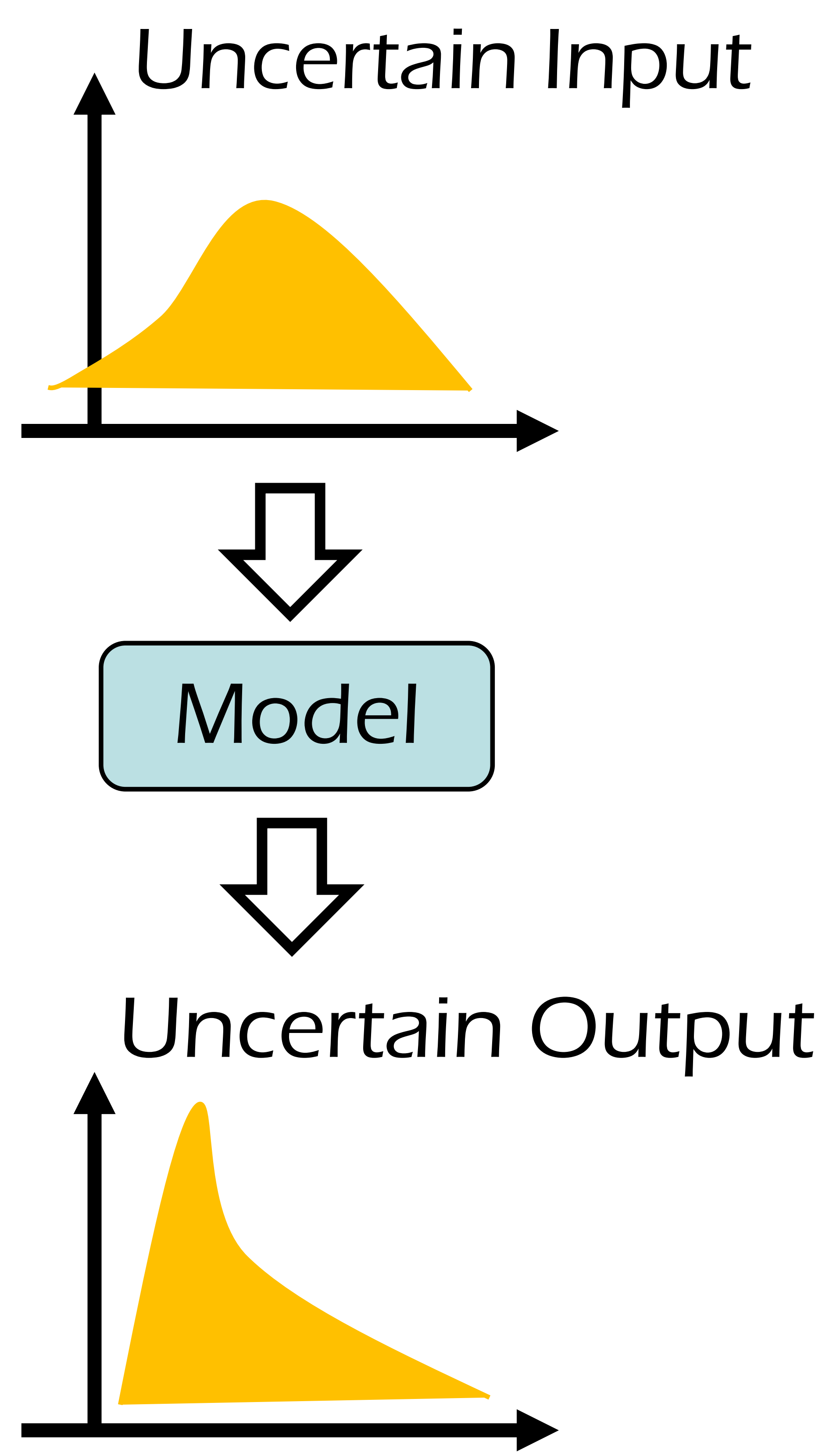

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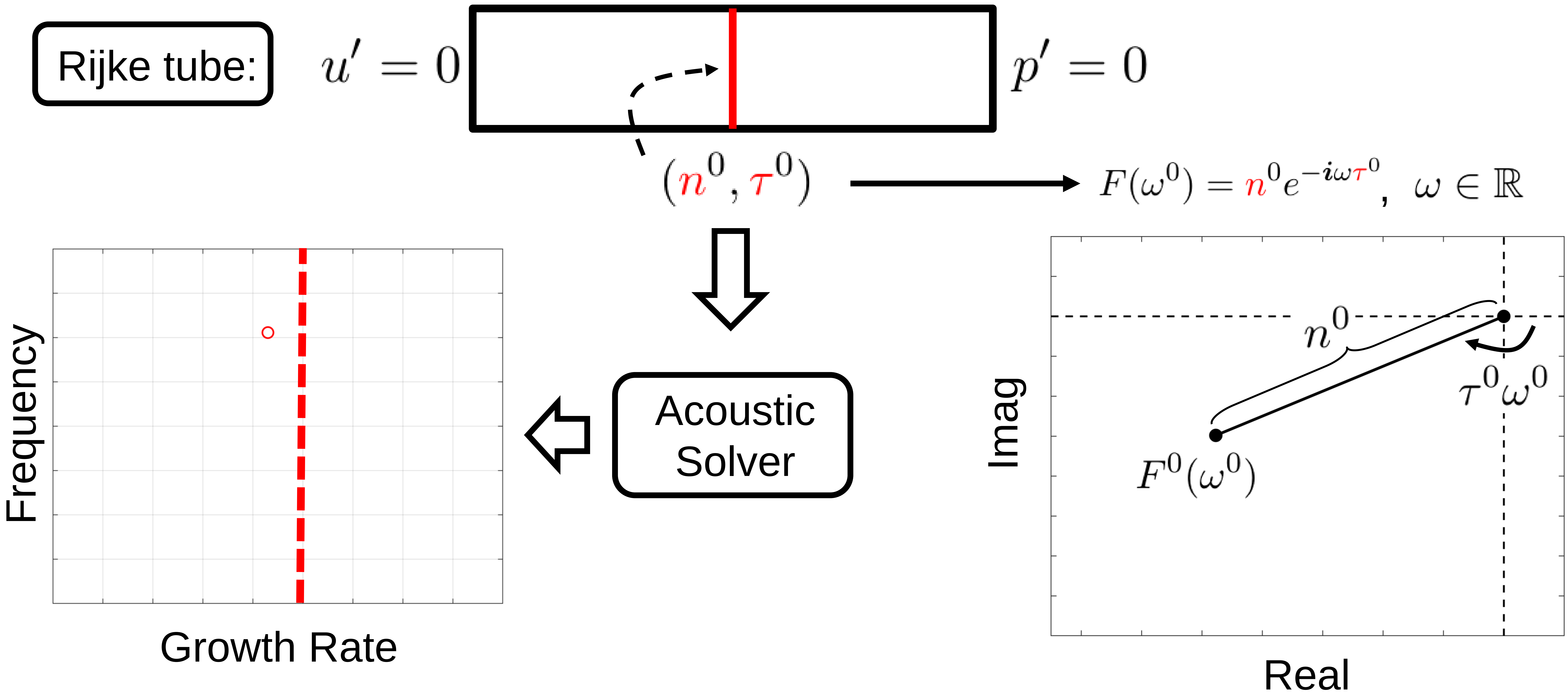
Flame model uncertainty needs to be considered for achieving more robust thermoacoustic instability prediction



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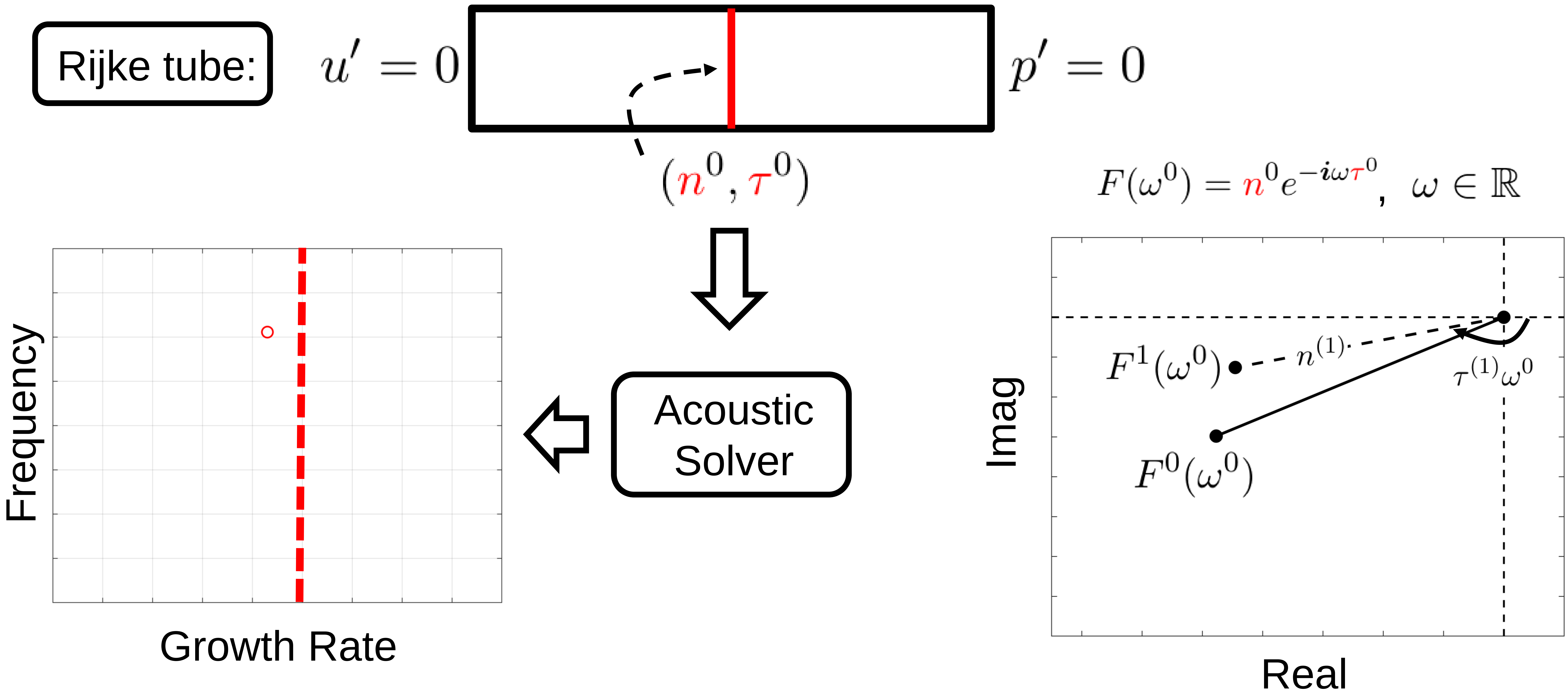

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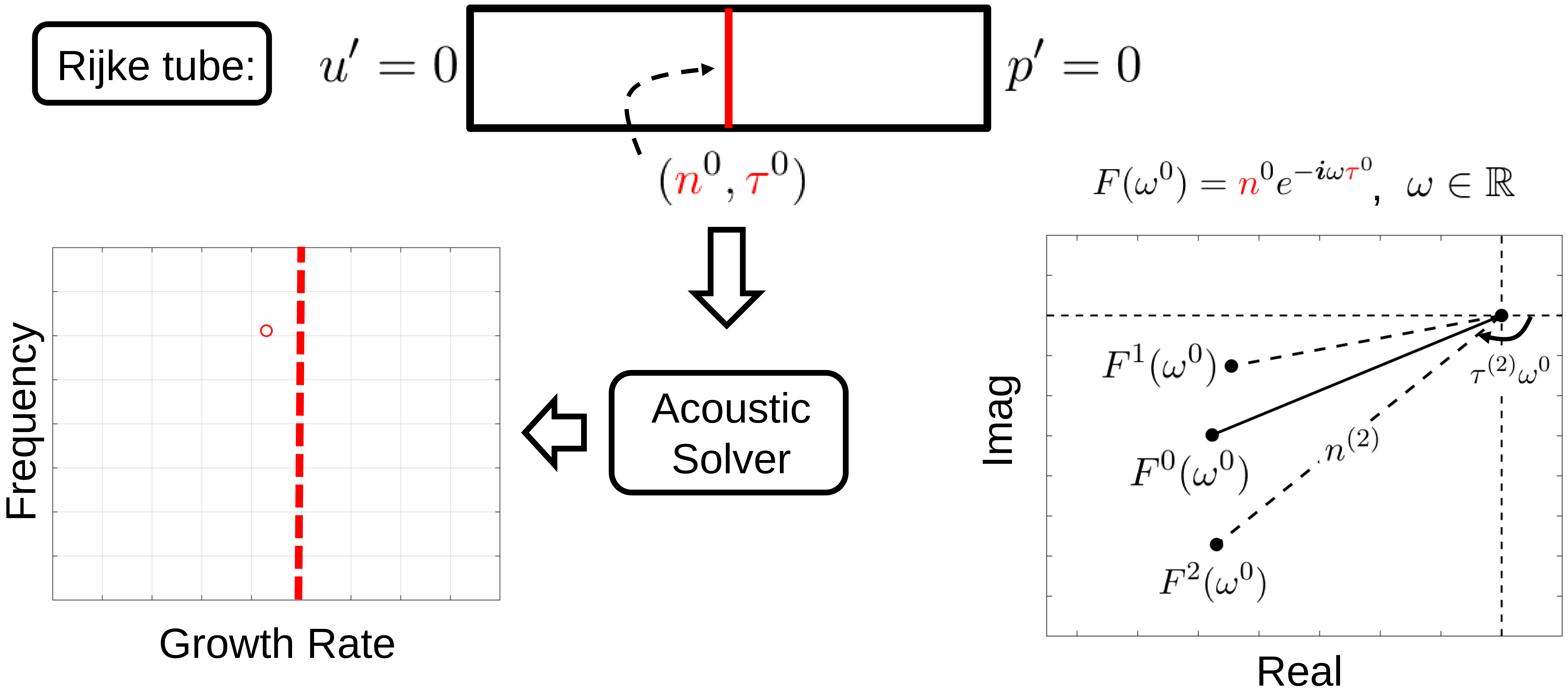
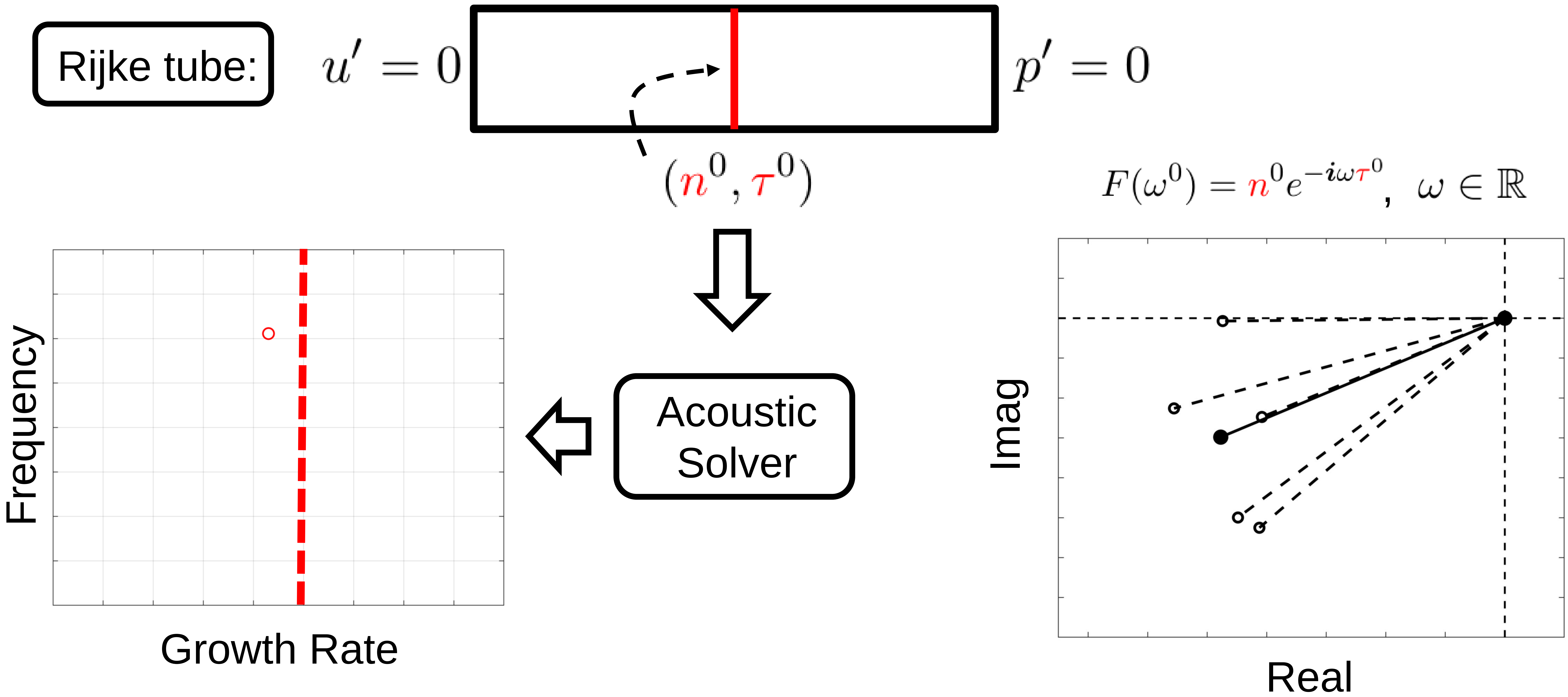


Flame model uncertainty needs to be considered for achieving more robust thermoacoustic instability prediction



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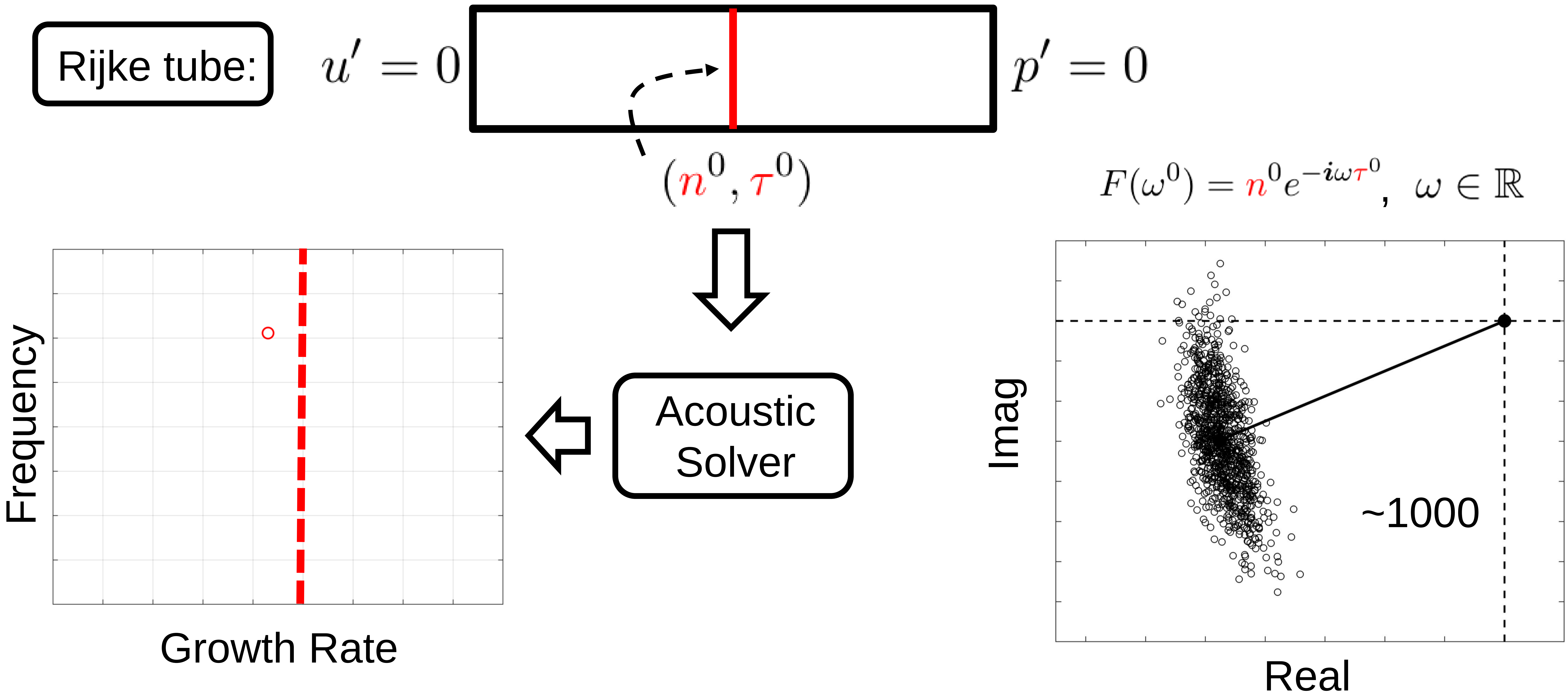

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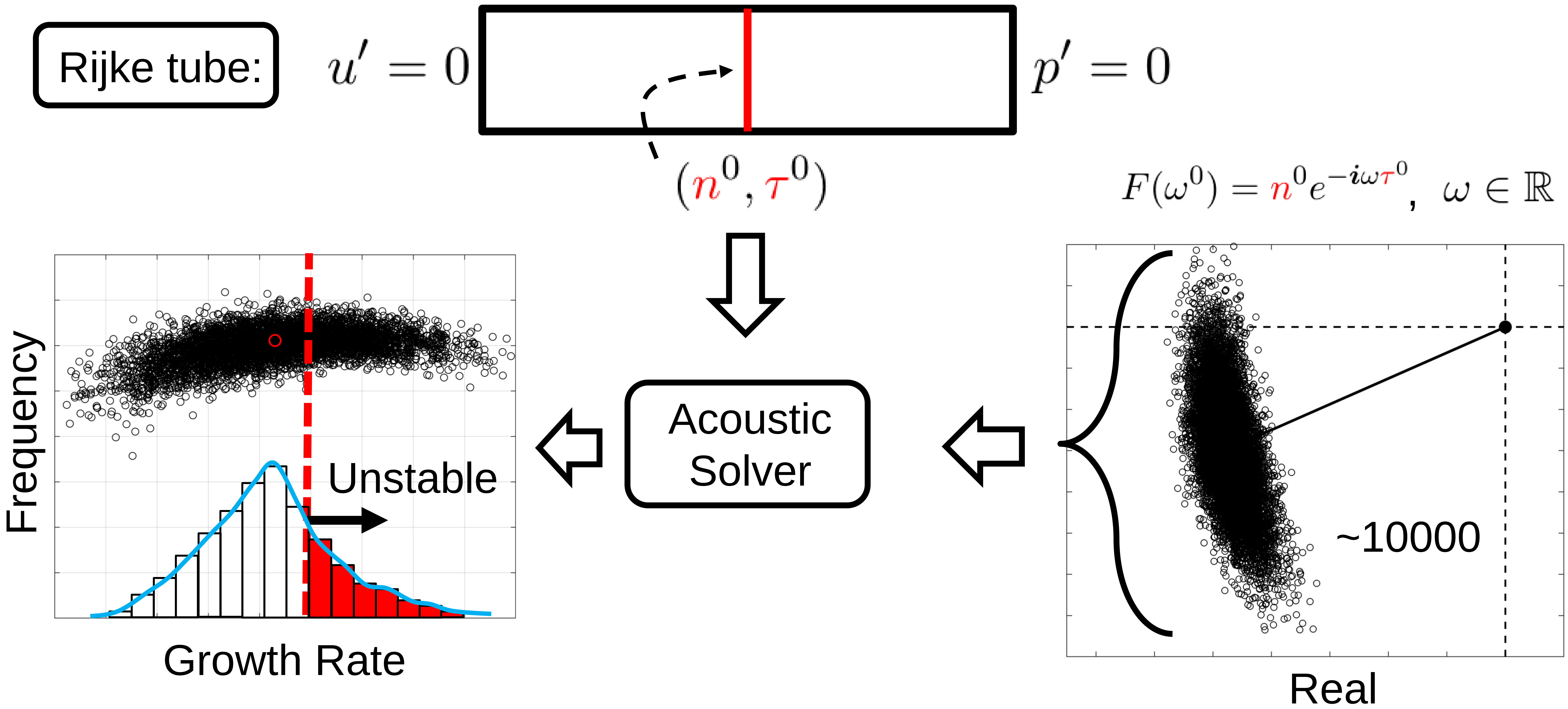

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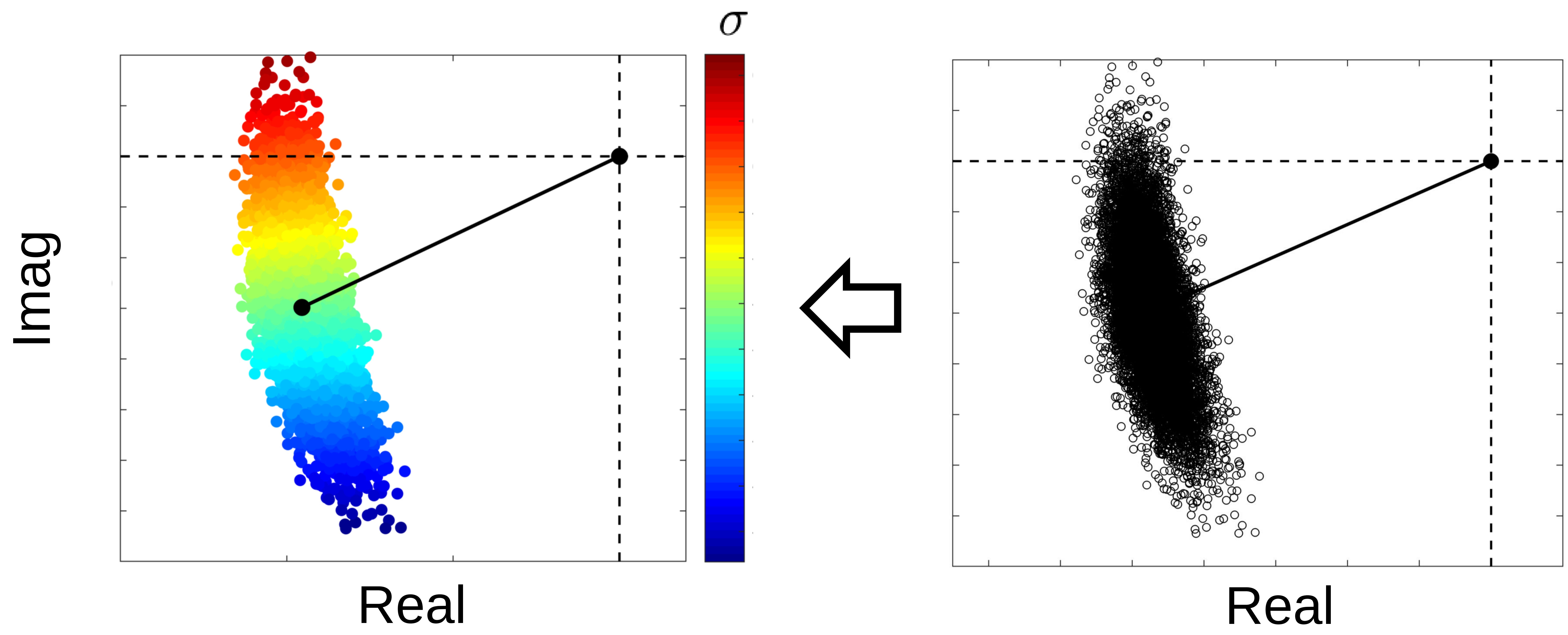
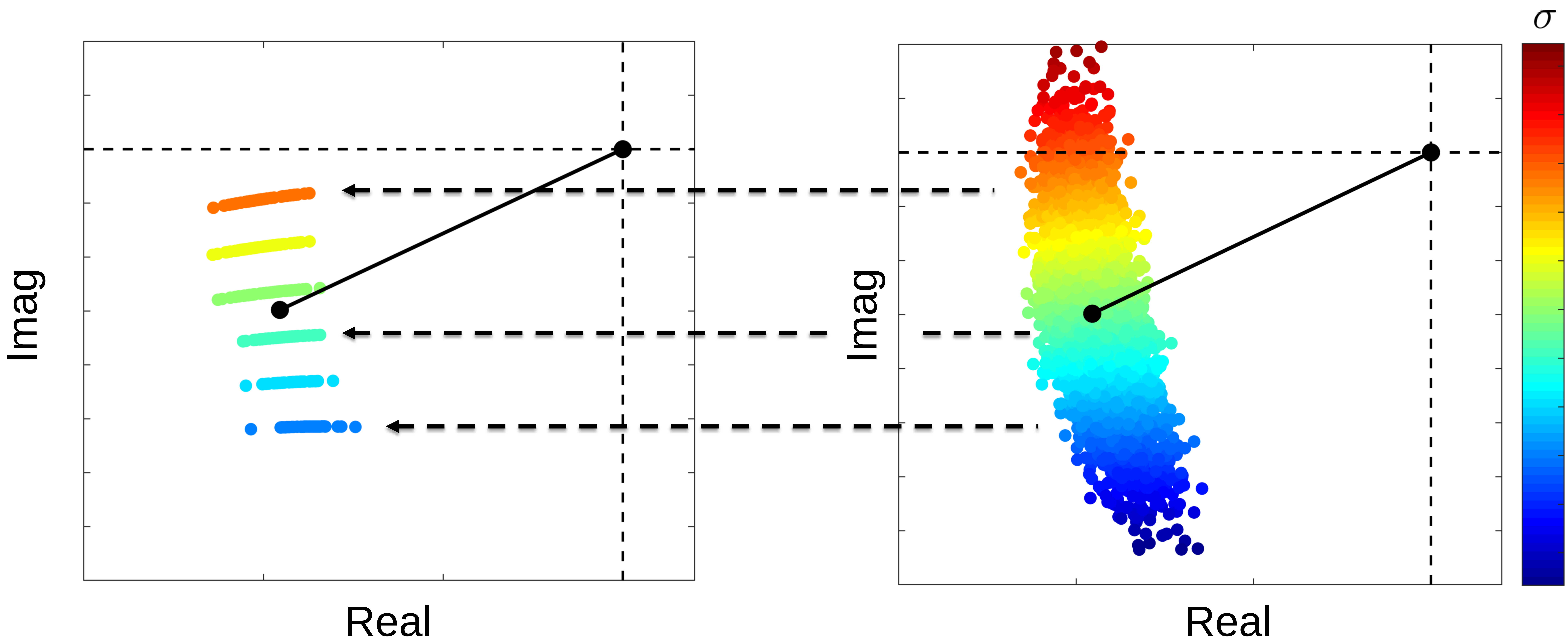


Growth rate contours are approximately parallel straight lines on the phasor plot of F for Rijke tube case



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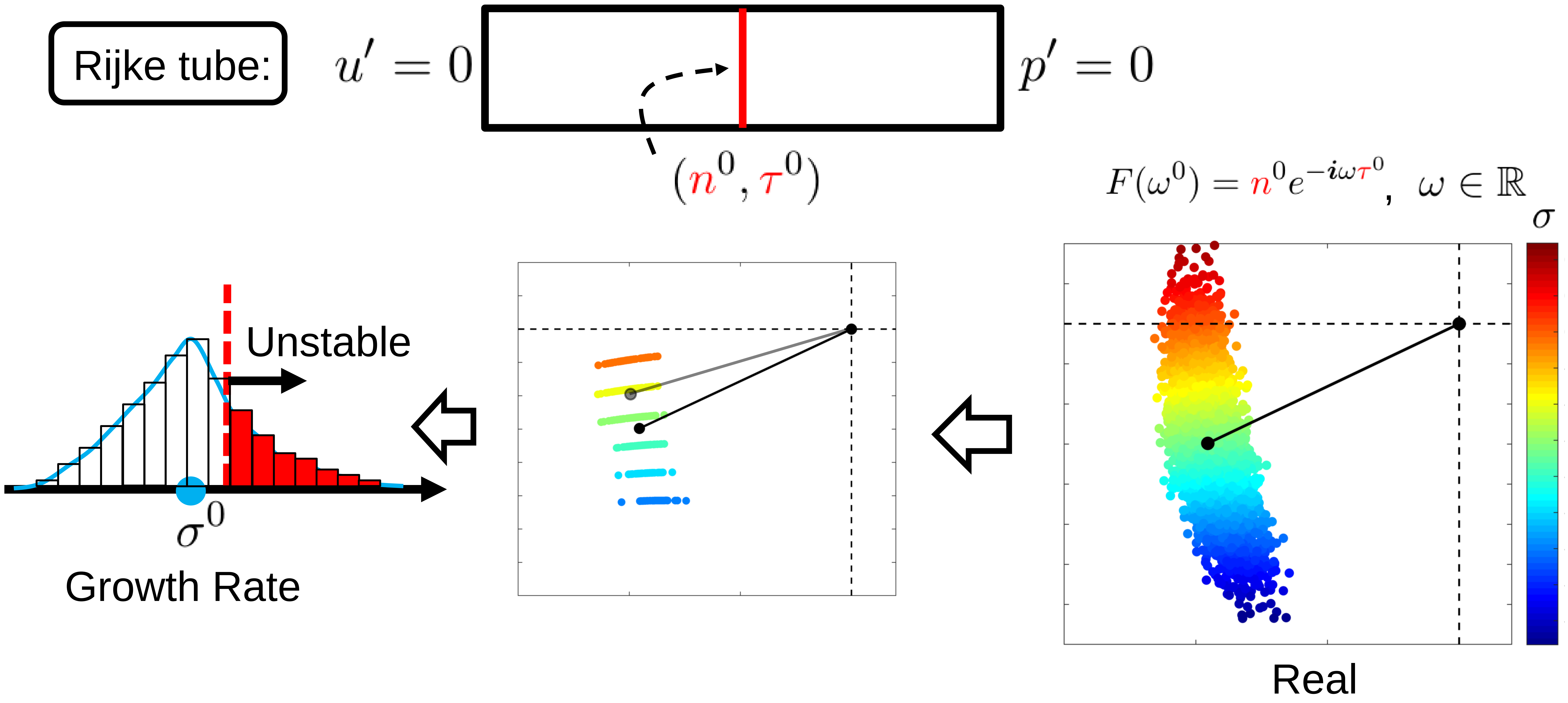
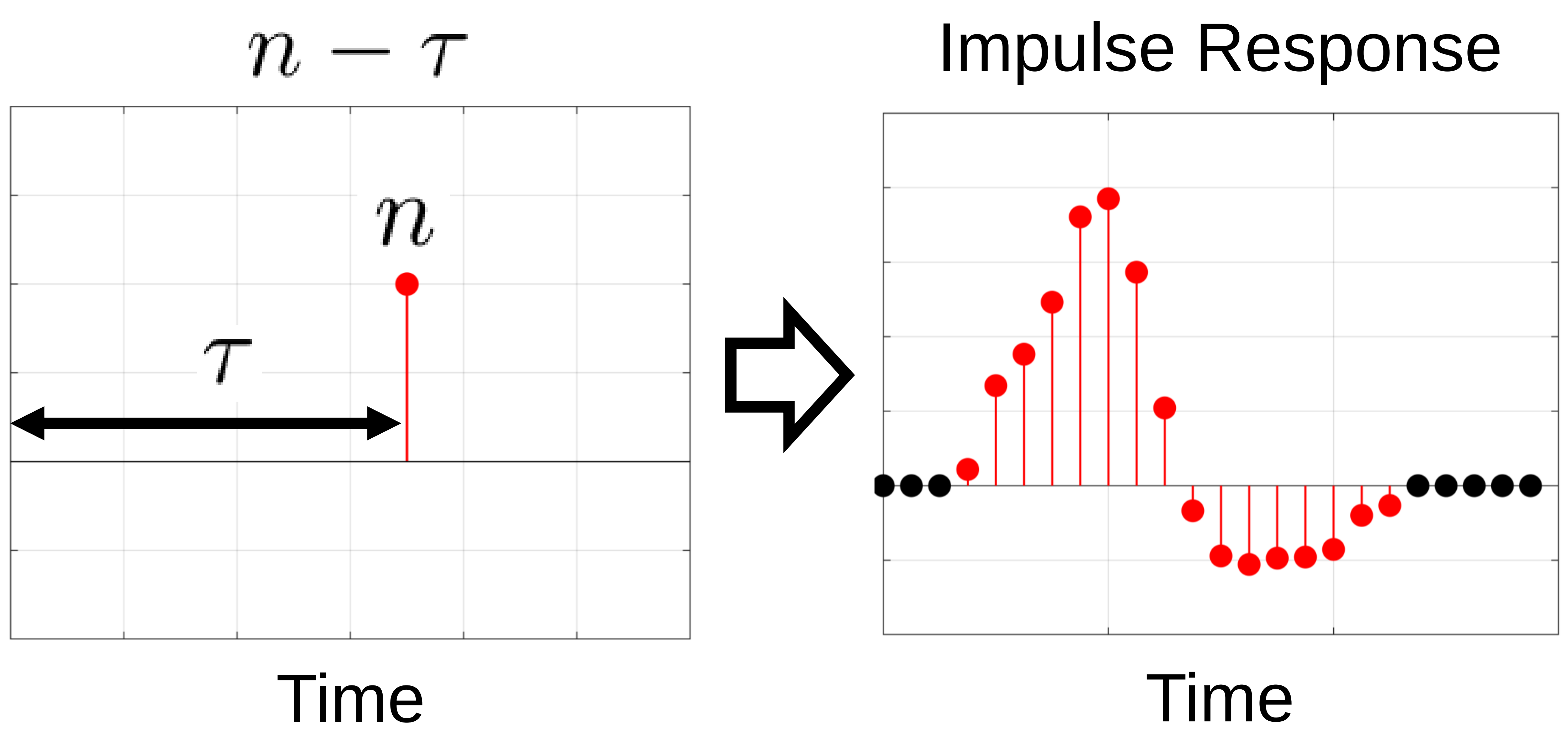
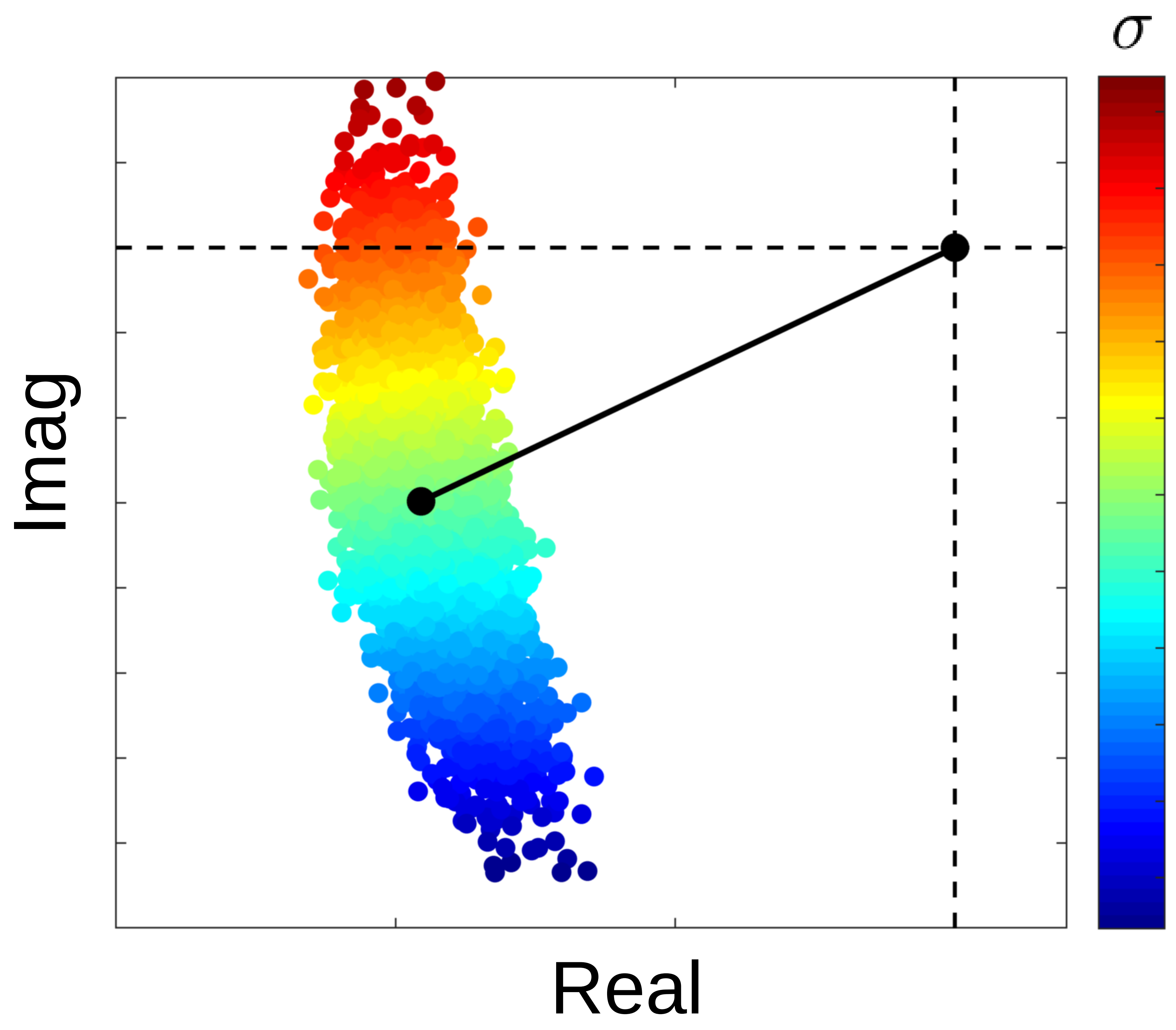


Questions:

- *What do growth rate contours look like in general?*

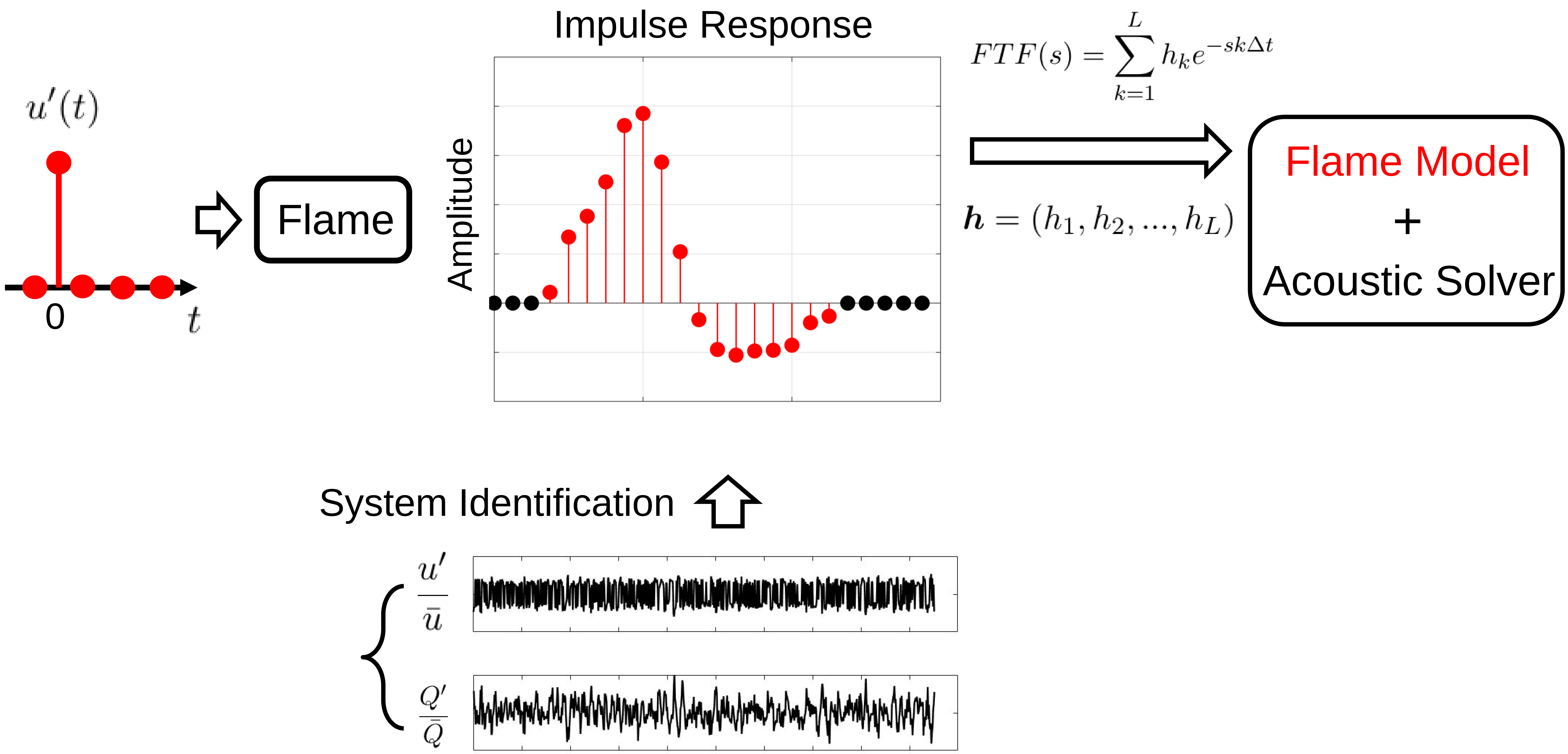




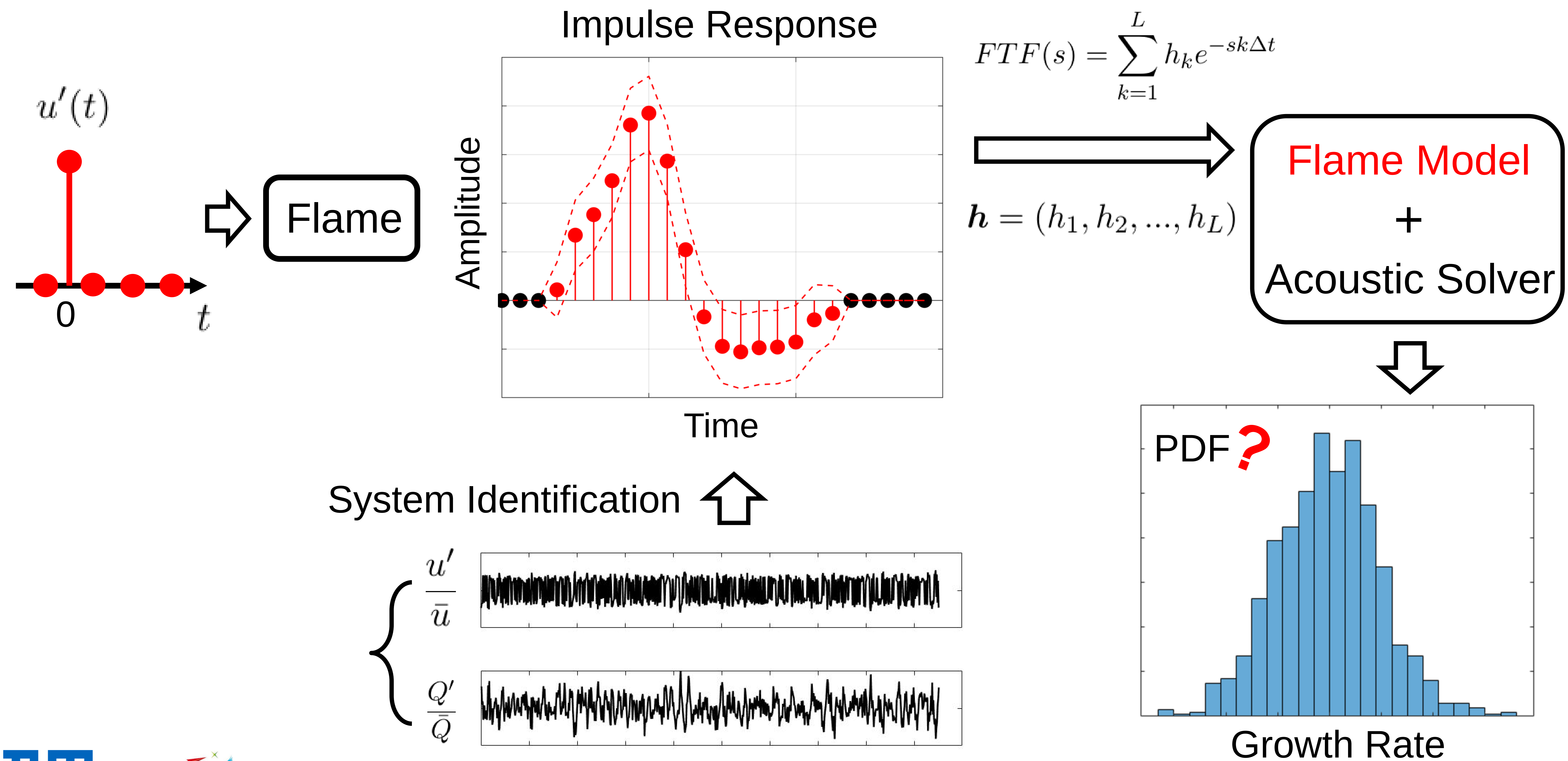
Presentation overview

- ☐ Motivation
- ☐ Visualization: from FIR to the phasor plot of FRF
- ☐ Analytical Results: what do growth rate contours look like
- ☐ UQ Strategy & Case Studies
- ☐ Conclusions & Outlook

Flame impulse response is a flexible representation of realistic flame dynamics in time domain



Flame impulse response identification contains uncertainty, which will influence the growth rate prediction



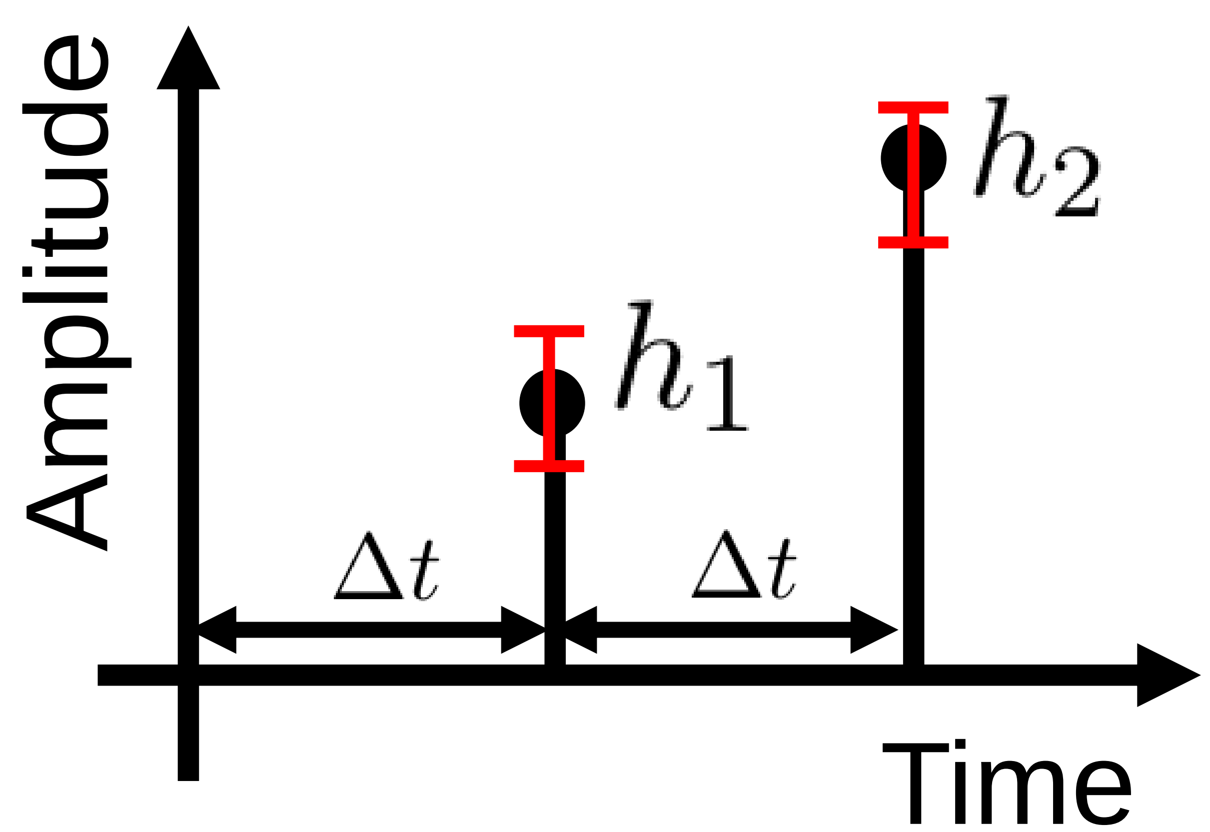
Results visualization: phasor plot of flame frequency response

Frequency Response

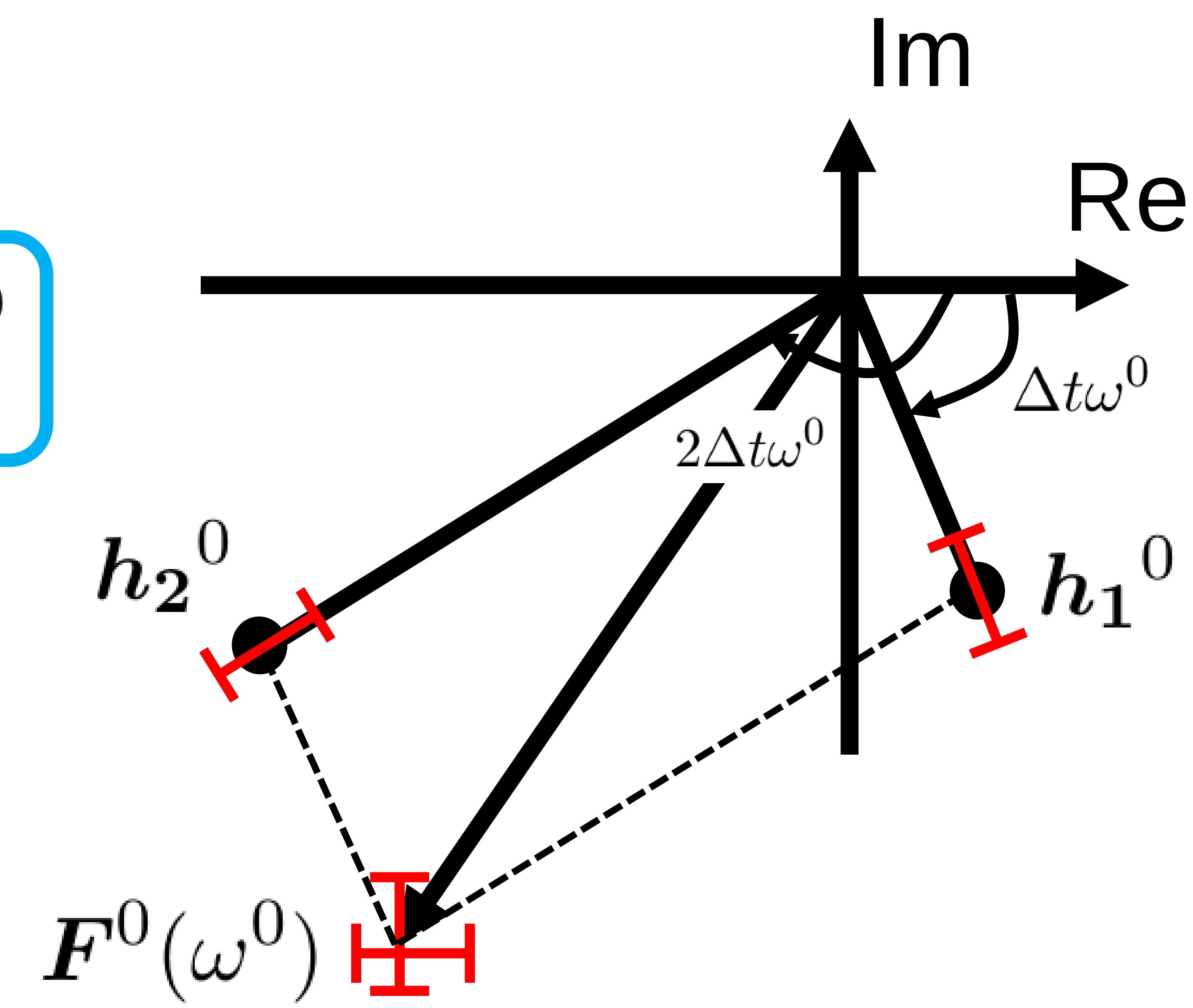
$$F(\omega) = \sum_{k=1}^{L} h_k e^{-\boldsymbol{i}k\Delta t\omega}, \quad \omega \in \mathbb{R}$$

Impulse Response

Example:



$$\boxed{F(\omega^0) = h_1 e^{-\boldsymbol{i}\Delta t\omega^0} + h_2 e^{-\boldsymbol{i}(2\Delta t)\omega^0}}$$



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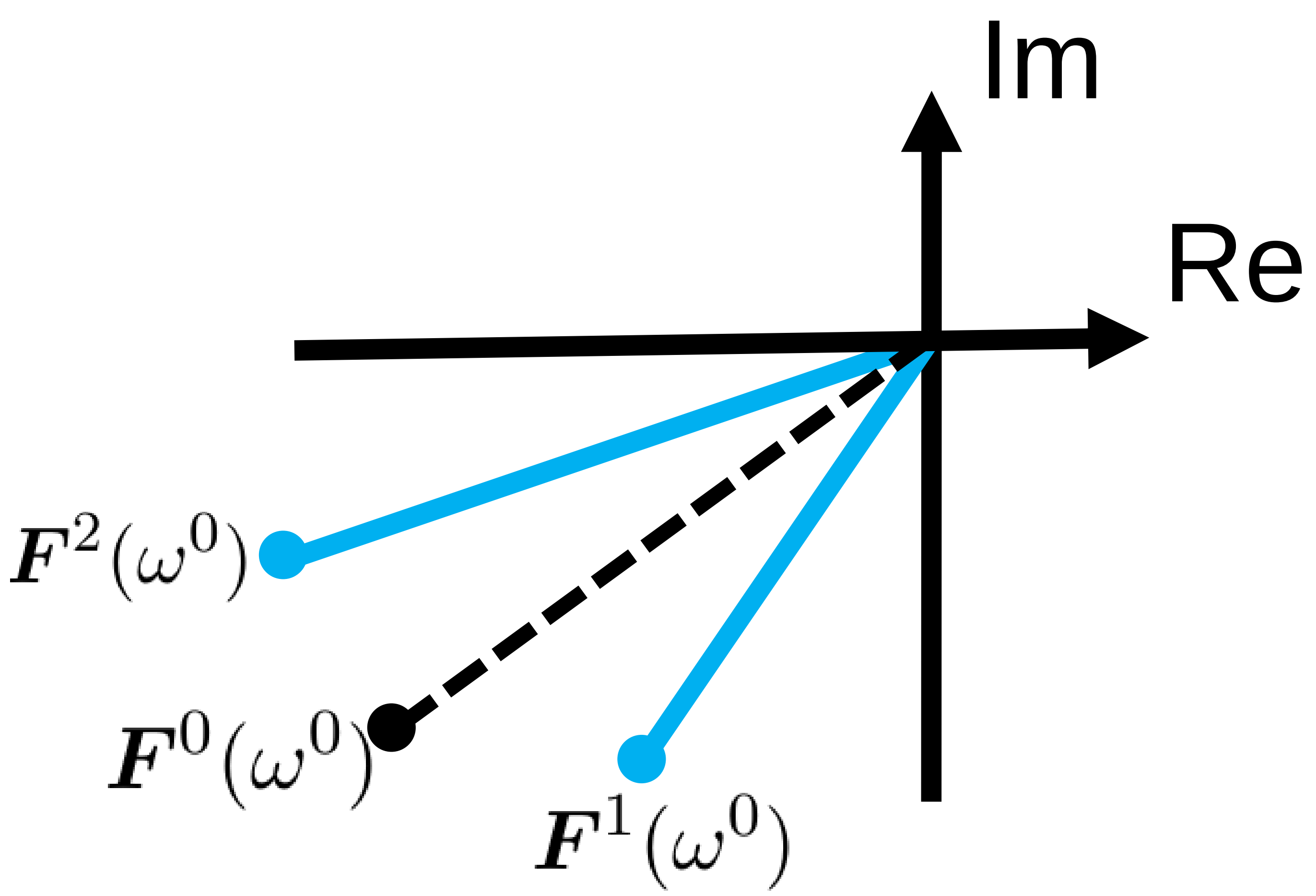
Analytical derivation: An overview

Goal

Geometric Relation?

$\boldsymbol{F}^1(\omega^0) \Longleftrightarrow \boldsymbol{F}^2(\omega^0)$



Analytical derivation: An overview

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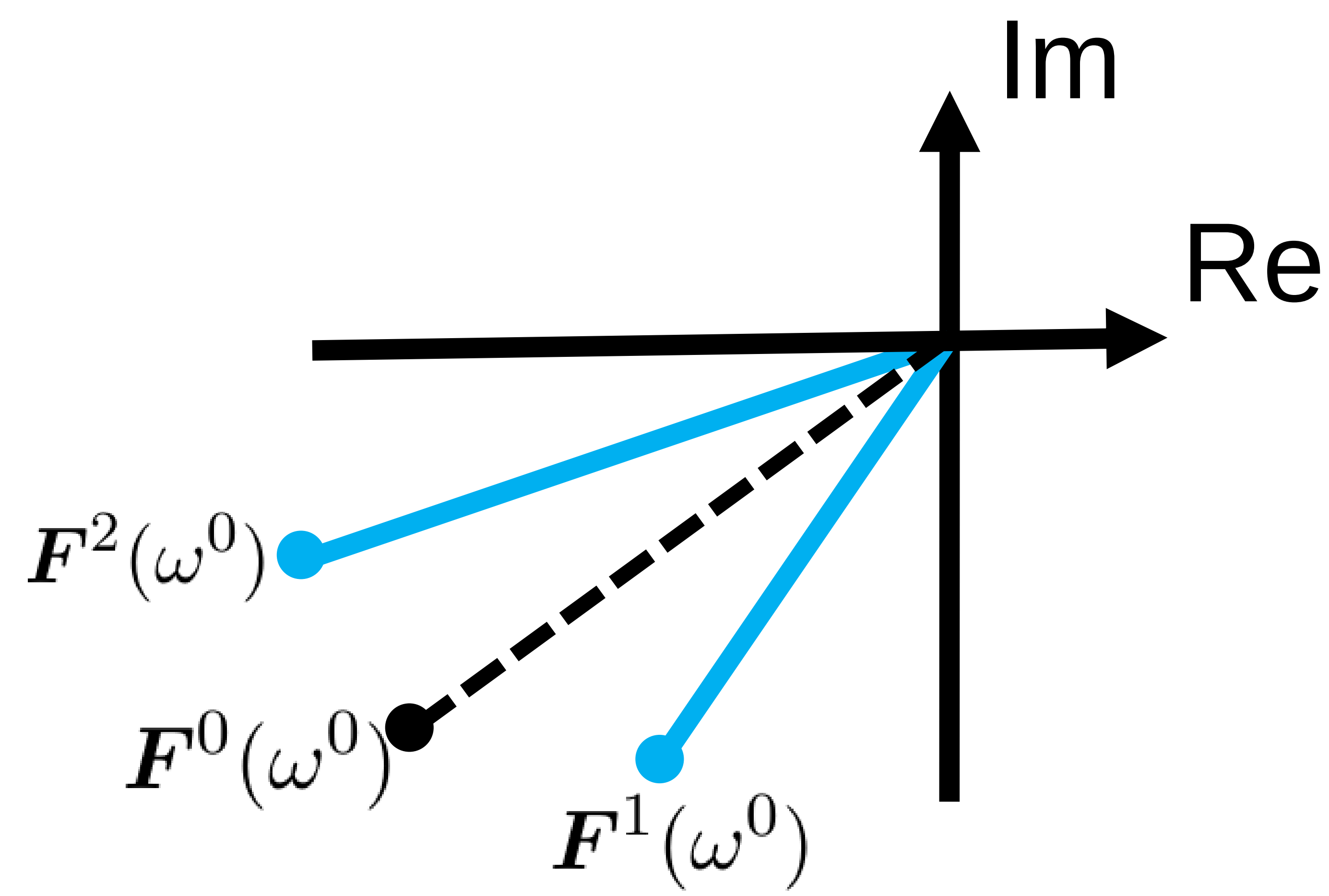
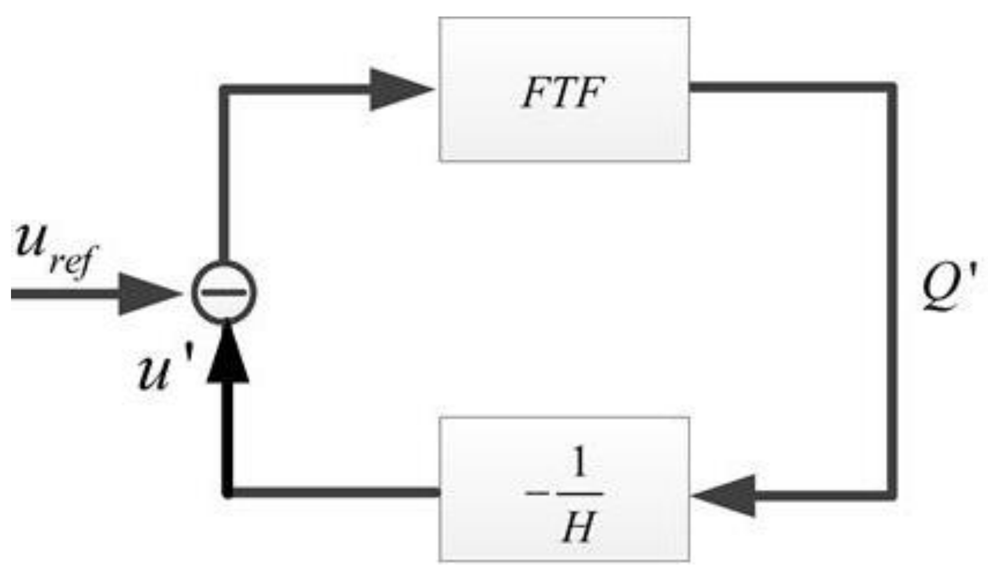




$$FTF(\omega - \boldsymbol{i}\sigma) = H(\omega - \boldsymbol{i}\sigma) \quad \omega, \sigma \in \mathbb{R}$$

$$\sum_{k=1}^{L} h_k e^{-\boldsymbol{i}k\Delta t(\omega - \boldsymbol{i}\sigma)}$$

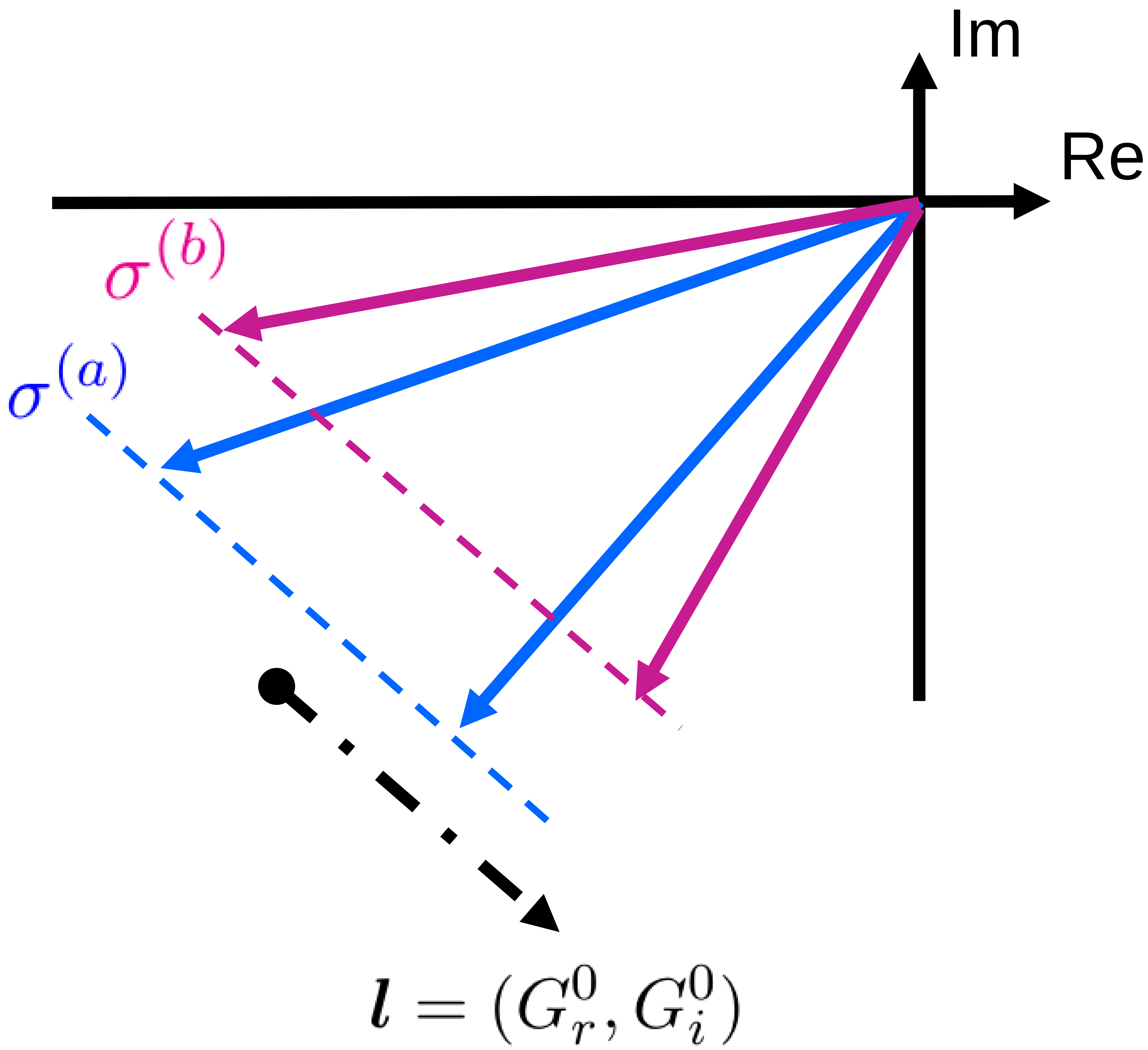
Assumption

- Uncertainty is not large
- Marginally stable mode

Method

First-order analysis

To first order, the growth rate contours are parallel straight lines on the phasor plot of F



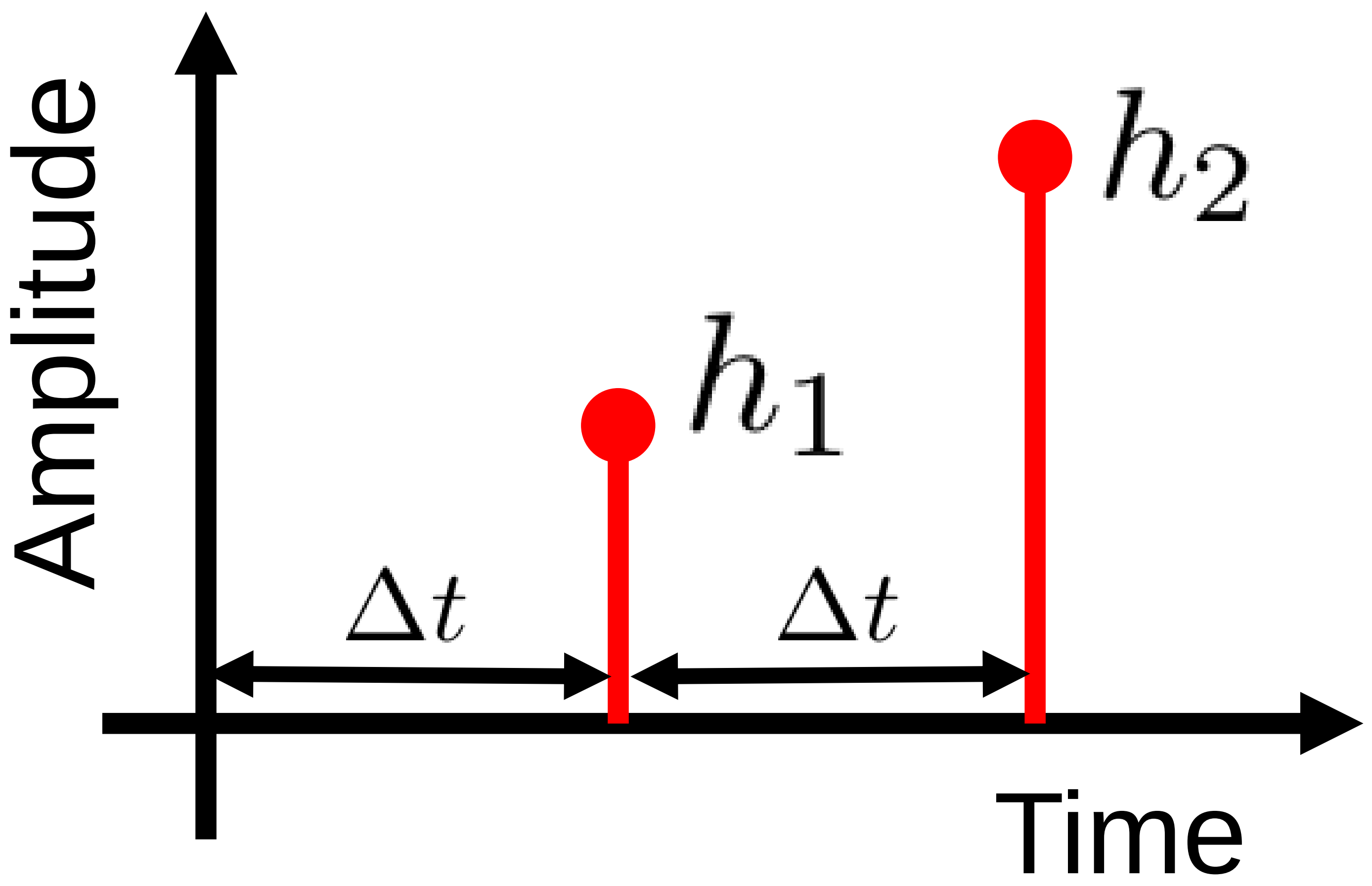
$$G_r^0 = \frac{\partial H_r}{\partial \omega}\bigg|_{\omega^0,\sigma^0} + \sum_{k=1}^{L} h_k^0 sin[k\Delta t\omega^0]k\Delta t$$

$$G_i^0 = \frac{\partial H_i}{\partial \omega}\bigg|_{\omega^0,\sigma^0} + \sum_{k=1}^{L} h_k^0 cos[k\Delta t\omega^0]k\Delta t$$

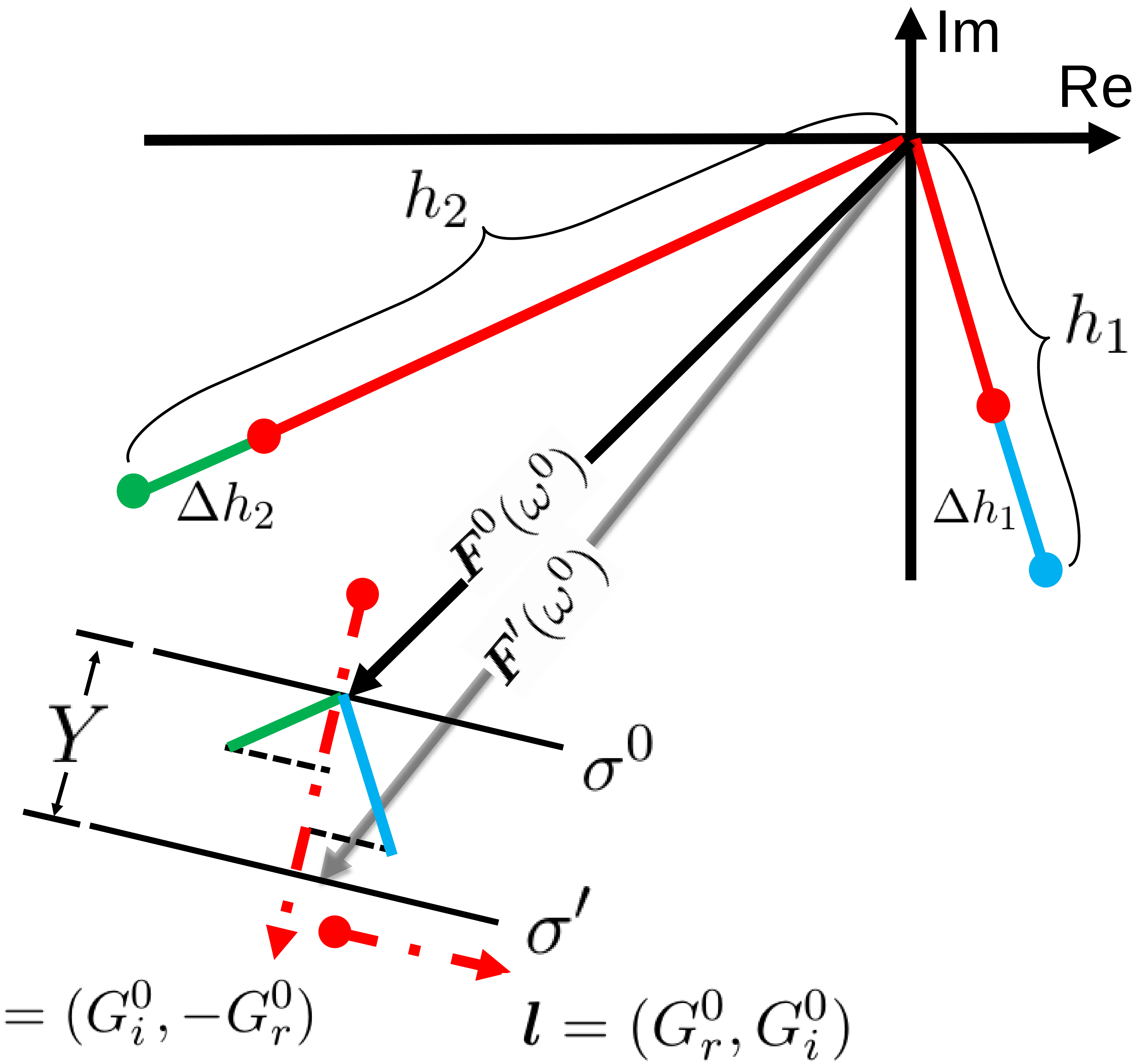
This direction is determined by:

- FIR model
- Acoustic transfer function
- Thermoacoustic mode

Analytical results can be leveraged to deliver a dimensionality reduction UQ strategy



$$Y = \sum_{k=1}^{L} (h_k - h_k^0) e^{-\boldsymbol{i}\Delta t \omega^0} \cdot \frac{\boldsymbol{n}}{|\boldsymbol{n}|}$$



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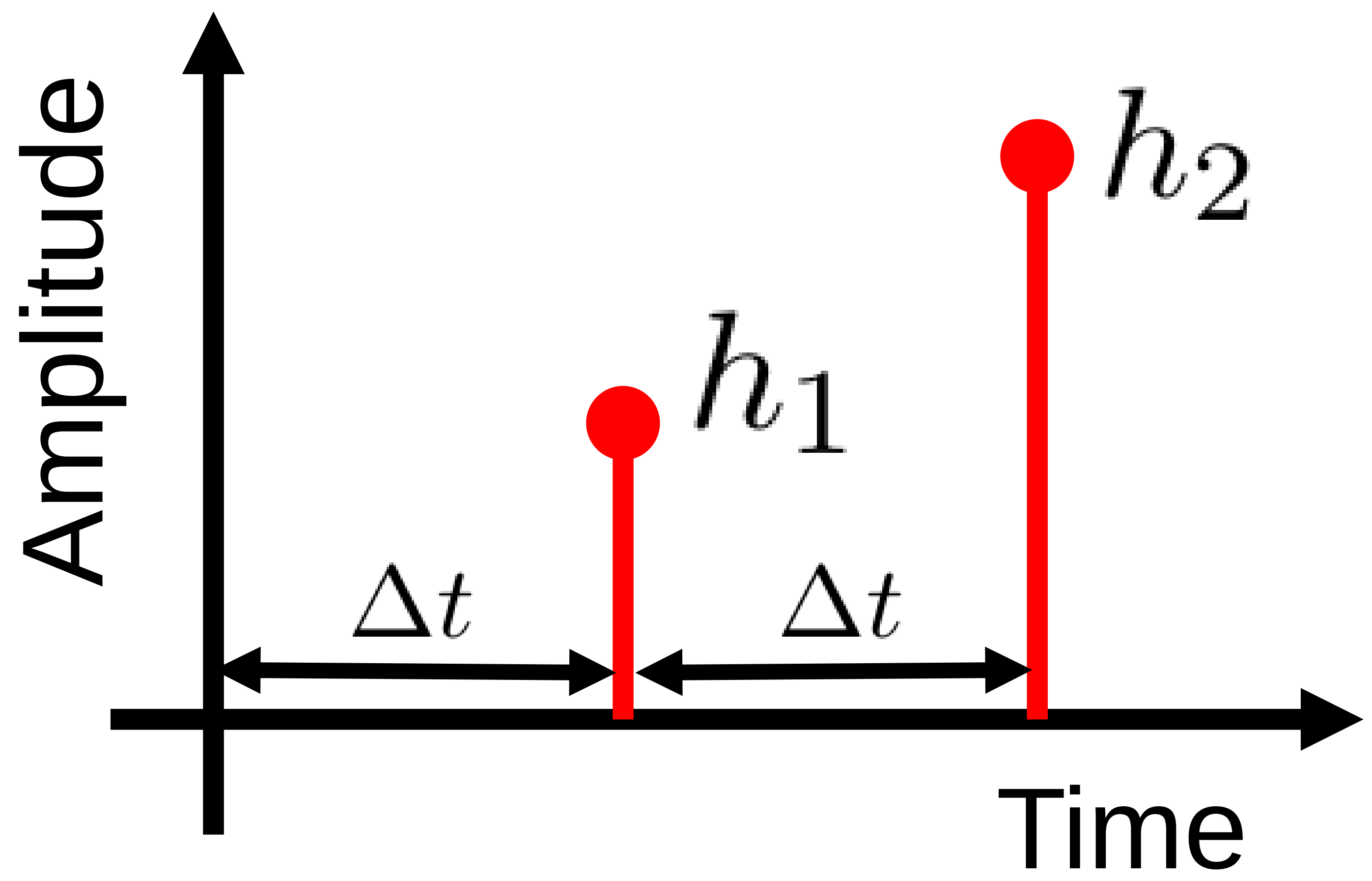
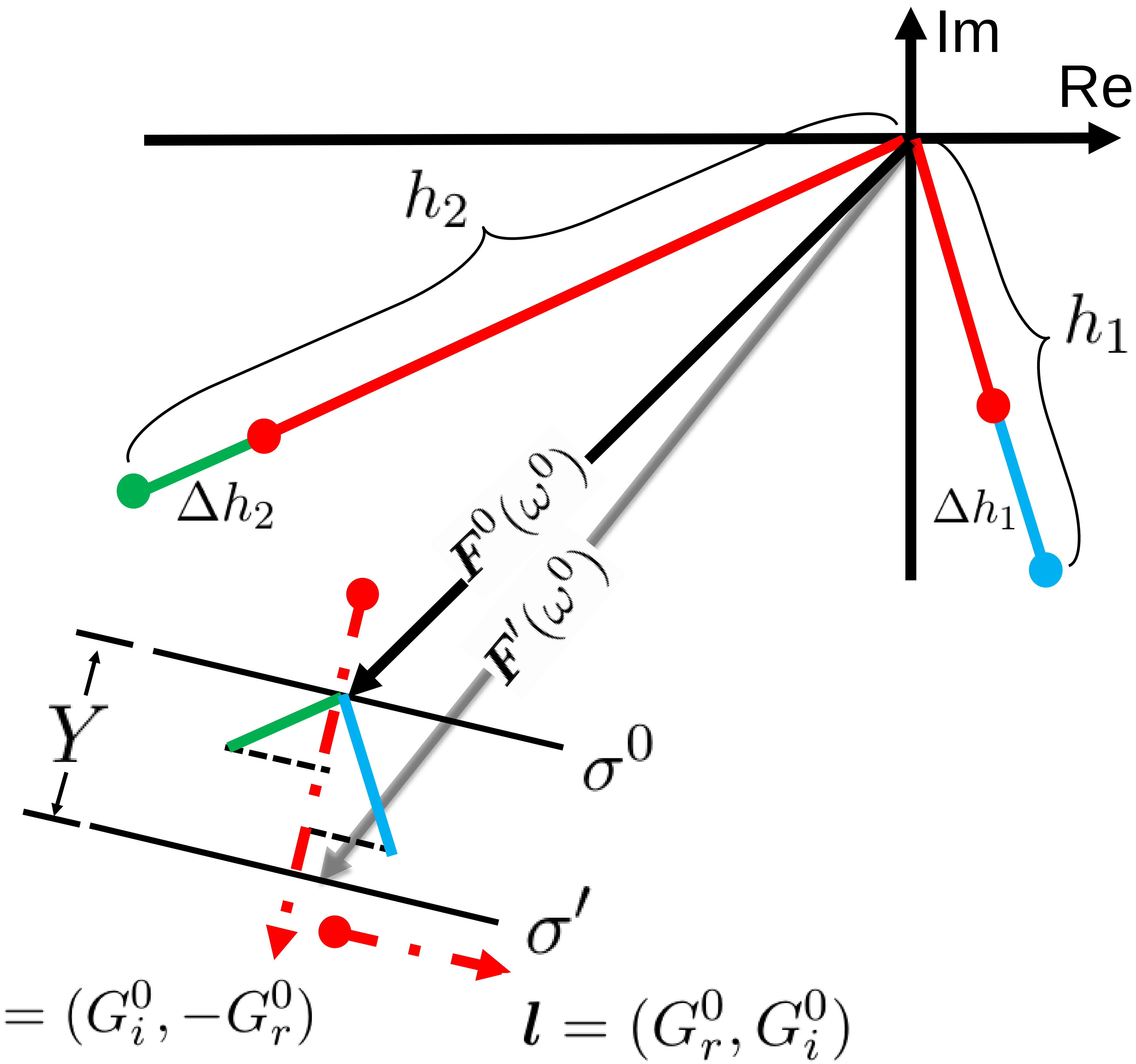




$$Y = \sum_{k=1}^{L} (h_k - h_k^0) e^{-\boldsymbol{i}\Delta t \omega^0} \cdot \frac{\boldsymbol{n}}{|\boldsymbol{n}|}$$

$$\sigma = f(Y) \approx poly(Y)$$

Dimensionality Reduction!

A dimensionality reduction UQ strategy leveraged on the distribution pattern of the growth rate contours

1 Calculate ω^0 and σ^0

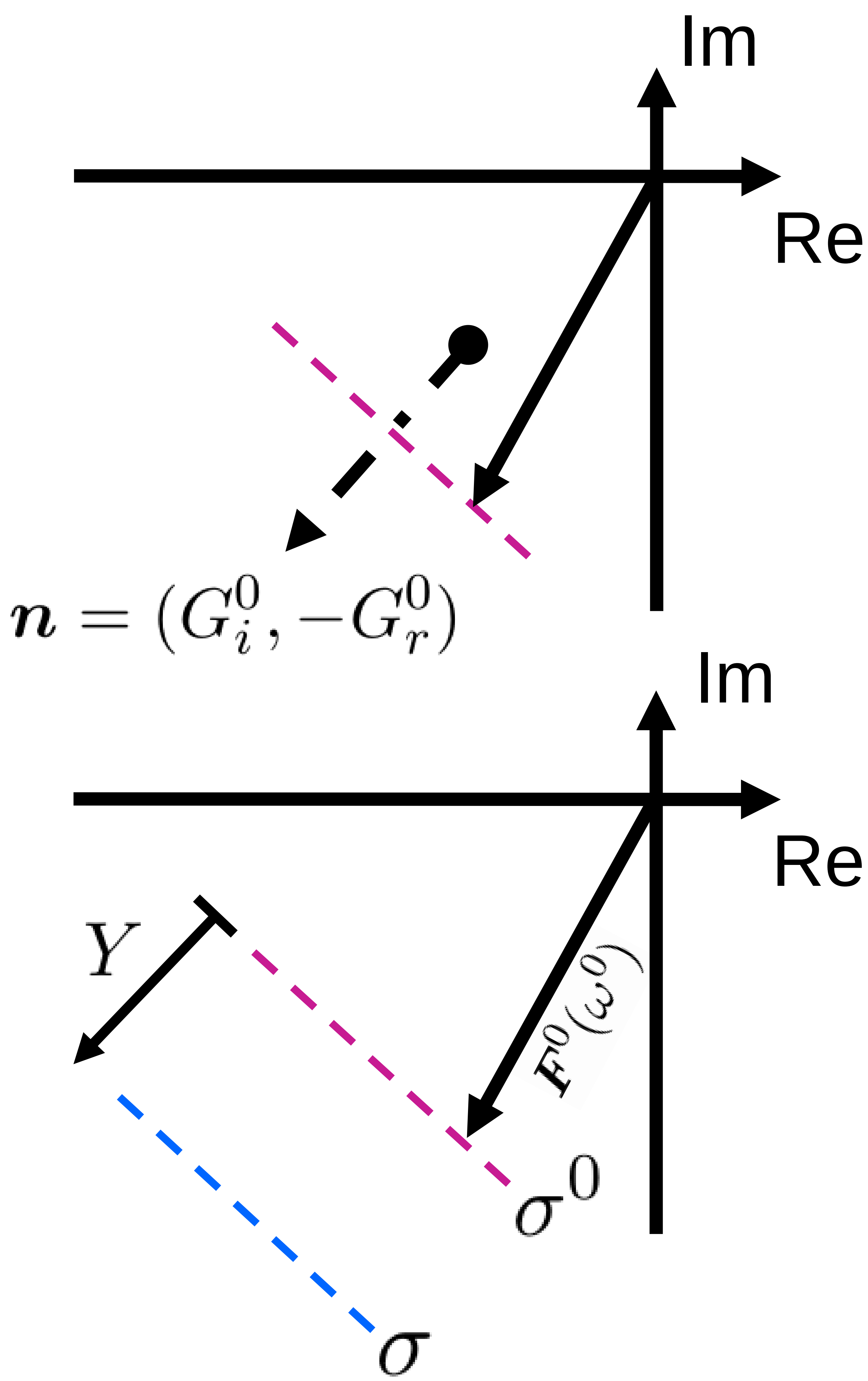
2 Calculate the gradient direction

$$G_r^0 = \frac{\partial H_r}{\partial \omega}\Big|_{\omega^0,\sigma^0} + \sum_{k=1}^{L} h_k^0 sin[k\Delta t\omega^0]k\Delta t$$

$$G_i^0 = \frac{\partial H_i}{\partial \omega}\Big|_{\omega^0,\sigma^0} + \sum_{k=1}^{L} h_k^0 cos[k\Delta t\omega^0]k\Delta t$$

3 Fit a polynomial function

$$Y = \sum_{k=1}^{L}(h_k - h_k^0)e^{-i\Delta t\omega^0} \cdot \frac{\boldsymbol{n}}{|\boldsymbol{n}|} \quad , \quad \sigma \approx poly(Y)$$



A dimensionality reduction UQ strategy leveraged on the distribution pattern of the growth rate contours

1. Calculate ω^0 and σ^0

2. Calculate the gradient direction

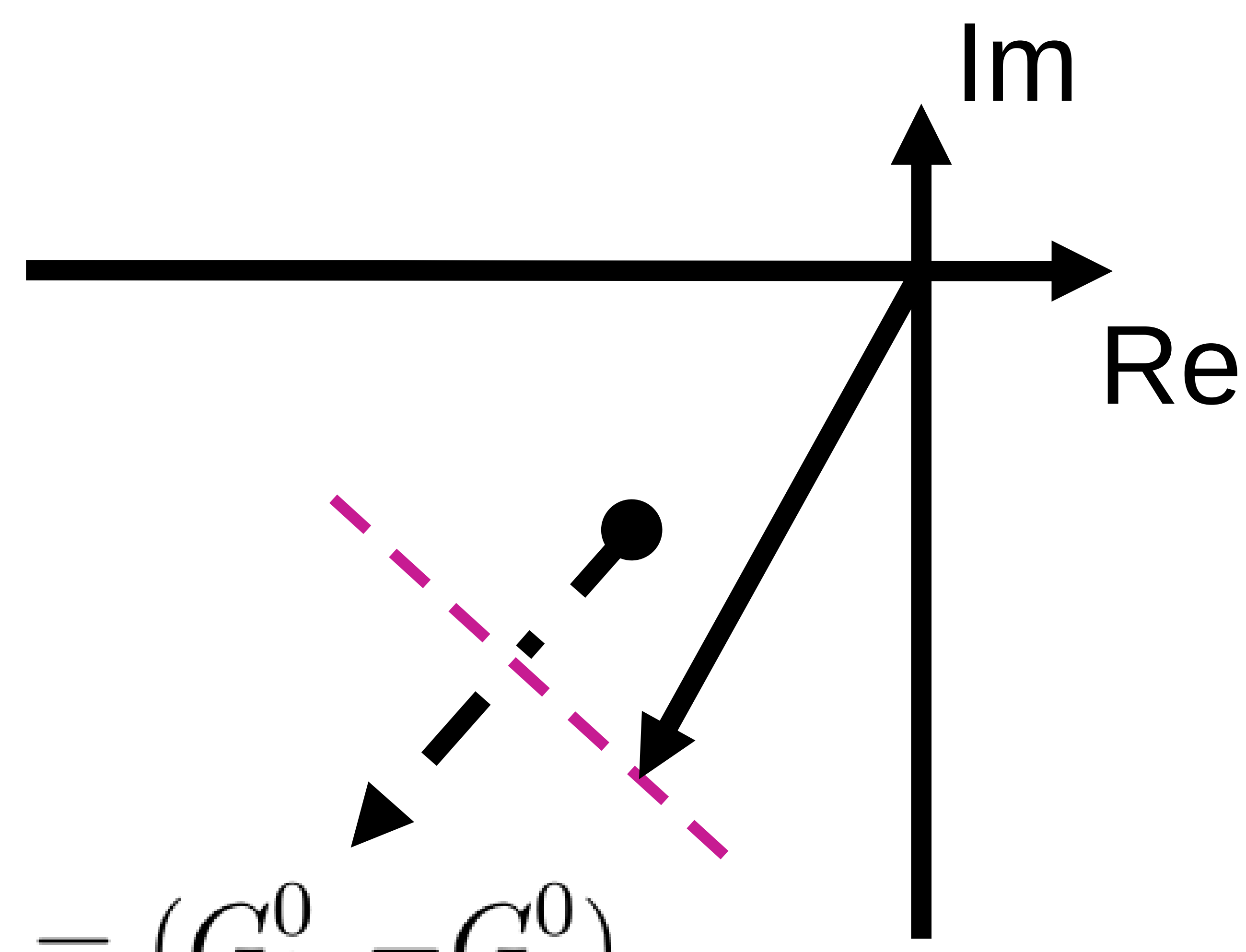
$$G_r^0 = \frac{\partial H_r}{\partial \omega}\Big|_{\omega^0,\sigma^0} + \sum_{k=1}^{L} h_k^0 sin[k\Delta t\omega^0]k\Delta t$$

$$G_i^0 = \frac{\partial H_i}{\partial \omega}\Big|_{\omega^0,\sigma^0} + \sum_{k=1}^{L} h_k^0 cos[k\Delta t\omega^0]k\Delta t$$

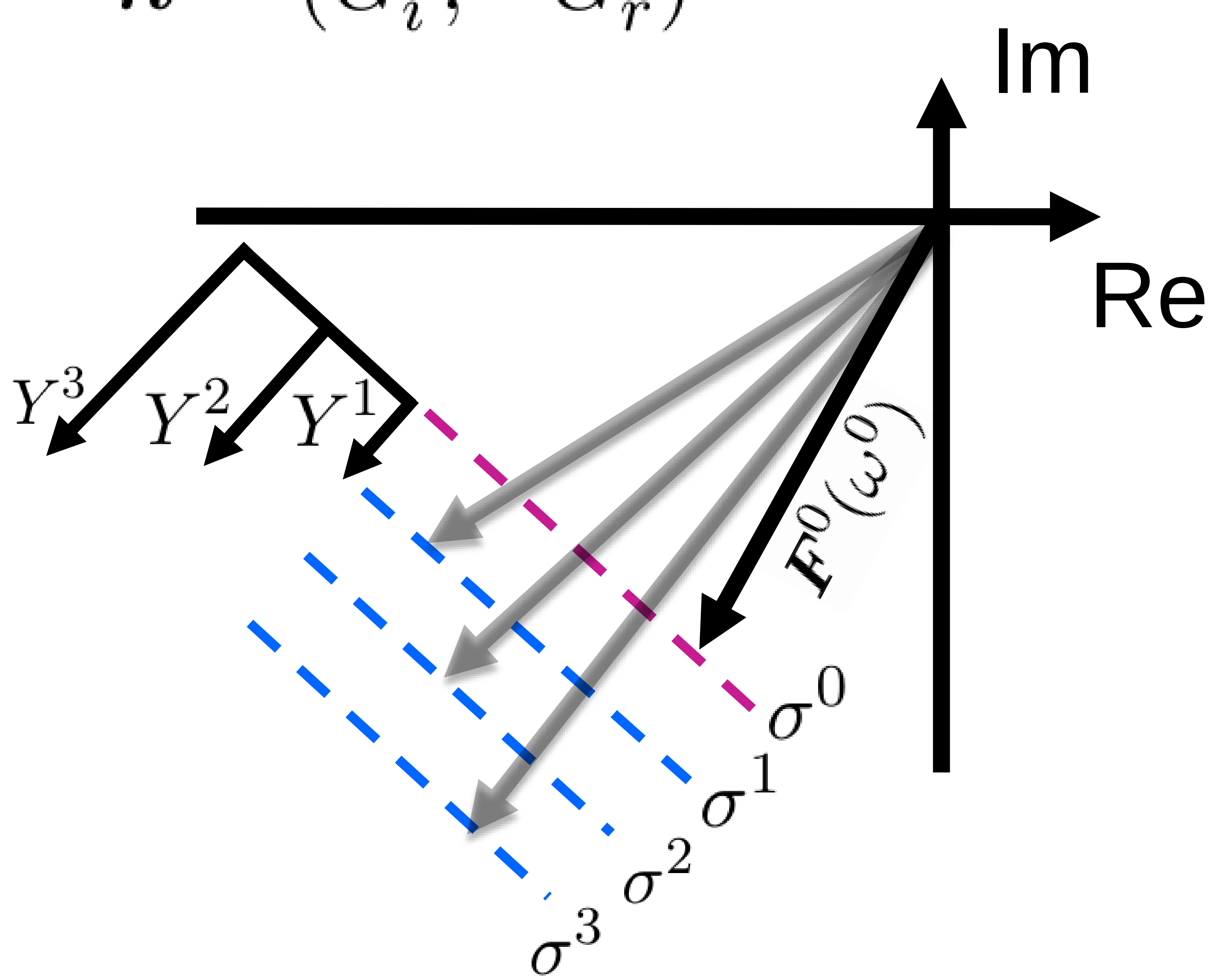
3. Fit a polynomial function (5~10 samples)

$$Y = \sum_{k=1}^{L}(h_k - h_k^0)e^{-\boldsymbol{i}\Delta t\omega^0} \cdot \frac{\boldsymbol{n}}{|\boldsymbol{n}|} \quad , \quad \sigma \approx poly(Y)$$

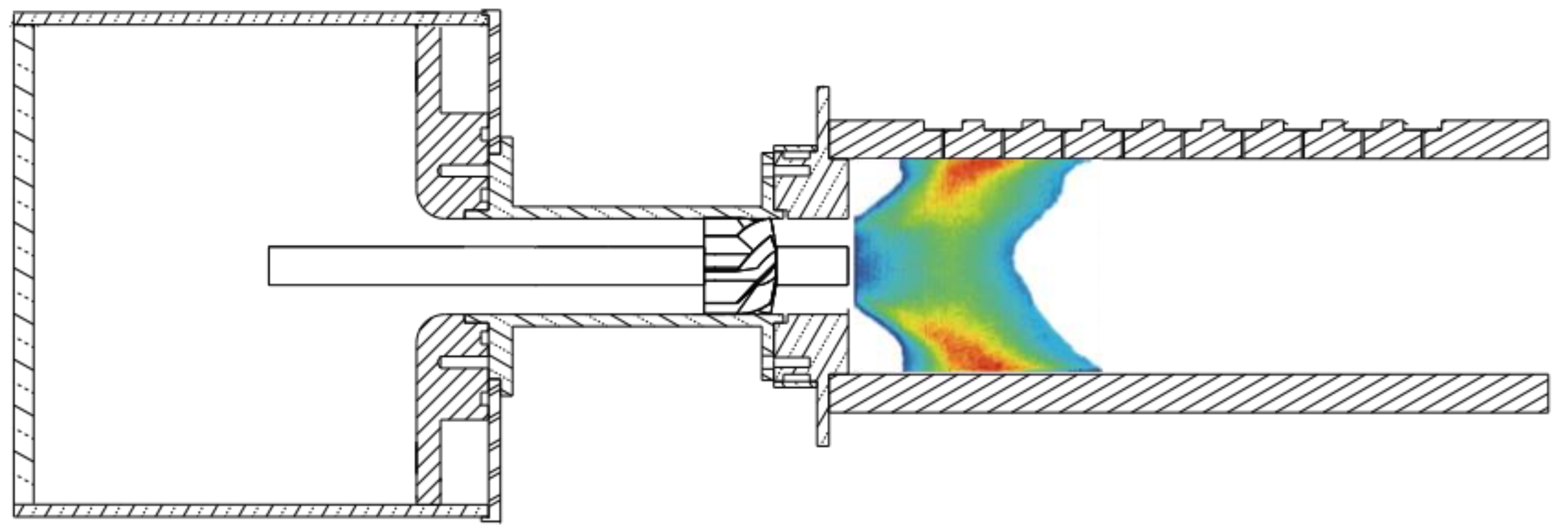
4. Monte Carlo simulation

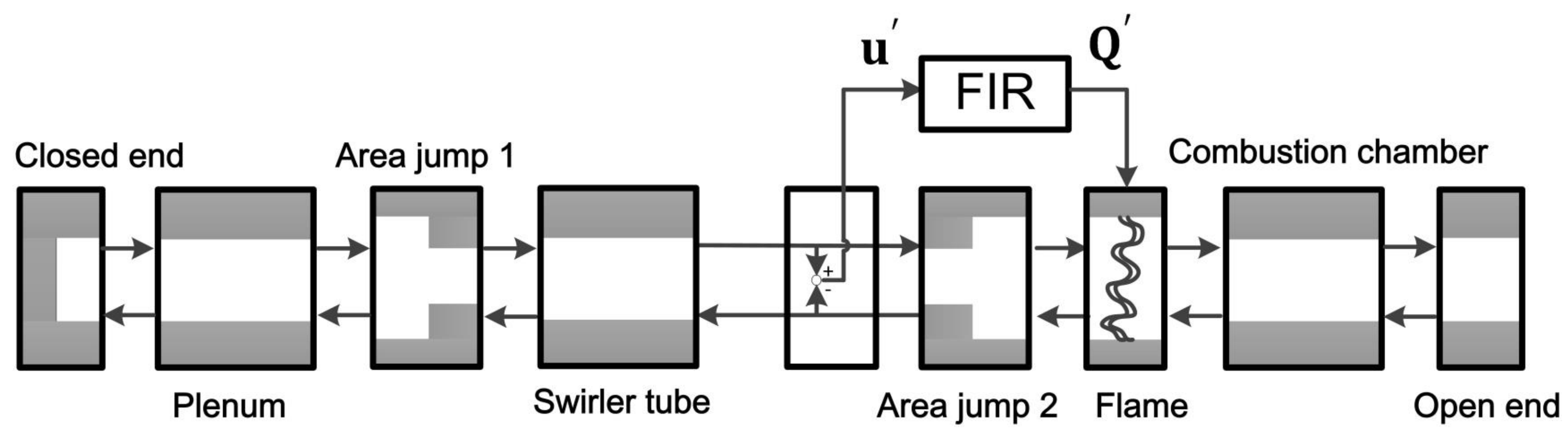


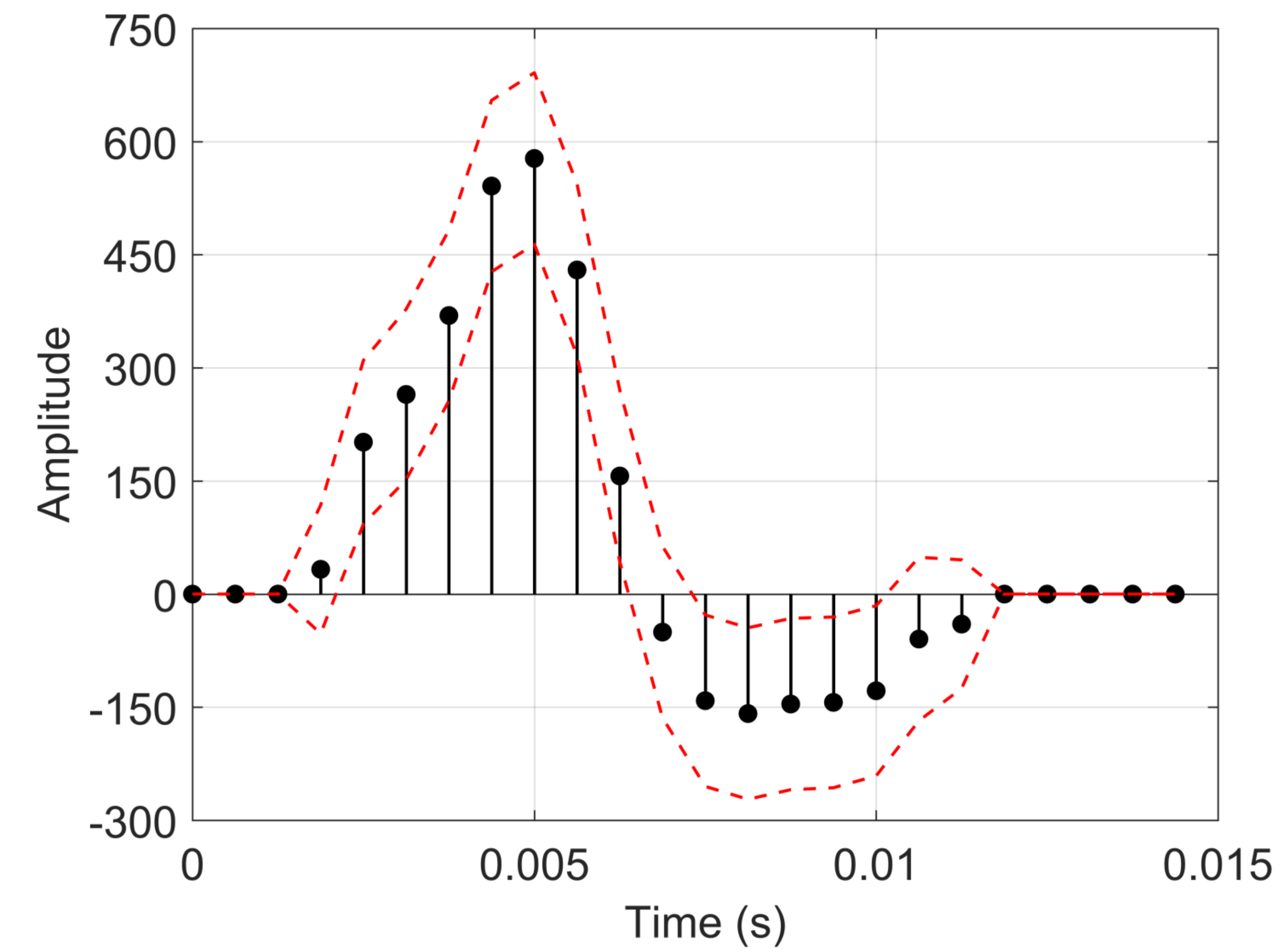
$$\boldsymbol{n} = (G_i^0, -G_r^0)$$

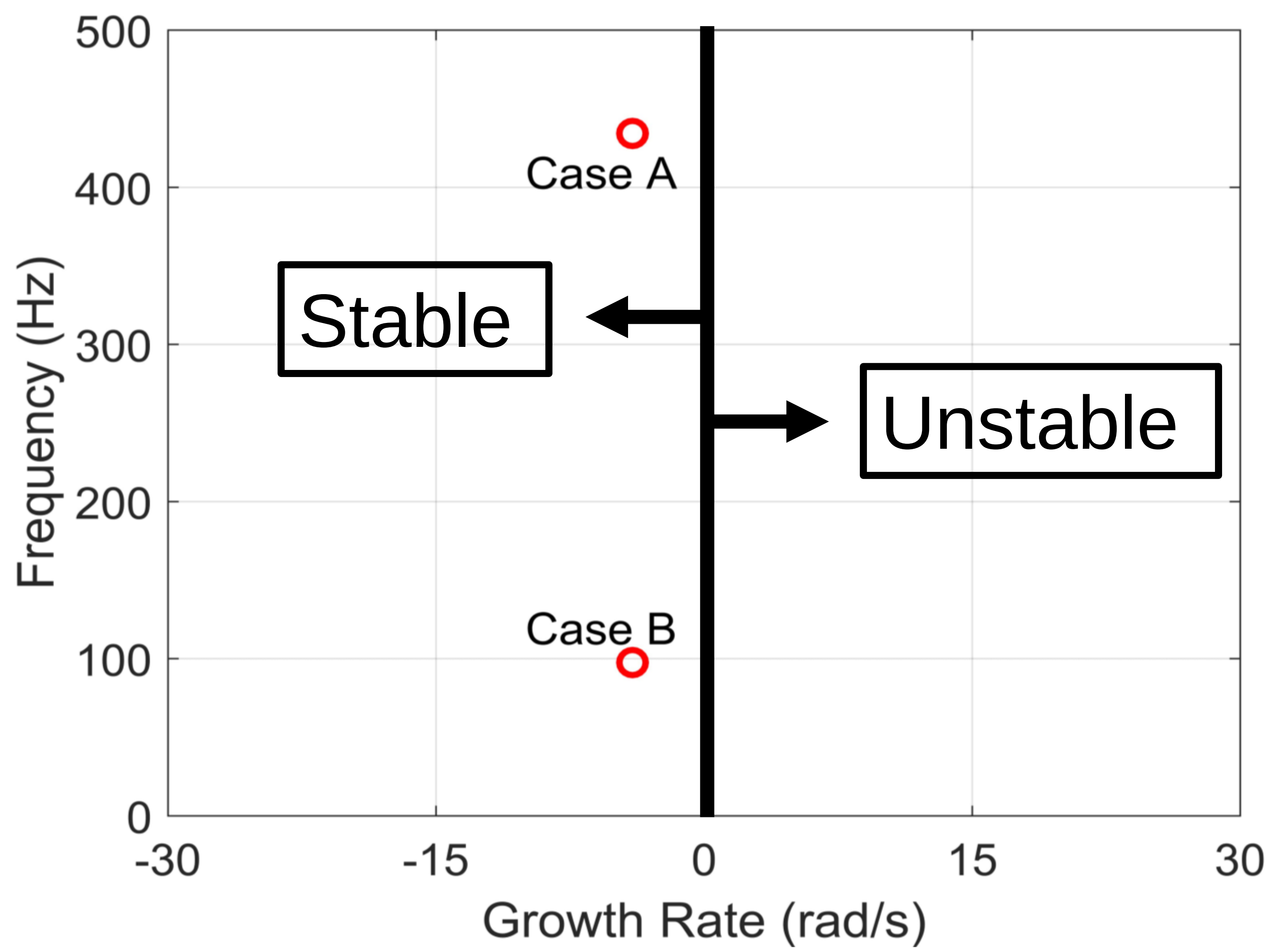


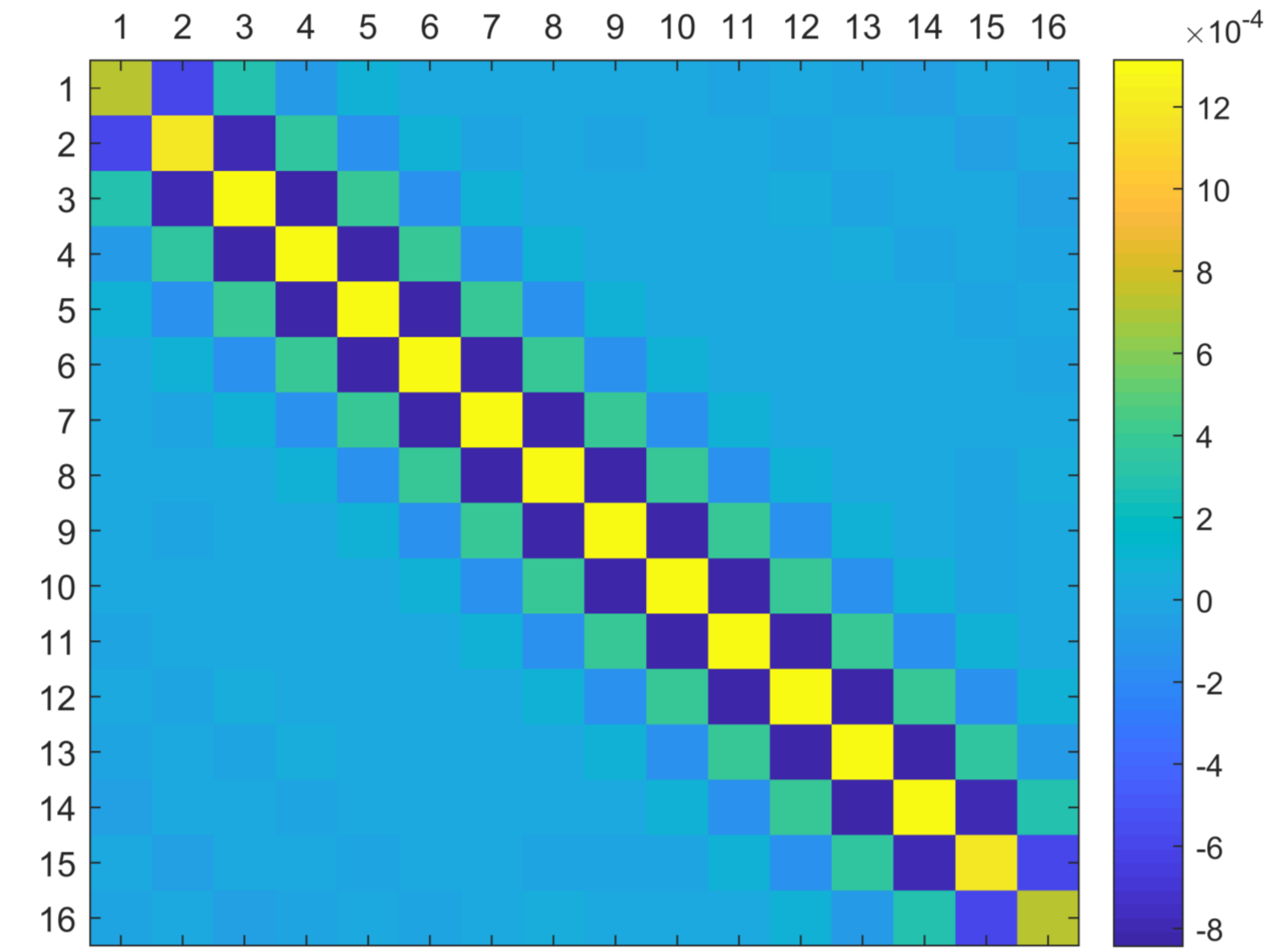
Case study: acoustic network model, impulse response identification and thermoacoustic mode specification



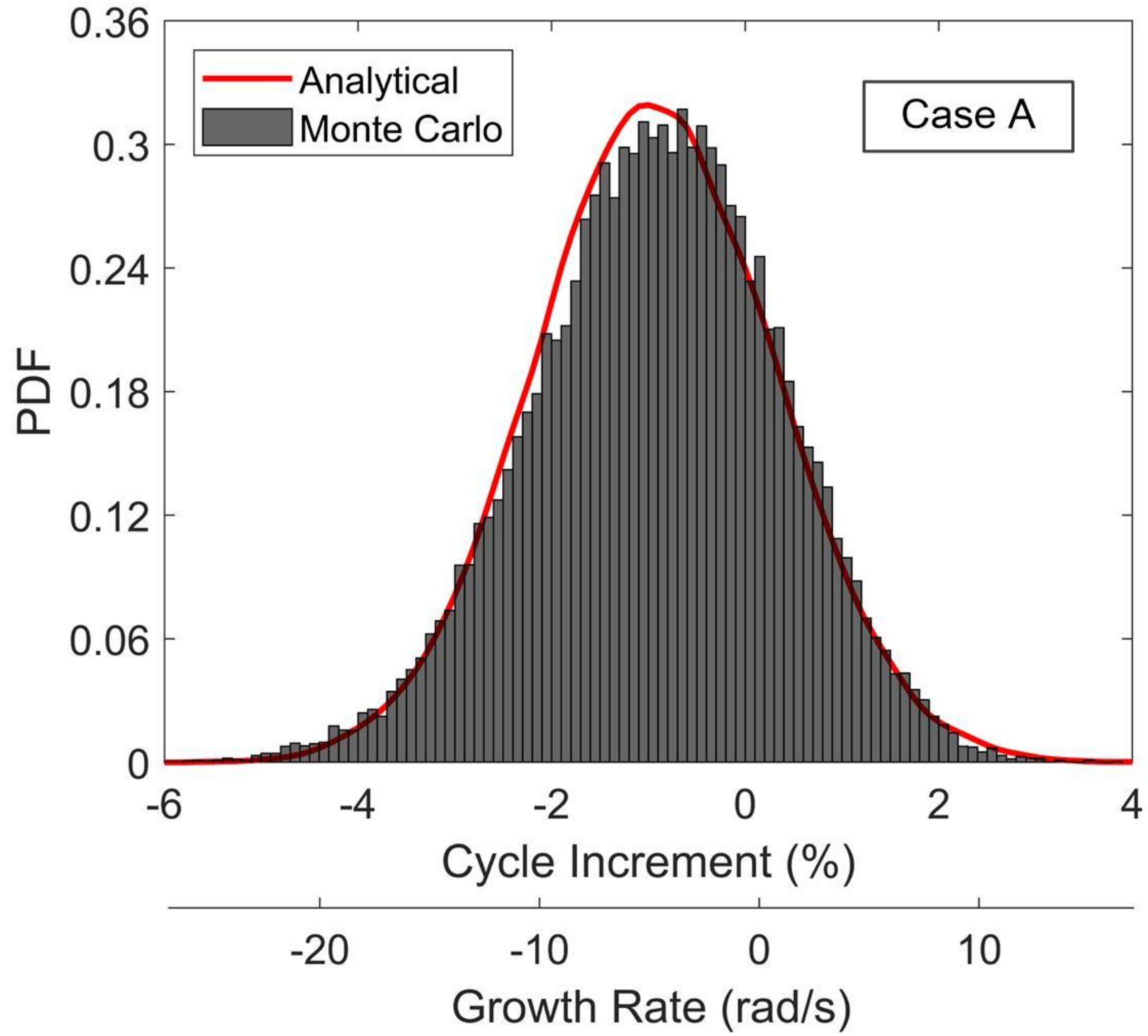


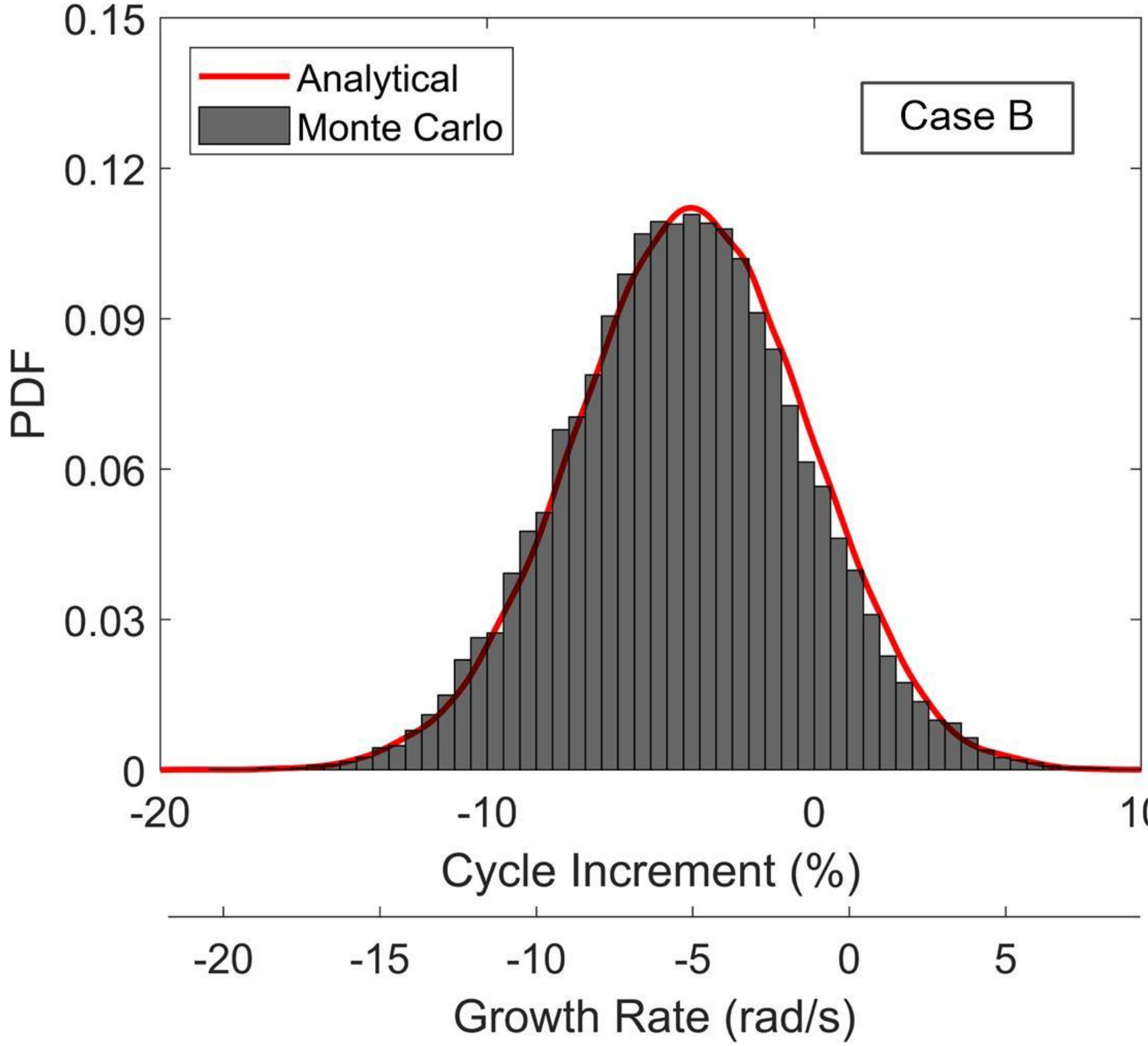






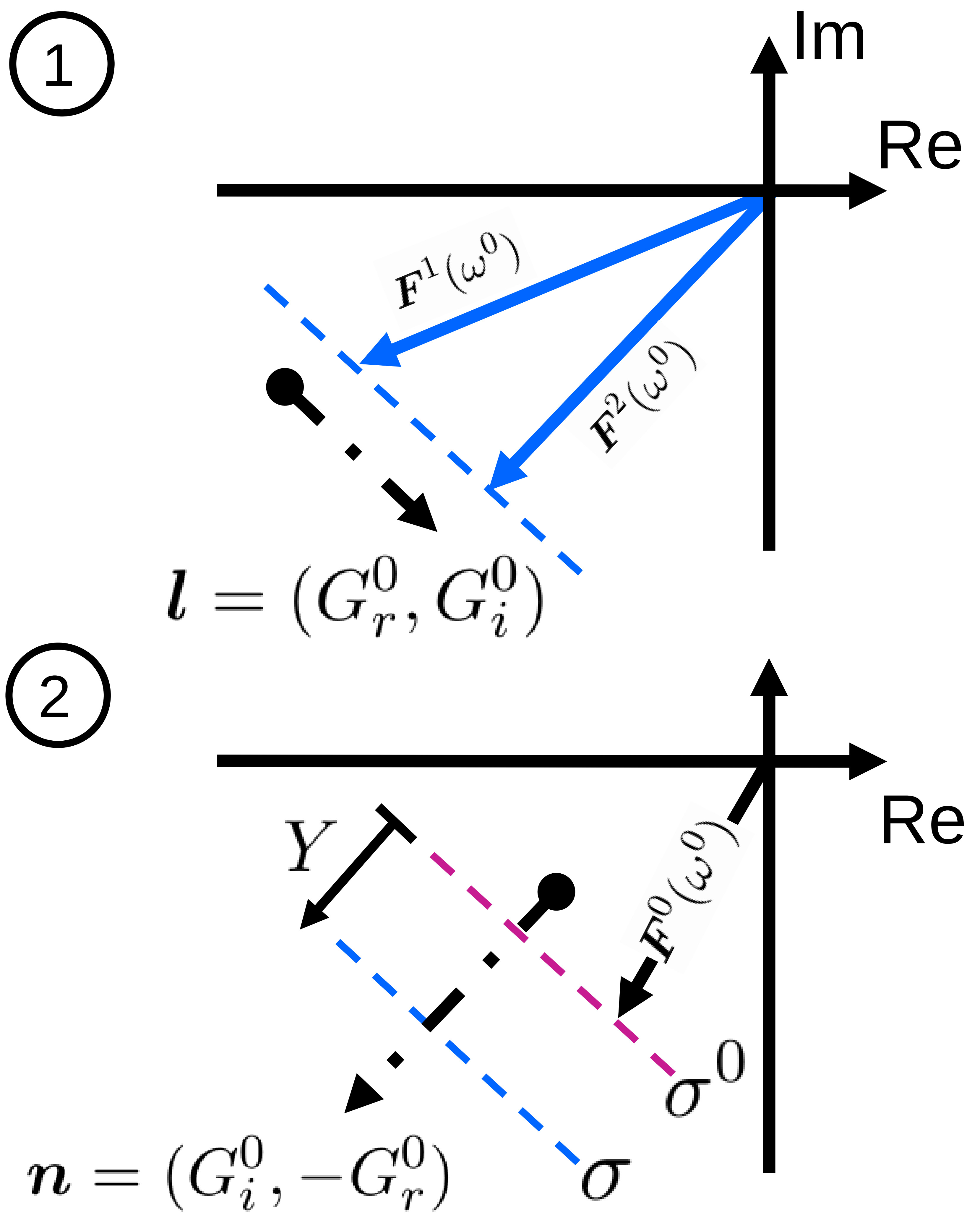
Our UQ strategy replicated the reference Monte Carlo results with 5000 times less computational cost

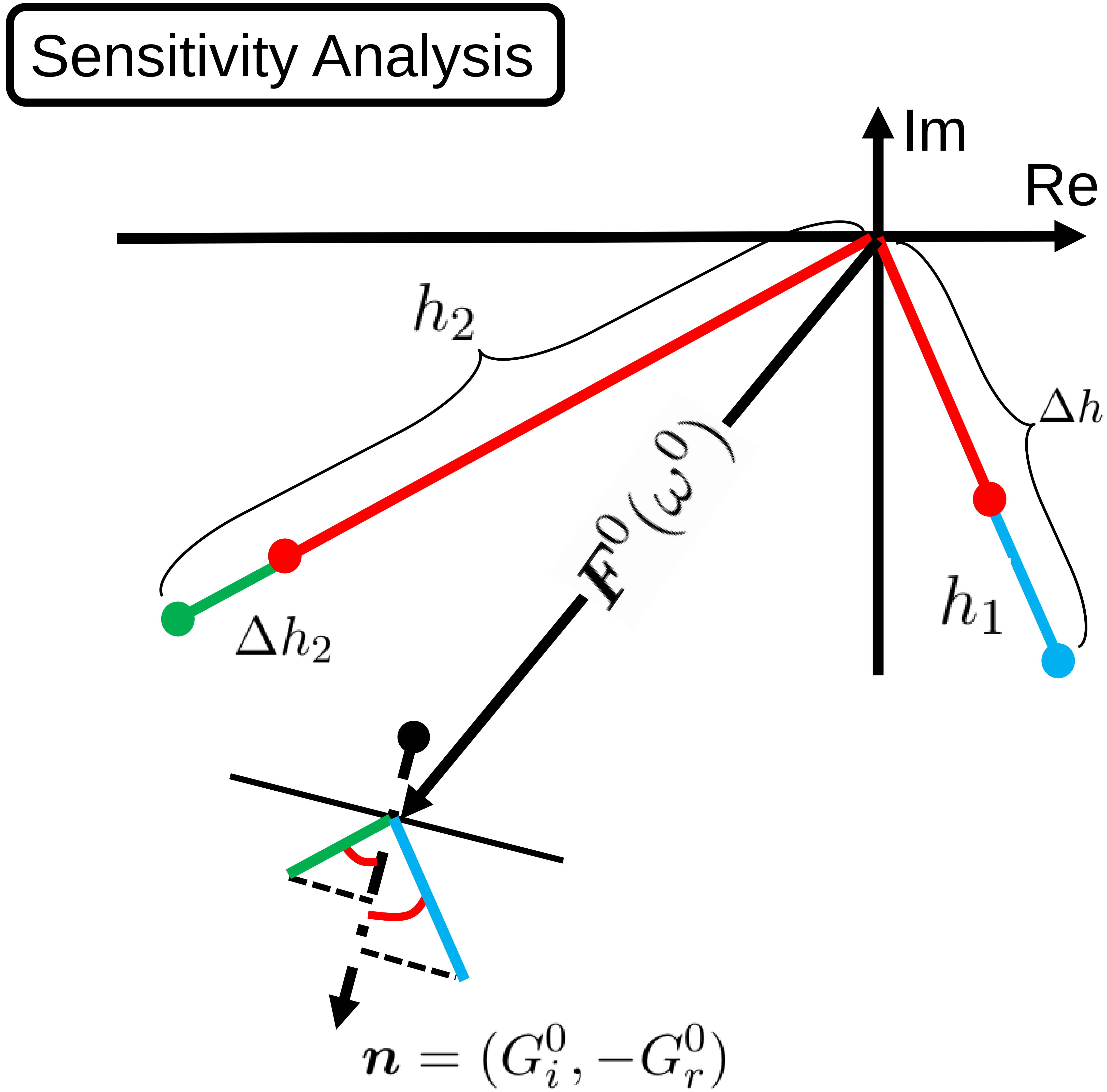




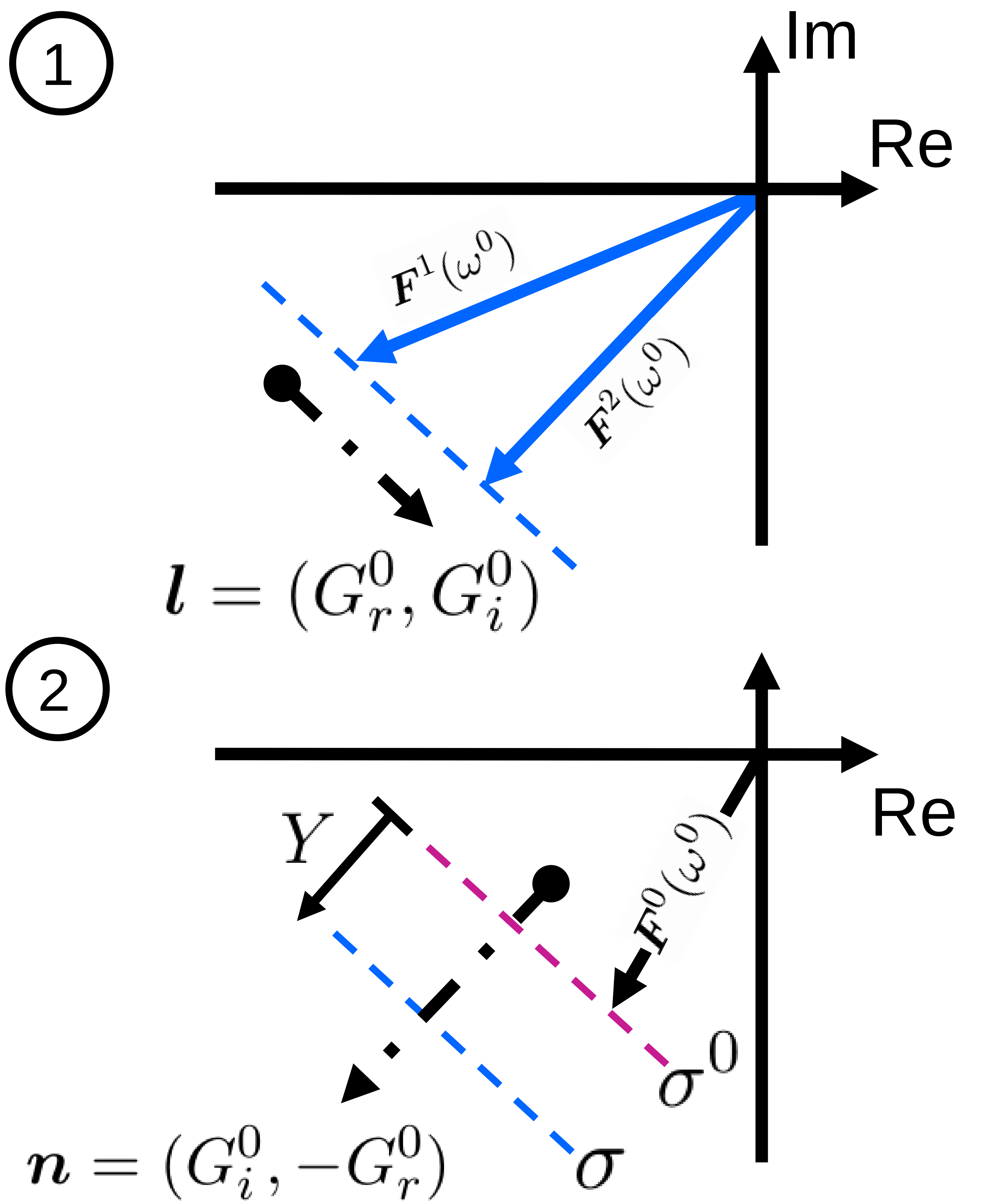
Computational Cost:
- Monte Carlo: 30000 acoustic network calculations
- Our strategy: 1+5 acoustic network calculations

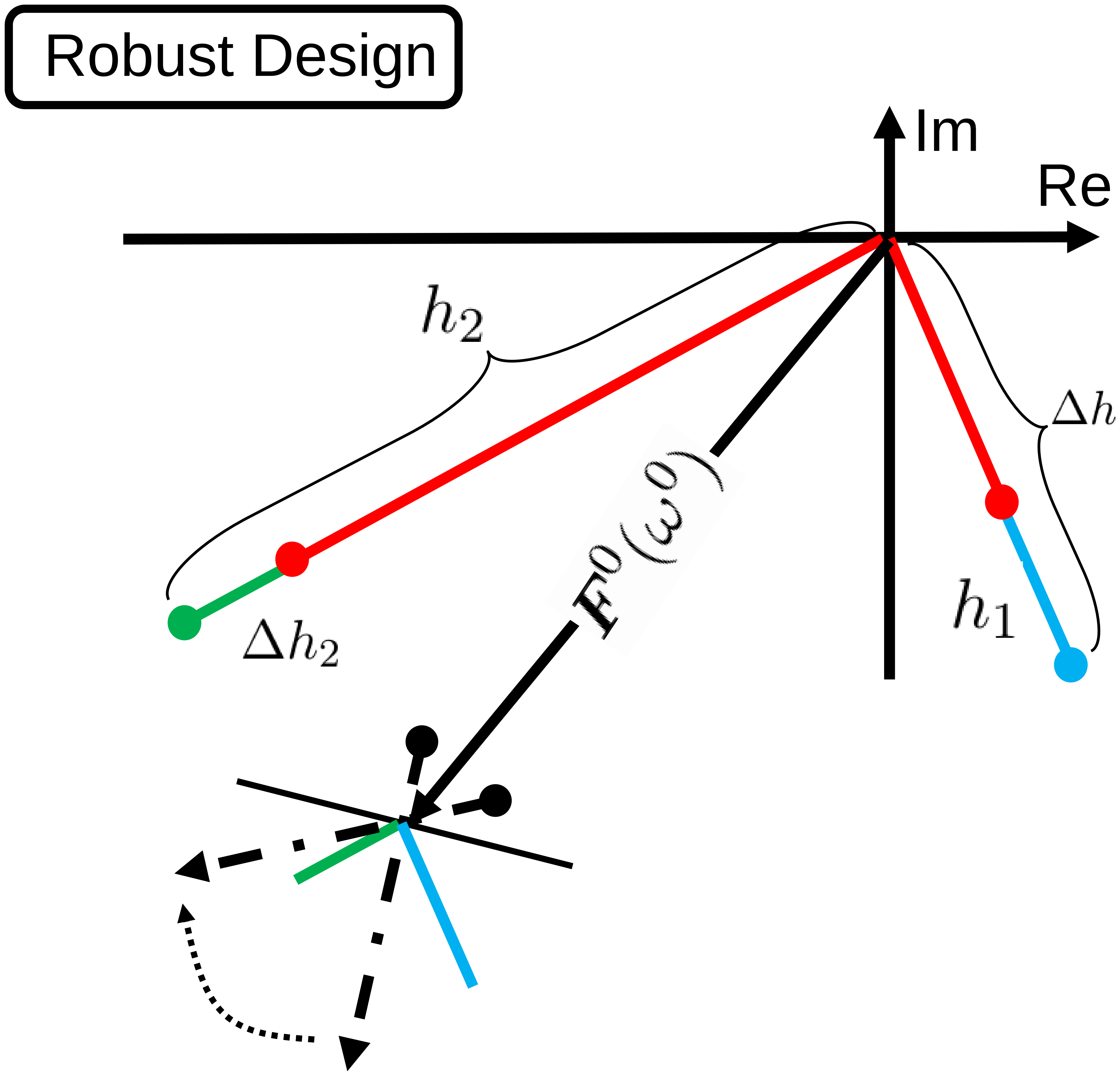
Conclusion & Outlook





Conclusion & Outlook





Back-up slides

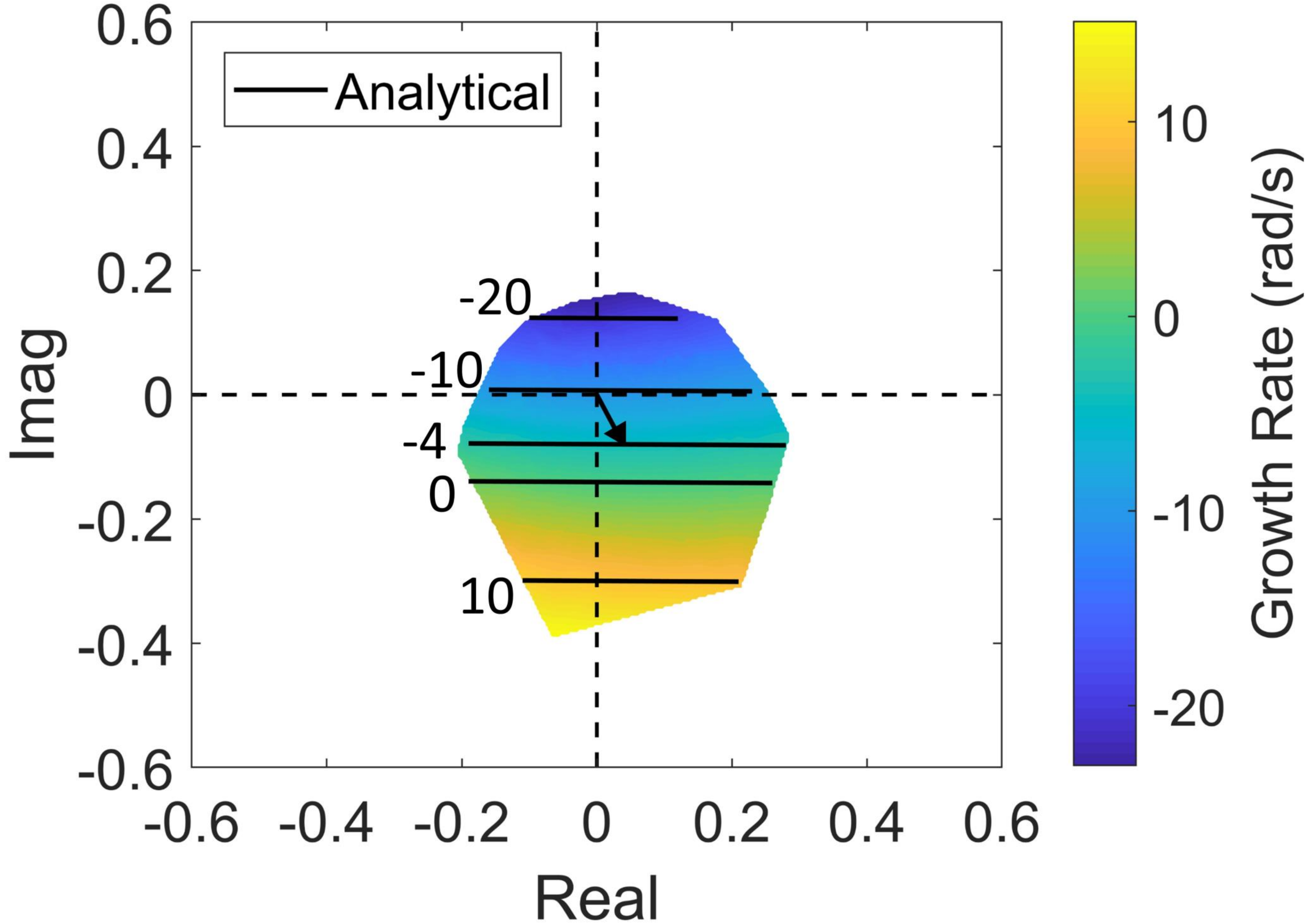
The assumption of marginal stable mode

$$FTF(\omega - i\sigma) = H(\omega - i\sigma) \quad \omega, \sigma \in \mathbb{R}$$

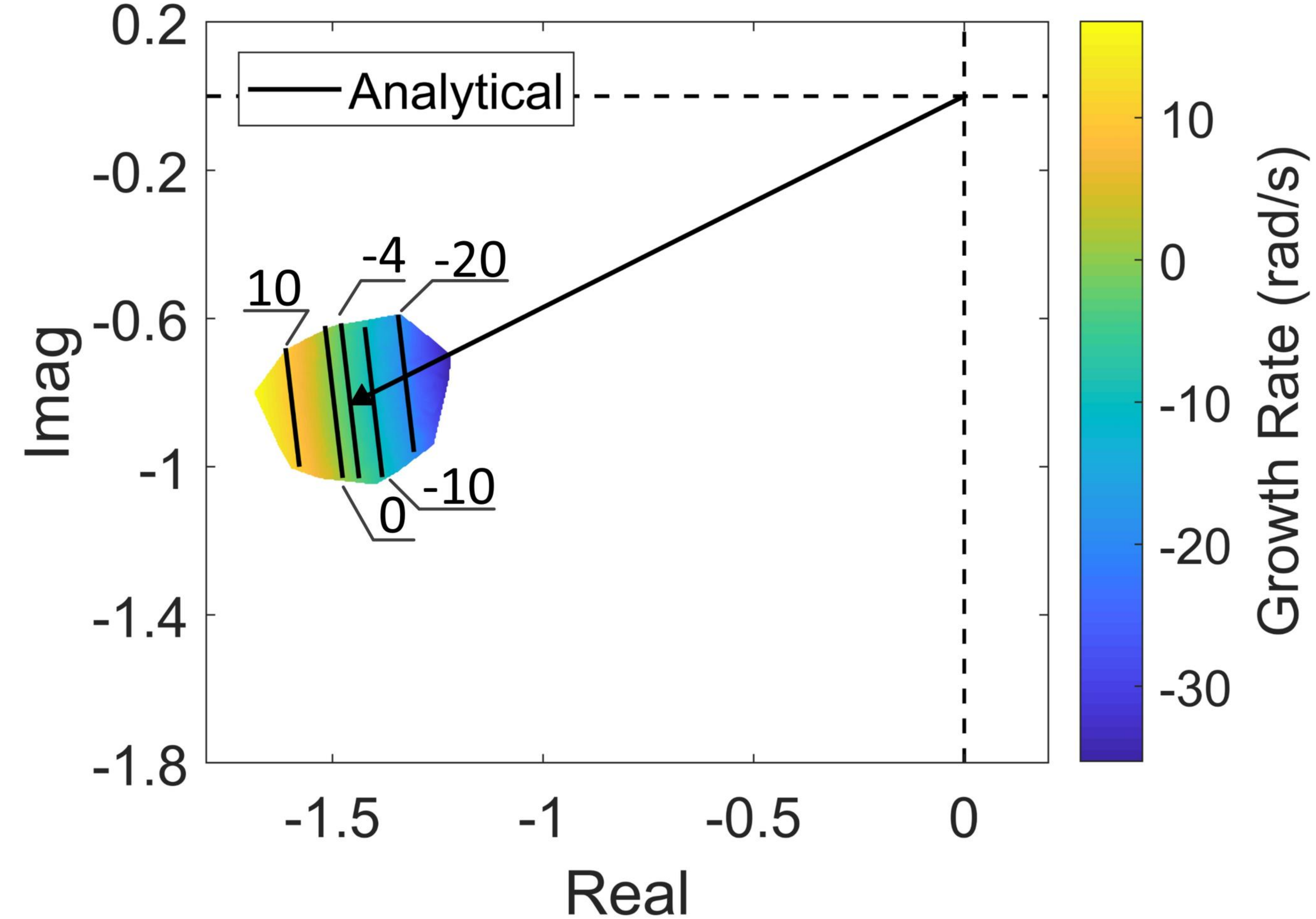
$$\sum_{k=0}^{L-1} h_k e^{-i(k+1)\Delta t(\omega - i\sigma)} = H(\omega - i\sigma)$$

$$e^{-(k+1)\Delta t\sigma}, k = 0...(L-1)$$

Growth rate contours for BRS burner

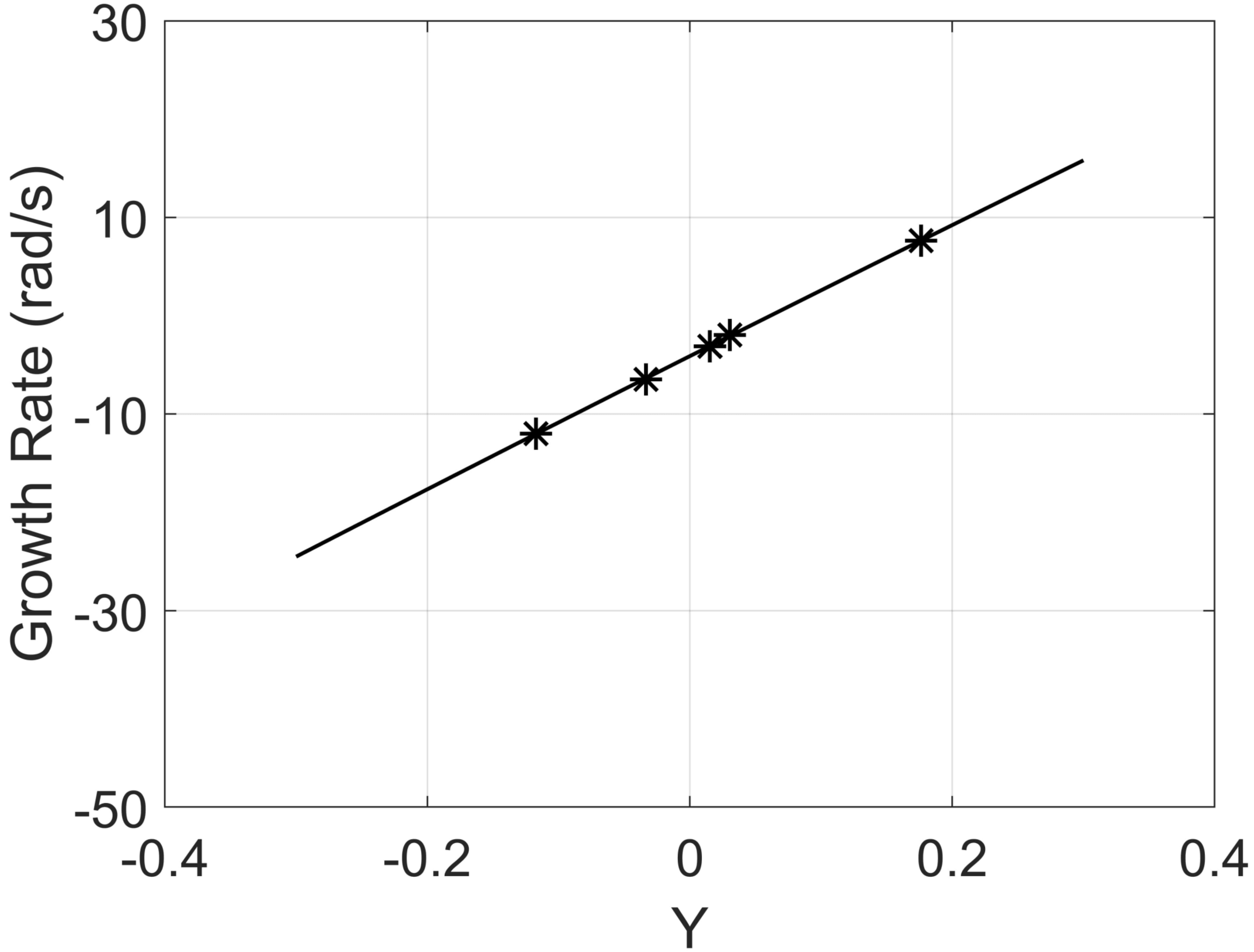


Case A



Case B

For each case, a quadratic function is fitted to link Y to modal growth rate



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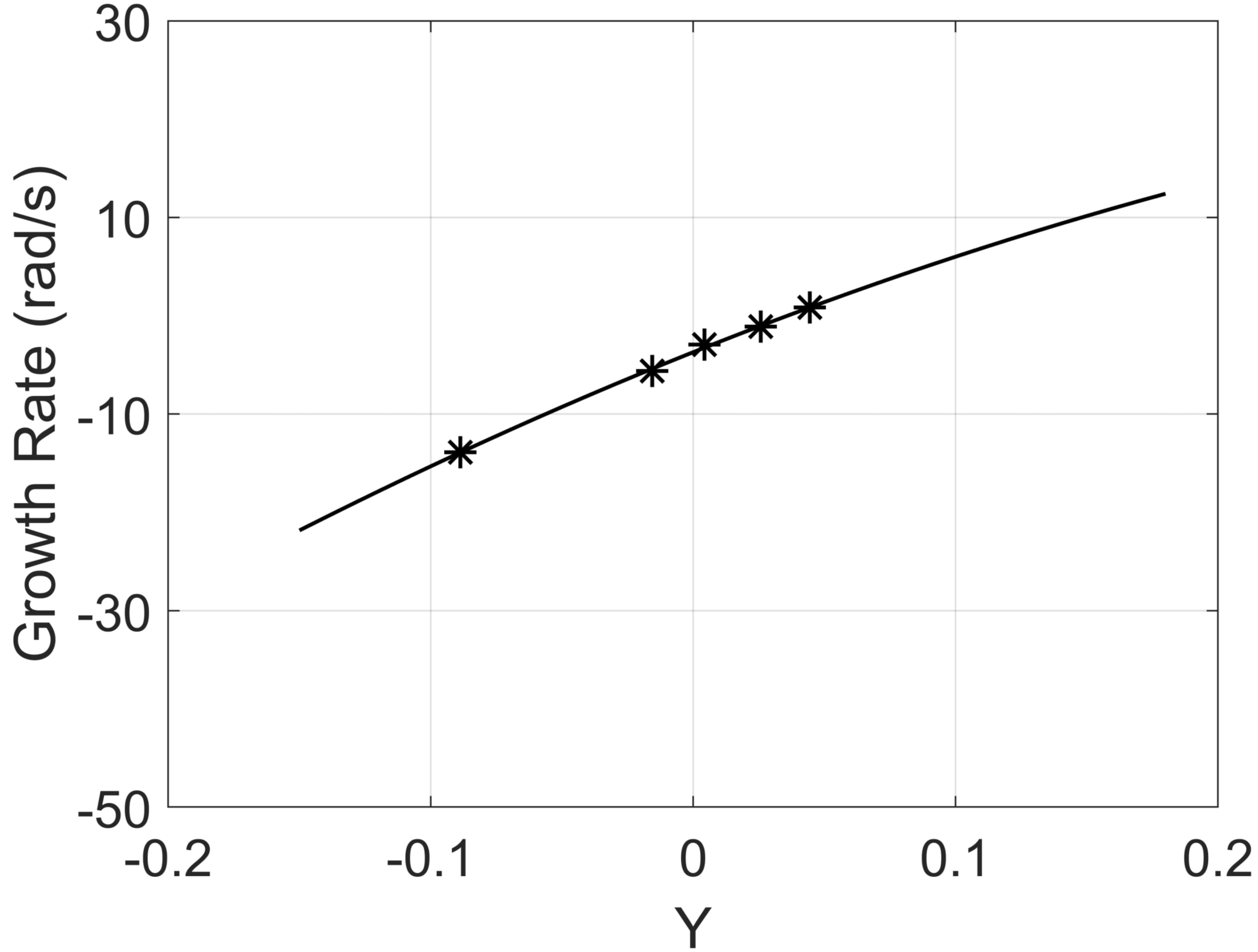

Case B