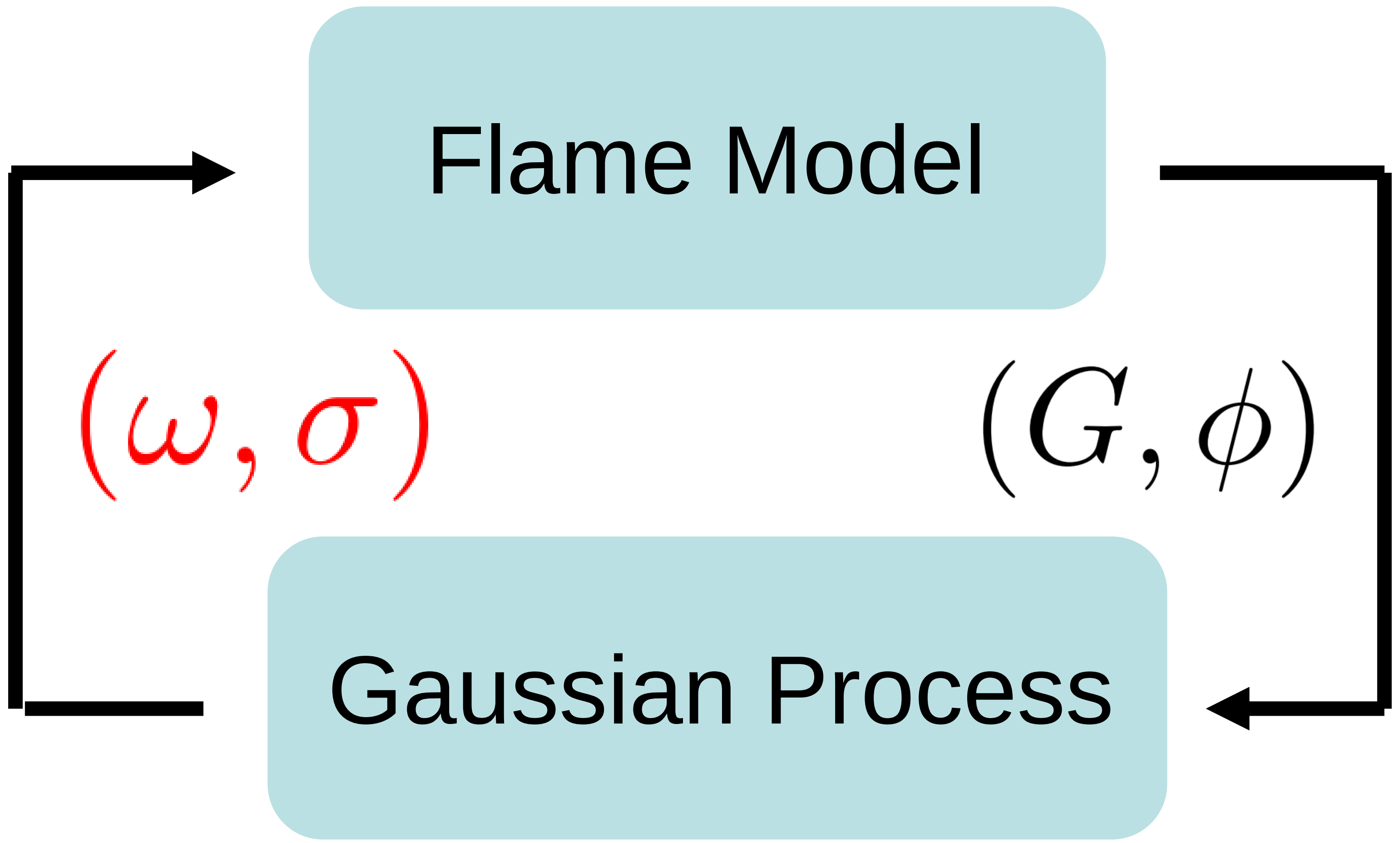
A Gaussian-Process-based framework for high-dimensional uncertainty quantification analysis in thermoacoustic Instability prediction

S. Guo, C. Silva, W. Polifke



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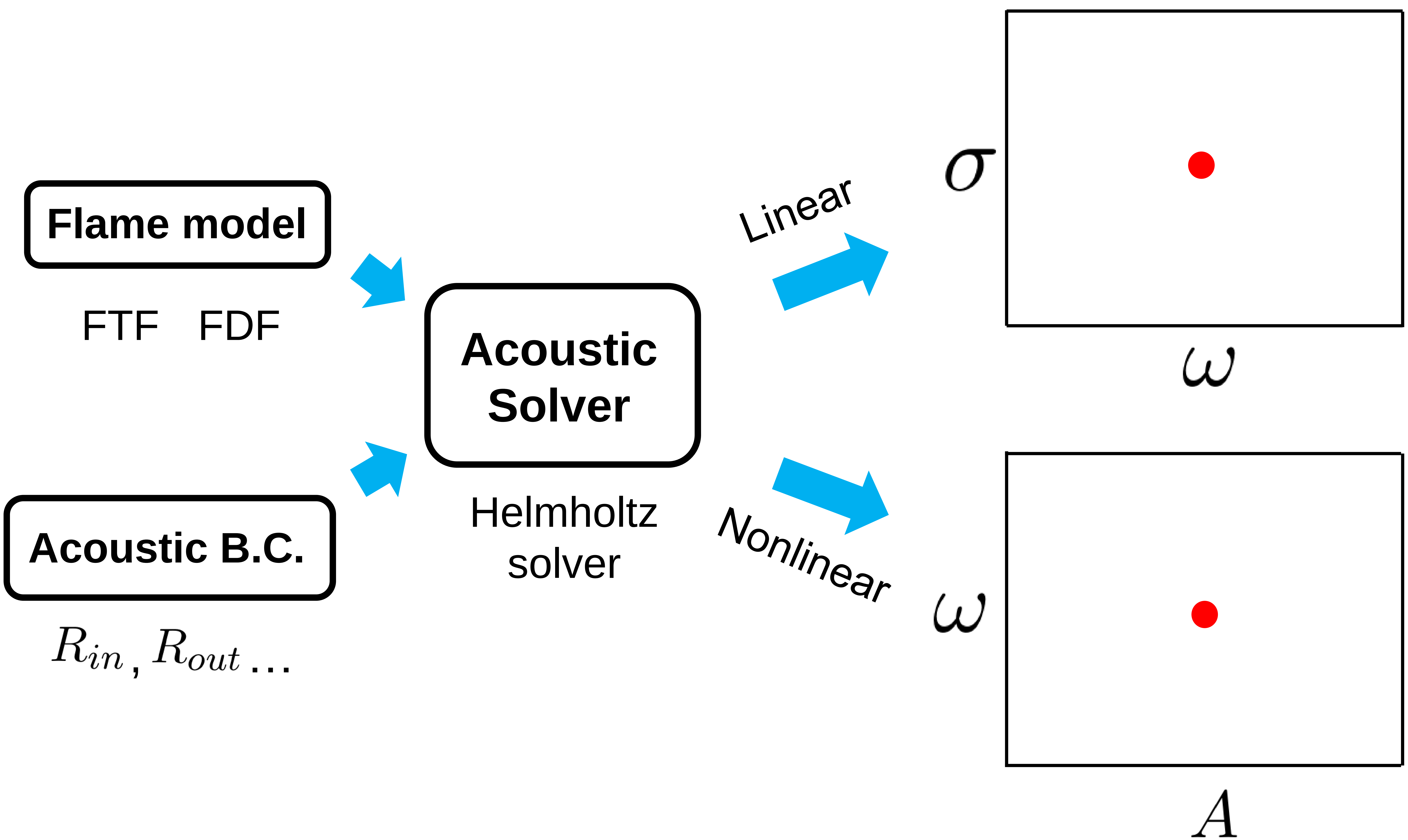
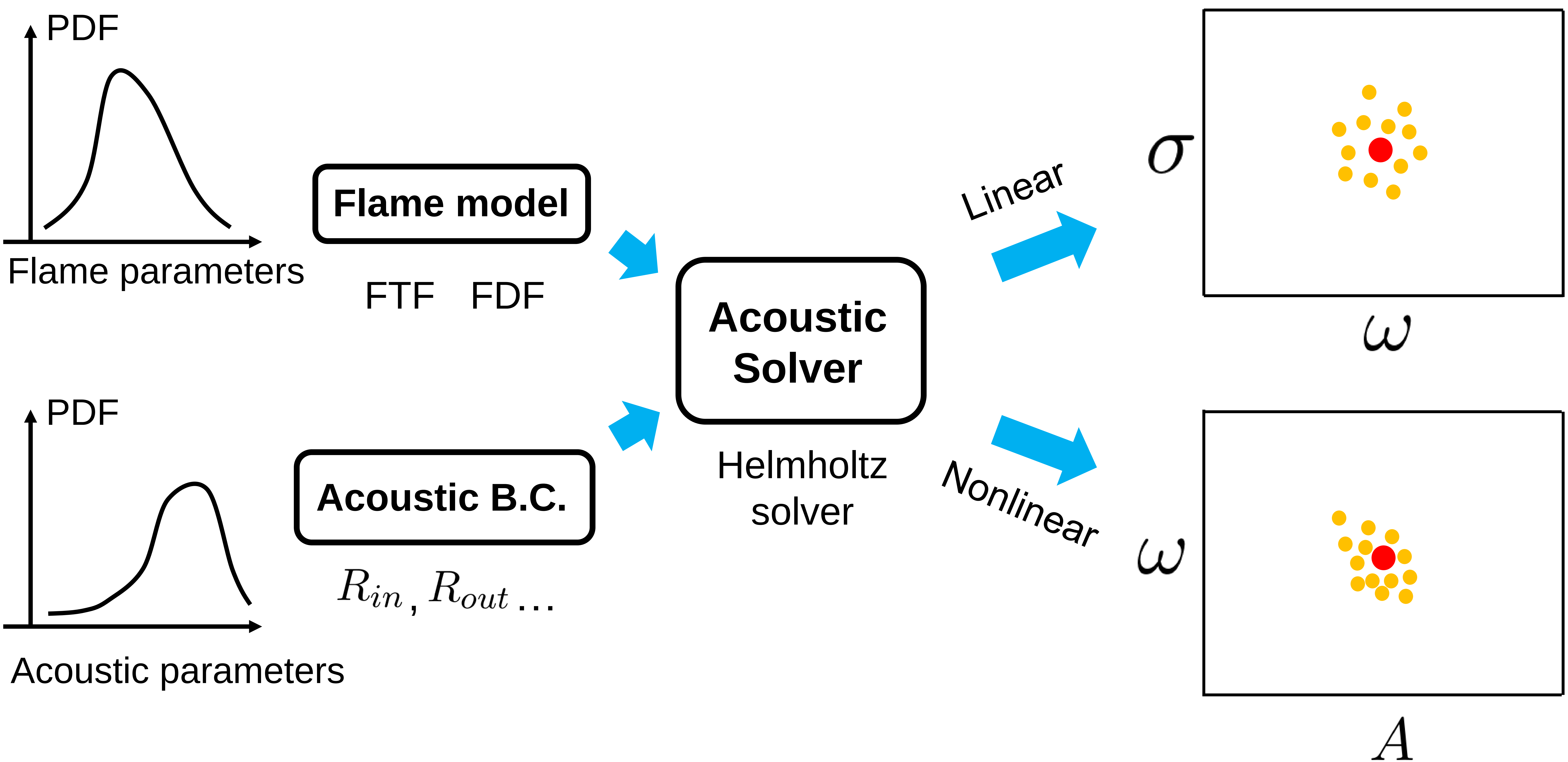
PROCI-D-19-01742

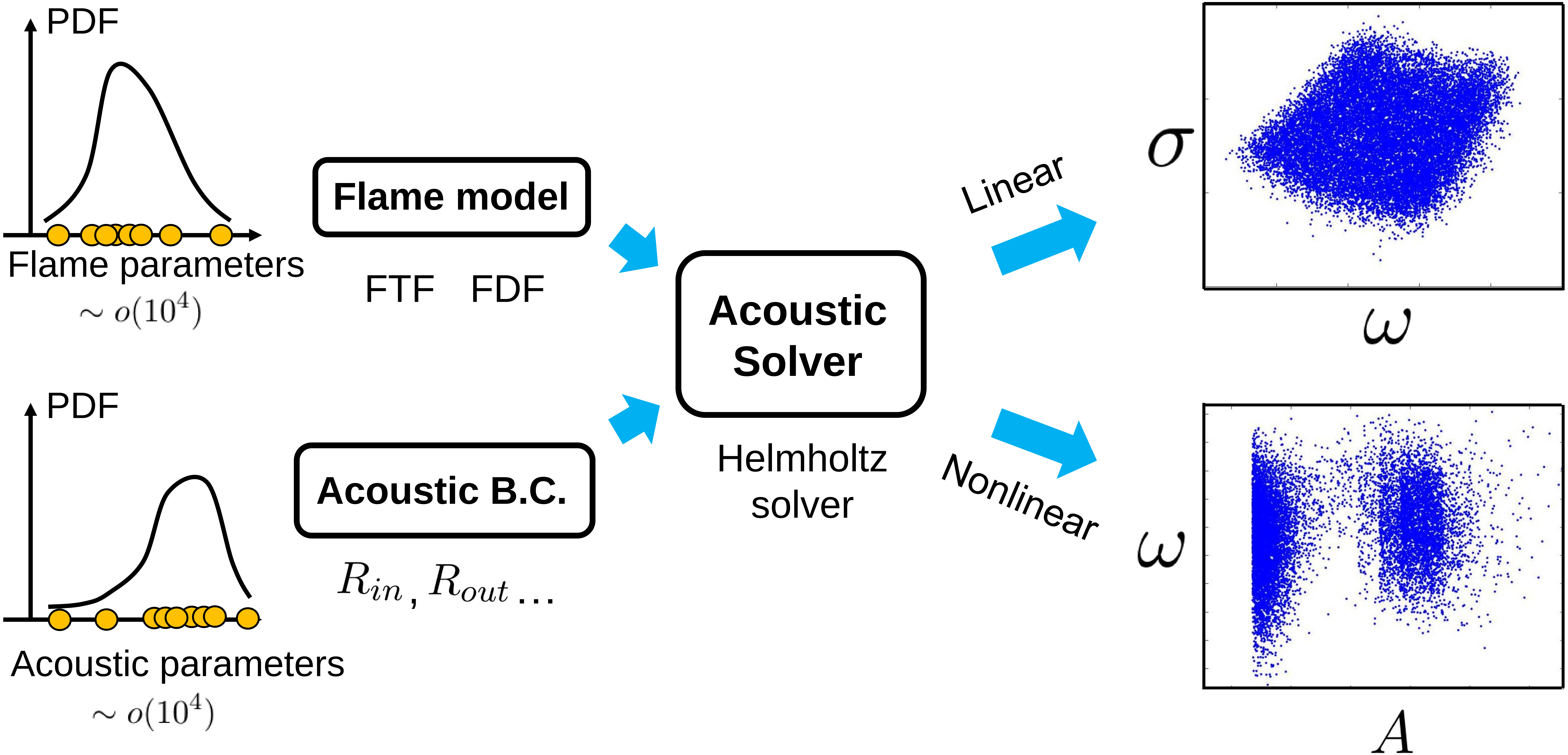
Combining a flame model with an acoustic solver is a popular way to predict thermoacoustic instability



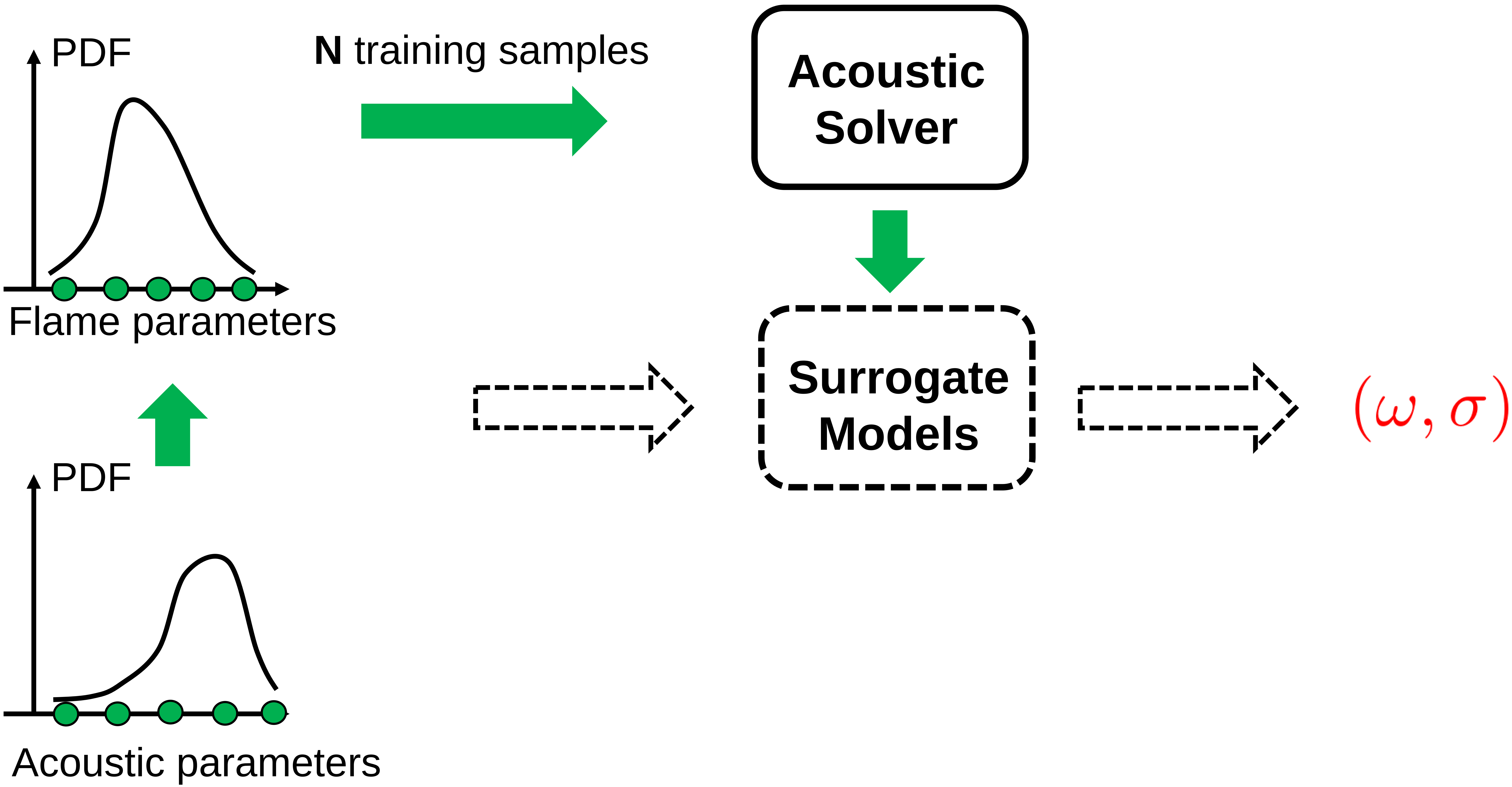
Uncertain inputs will lead to unreliable thermoacoustic instability calculations



Uncertainty quantification (UQ) requires running the acoustic solver many times



Surrogate modeling technique can greatly improve the UQ efficiency



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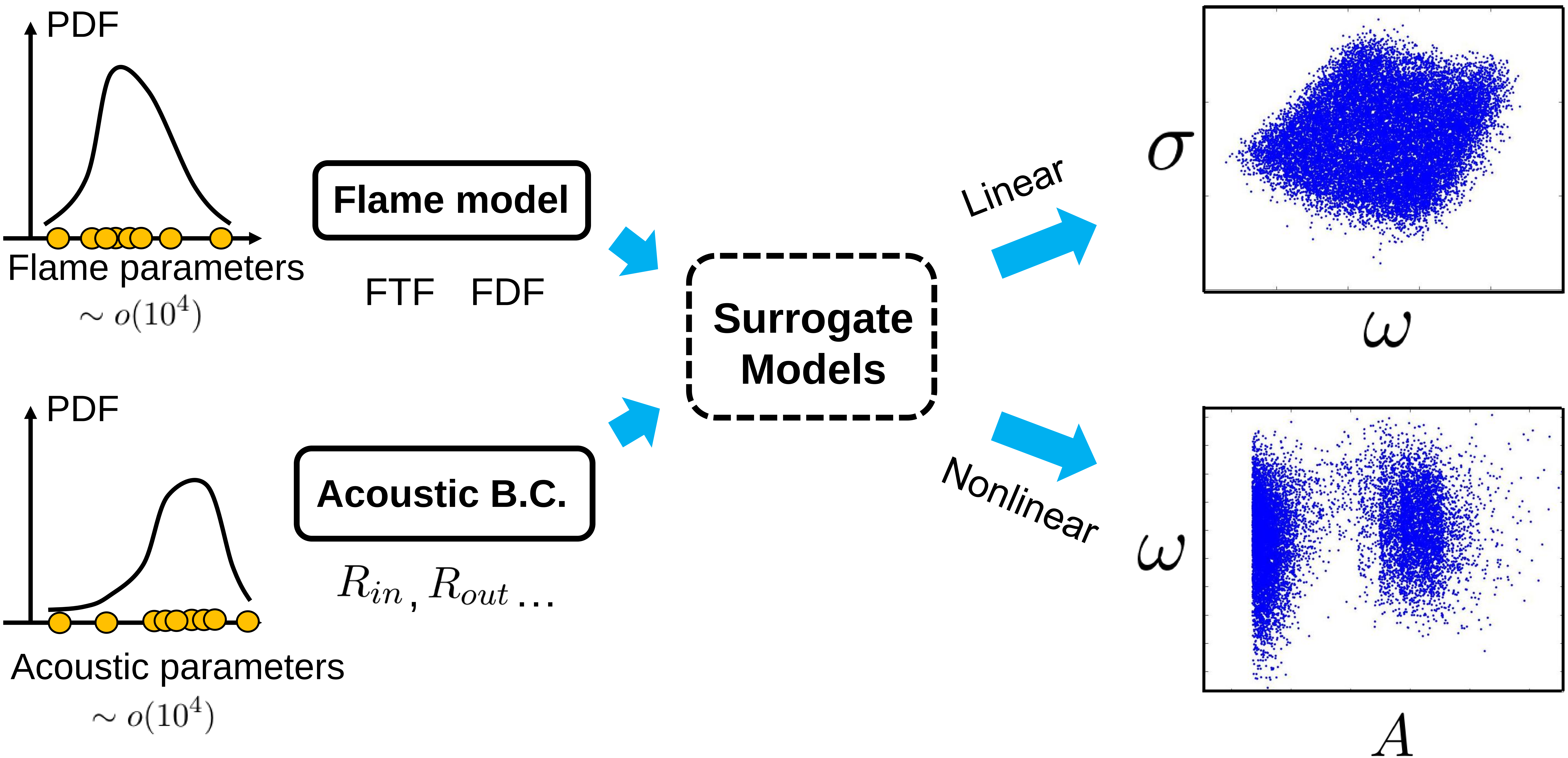
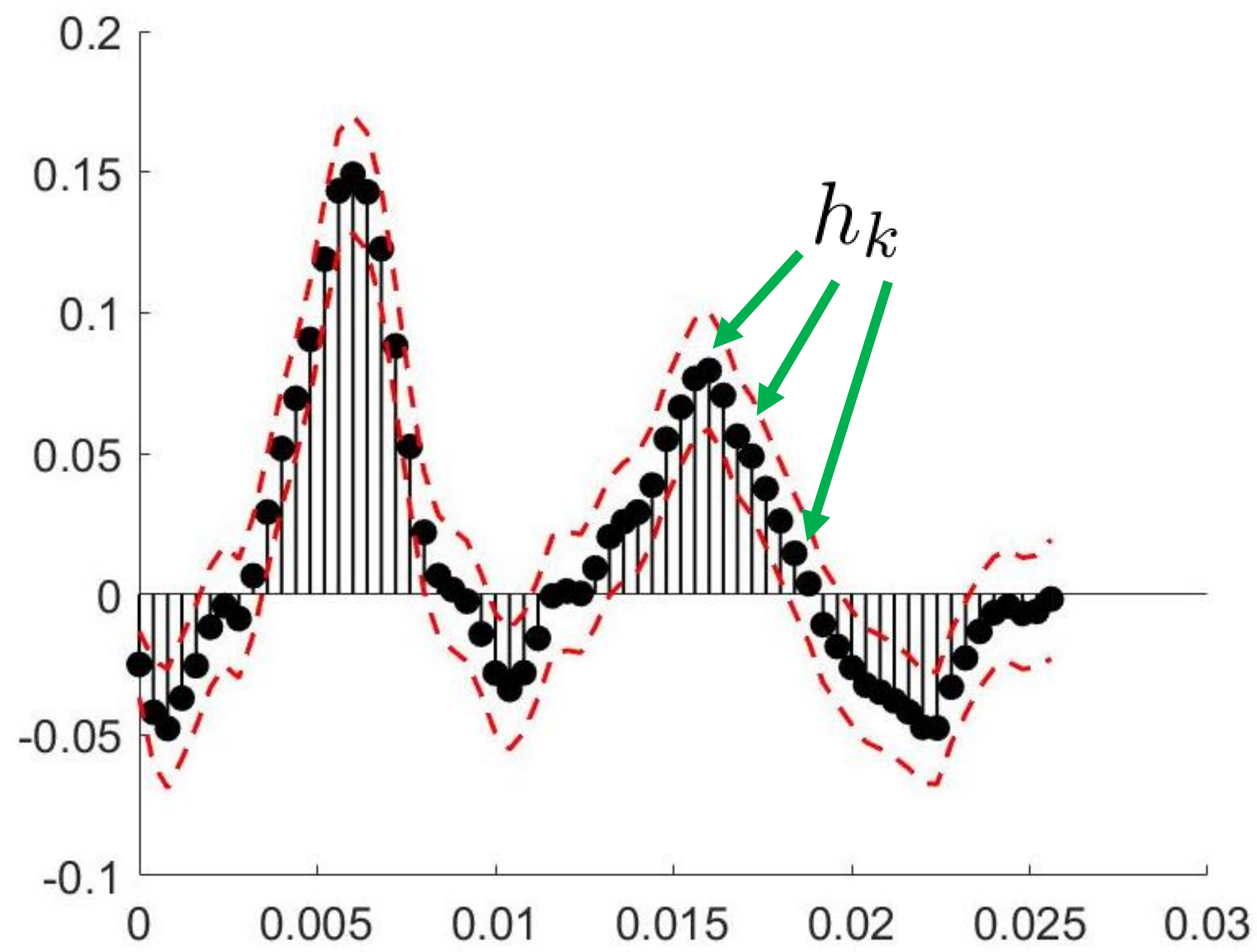


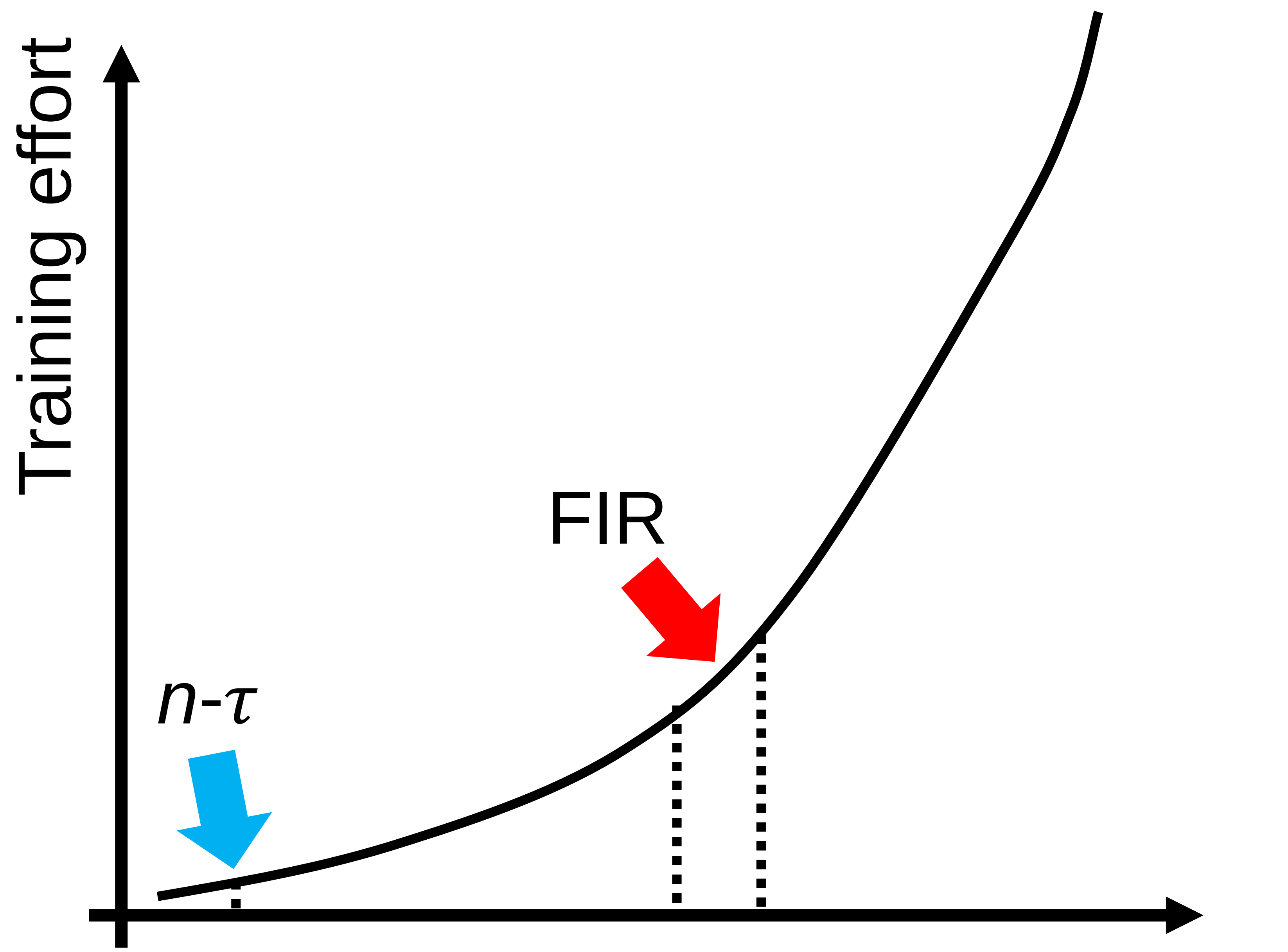
Training surrogate models becomes expensive as more flame parameters need to be considered

Flame impulse response model



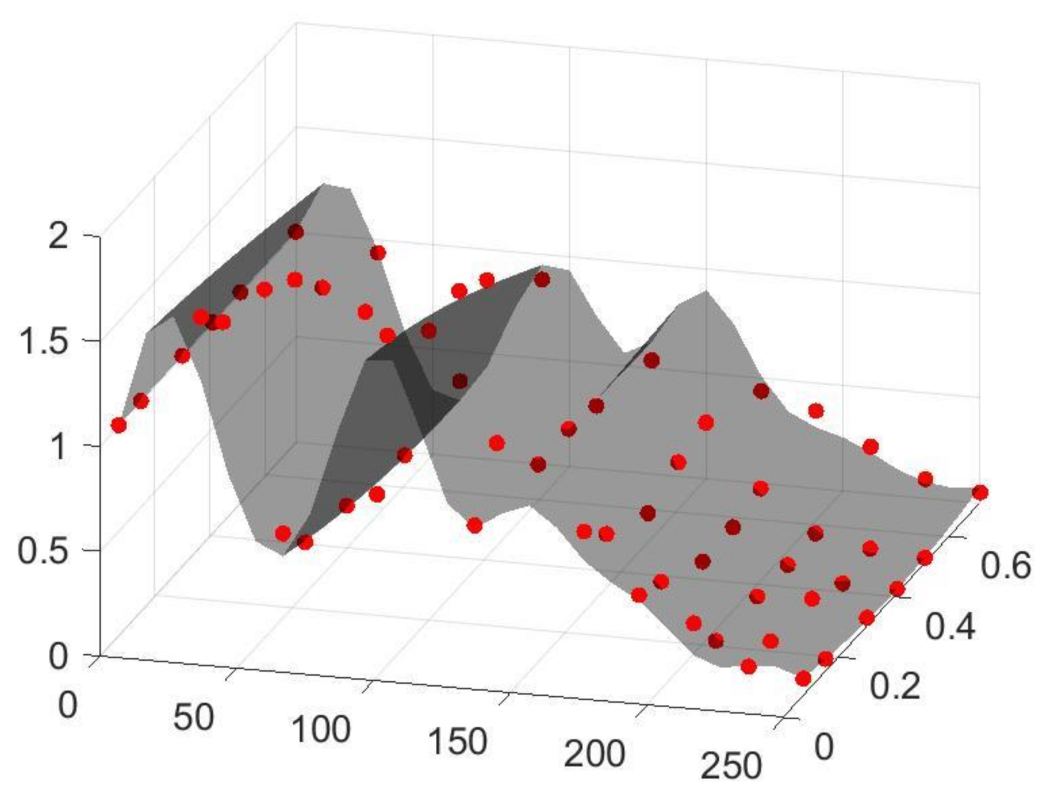
$$f^{(\omega)}(h_1, ..., h_N; \text{acoustic}) \approx \omega$$

$$f^{(\sigma)}(h_1, ..., h_N; \text{acoustic}) \approx \sigma$$

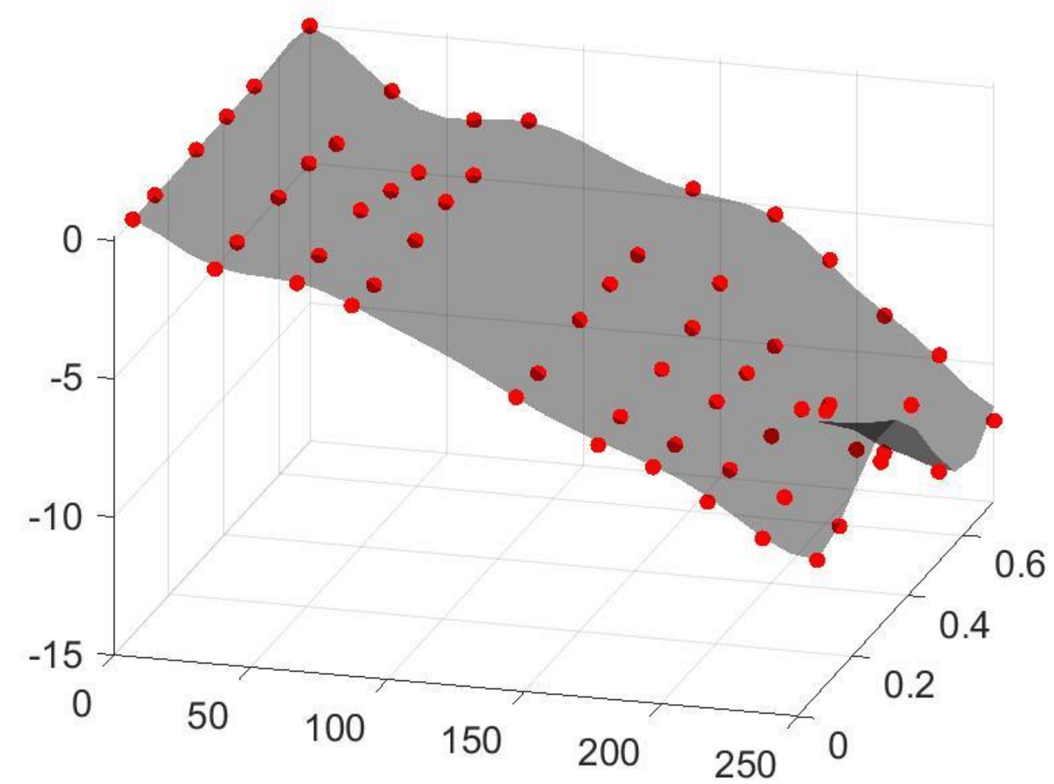


Training surrogate models becomes expensive as more flame parameters need to be considered

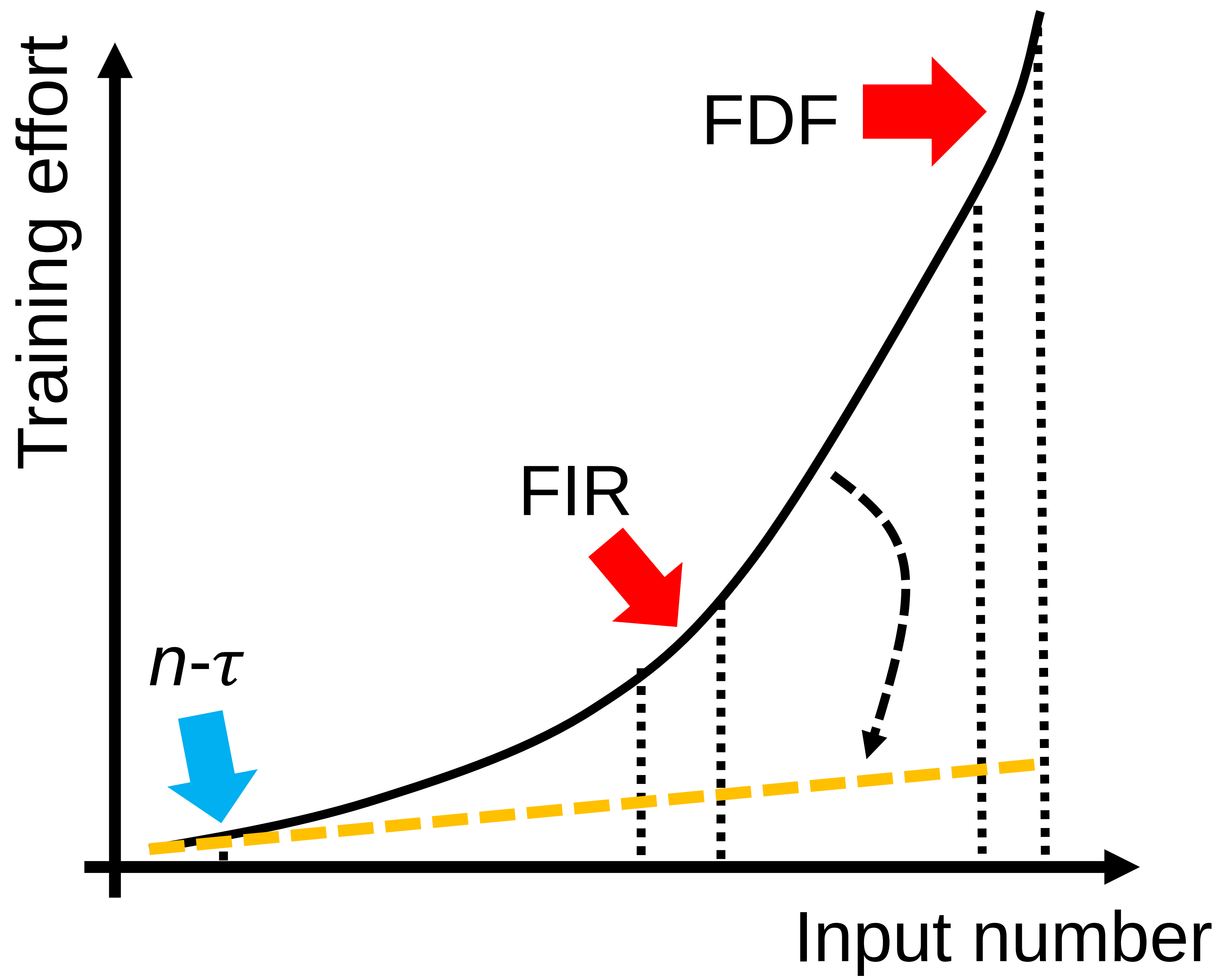
Flame describing function



Gain



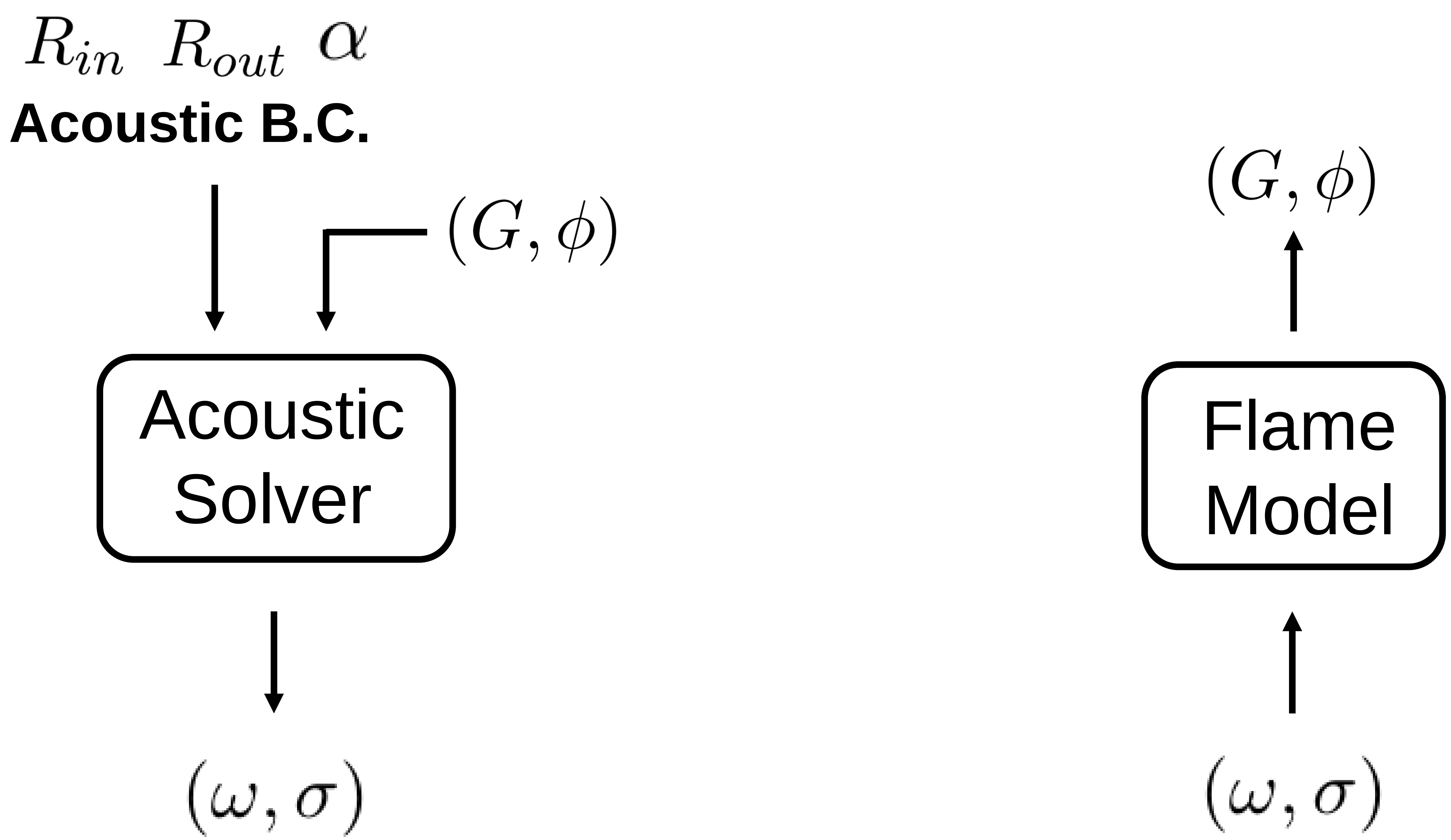
Phase



Presentation Overview

- Motivation
- A surrogate-based scheme for thermoacoustic UQ
- Gaussian Process
- Case studies
  - Thermoacoustic framework
  - Linear case: FIR uncertainty
  - Nonlinear case: FDF uncertainty

Thermoacoustic eigenmode calculations require a coupling between a flame model and an acoustic solver



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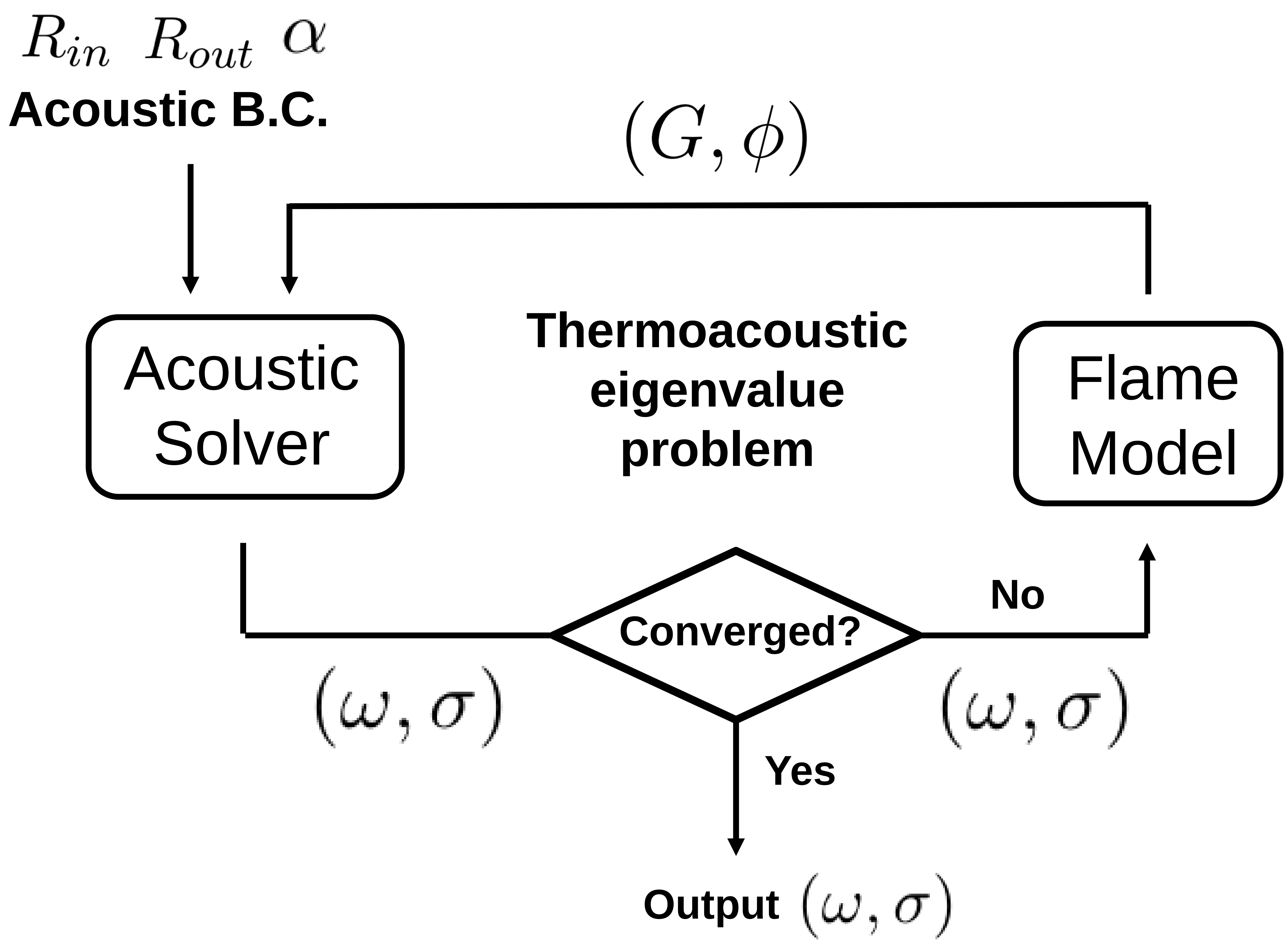

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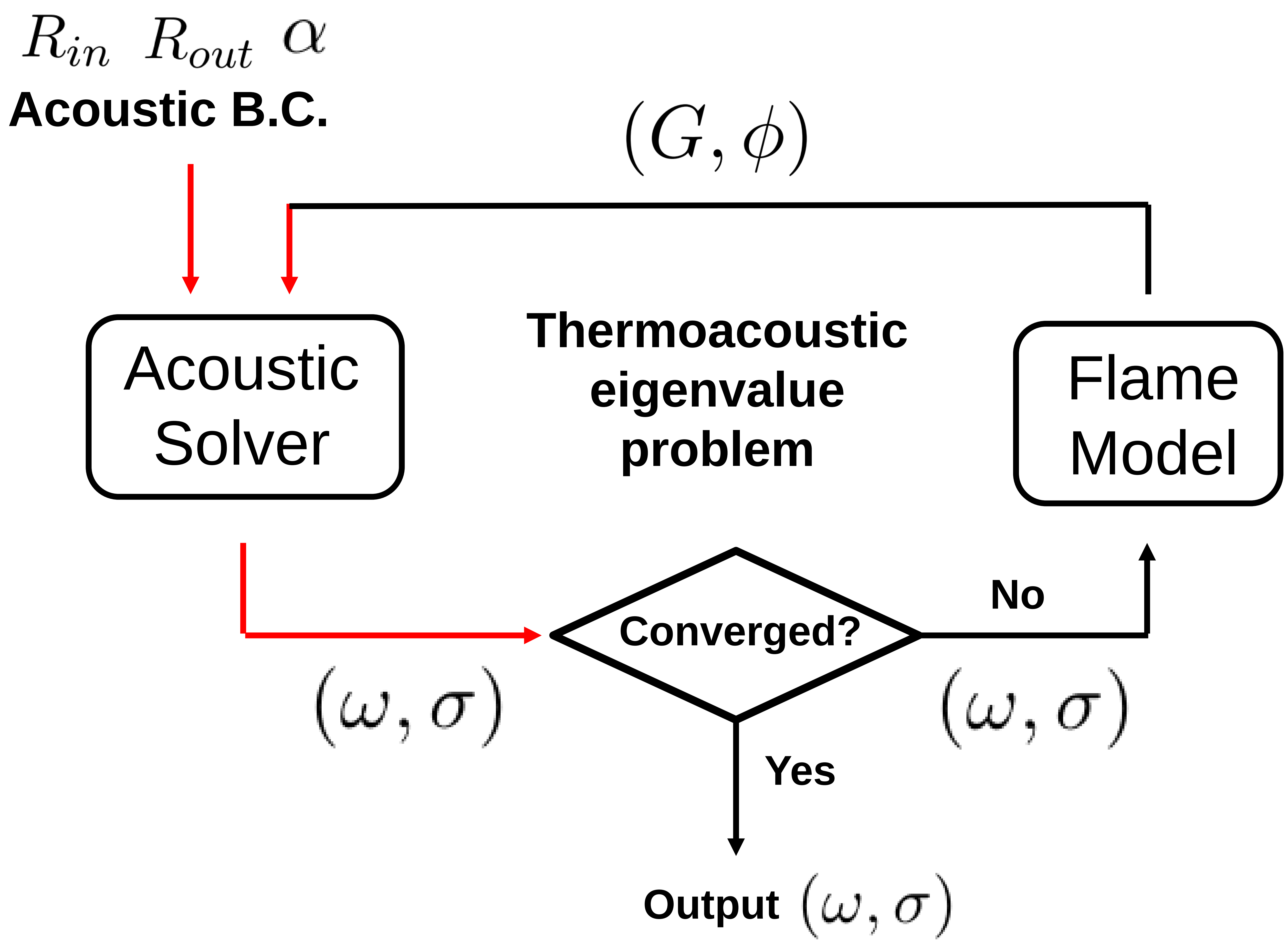
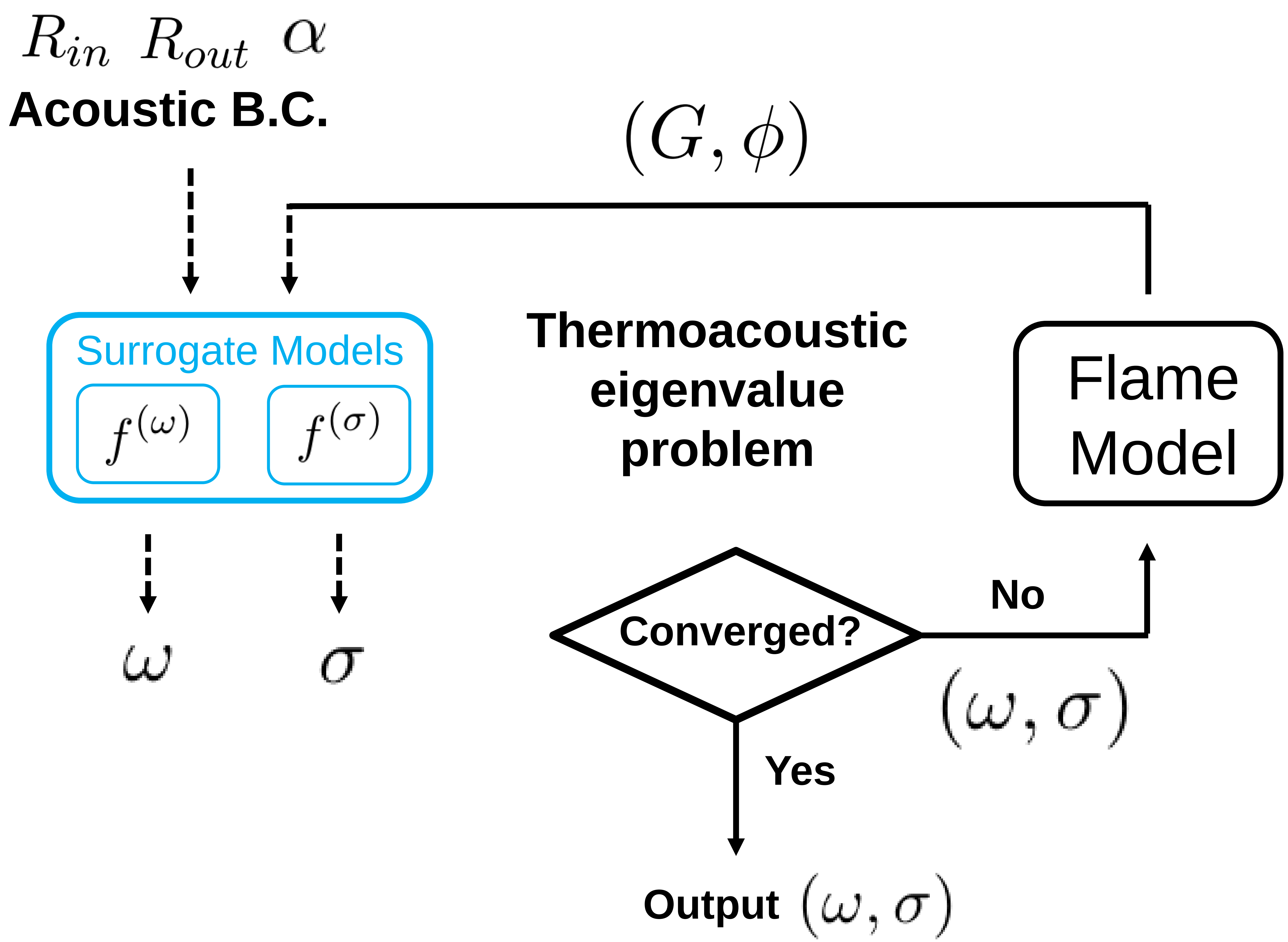


Surrogate models can be built to approximate acoustic solver to improve the efficiency



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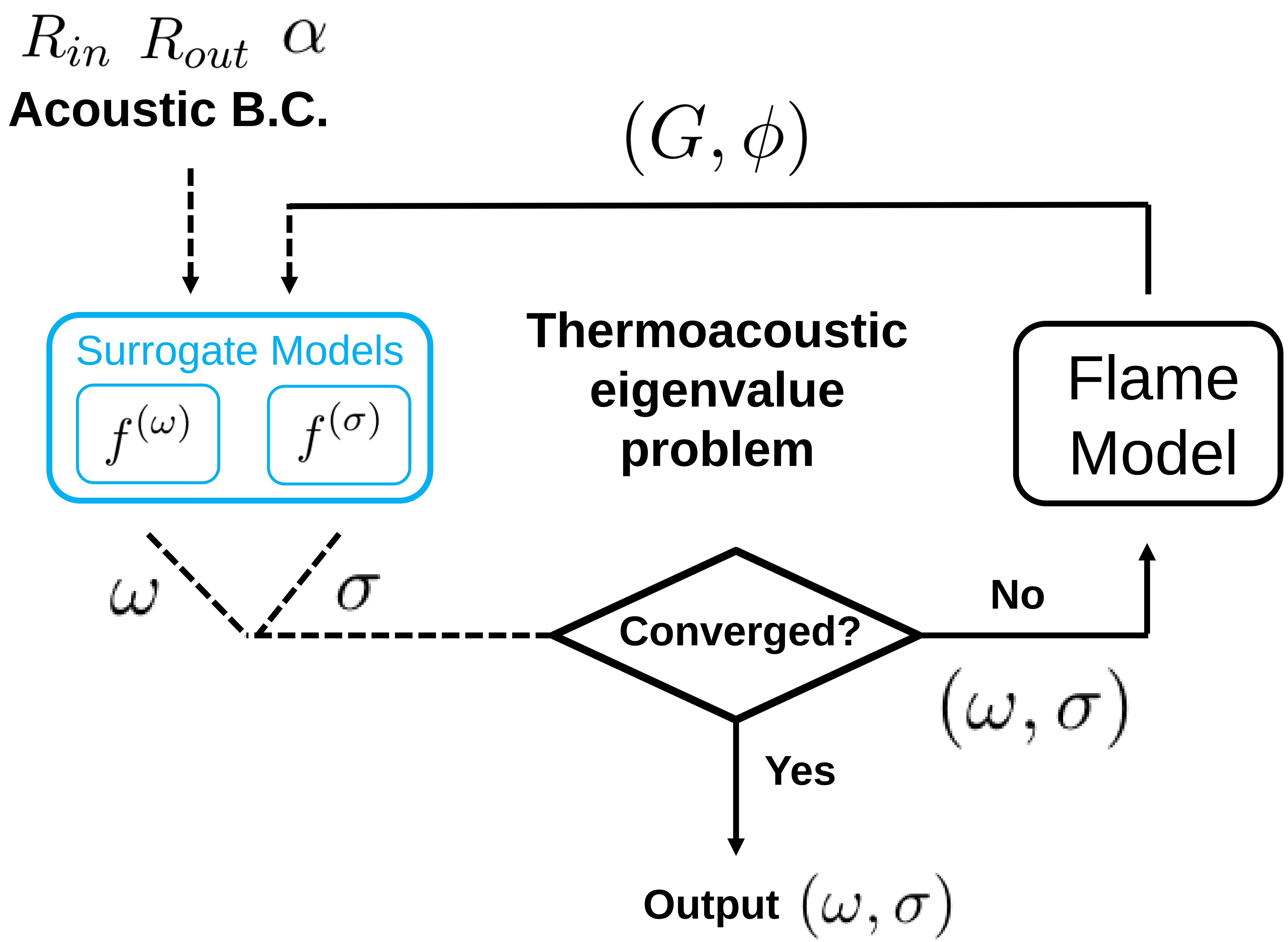

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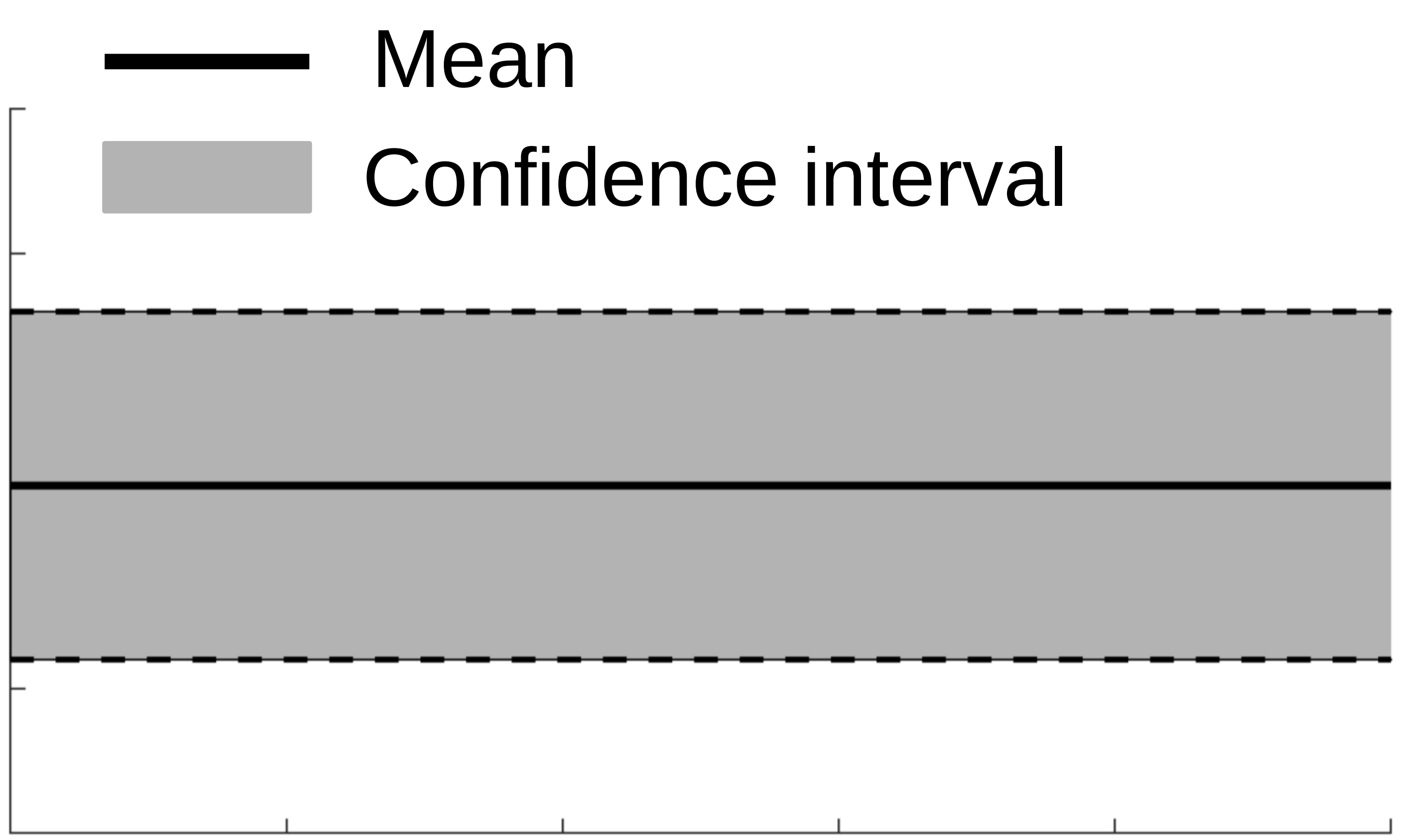
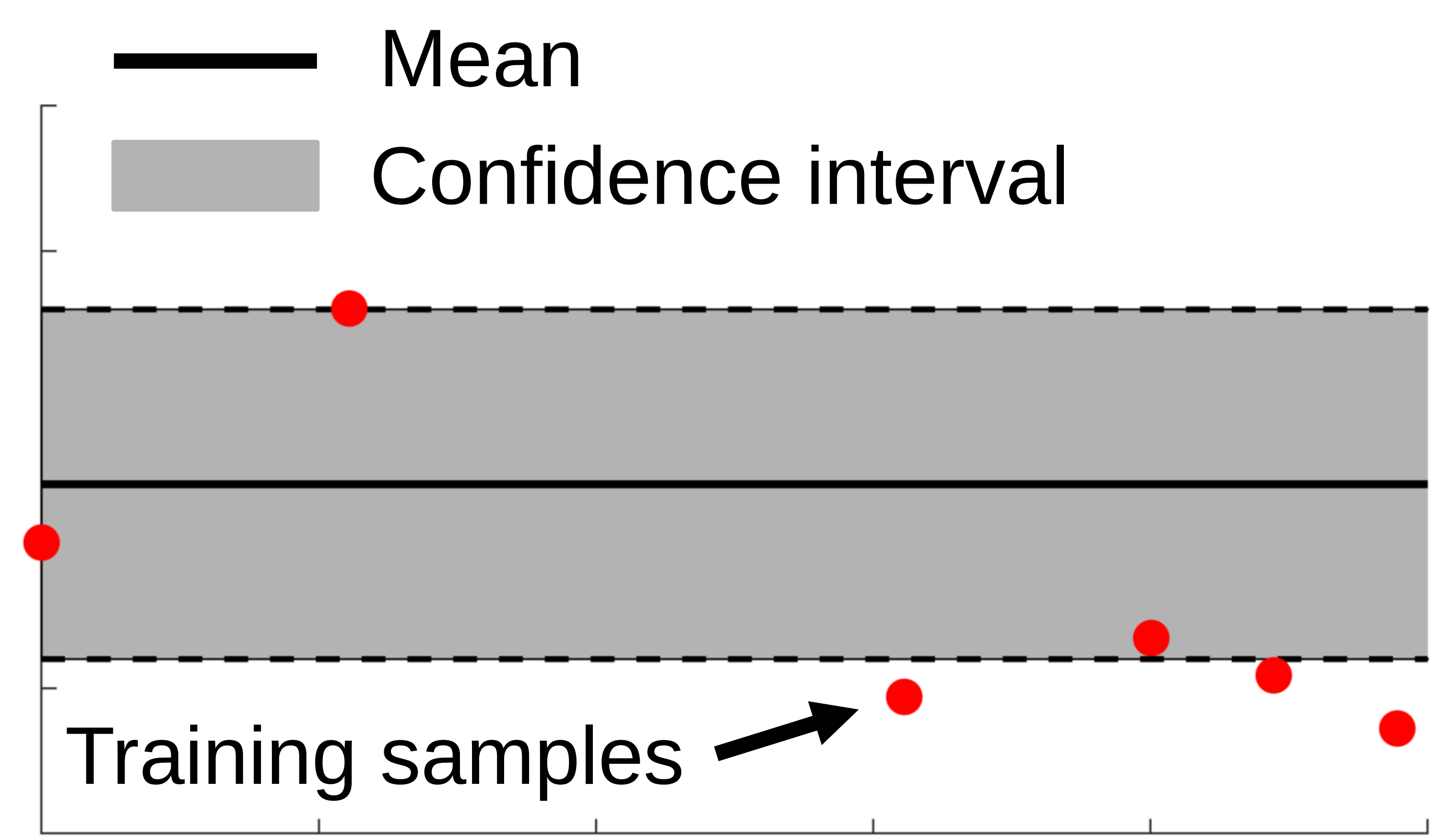
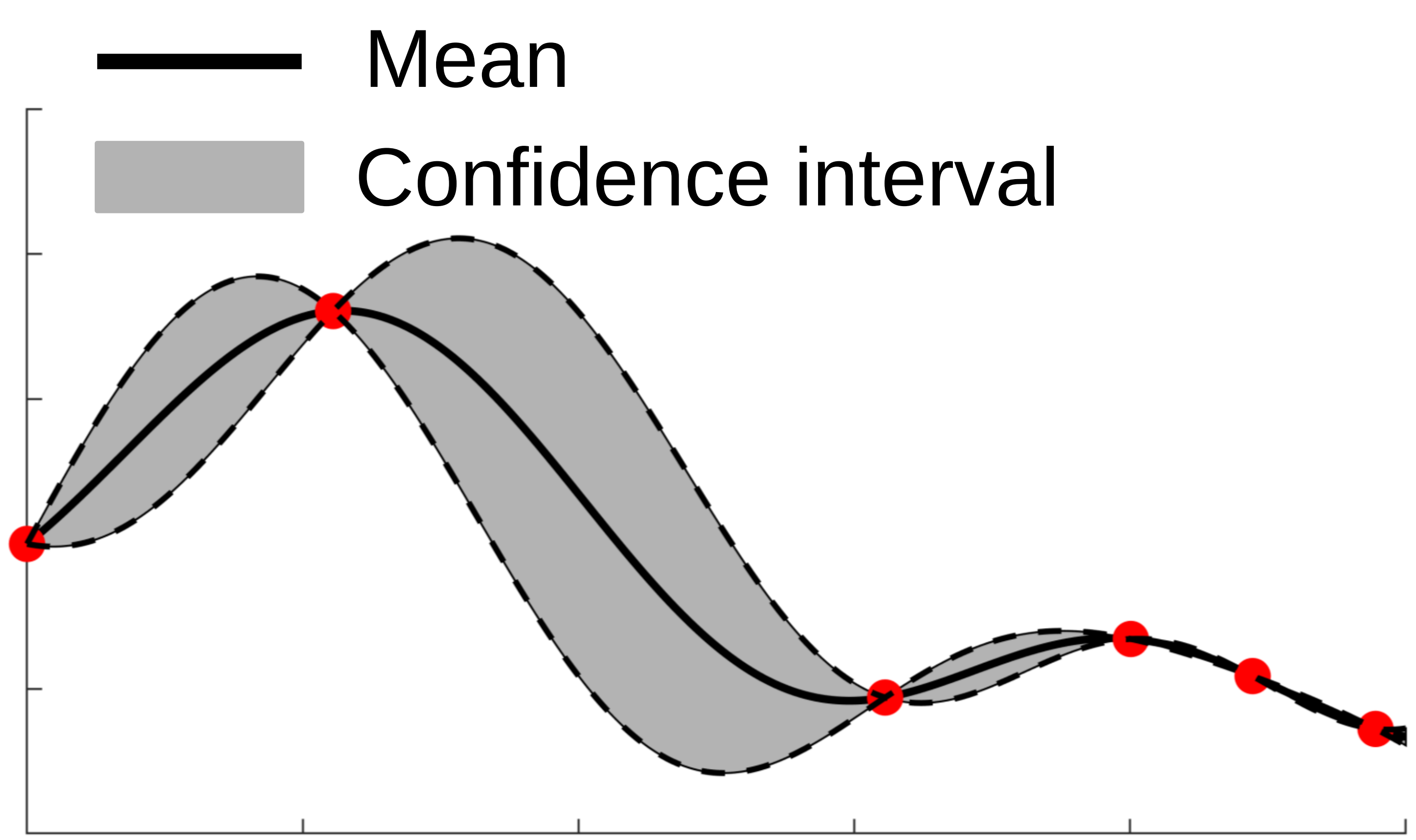
Gaussian process not only yields prediction, but also estimates prediction uncertainty







Prior **Data** **Posterior**

$$f(x) \sim \mathcal{GP}(\beta, k(x, x')) \quad \Longrightarrow \quad \hat{f}(x) \sim \mathcal{N}(\mu(x), \sigma^2(x))$$

β : Constant

$k(x, x') = \sigma^2 \text{exp}(-\theta|x - x'|^2)$: Kernel

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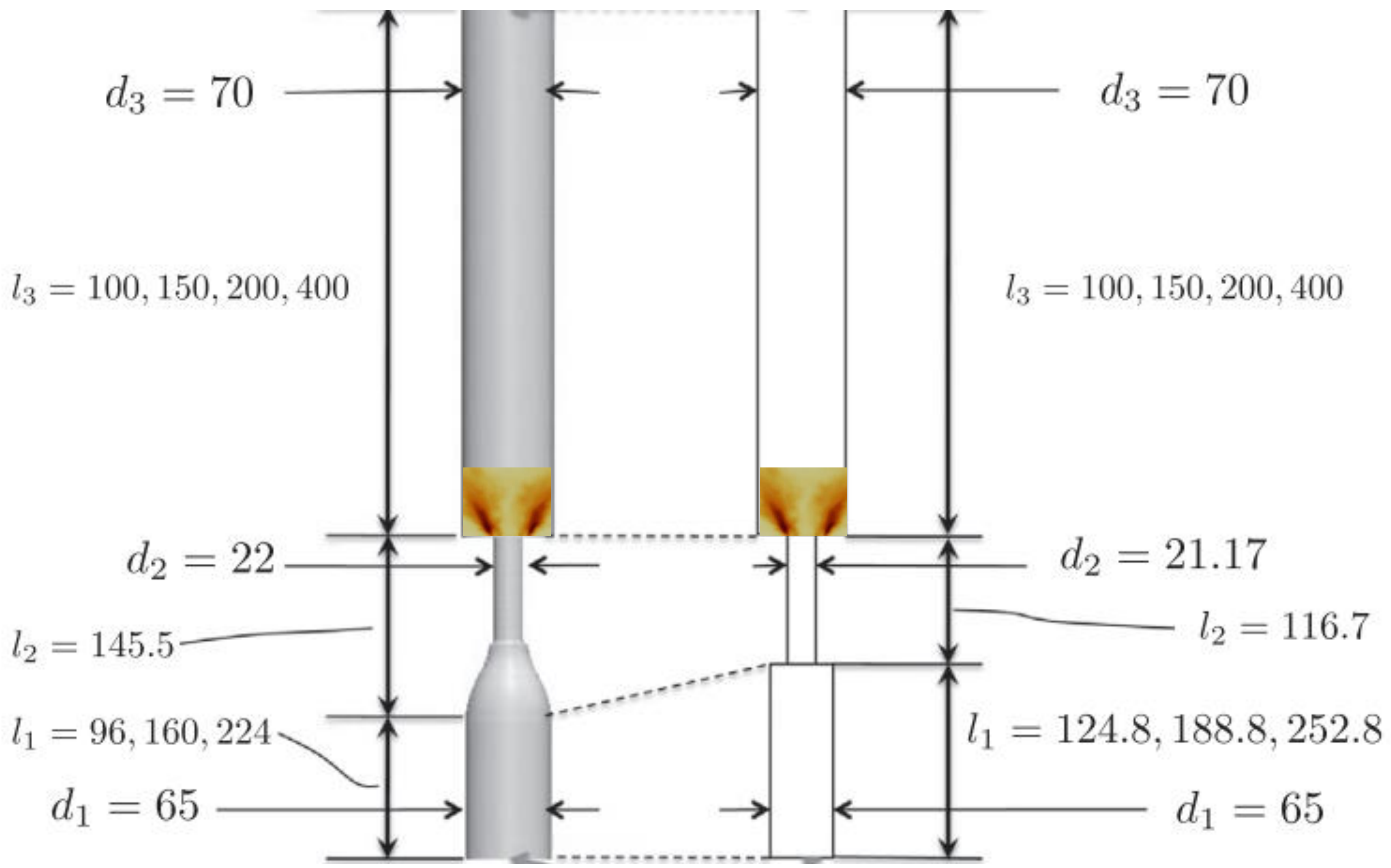
Thermoacoustic problem settings



- Configuration: EM2C C11[1]
- Helmholtz solver
- First acoustic mode

[1] C. F. Silva et al. Combustion and Flame, 2013.

We train two separate GP models to approximate modal frequency and growth rate

Goal

$$\omega \approx \mathsf{GP}^{(\omega)}(G, \phi, R_{in}, R_{out}, \alpha)$$

$$\sigma \approx \mathsf{GP}^{(\sigma)}(G, \phi, R_{in}, R_{out}, \alpha)$$

⇦ **150 samples**

Range

Parameter	Range
G	$0.5 \sim 3$
ϕ	$0 \sim \pi$
$\lvert R_{in} \rvert$	$0.7 \sim 1$
$\lvert R_{out} \rvert$	$0.6 \sim 1$
α[1]	$100 \sim 160$

[1] C. F. Silva et al. Combustion and Flame, 2013.

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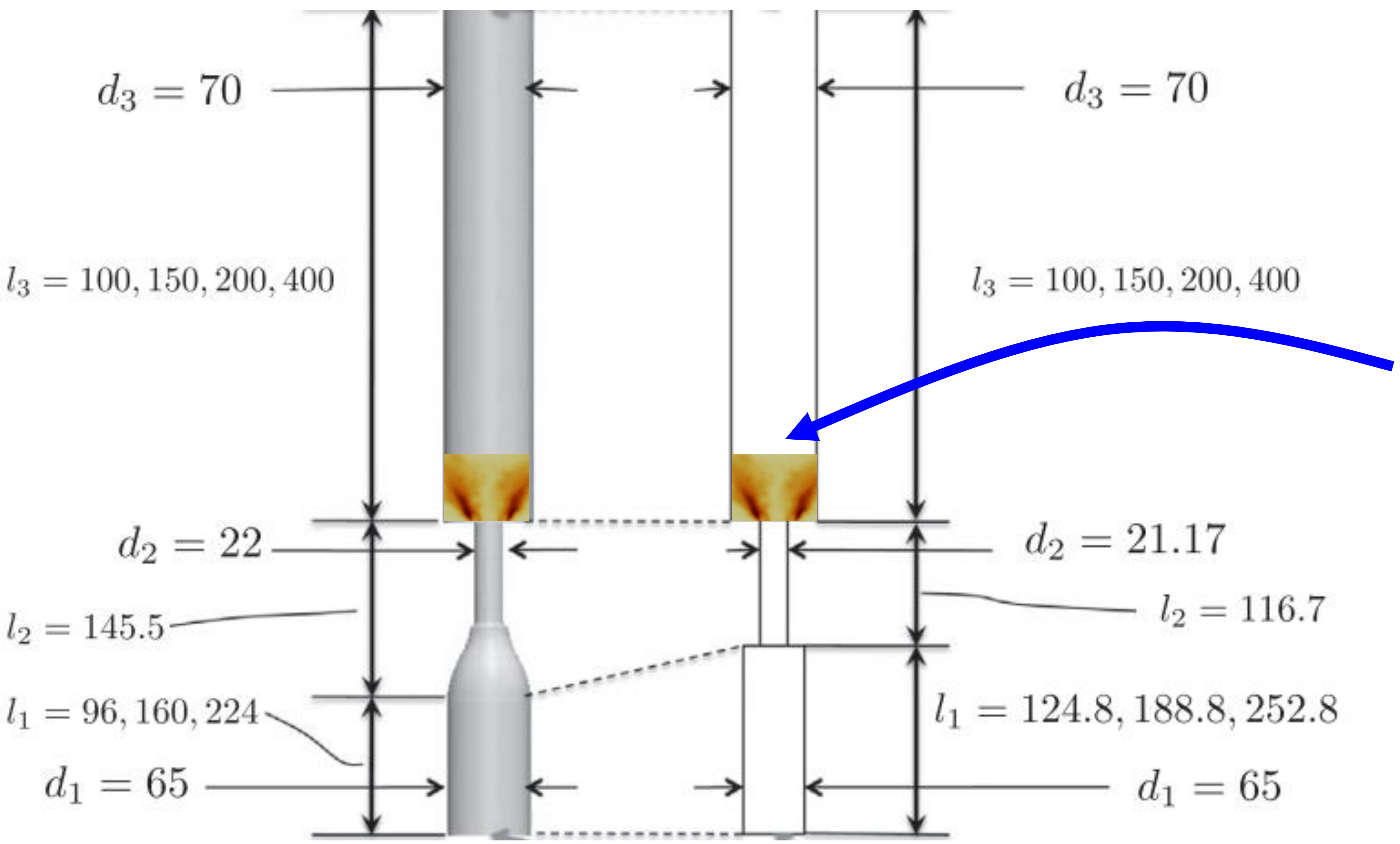
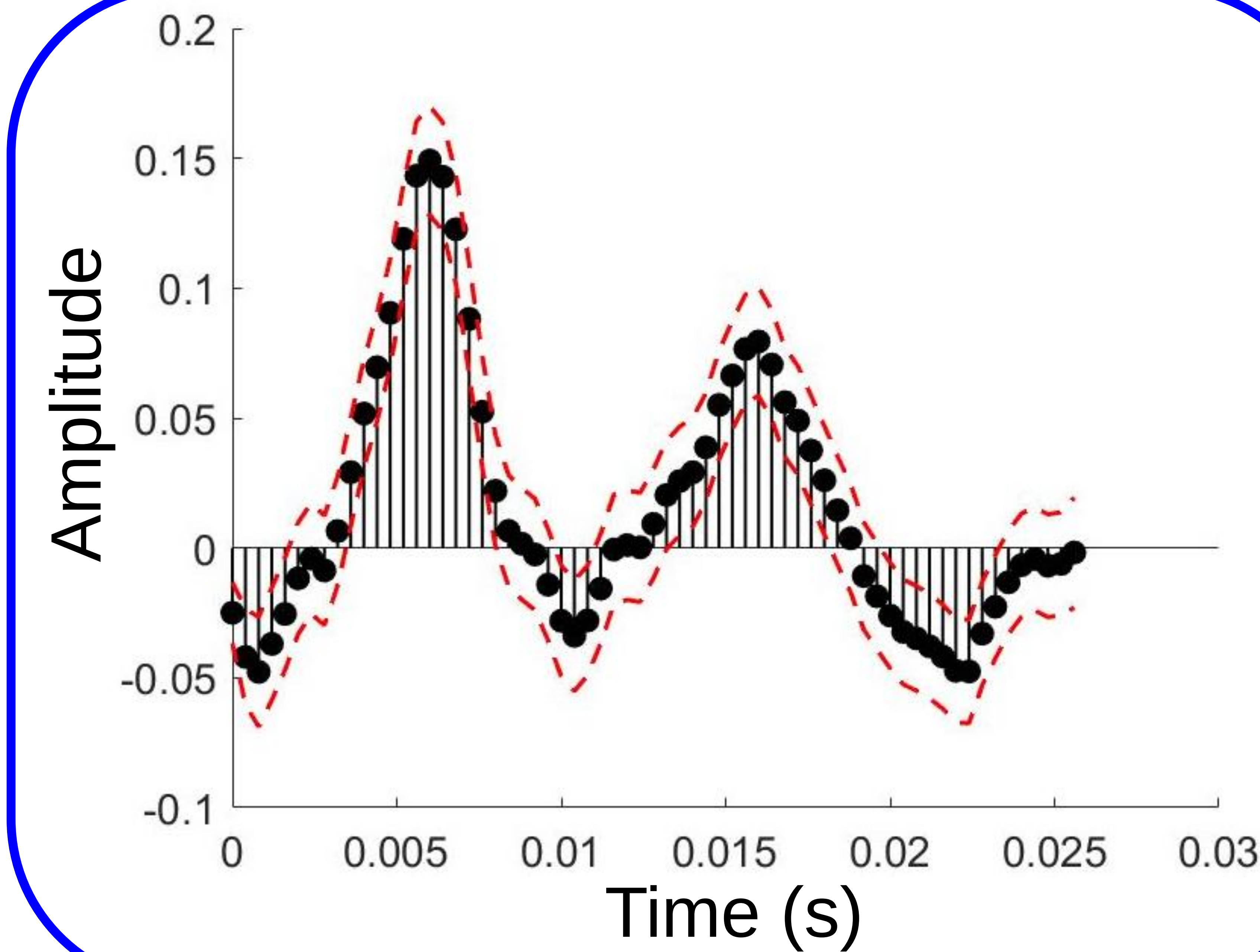
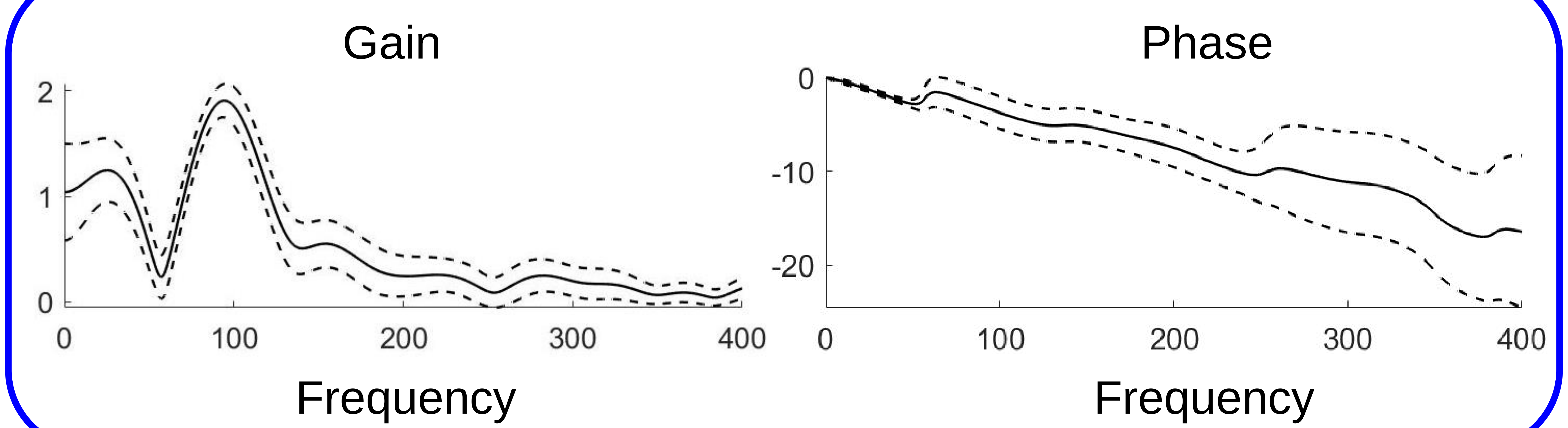
The linear case study involves an uncertain FIR model

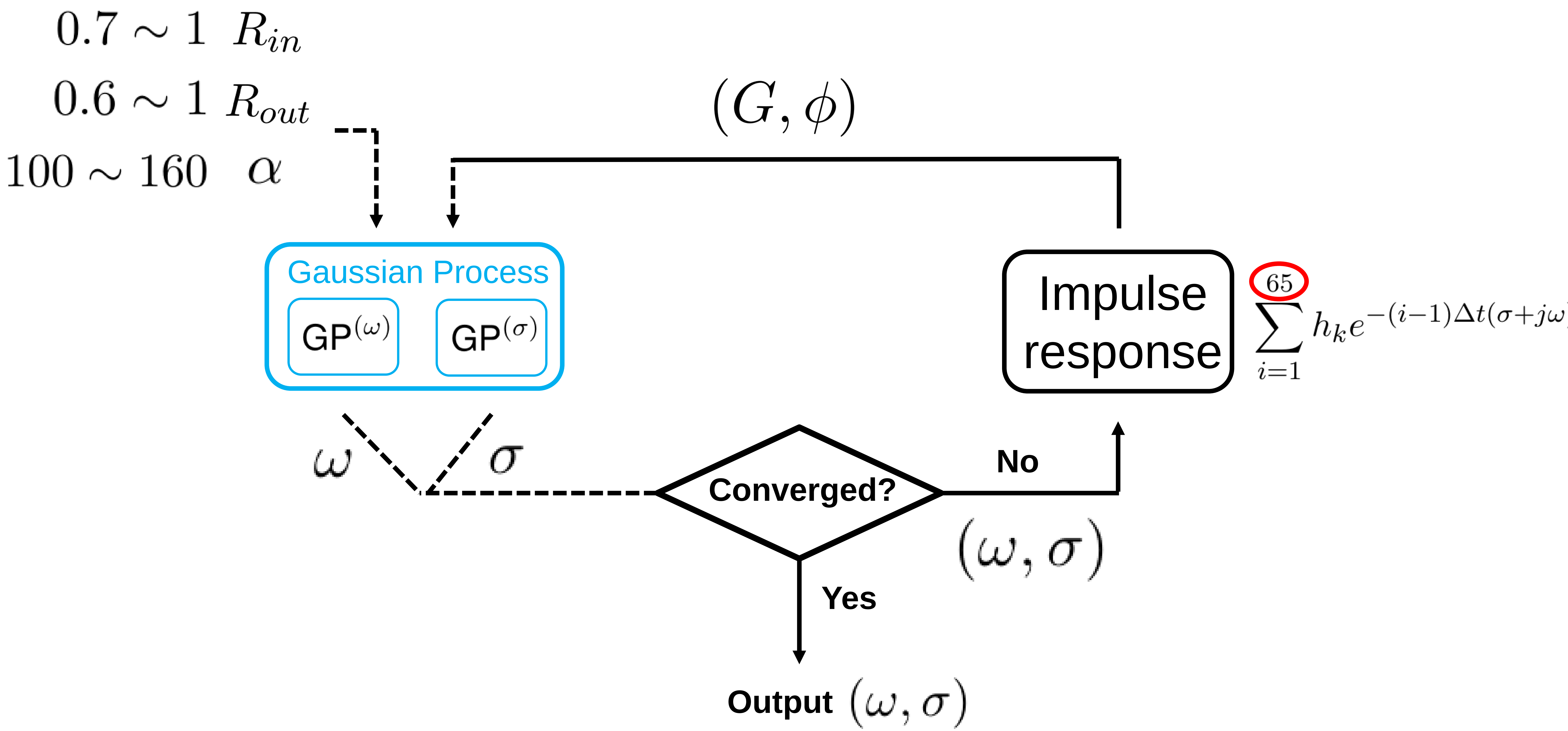




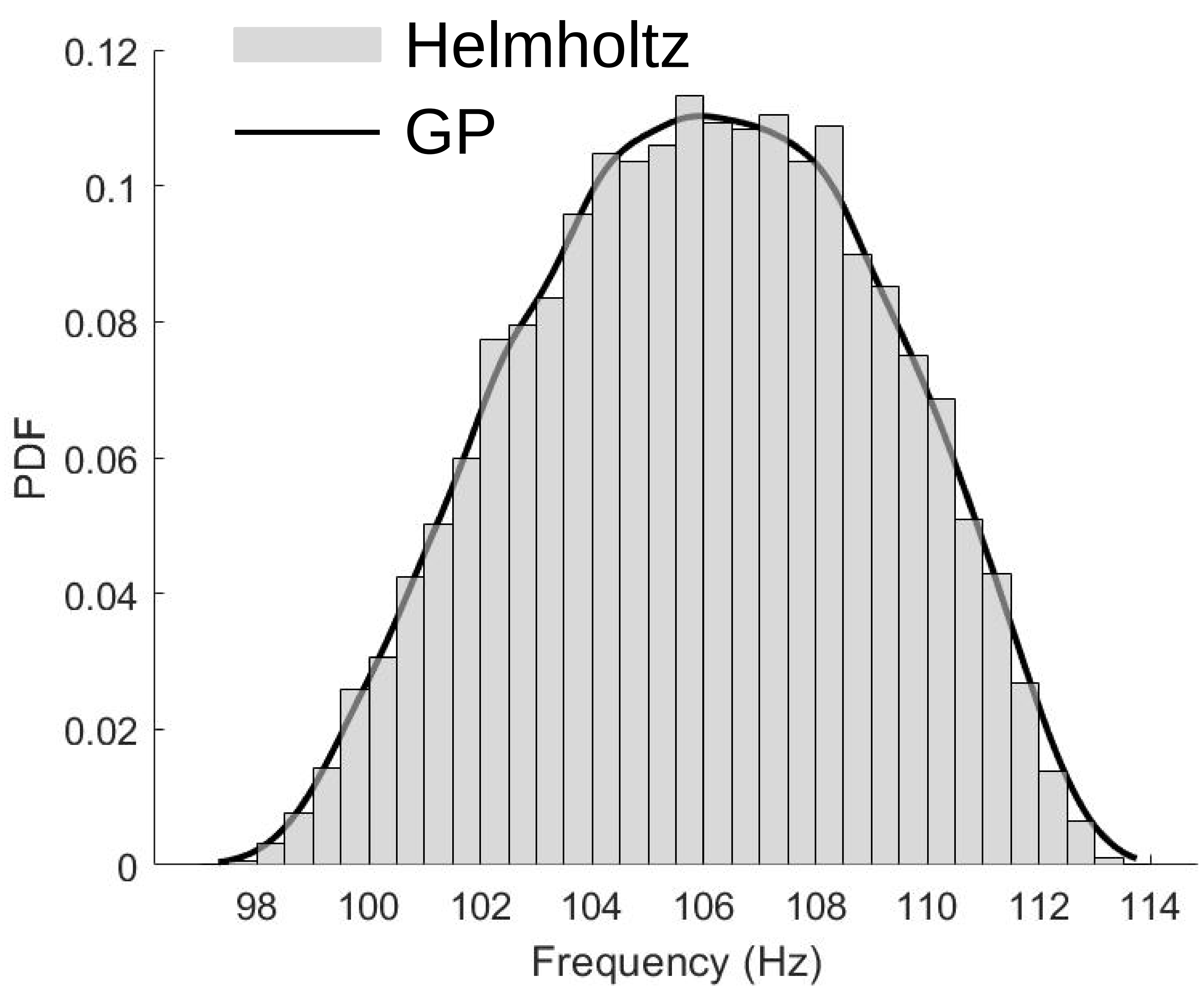
- ☑ Configuration: EM2C C11
- ☑ Helmholtz solver
- ☑ First acoustic mode

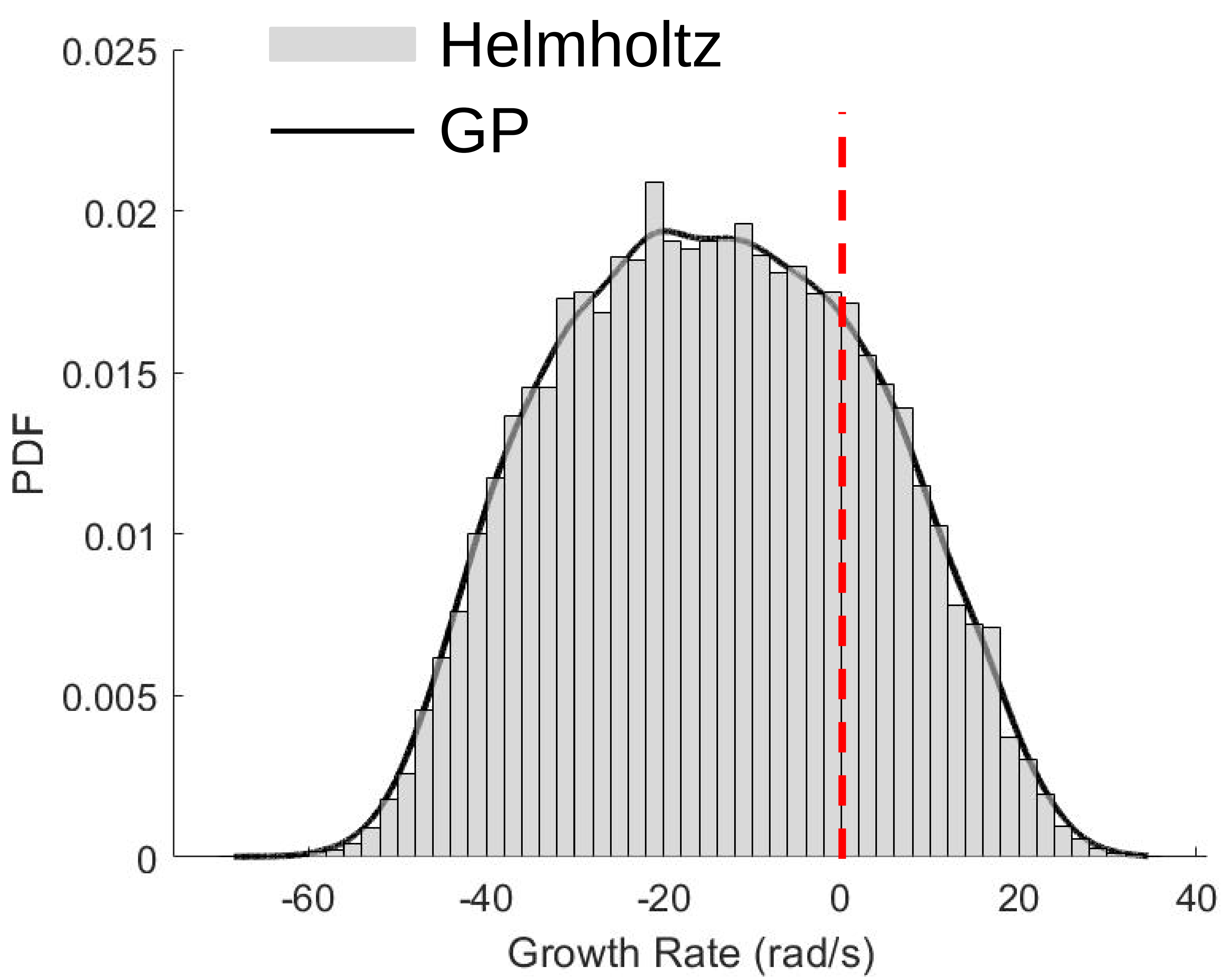


We use GP-based scheme to propagate FIR uncertainties



GP-based scheme achieved highly accurate and efficient linear thermoacoustic UQ analysis





Computation Time (single core)

GP	463 s	19.6 times faster
Helmholtz	9094 s	

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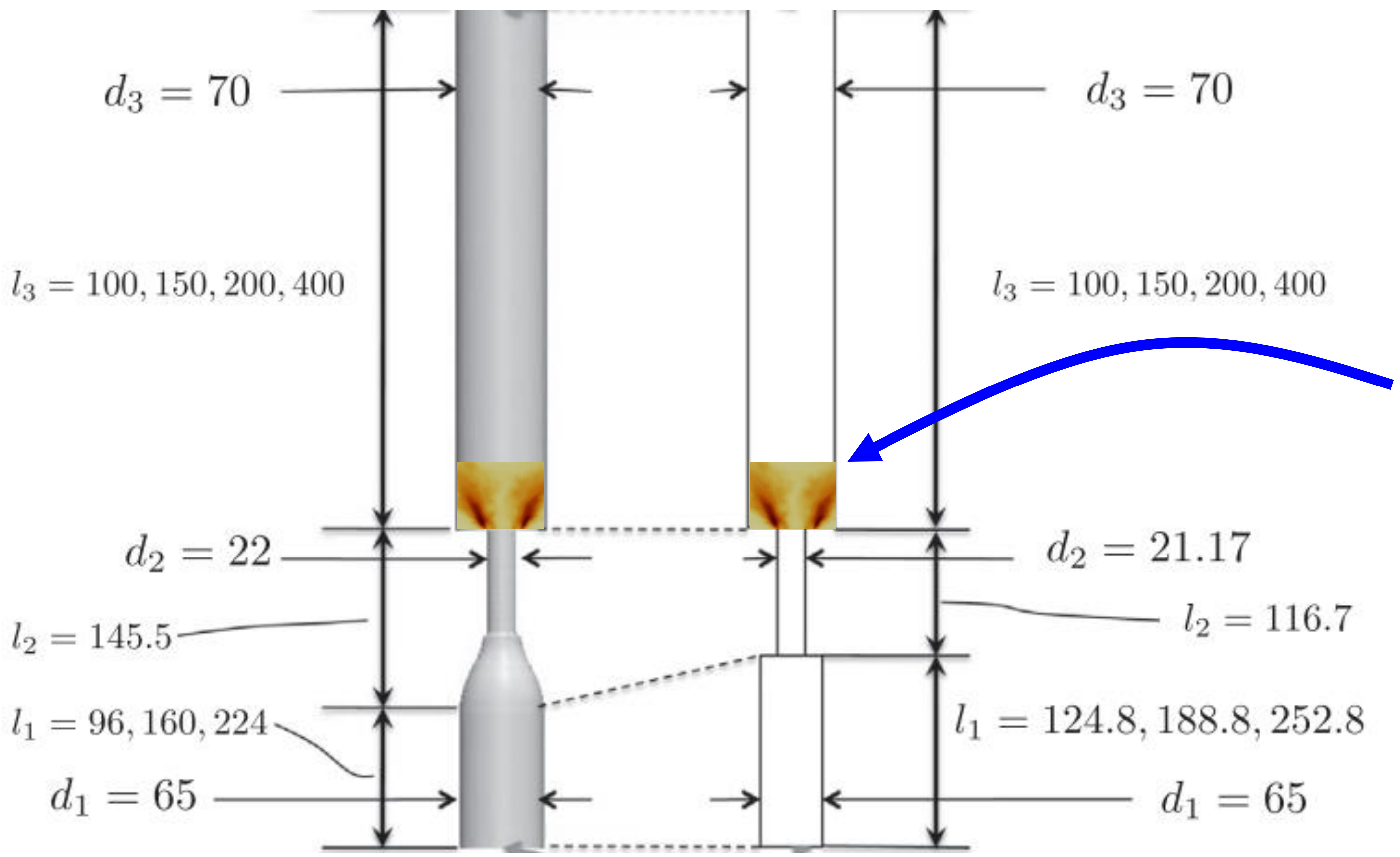
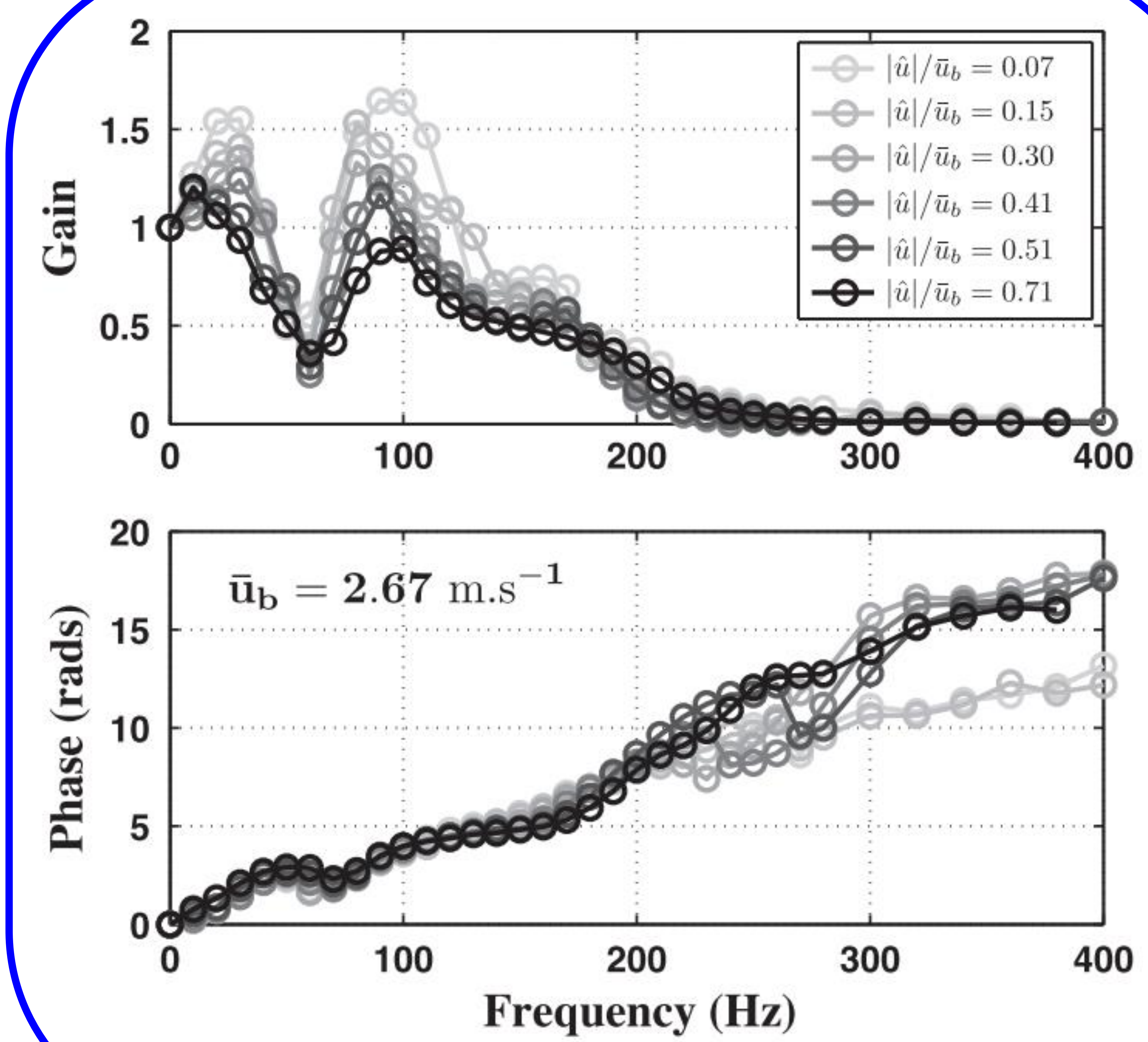
The nonlinear case study involves an uncertain FDF model

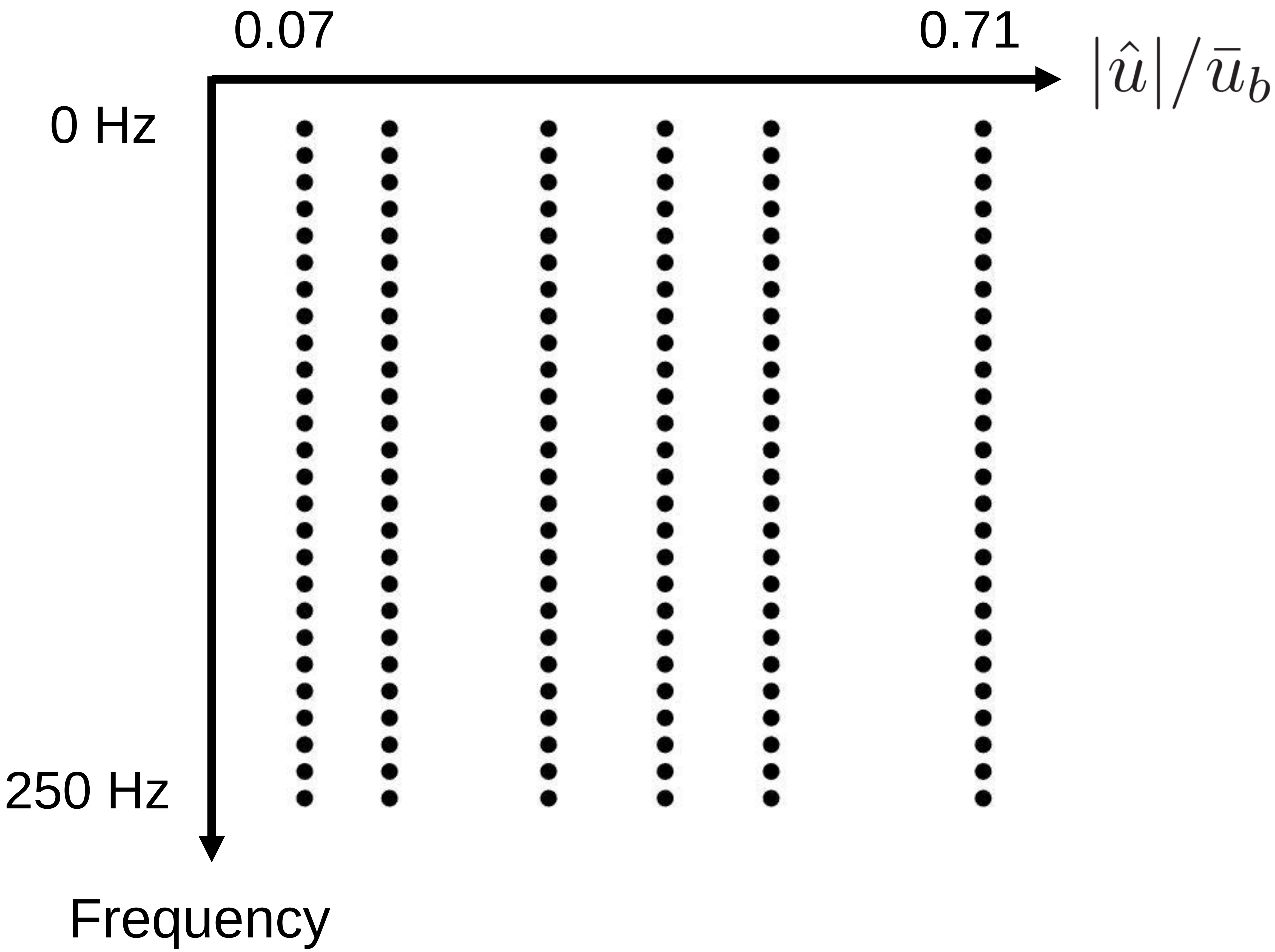




- Configuration: EM2C C11
- Helmholtz solver
- First acoustic mode

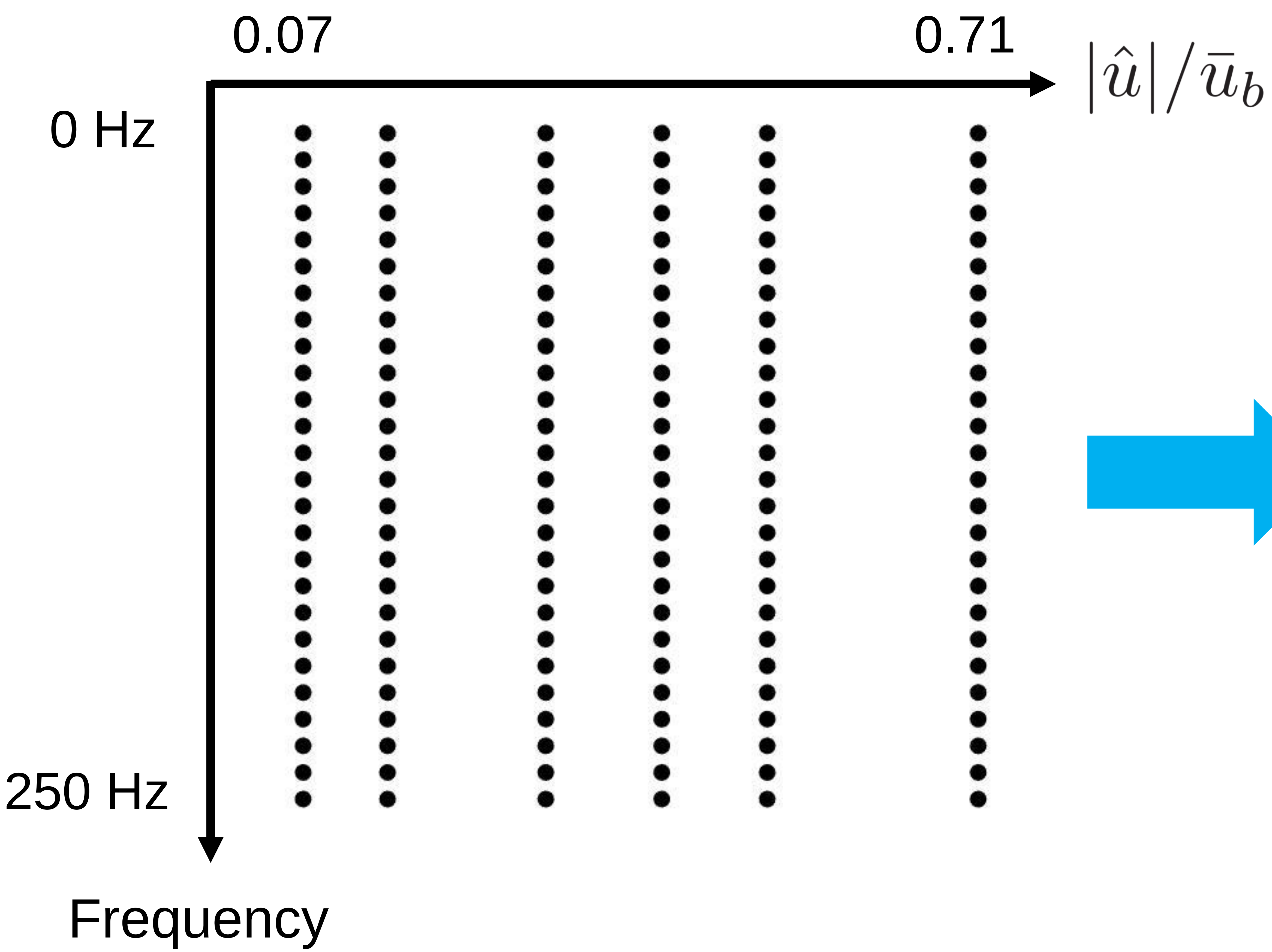
[1] P. Palies et al. Combustion and Flame, 2011.

FDF uncertainty comes from limited and noisy measurements

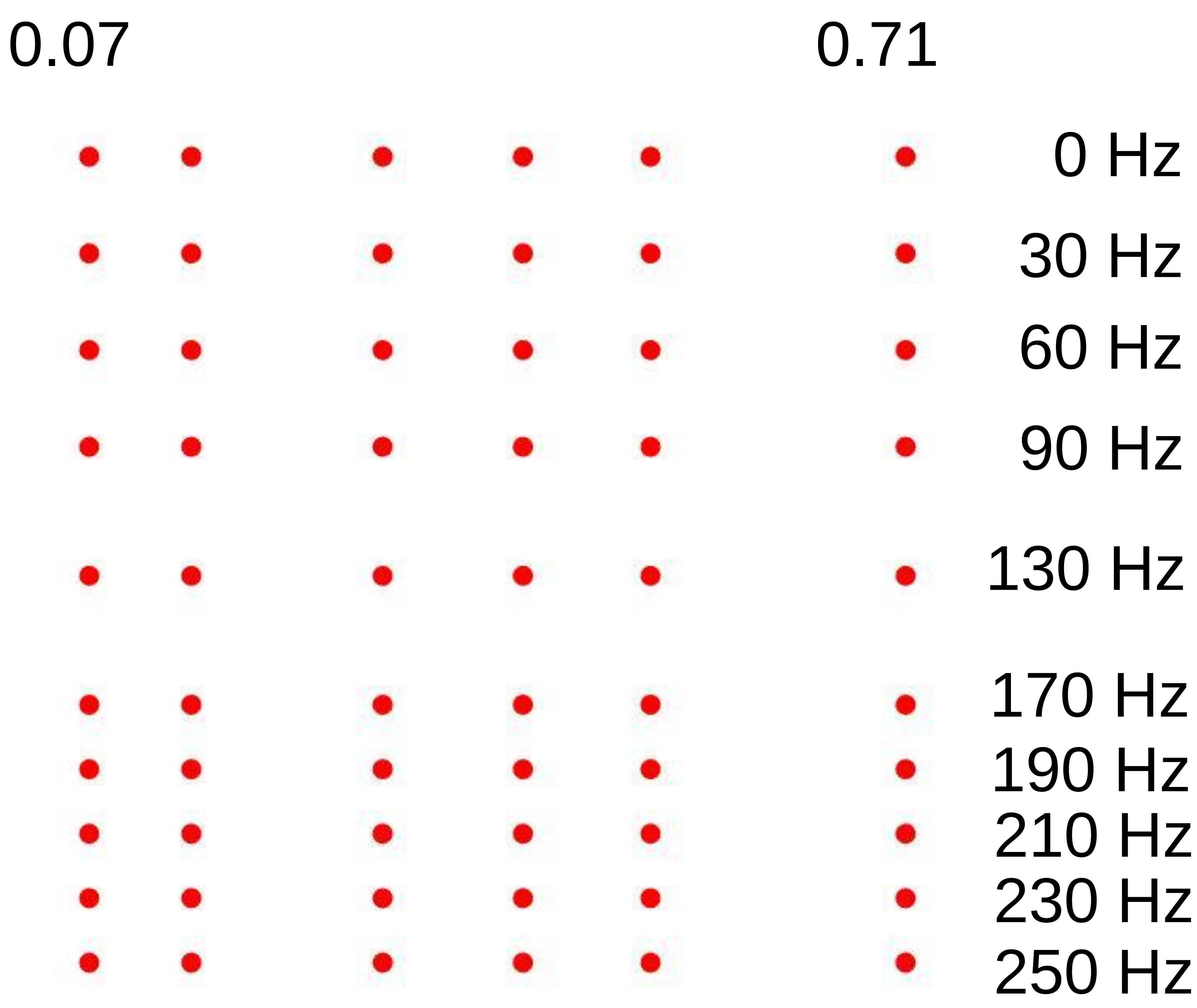


Experimental FDF data

We assume limited and noisy measurements to introduce FDF uncertainty

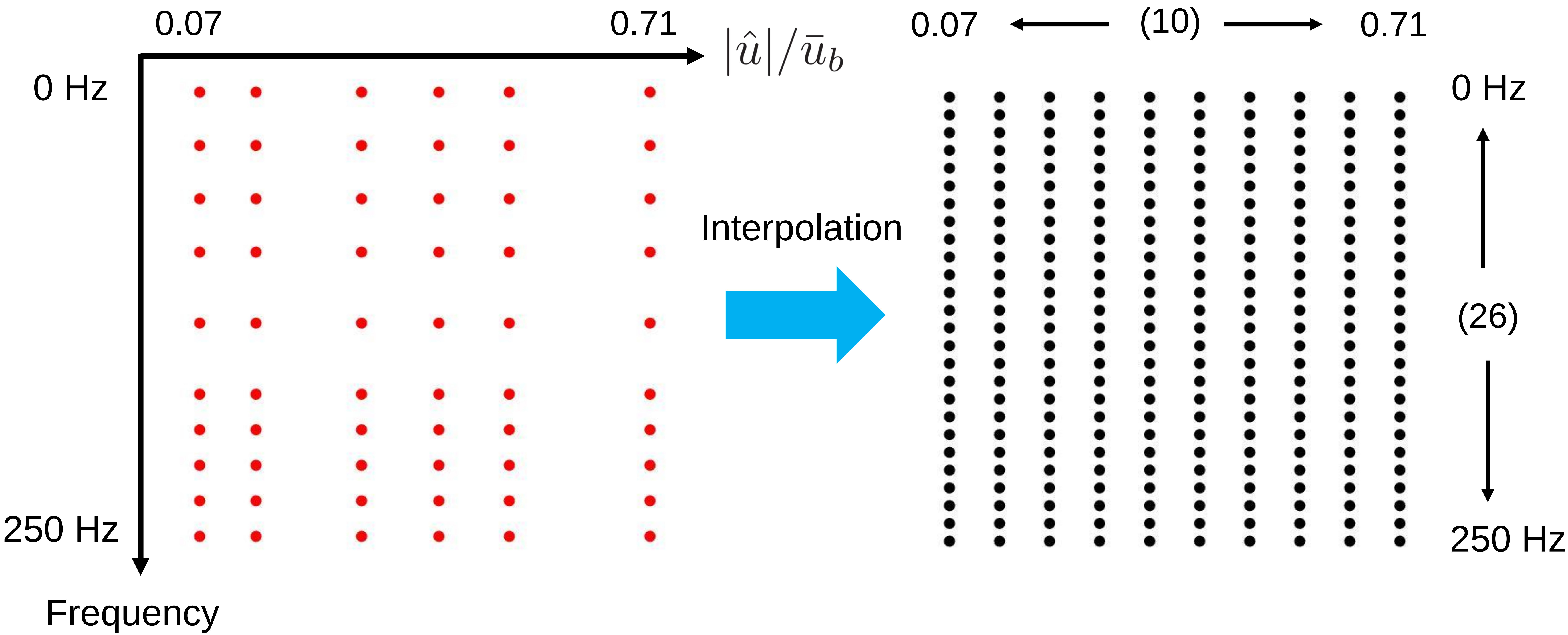


Experimental FDF data



Uncertain FDF data

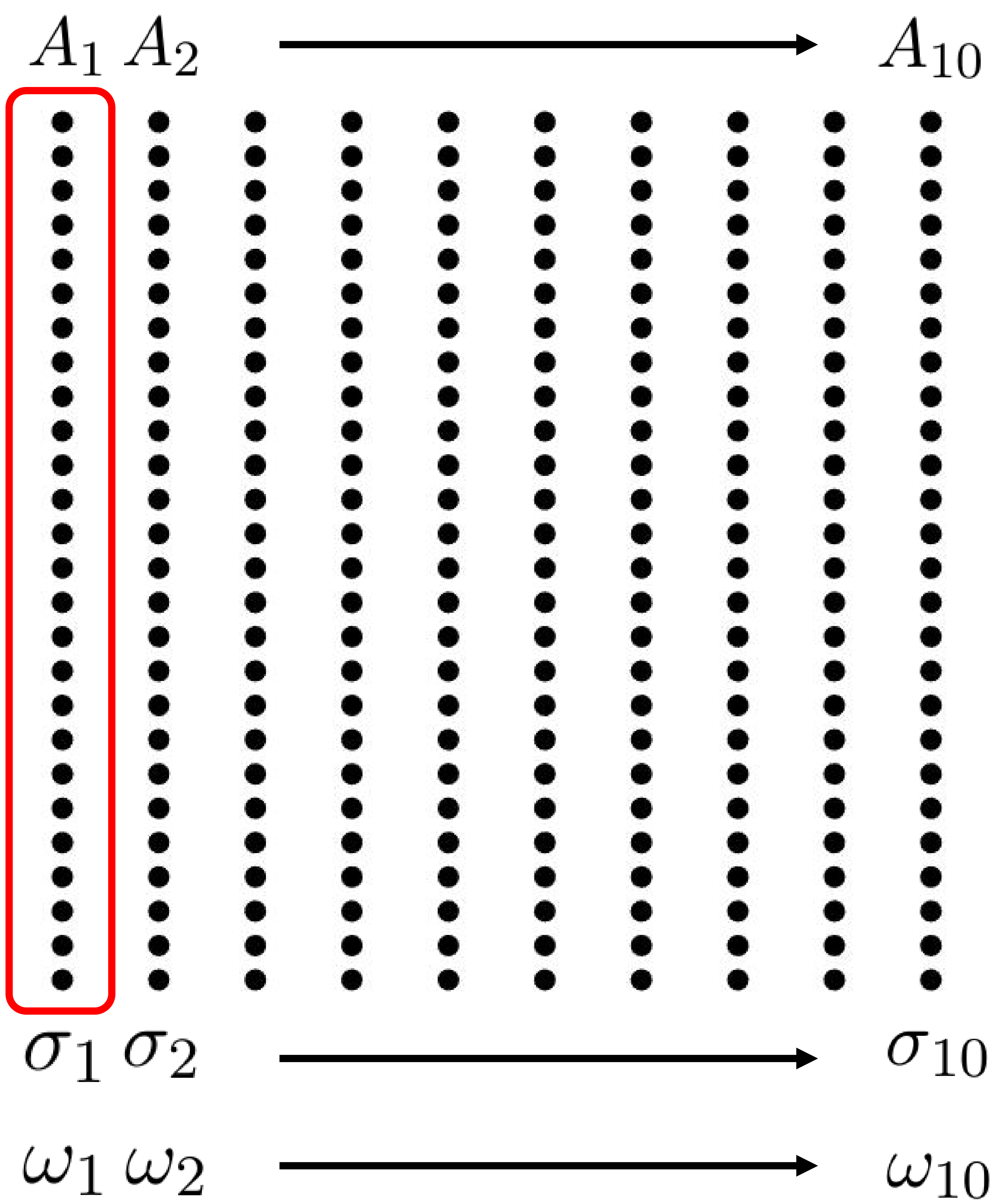
We interpolate FDF data to a finer grid to facilitate limit cycle calculation



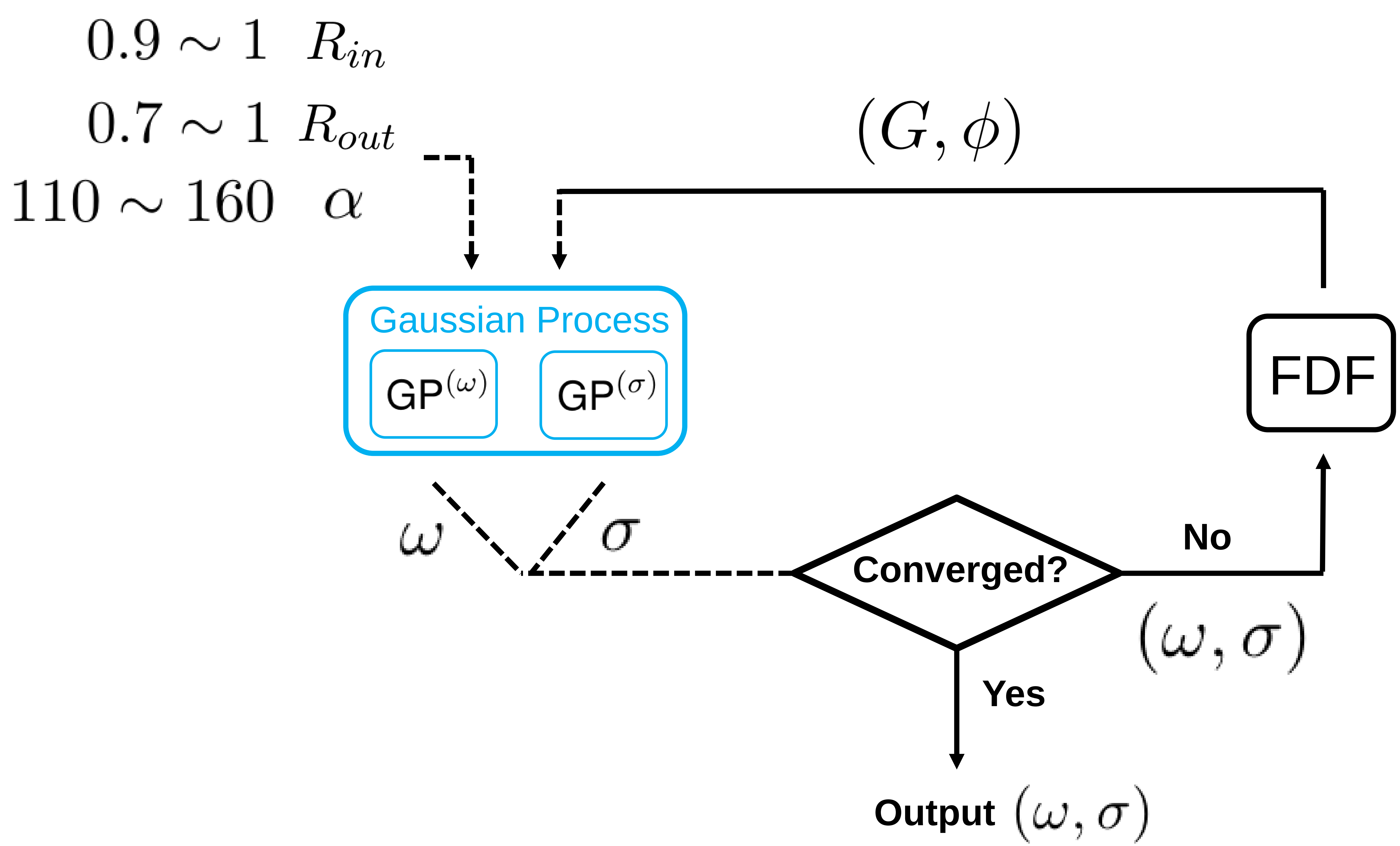
Uncertain FDF data

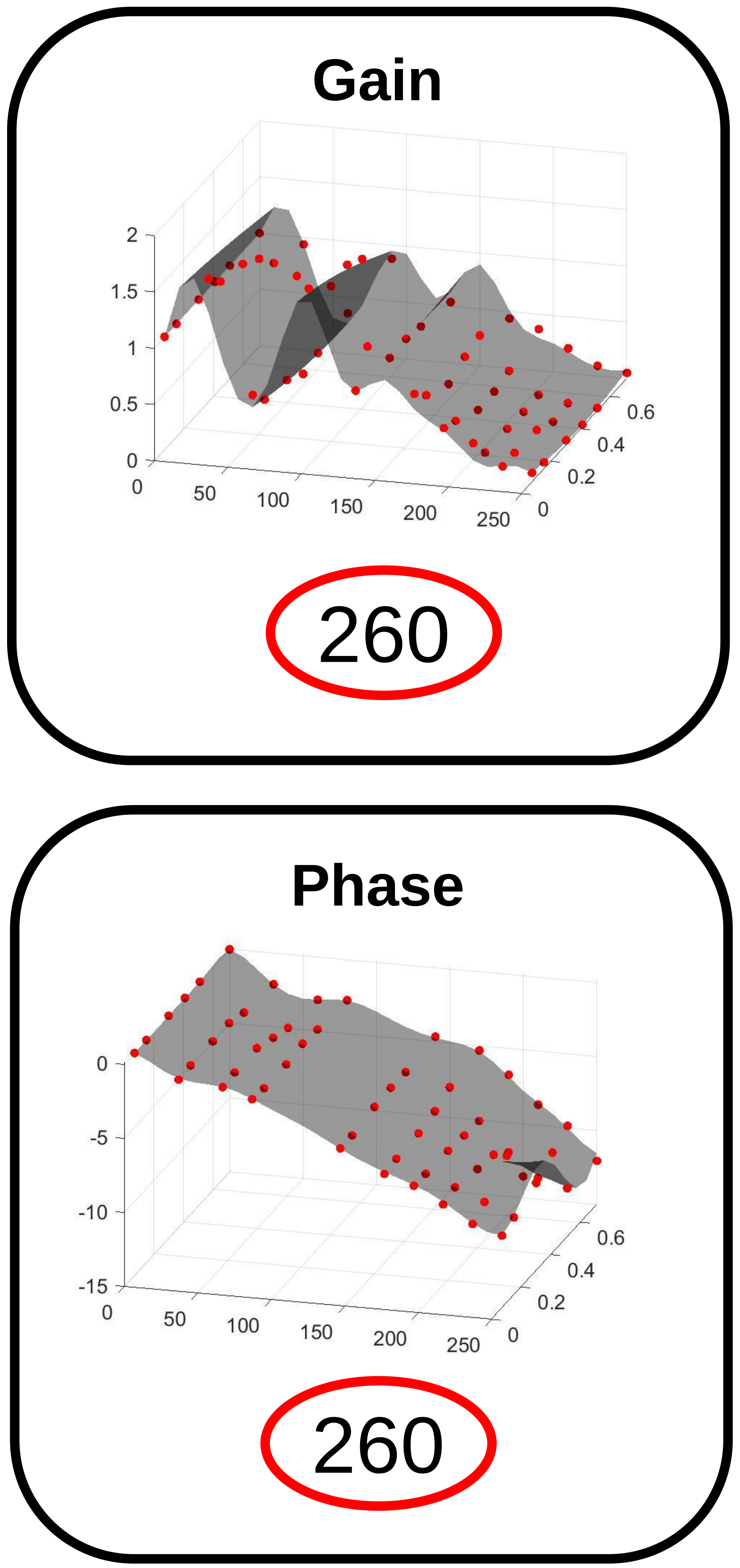
Interpolated FDF data

We calculate growth rate for each amplitude and determine limit cycle based on growth rate trajectory

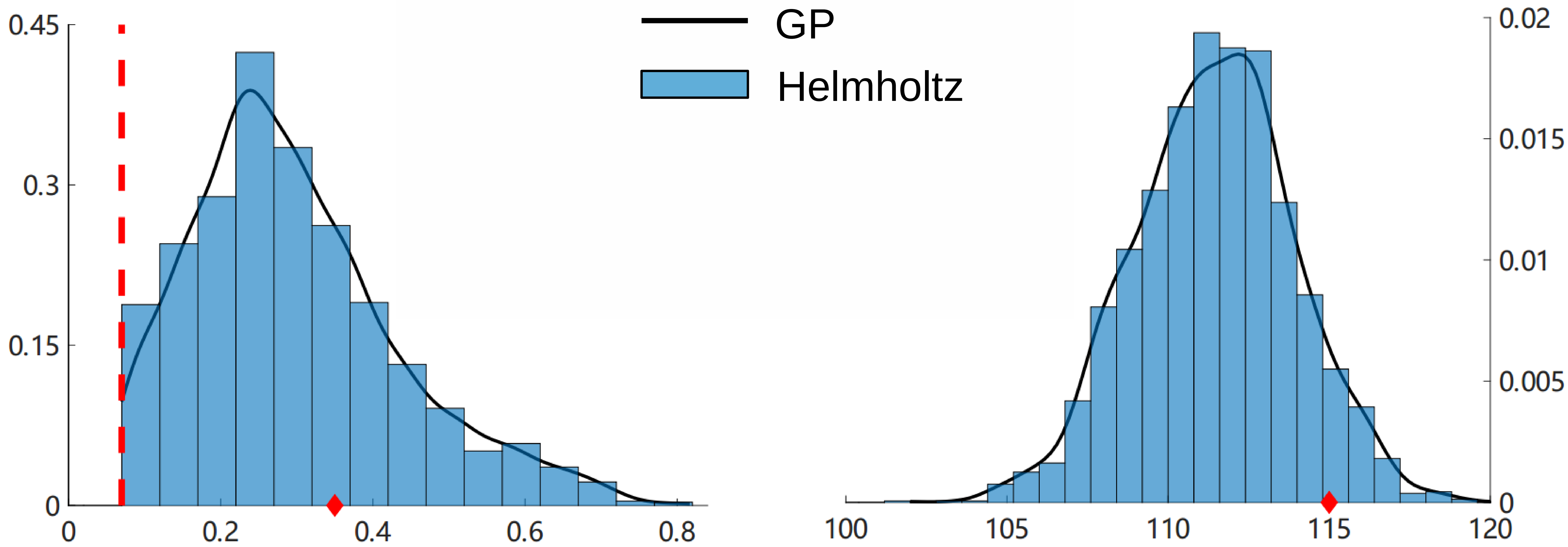


We use GP-based scheme to propagate FDF uncertainties





GP-based scheme achieved highly accurate and efficient nonlinear thermoacoustic UQ analysis



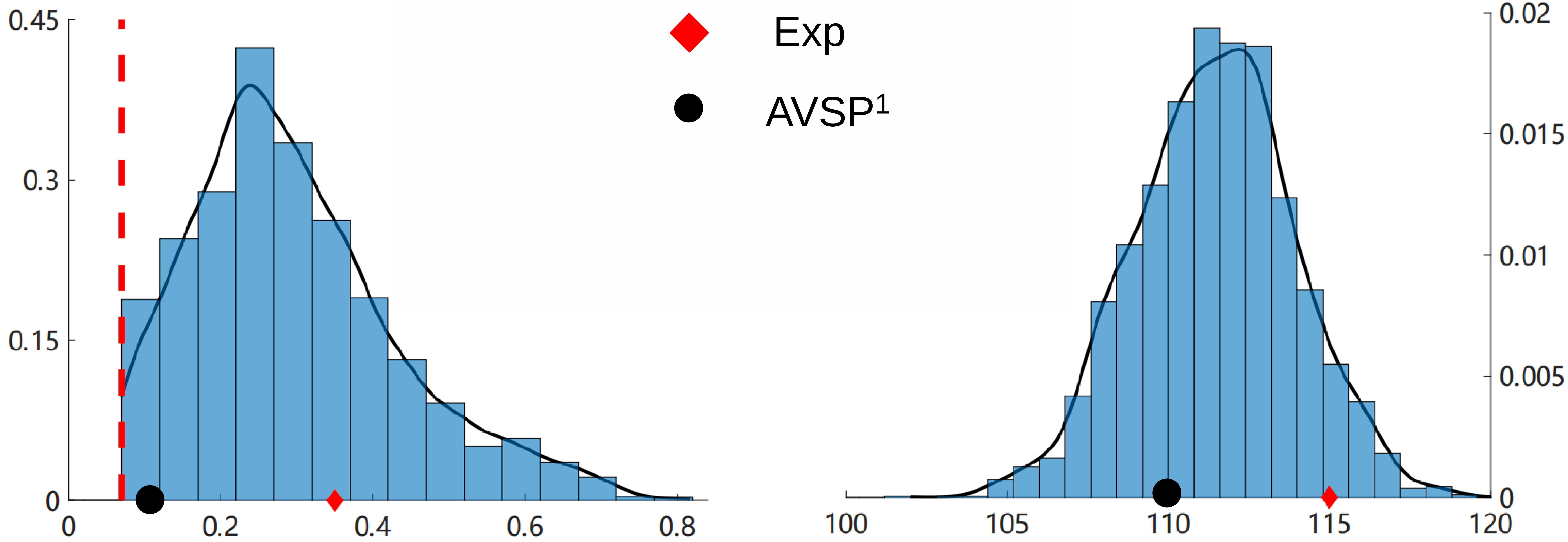
(a) Limit cycle amplitude

(b) Limit cycle frequency

Computation Time (4 cores)

GP	2800 s	15.3 times faster
Helmholtz	42700 s	

Mismatch between numerical and experimental results may be induced by the uncertain FDF data and acoustic boundary values



(a) Limit cycle amplitude

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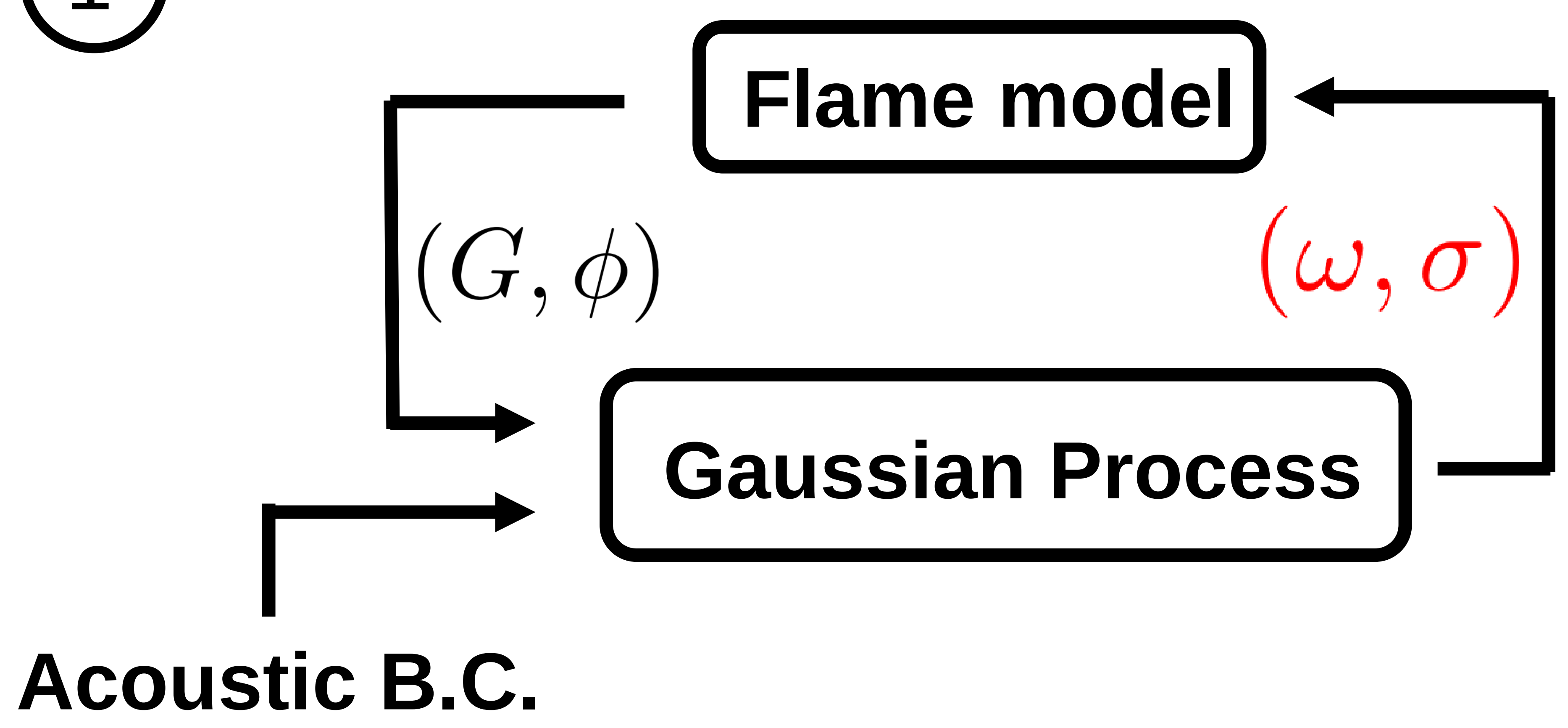
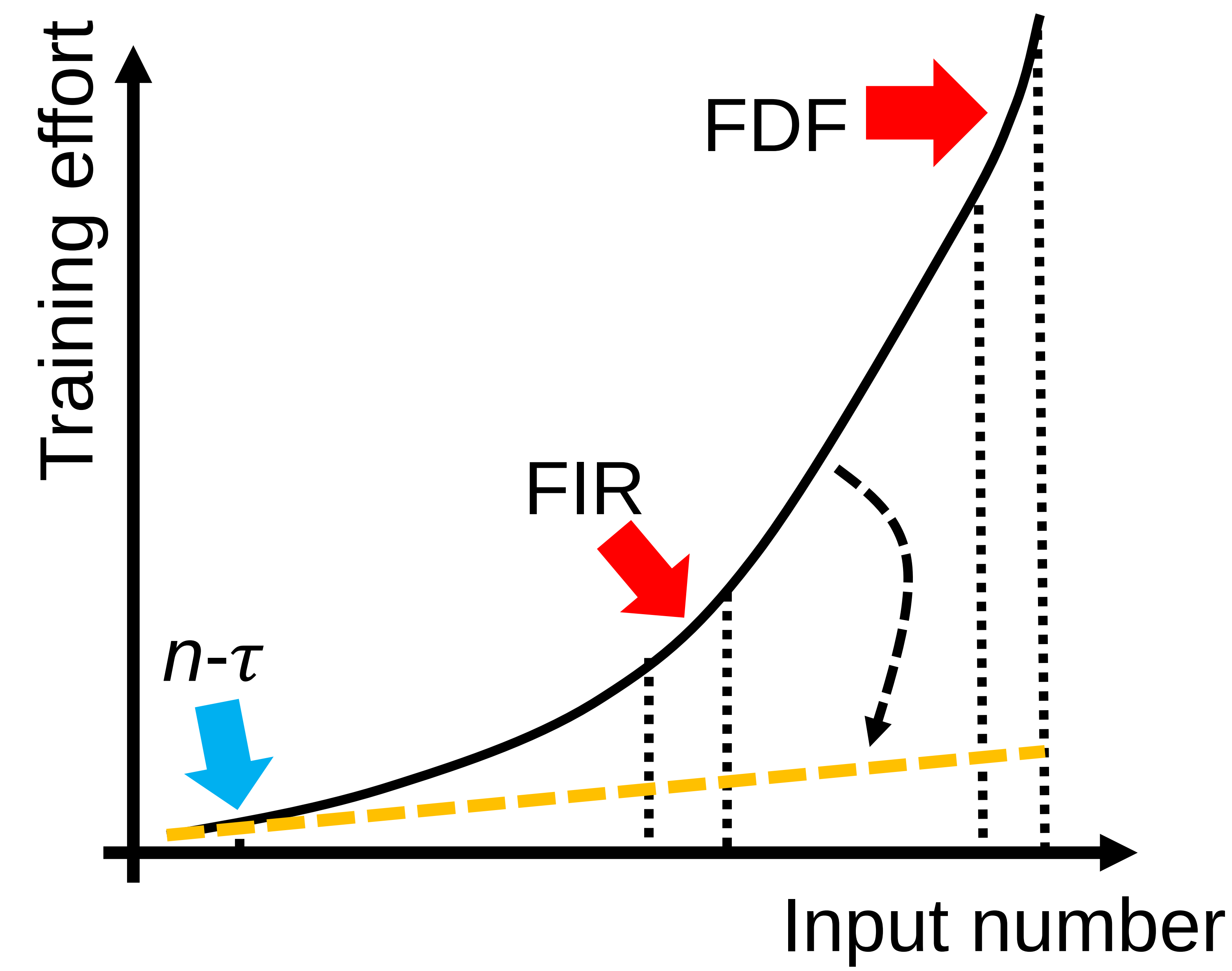
[1] C. F. Silva et al. Combustion and Flame, 2013.

Conclusion & Outlook

1



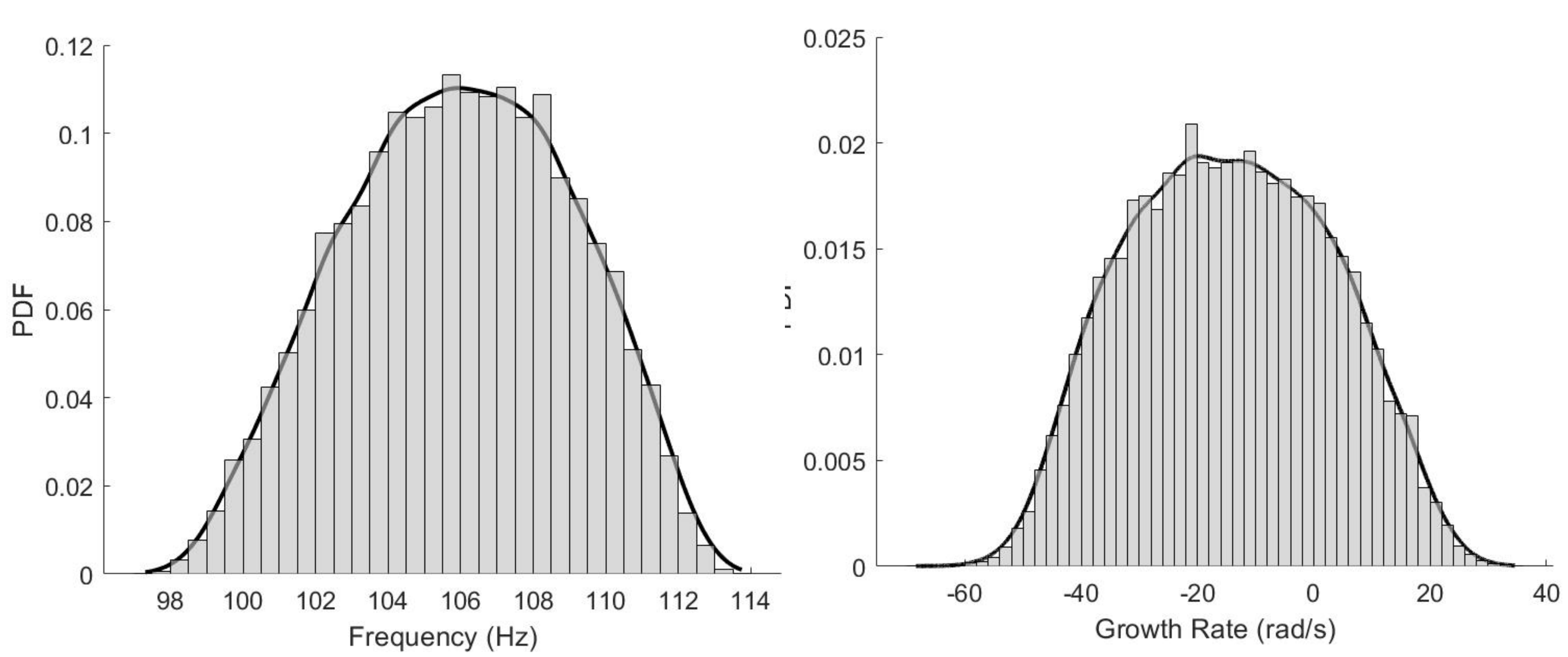


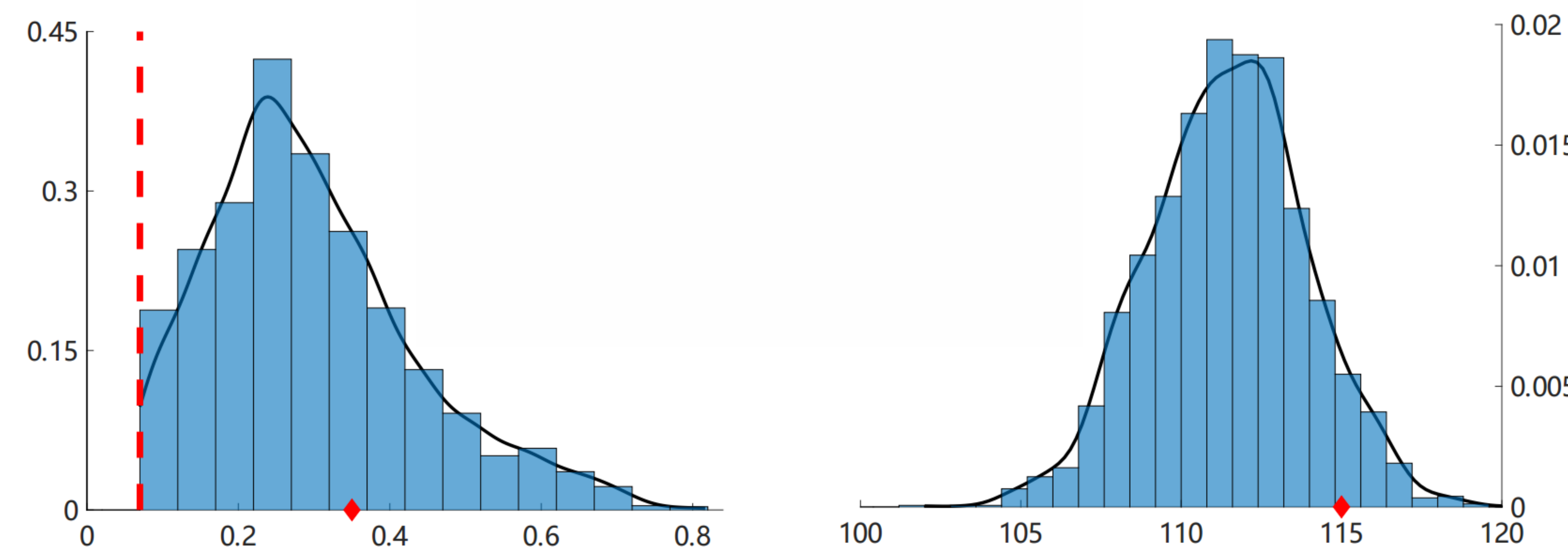
3

2

Frequency

Growth rate





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Conclusion & Outlook

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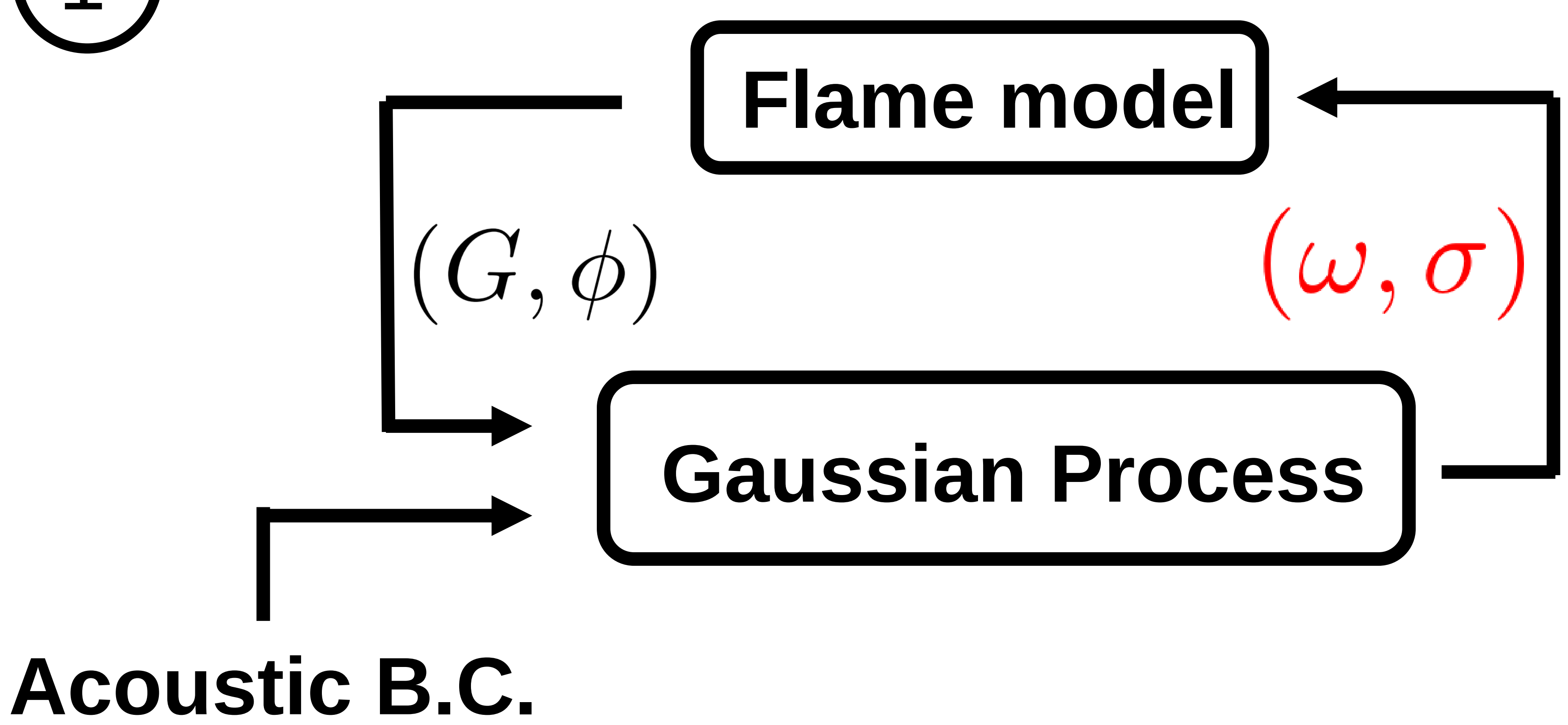

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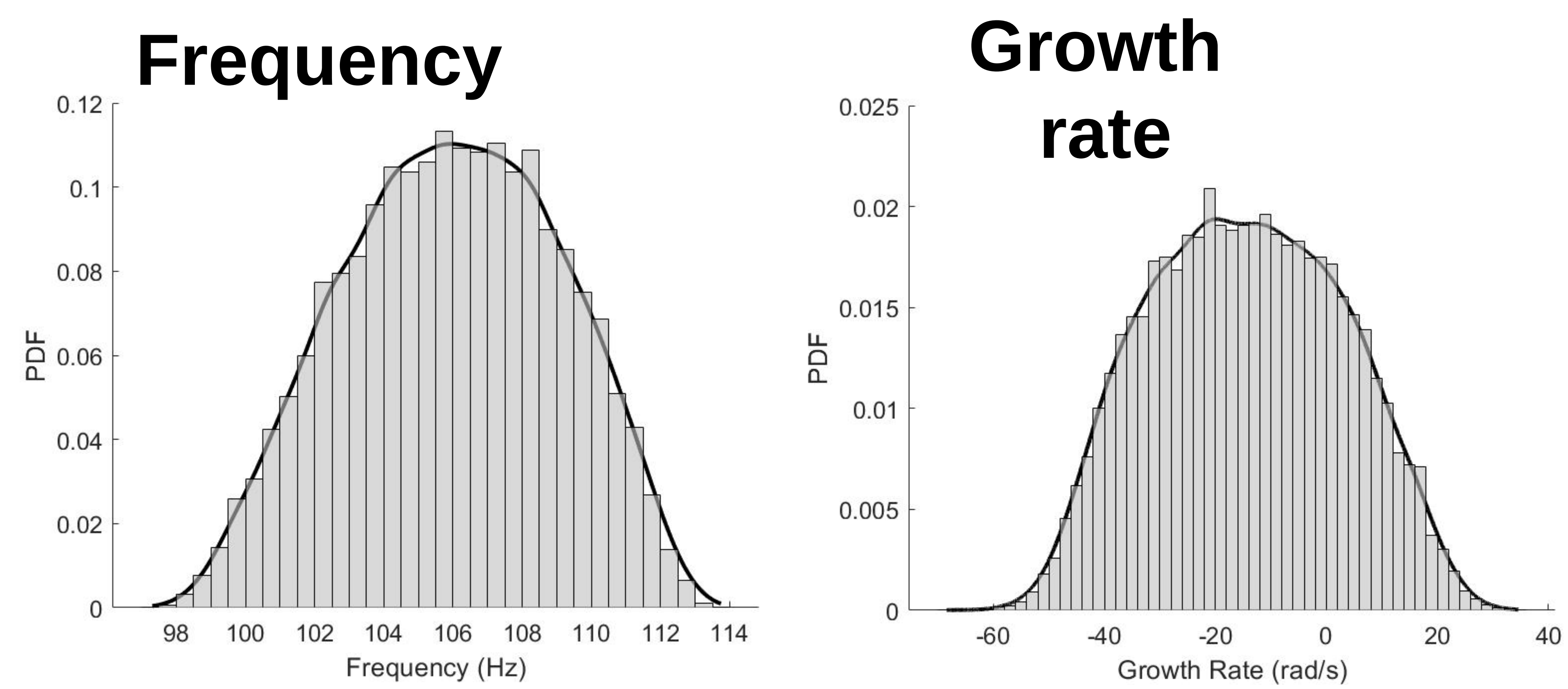

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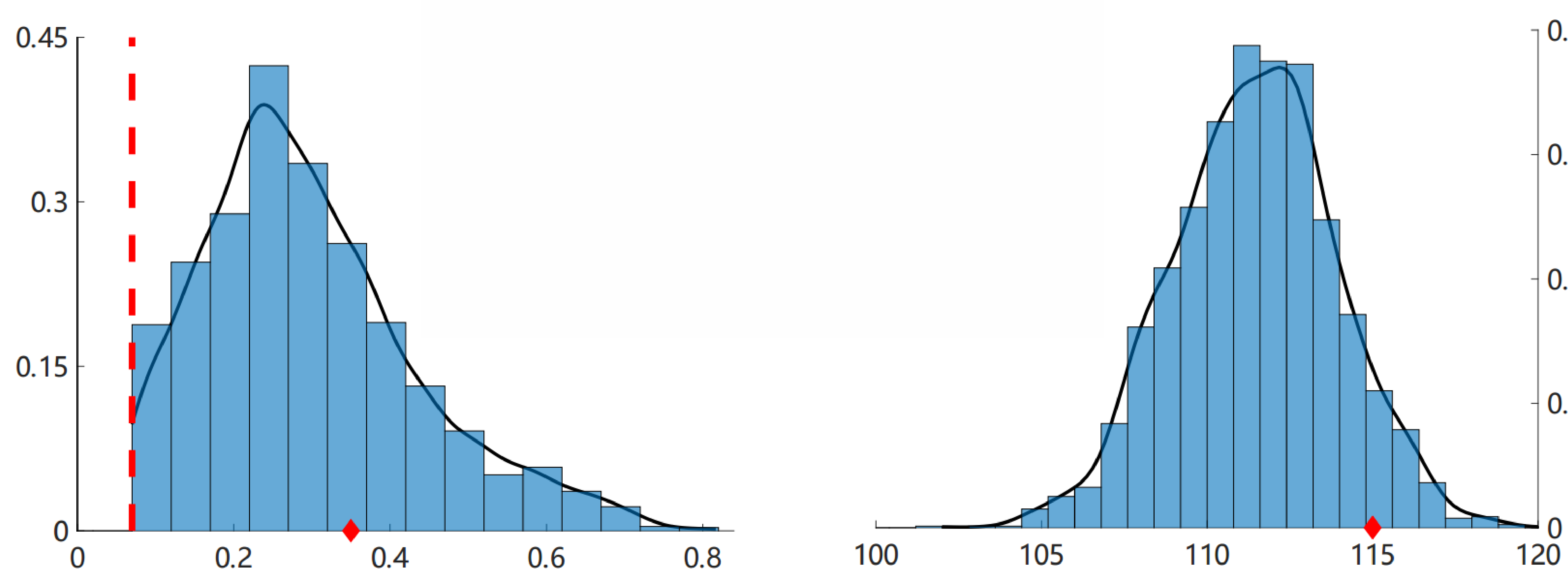

(a) Limit cycle amplitude

(b) Limit cycle frequency

- Calibrate Input parameters
- Global sensitivity analysis
- Robust design to achieve limit-cycle-free