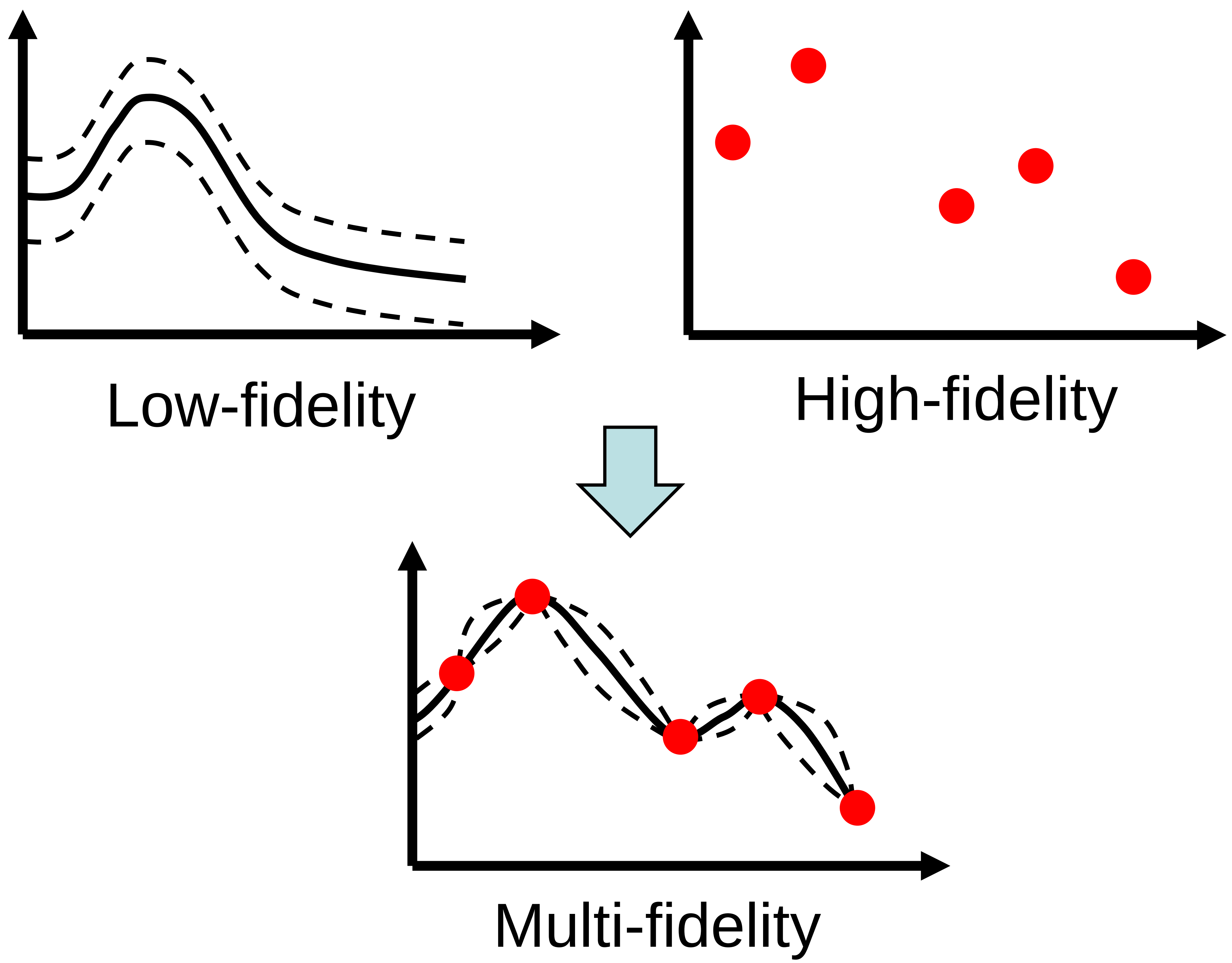
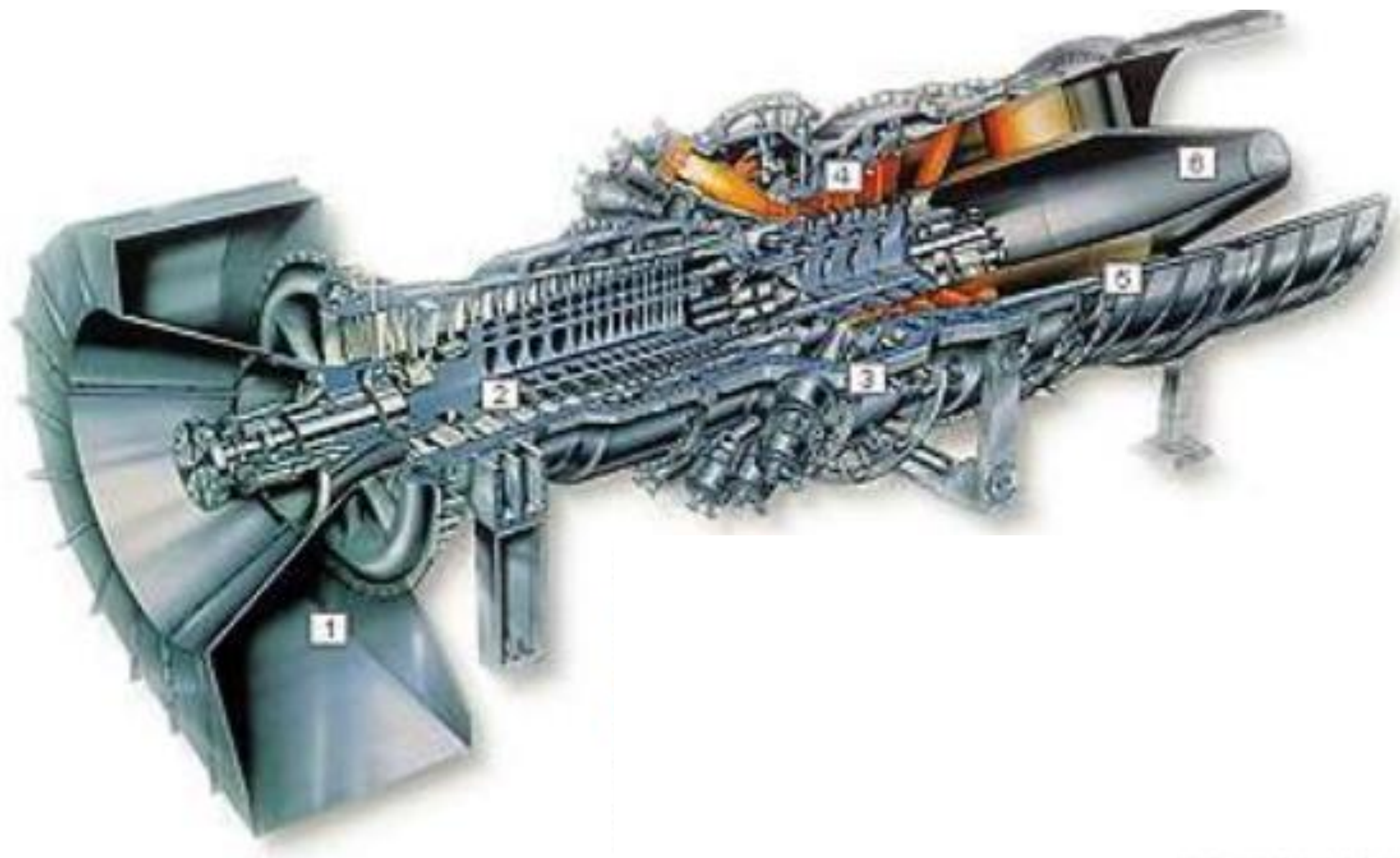
Robust Flame Frequency Response Identification via a Multi-Fidelity Approach

S. Guo, C. F. Silva, W. Polifke



CM^{4}P, Porto, Portugal, 2019

Combustion instability threatens the stable operation of a gas turbine



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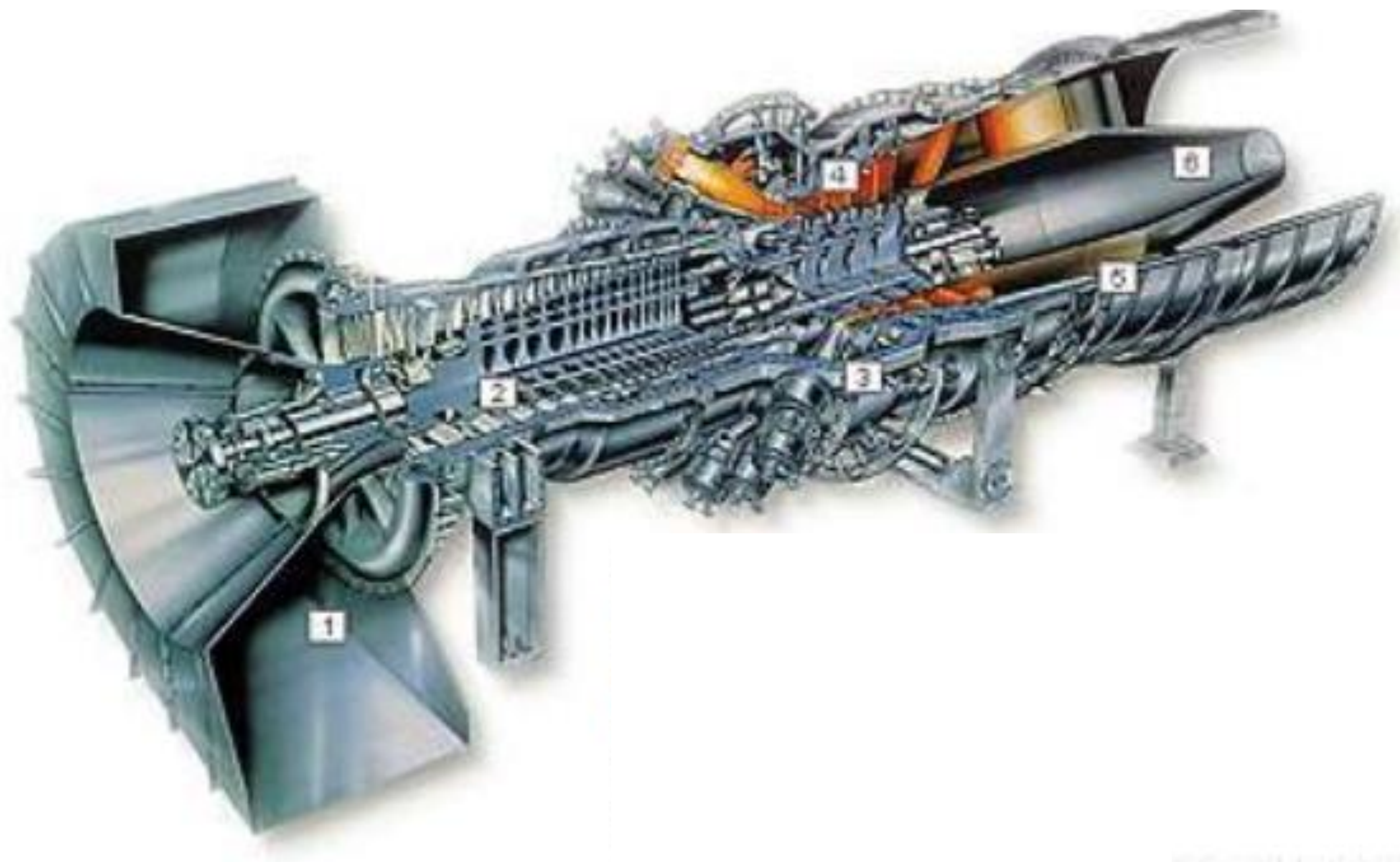

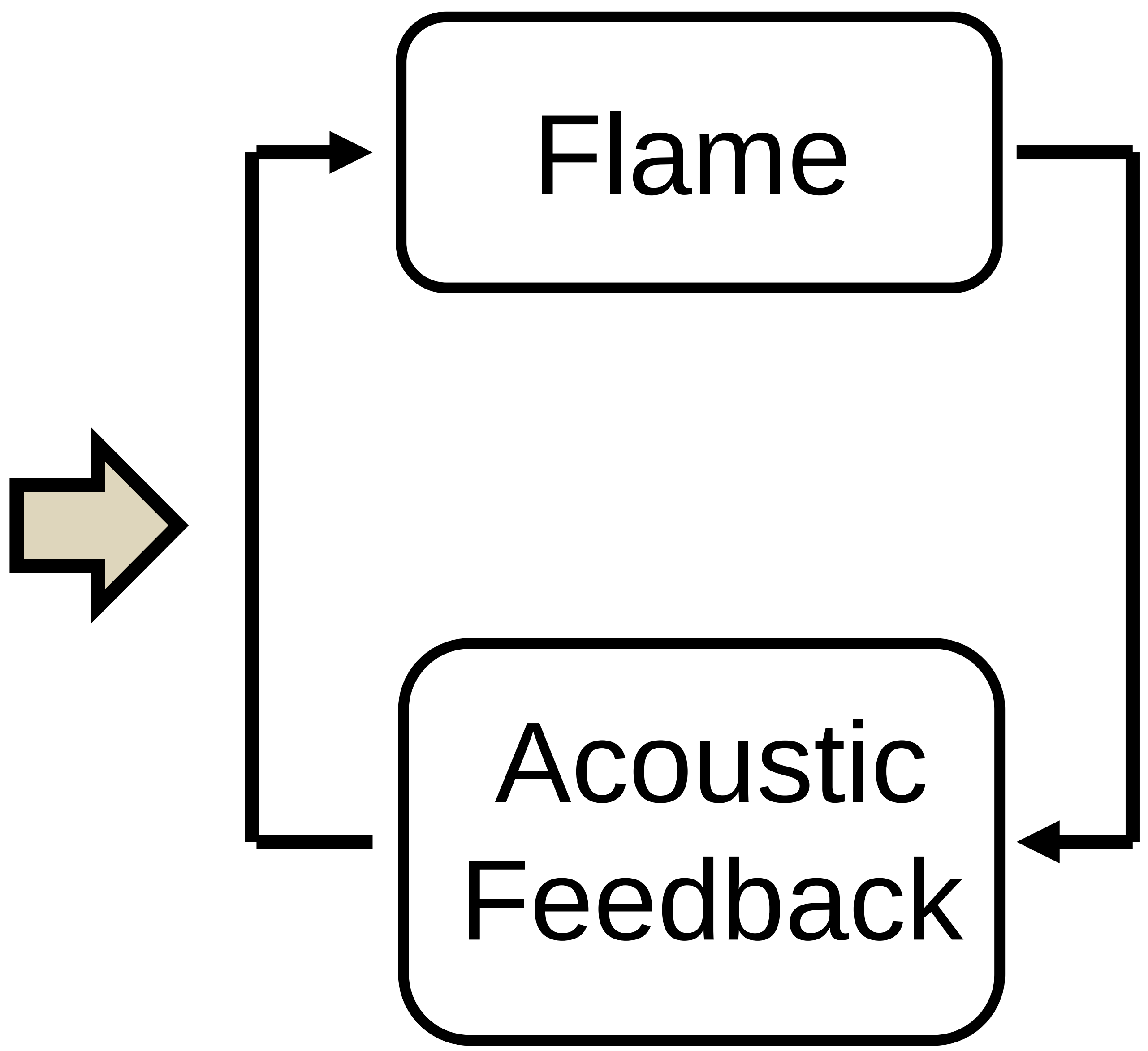

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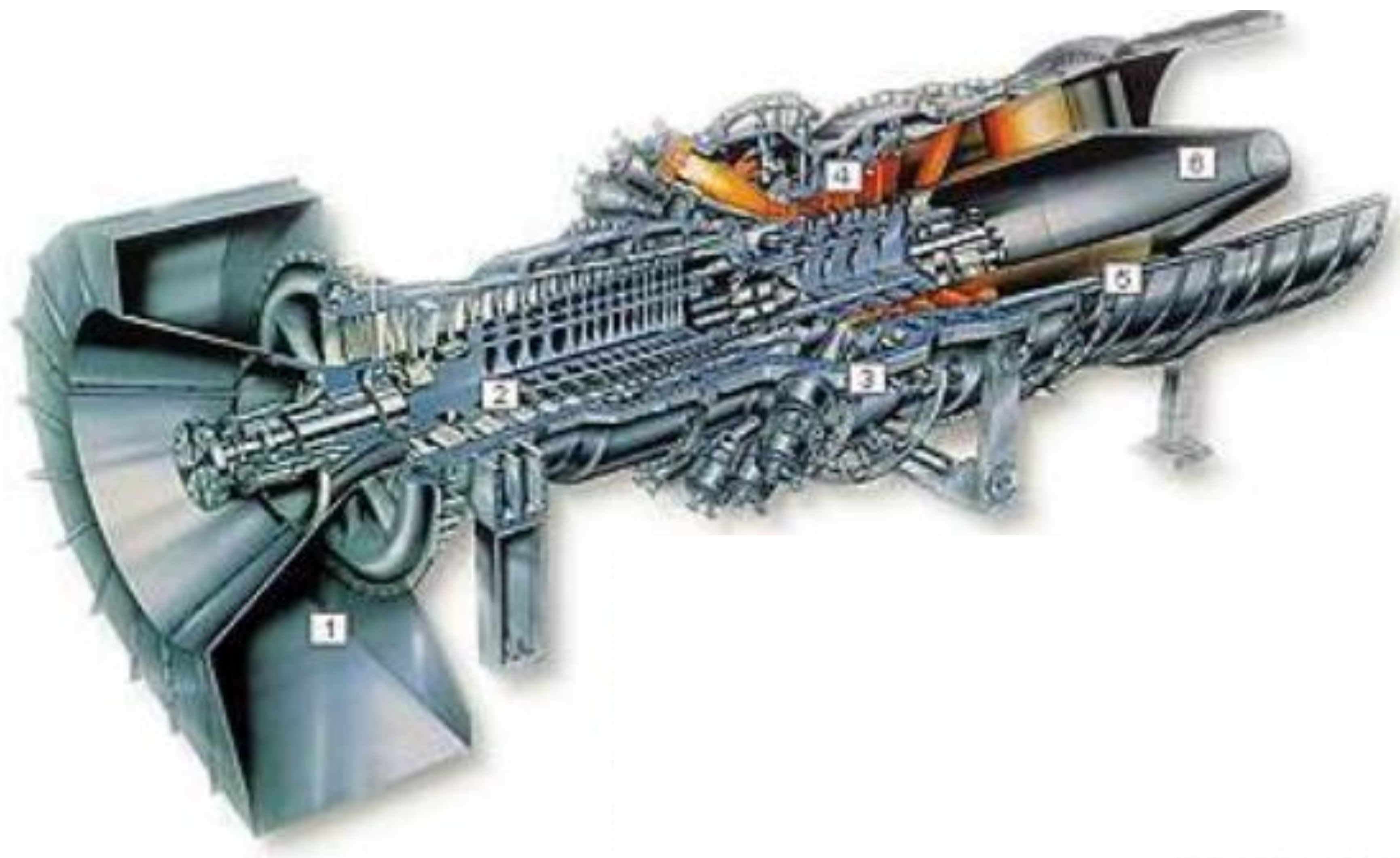


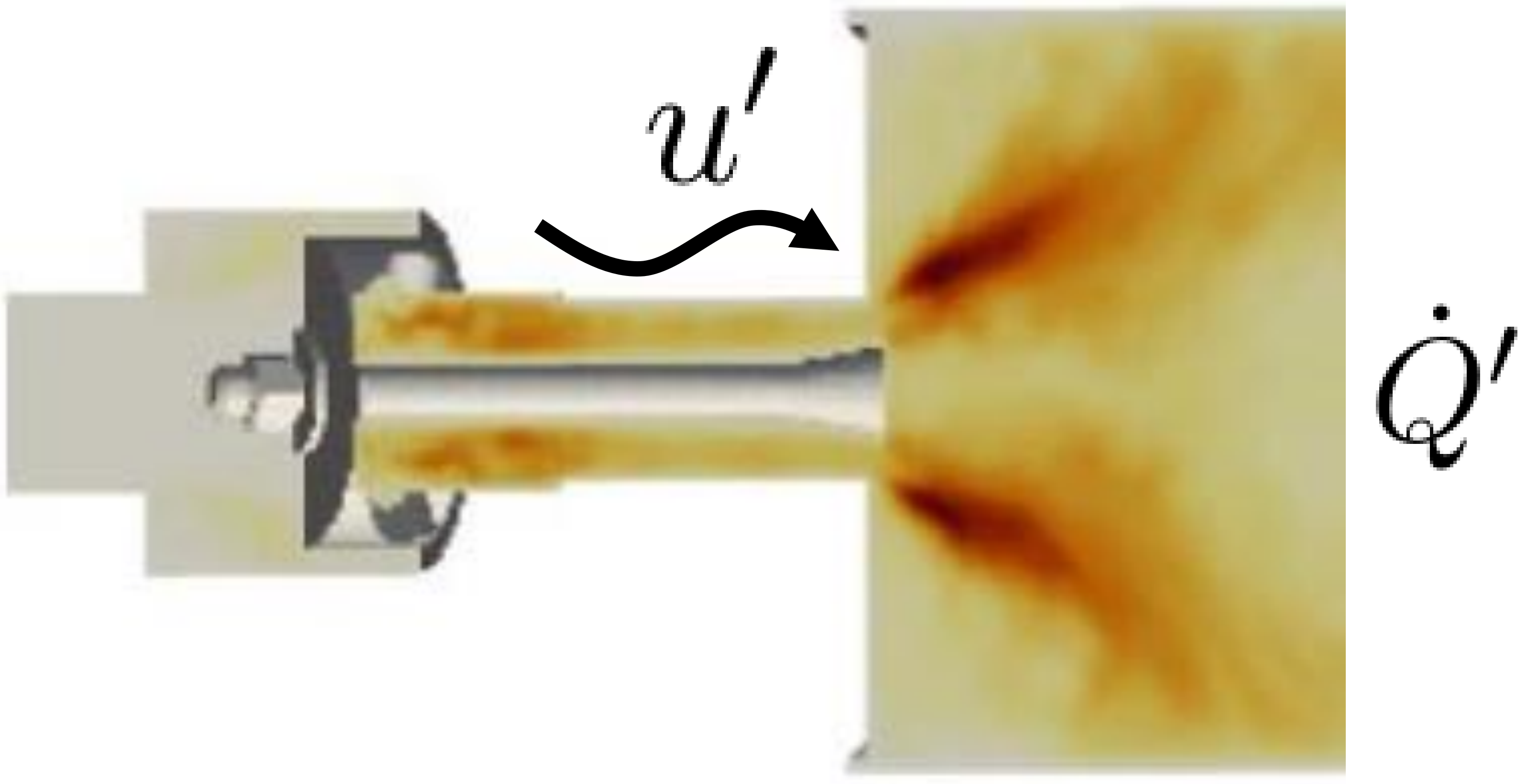
Flame

Acoustic Feedback

Emmert, 2016[1]

[1] T. Emmert. State Space Modeling of Thermoacoustic System with Application to Intrinsic Feedback. PhD thesis, TUM

Flame frequency response plays a key role in investigating combustion instability



Merk, 2018[2]

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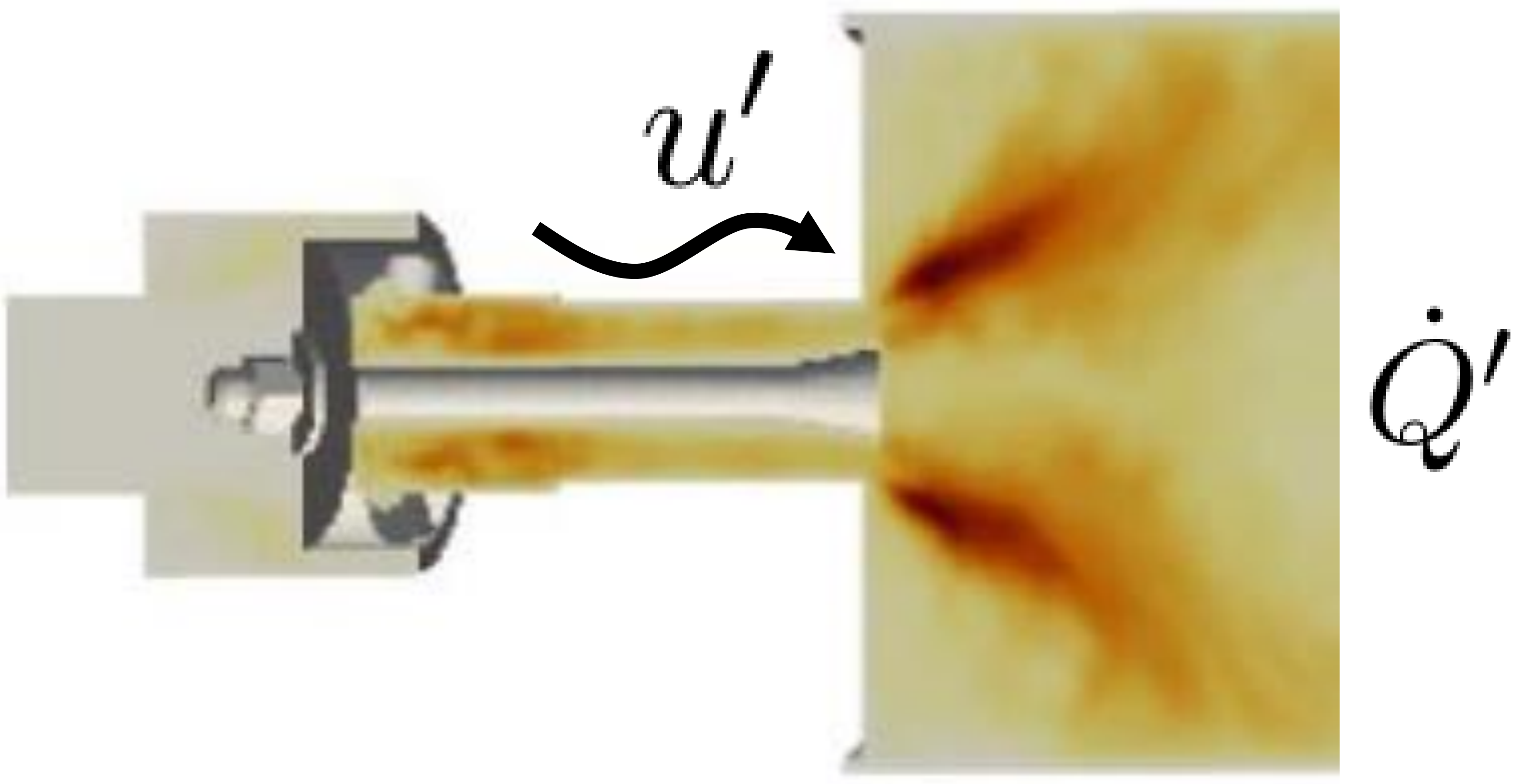

Merk, 2018[2]

$$F(\omega) = \frac{\dot{Q}'(\omega)/\bar{\dot{Q}}}{u'(\omega)/\bar{u}}$$

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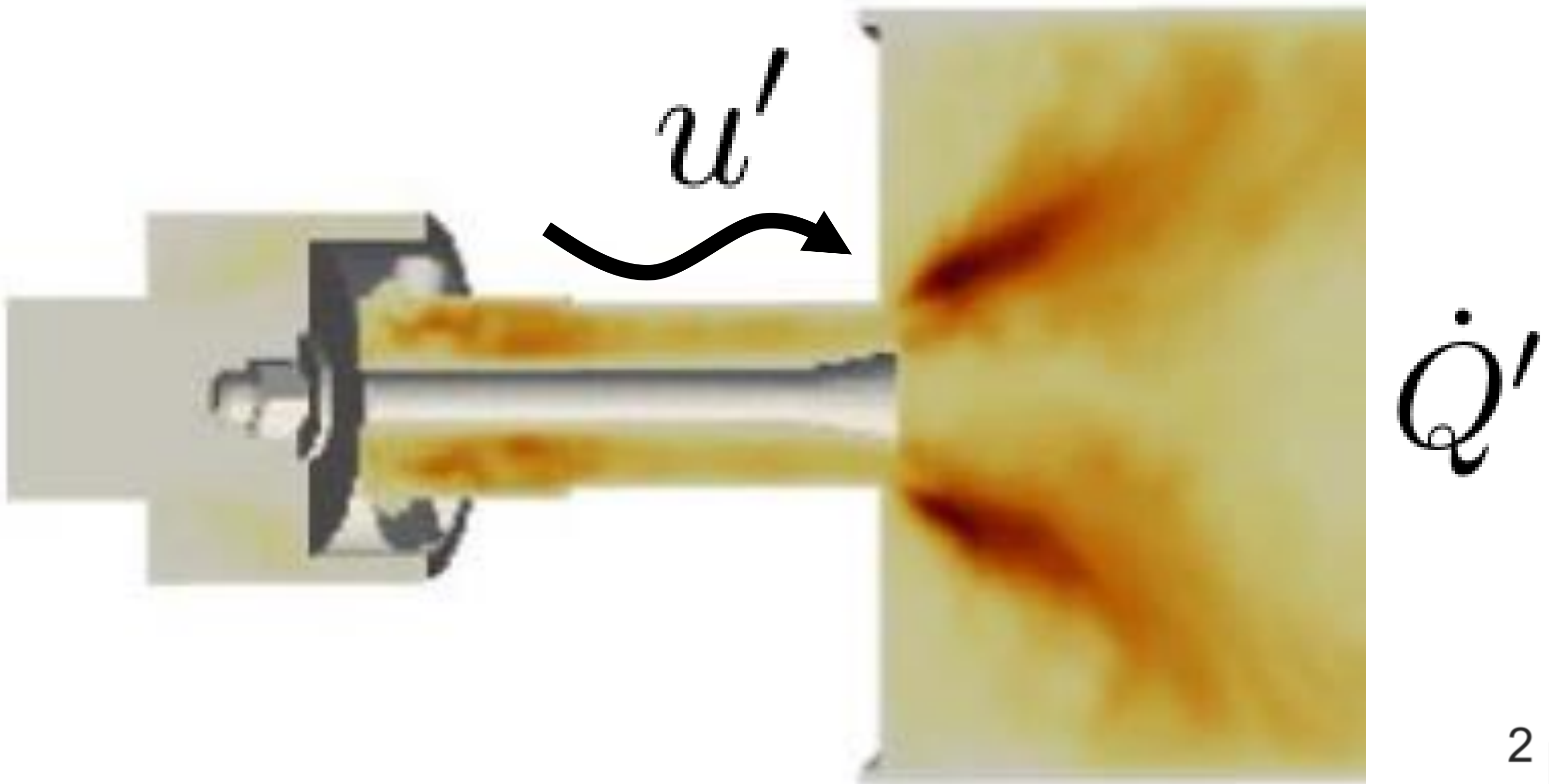

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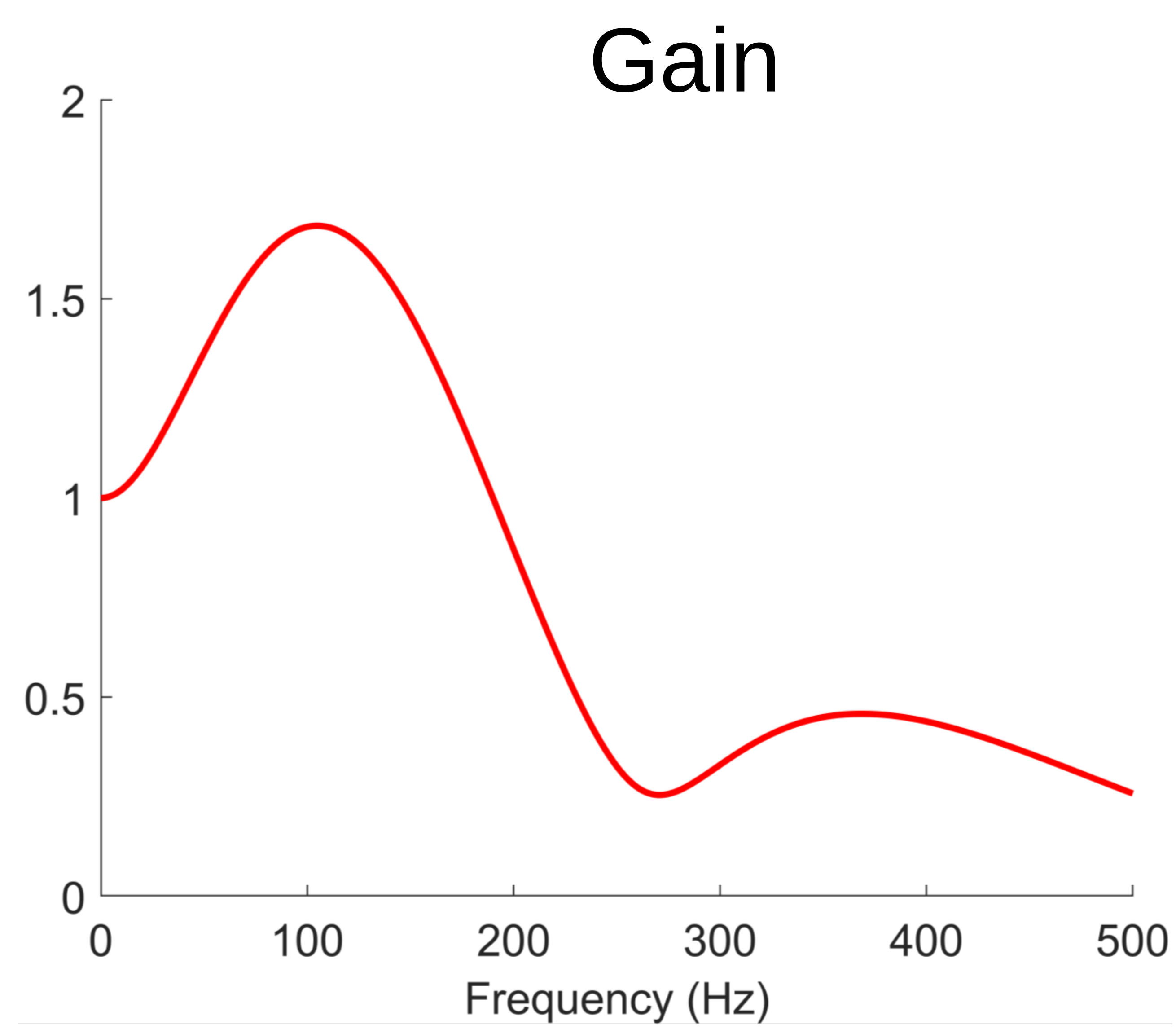

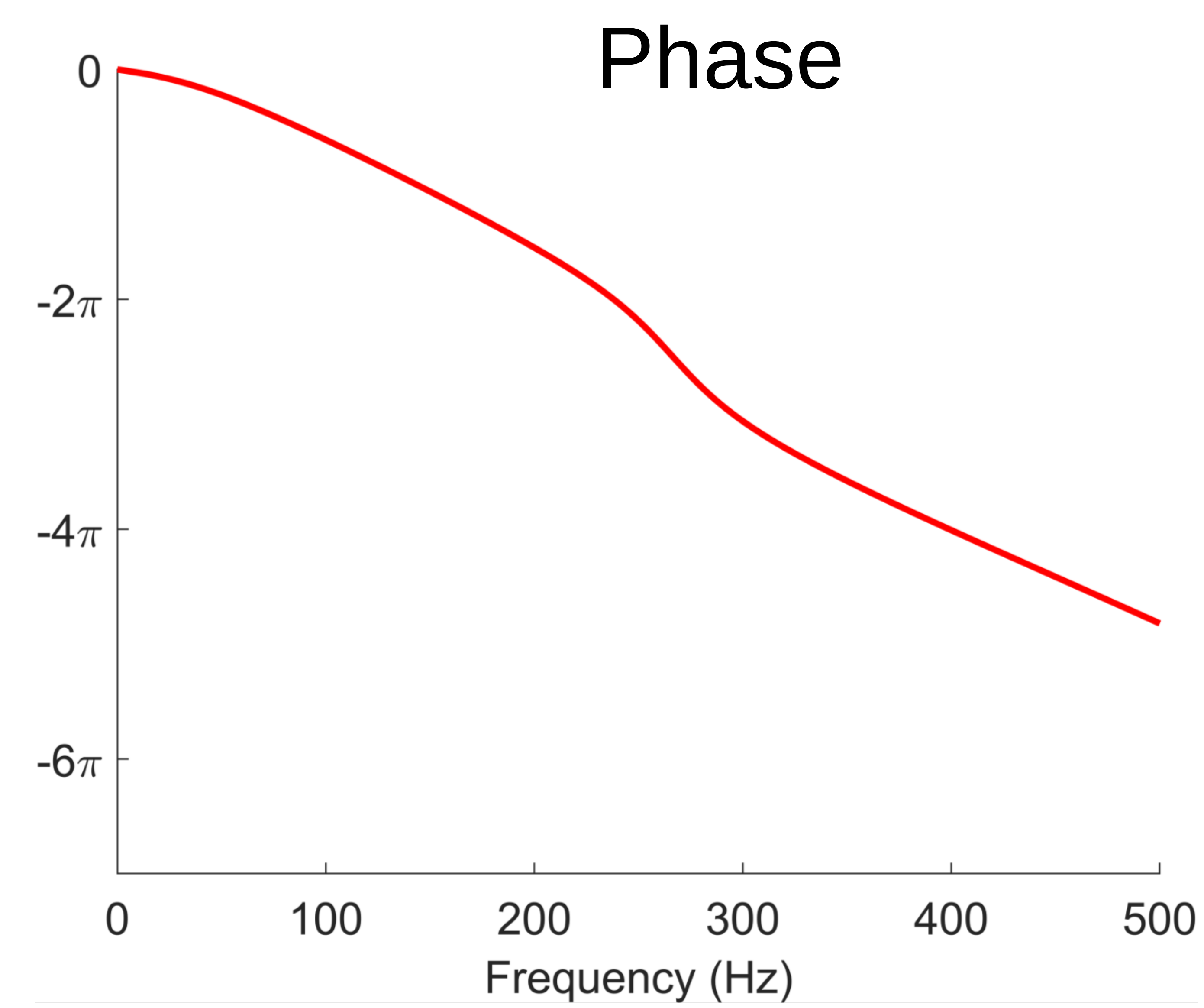

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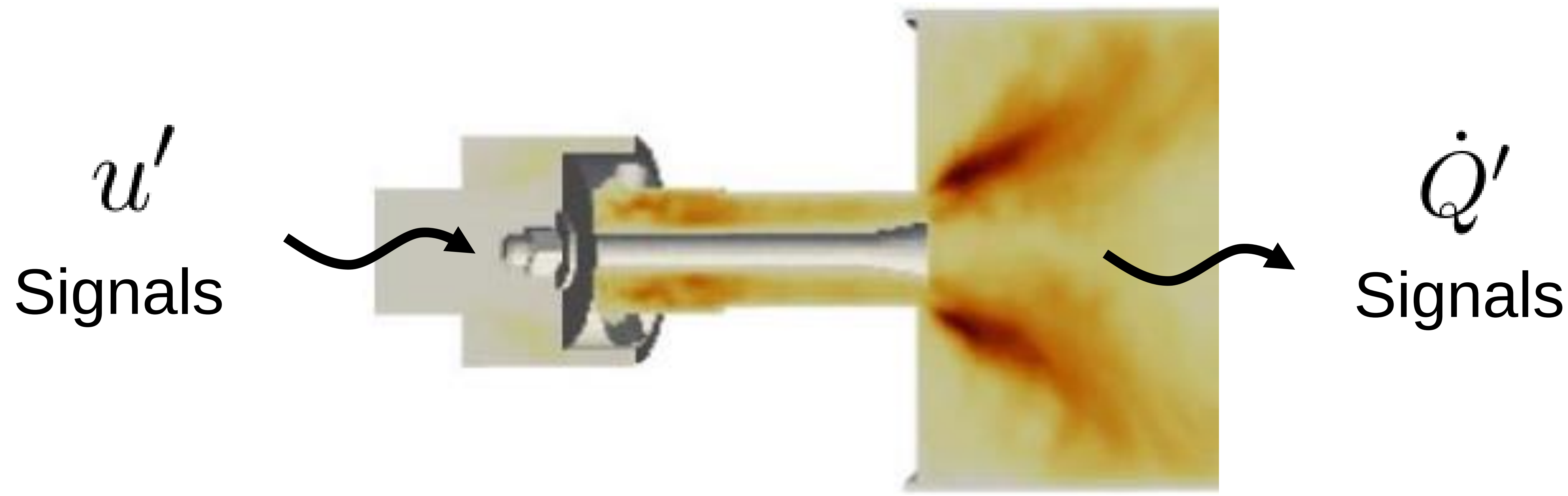
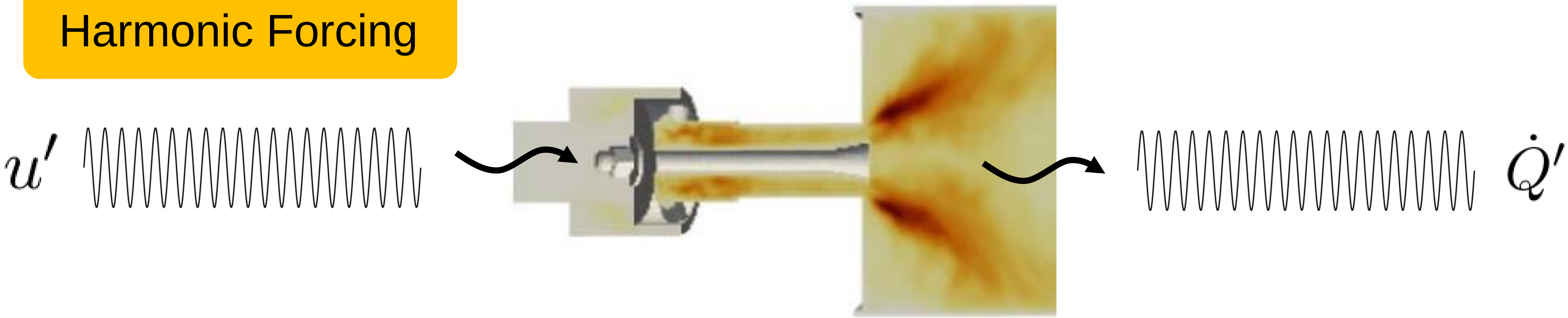
Flame frequency response can be derived from CFD simulations



Harmonic forcing is highly accurate but only provides point-estimation

Harmonic Forcing



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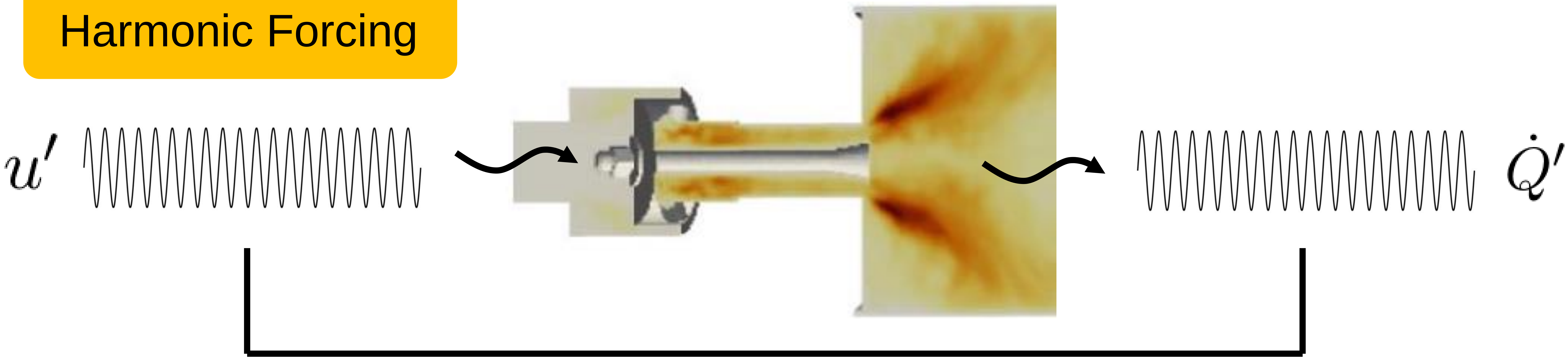

Fourier Analysis

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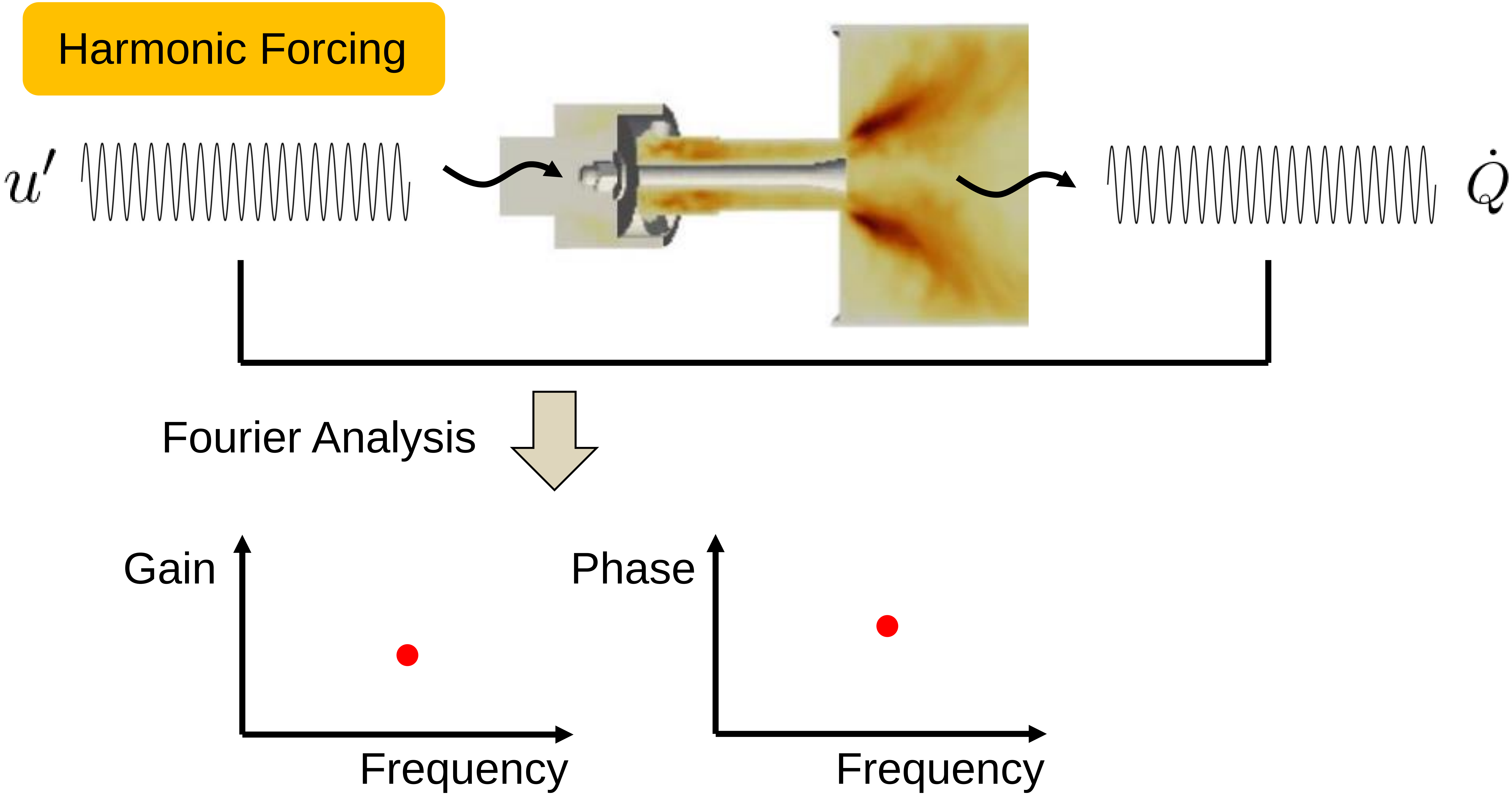

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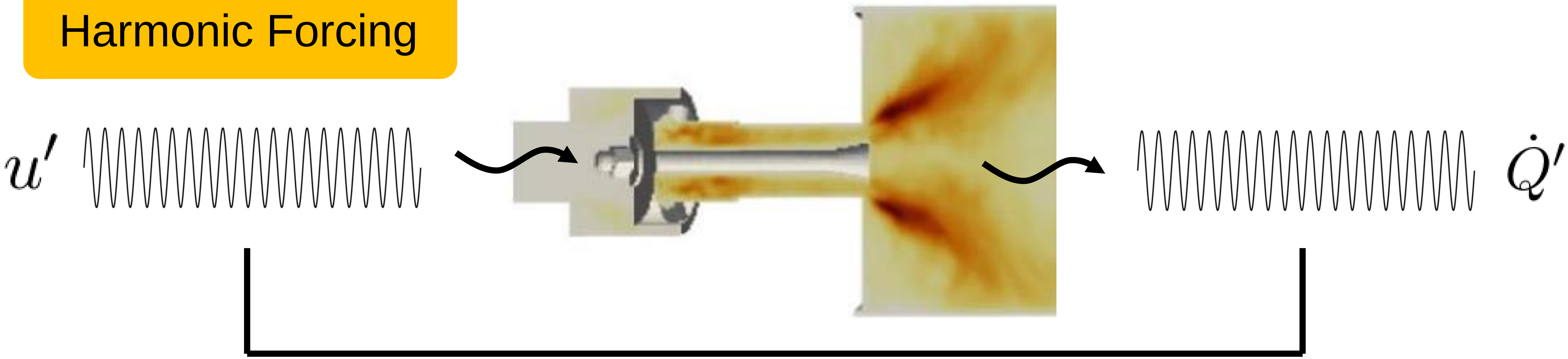

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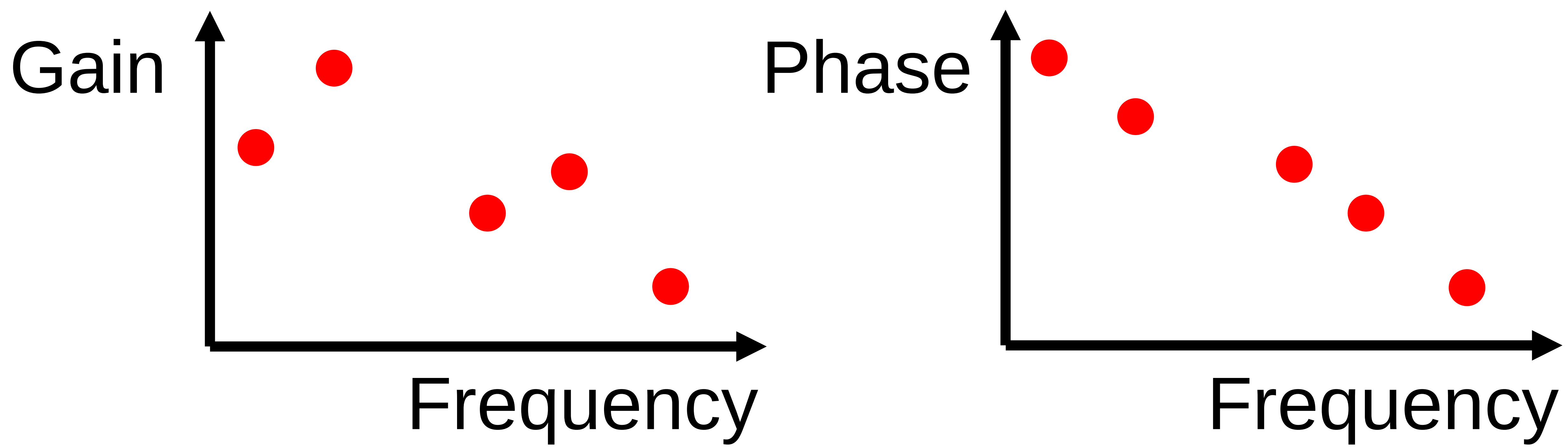

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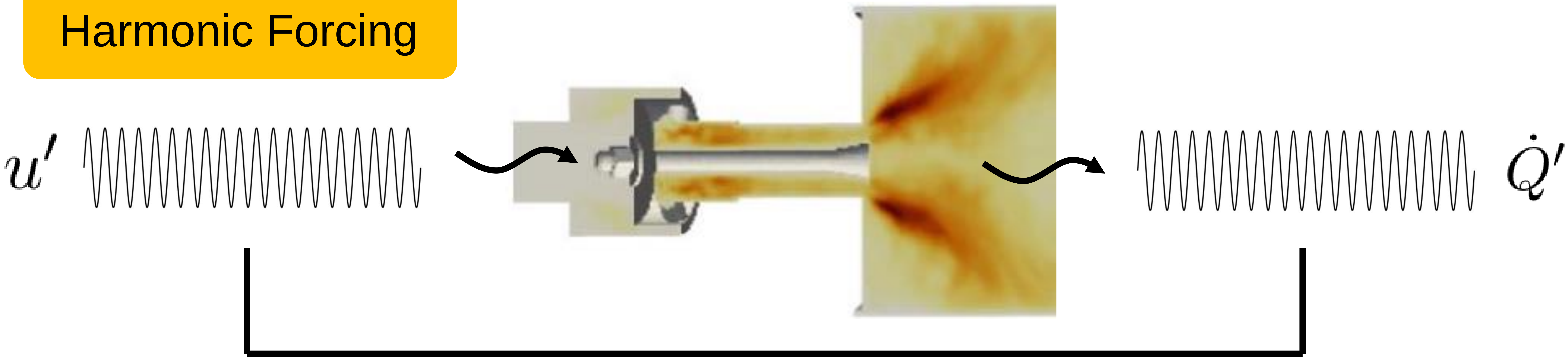

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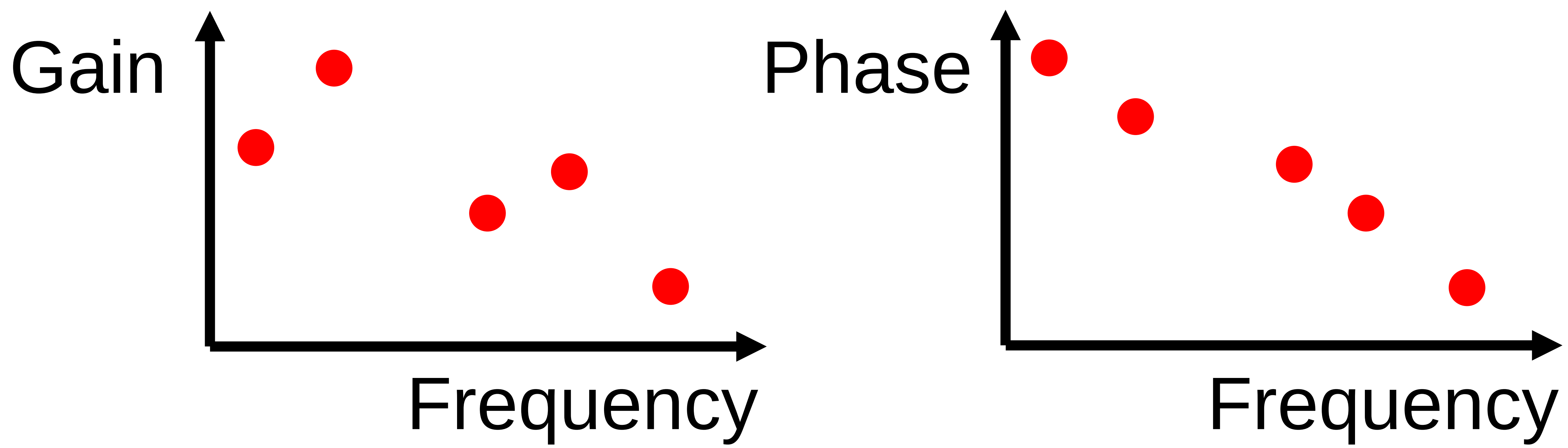
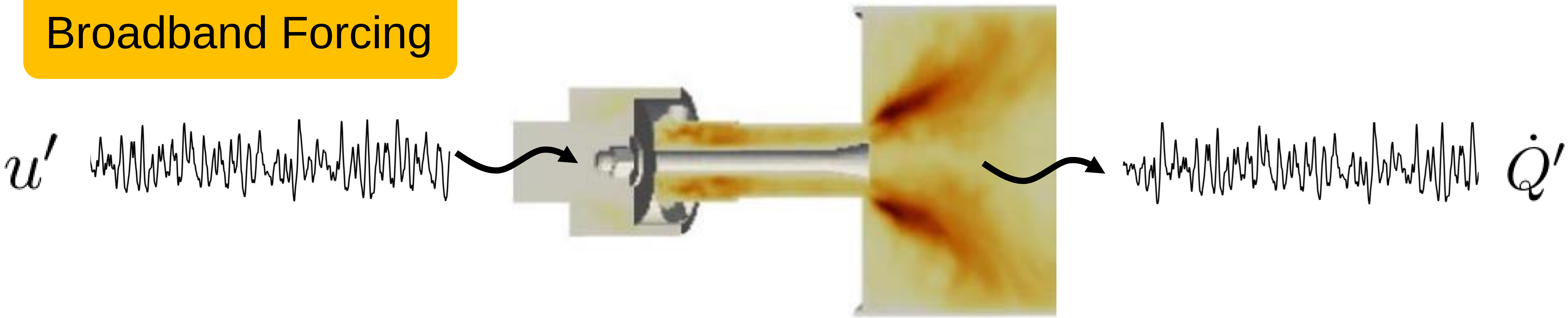


Time series length =

$$\sum_{i=1}^{n} \frac{1}{f_i} \times \text{excitation cycles}$$

Broadband forcing provides complete frequency response but with uncertainty

Broadband Forcing



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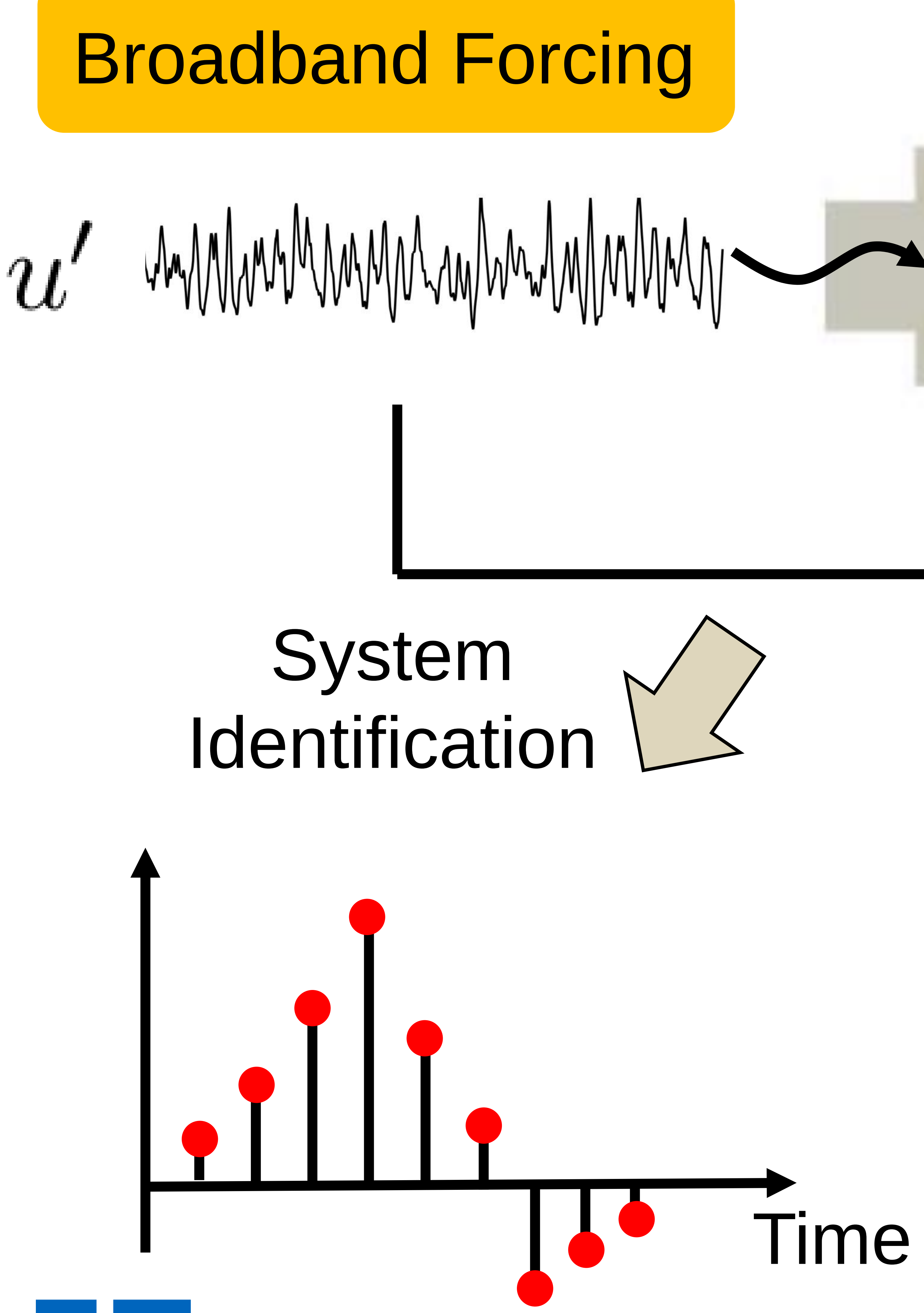

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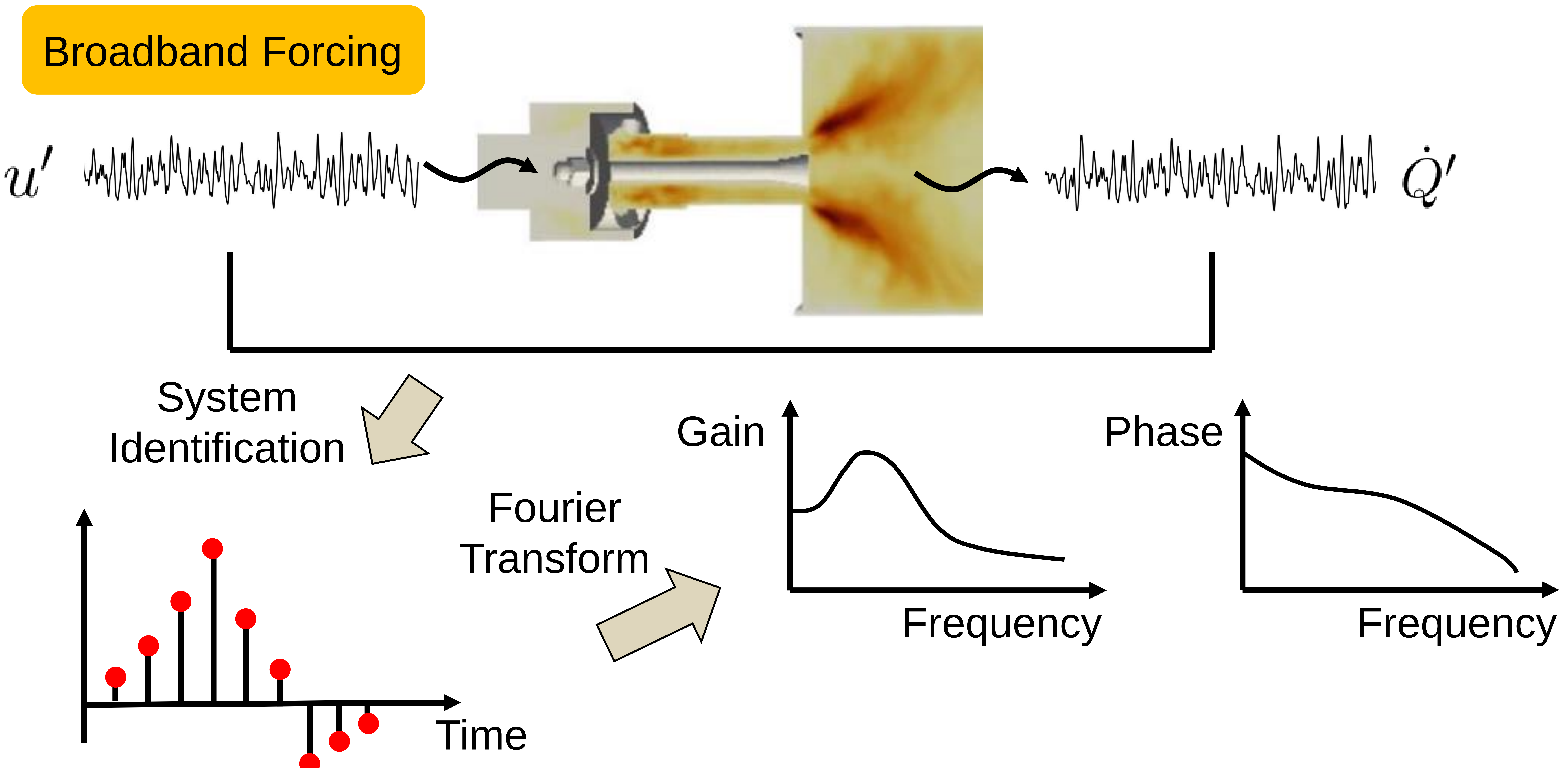

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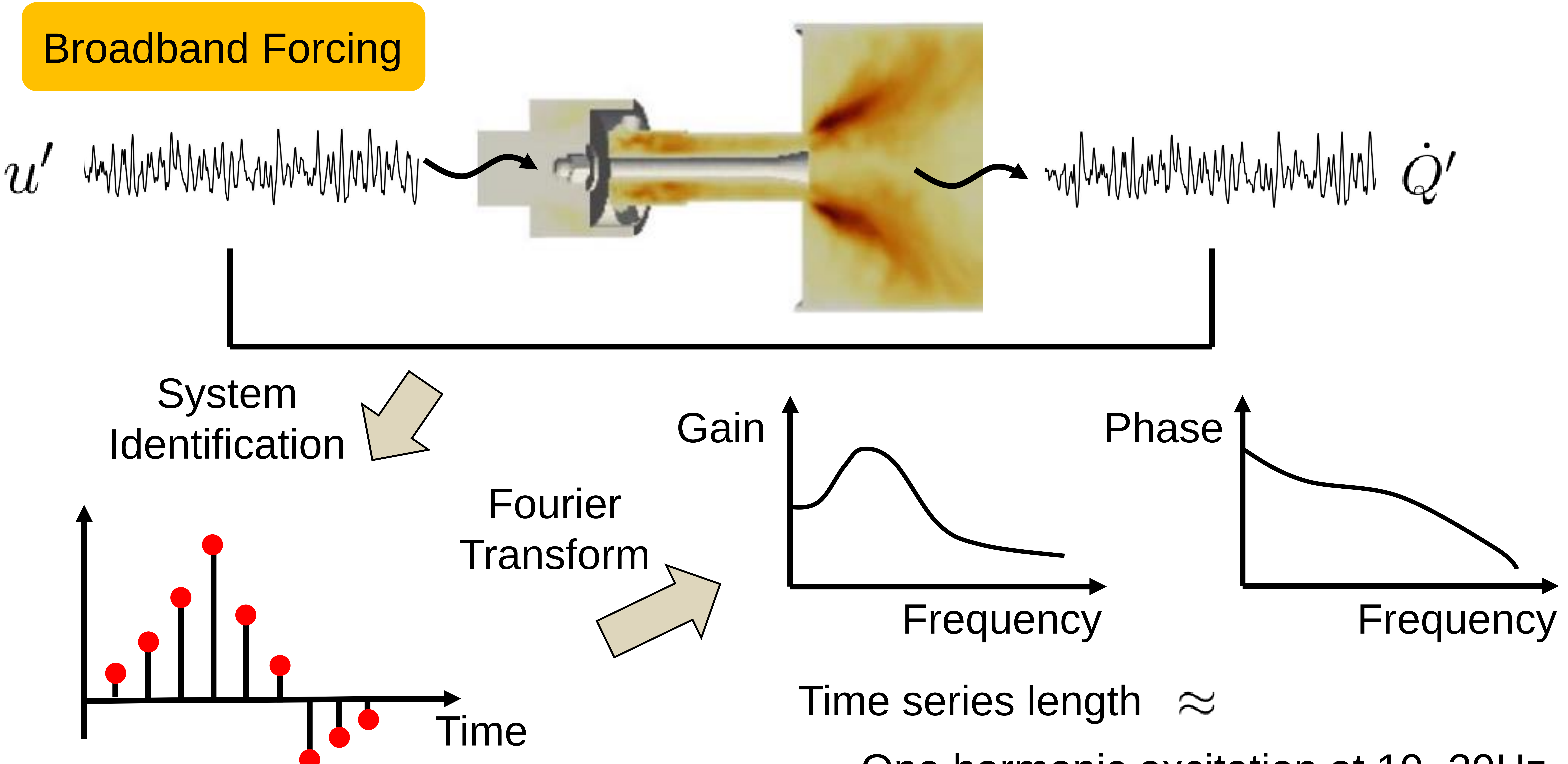

Time series length $\approx$

One harmonic excitation at 10~30Hz

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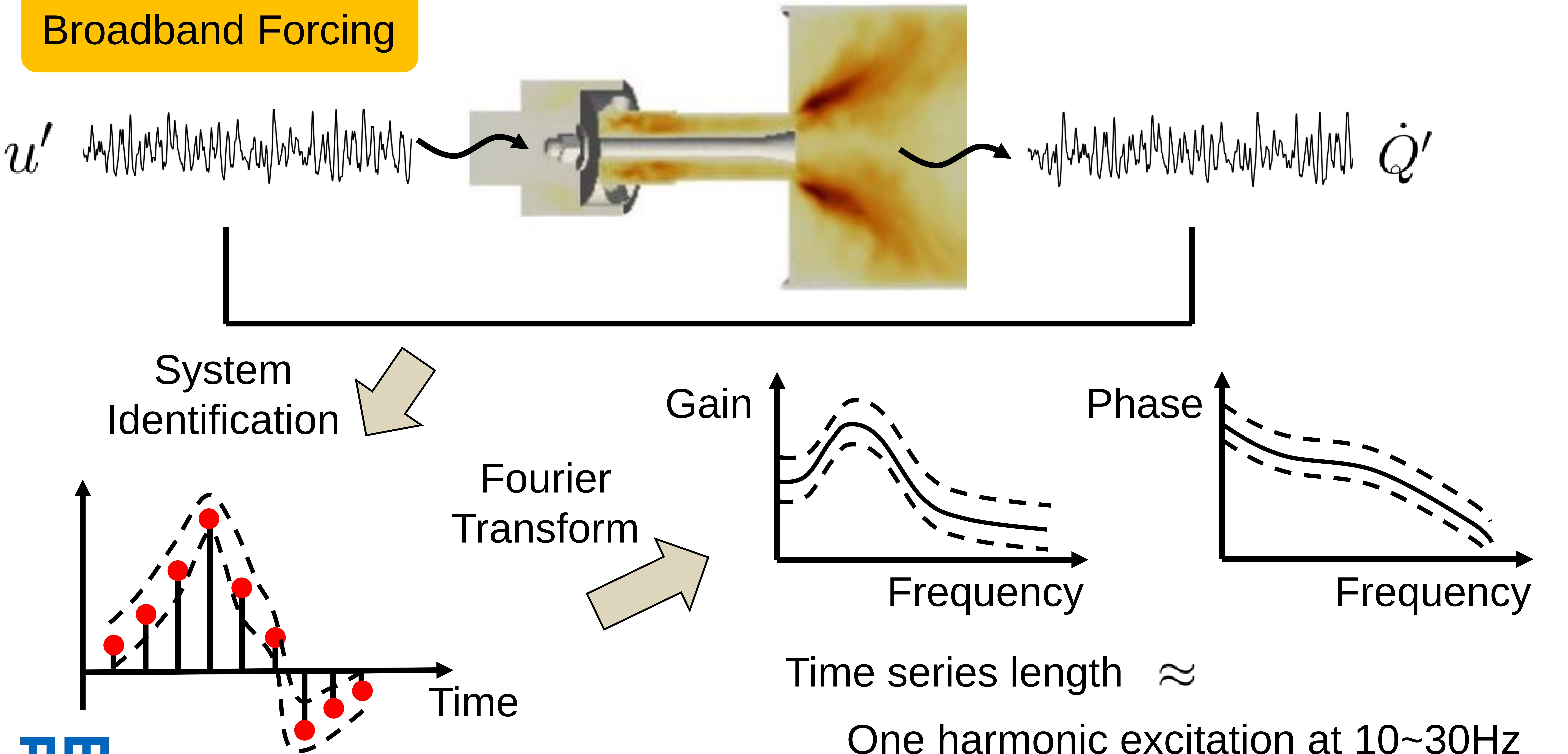

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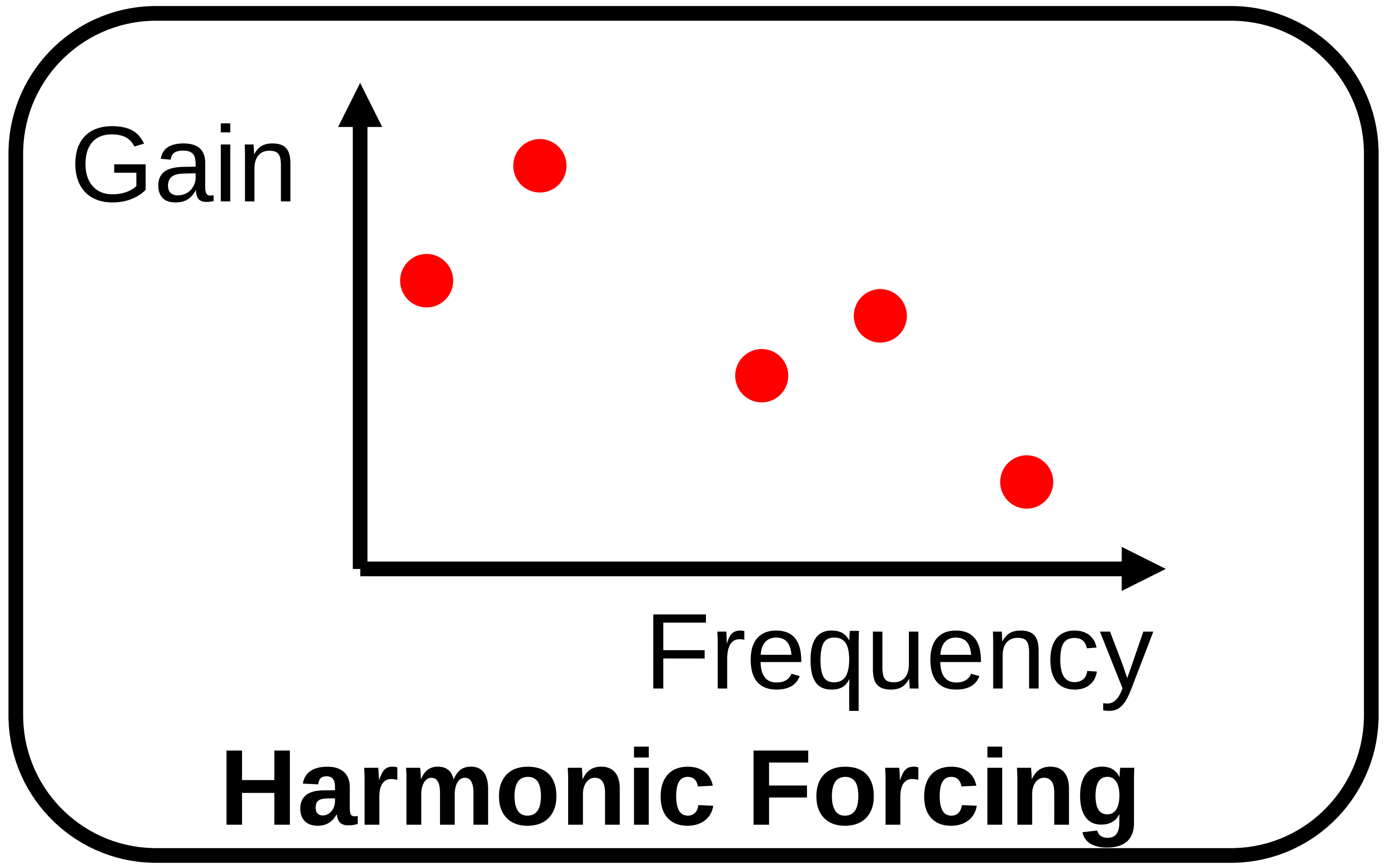
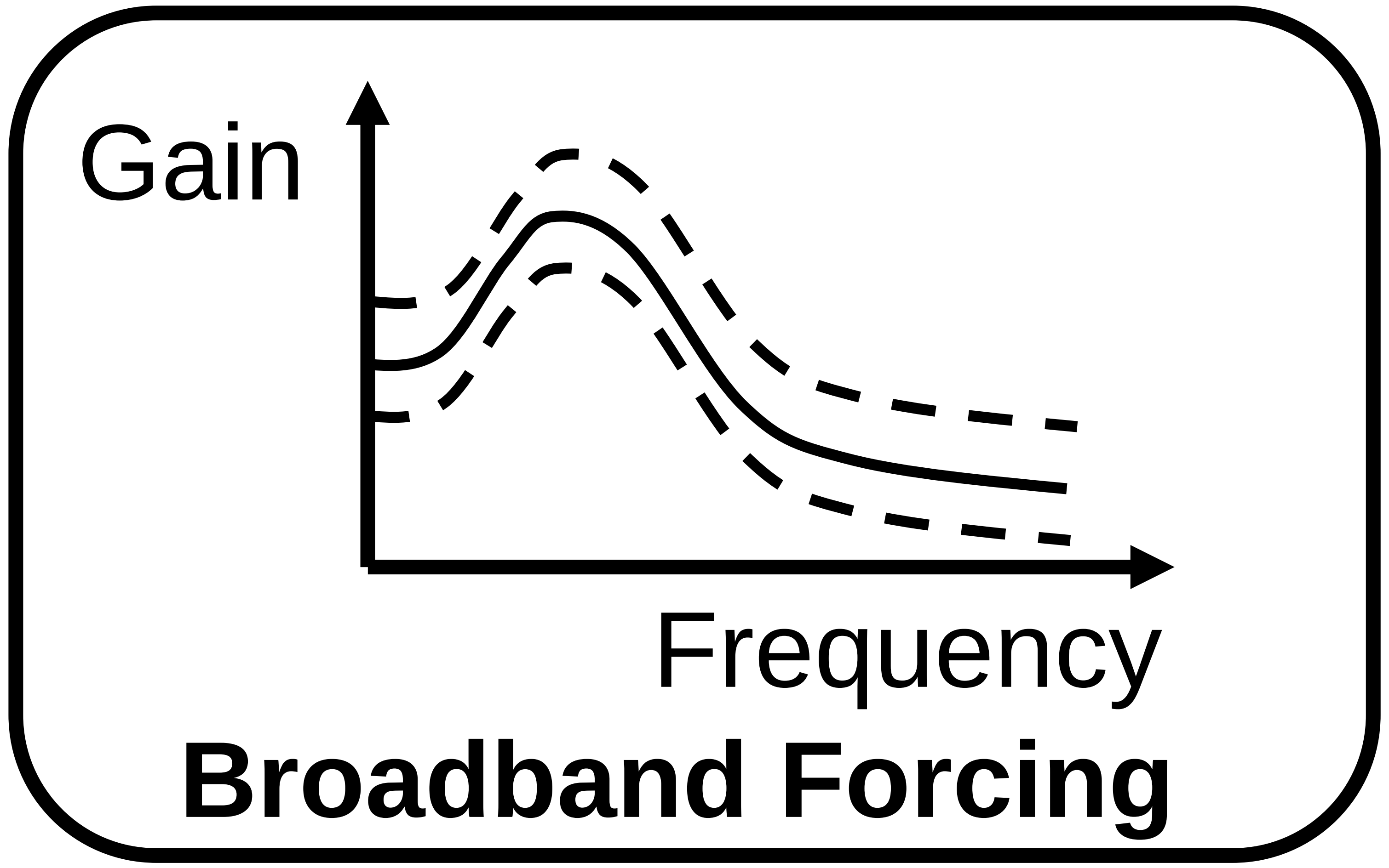
Combining two methods may yield more accurate and robust frequency response identification





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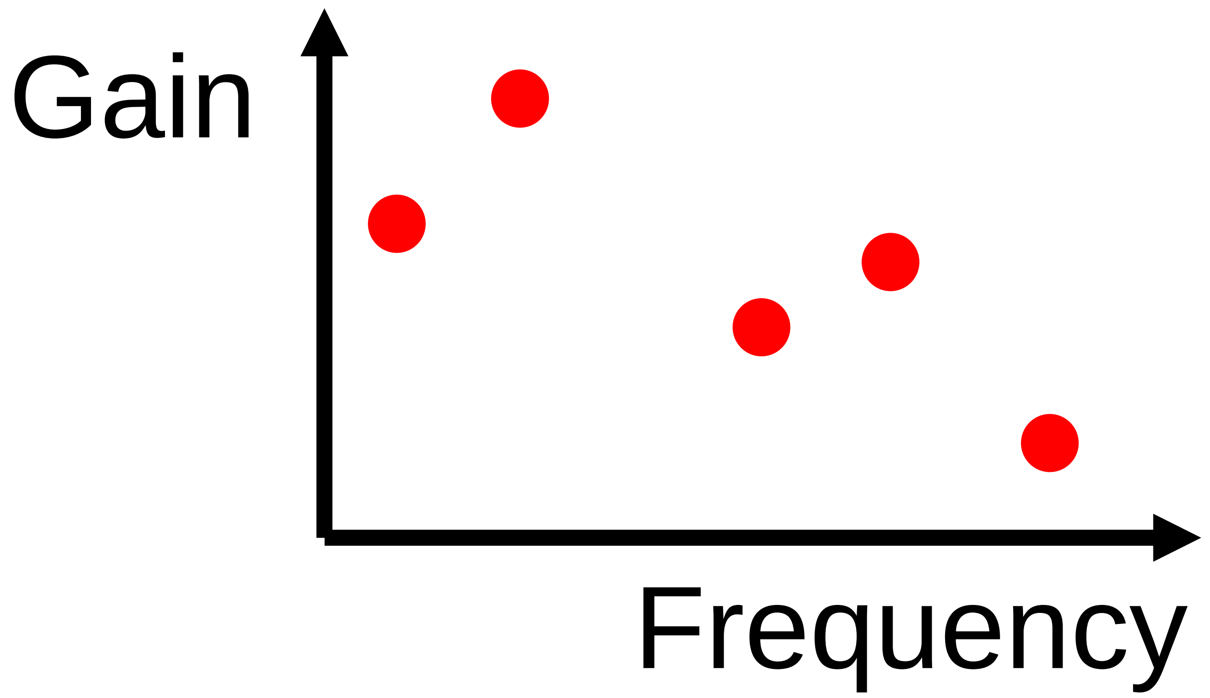

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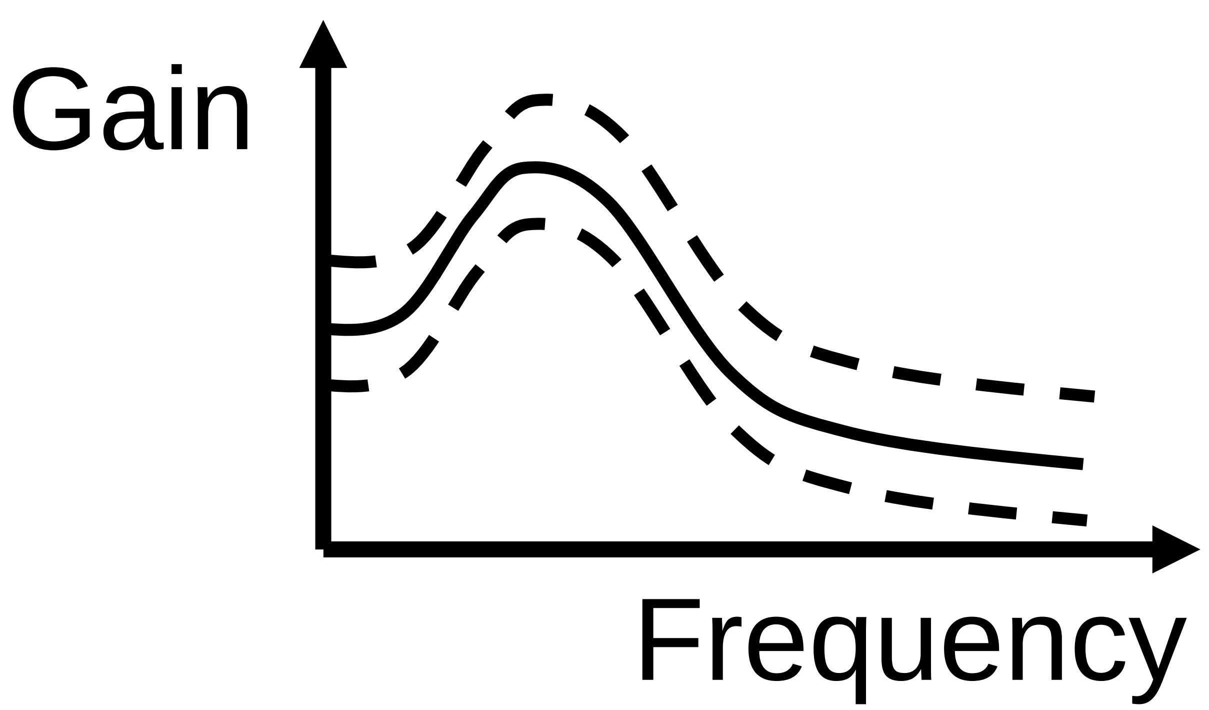

Broadband Forcing

→A short time
(1/4 of recommendation)

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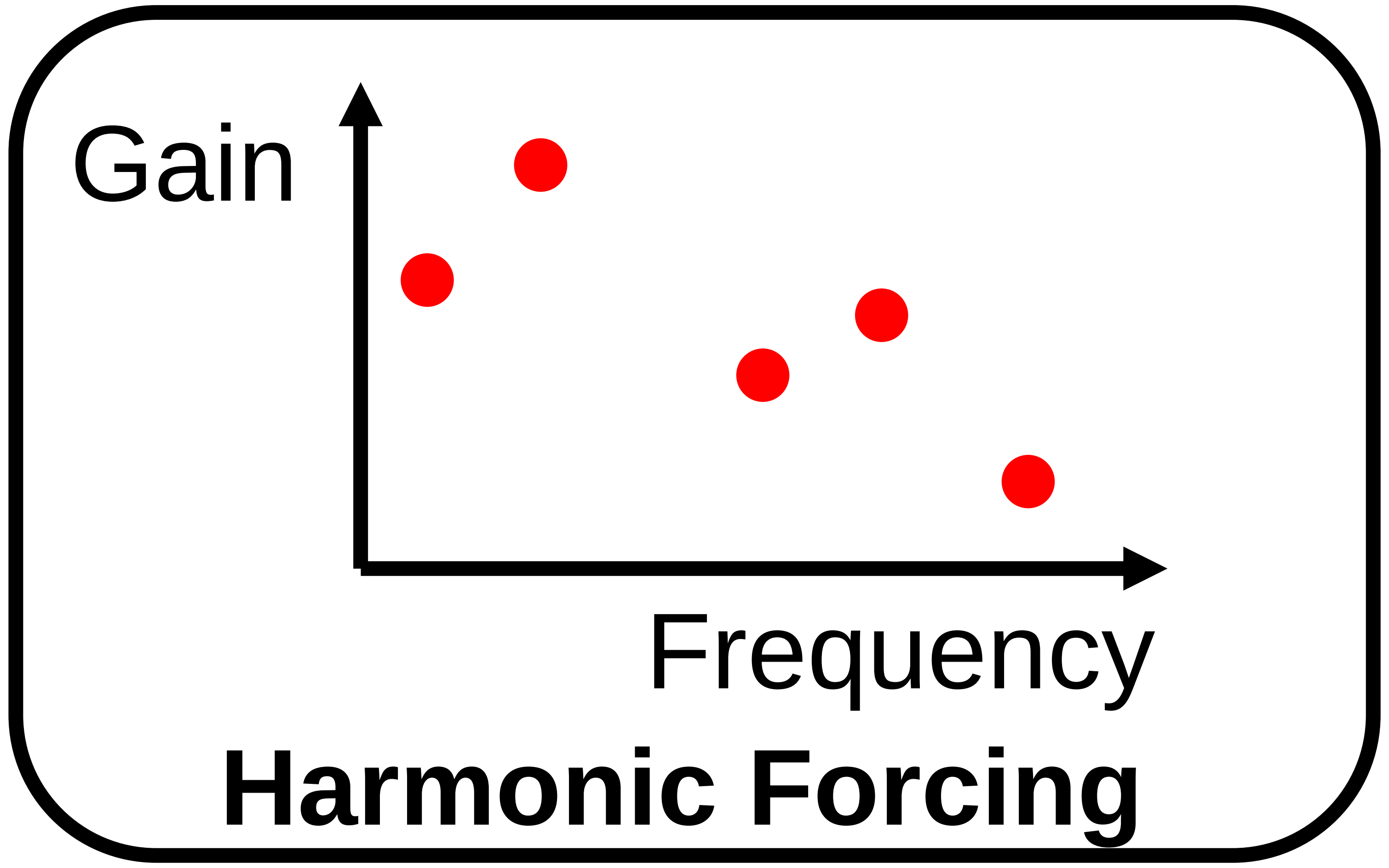

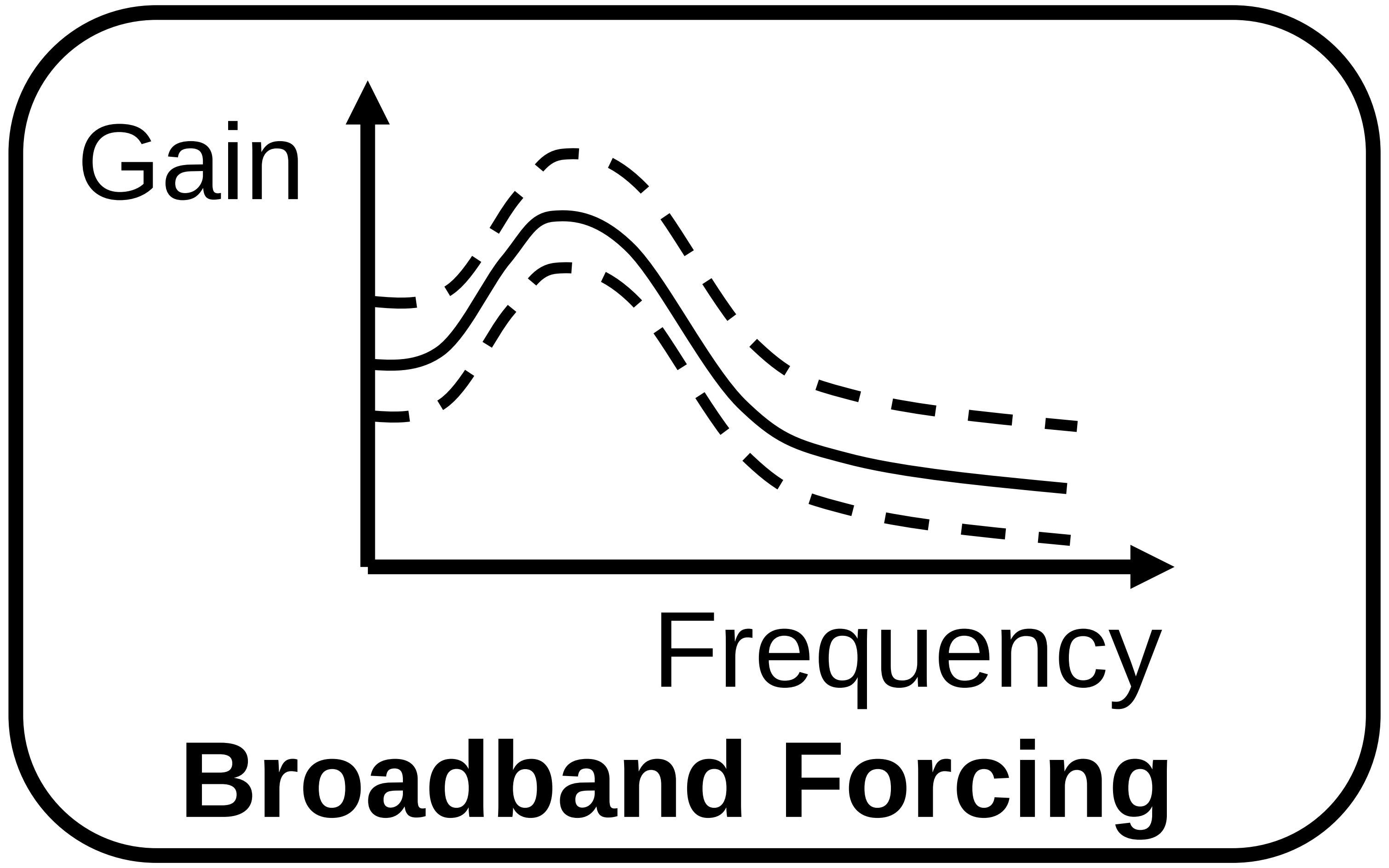

Low-fidelity

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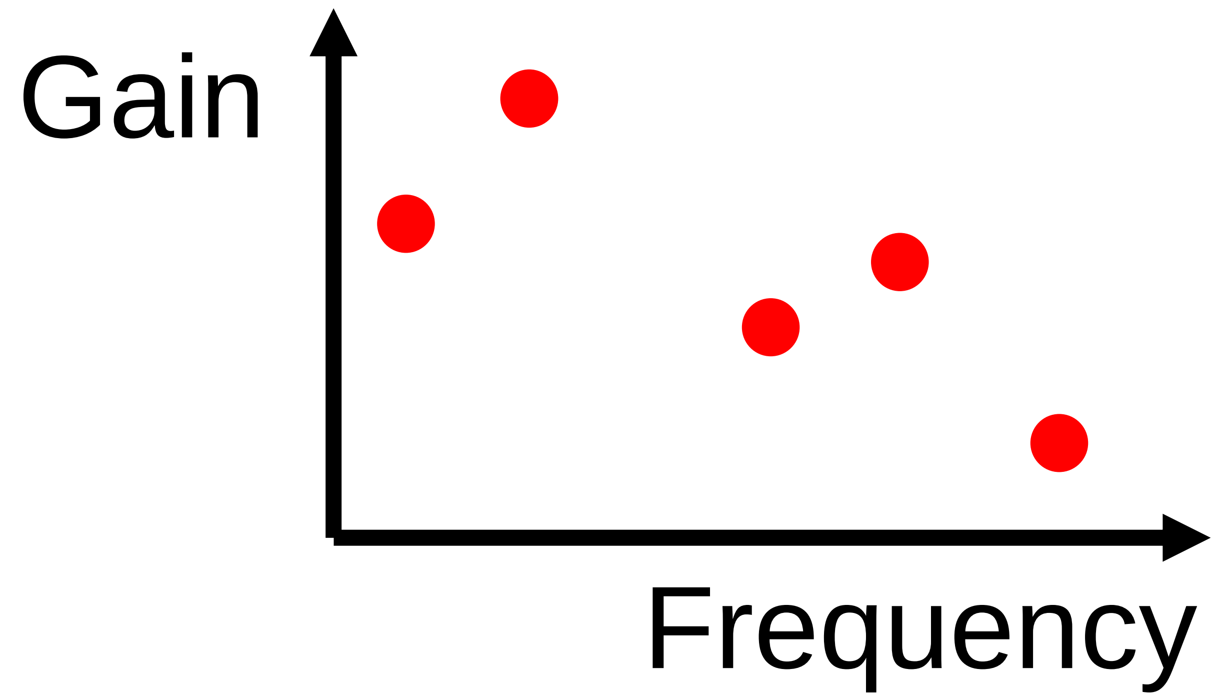

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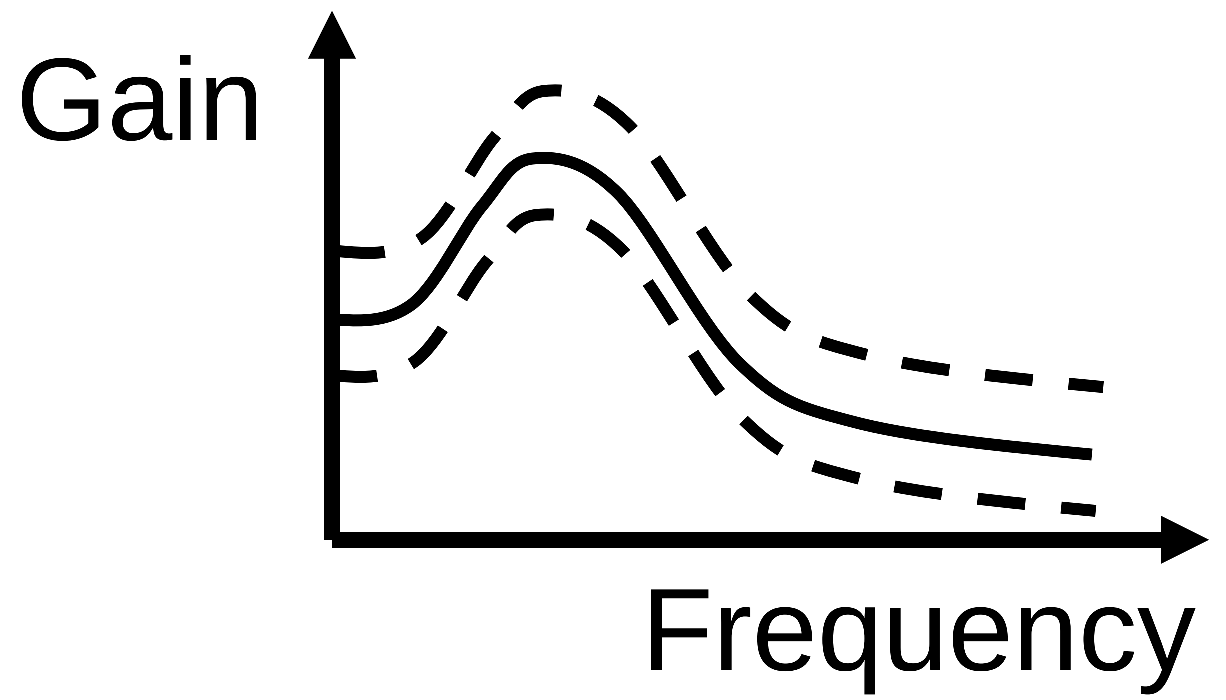
→ A few frequencies



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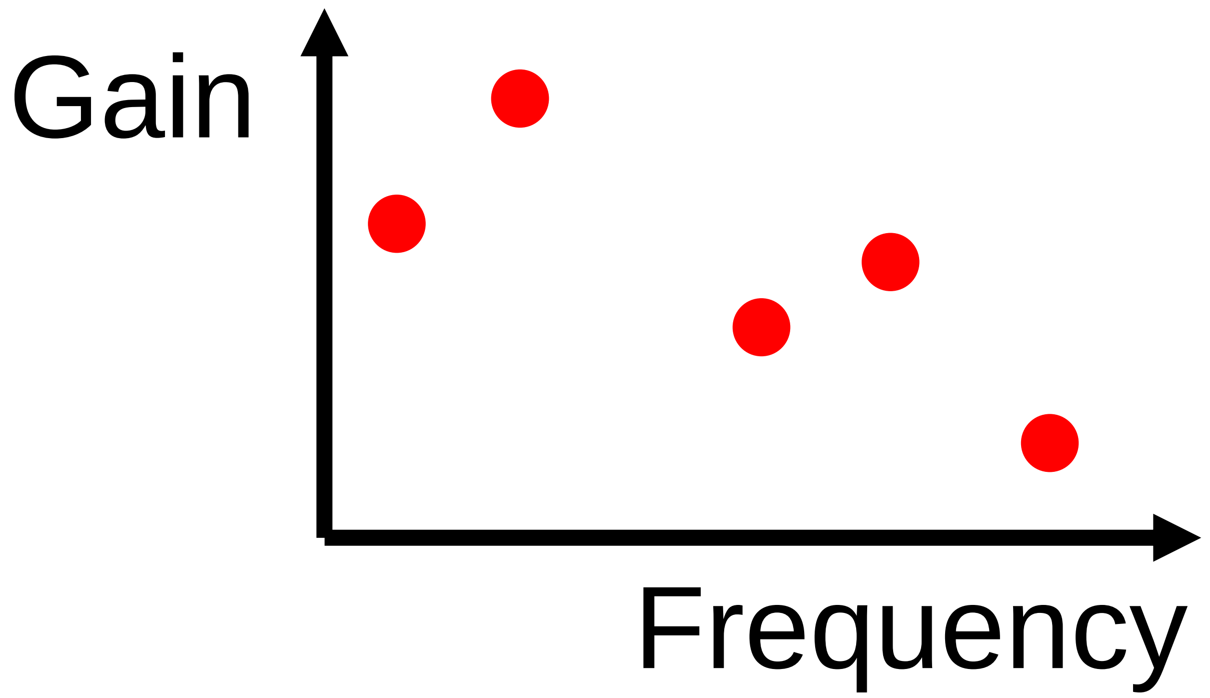

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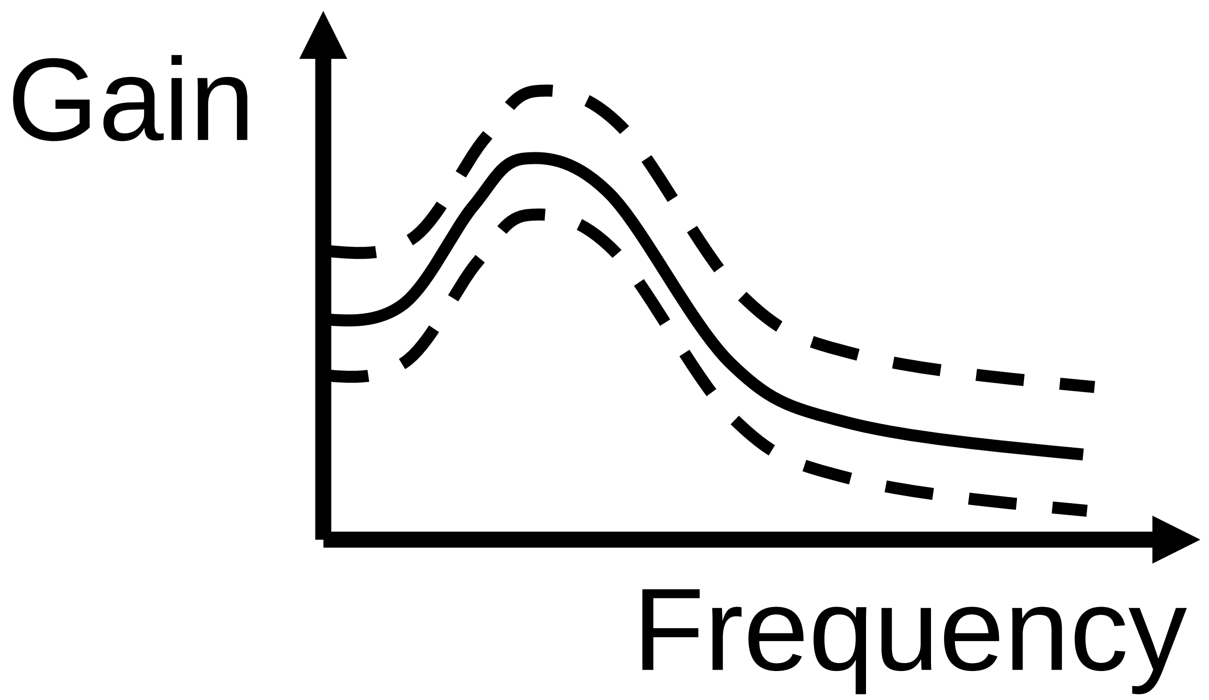

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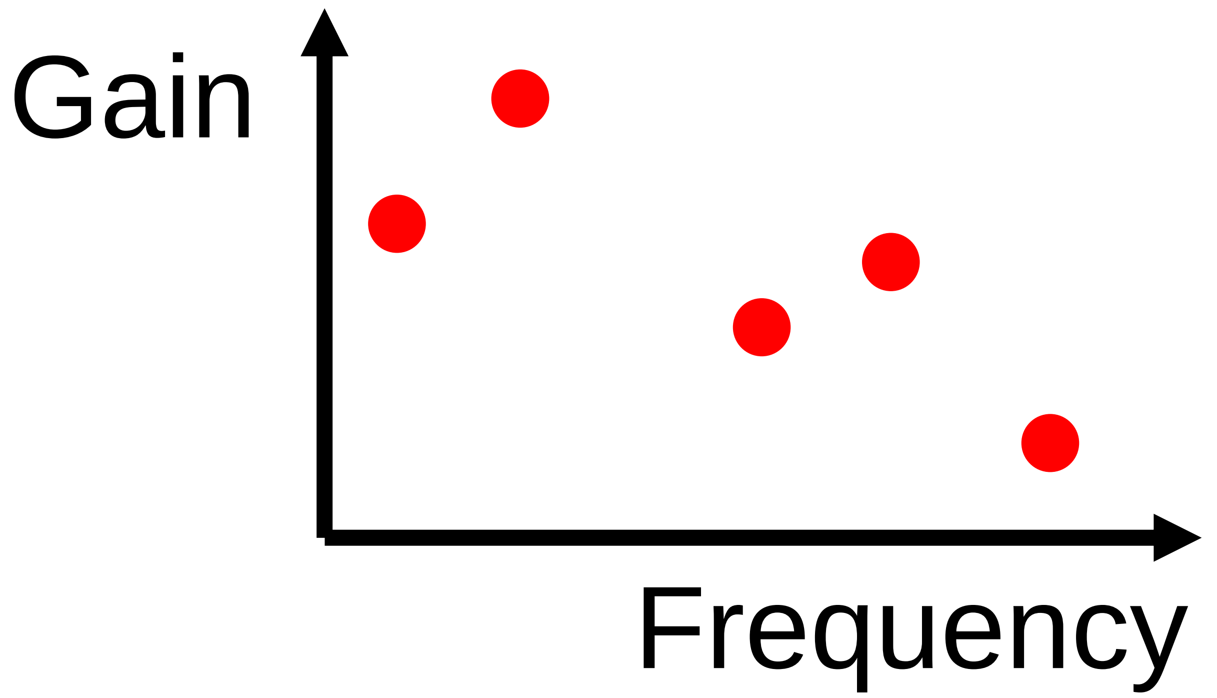

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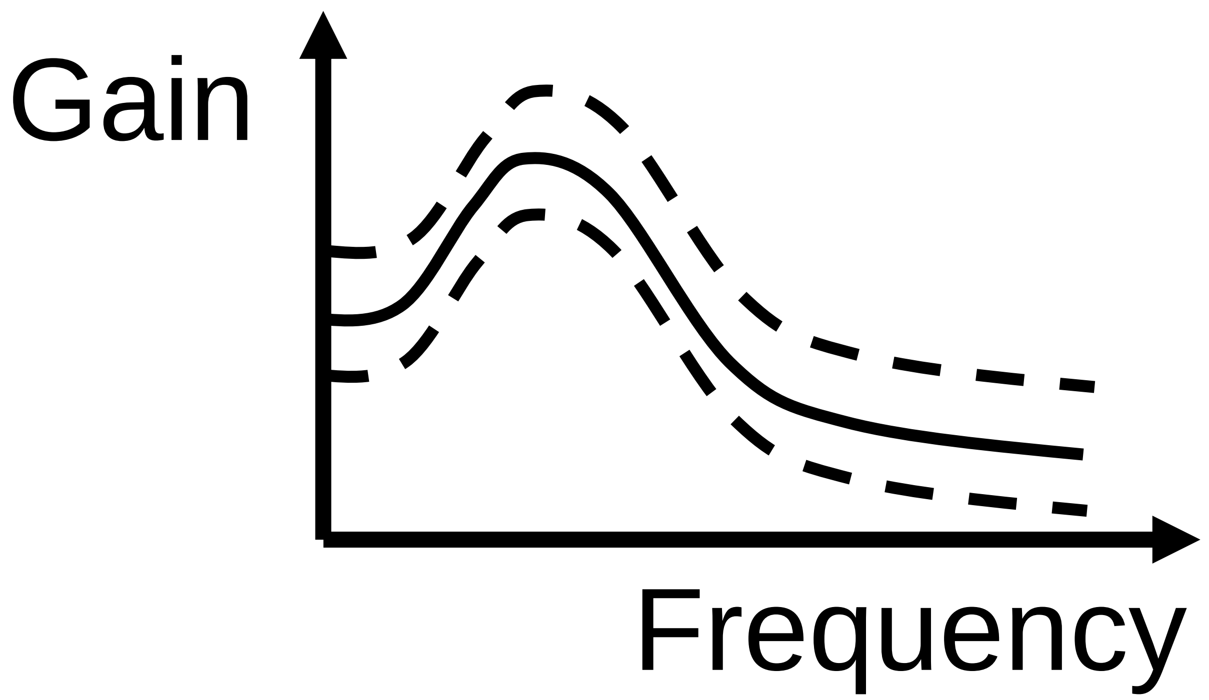

Broadband Forcing

→ A few frequencies

High-fidelity

Aggregate → Accurate? Robust?

Low-fidelity

→ A short time
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- Multi-fidelity Gaussian Process

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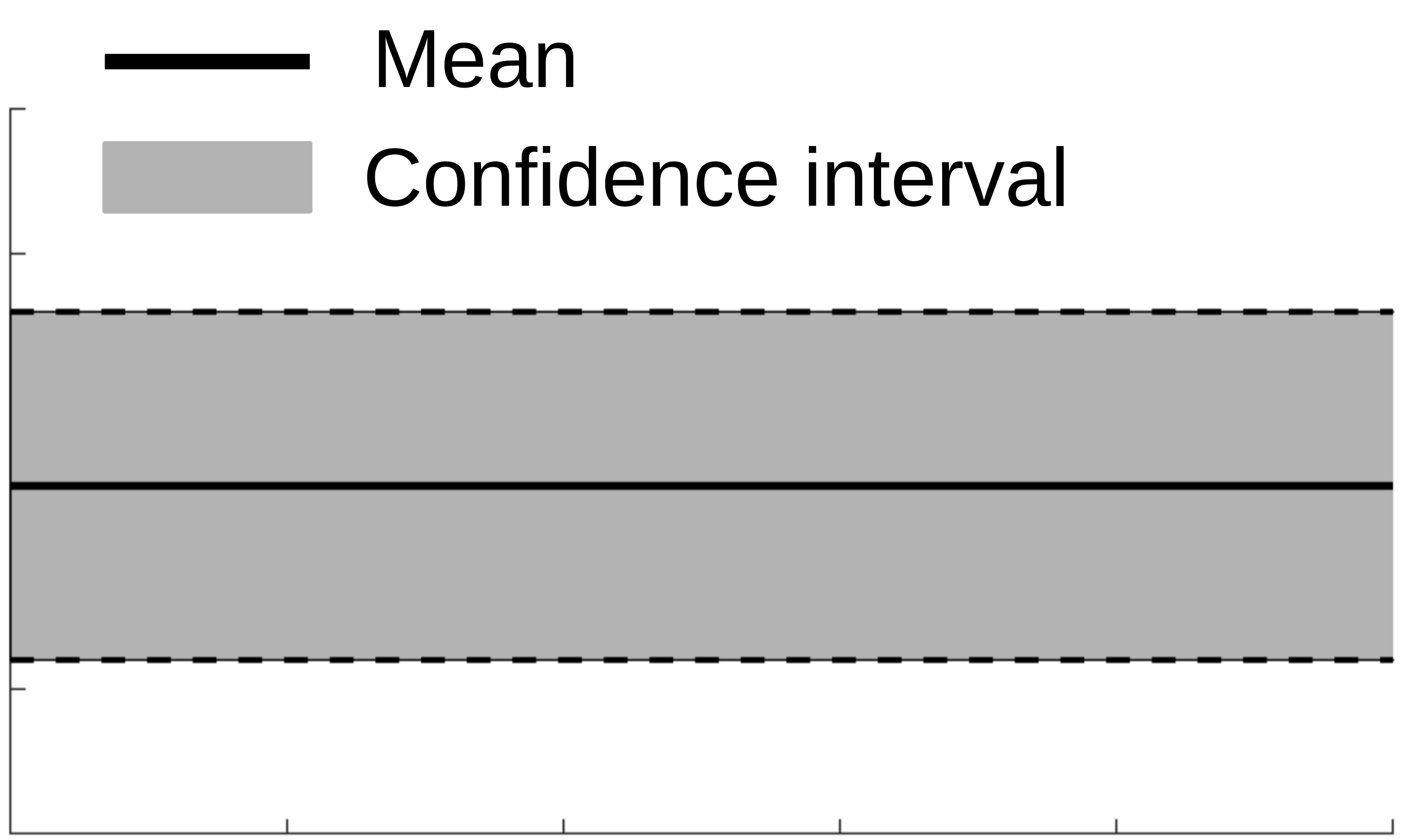
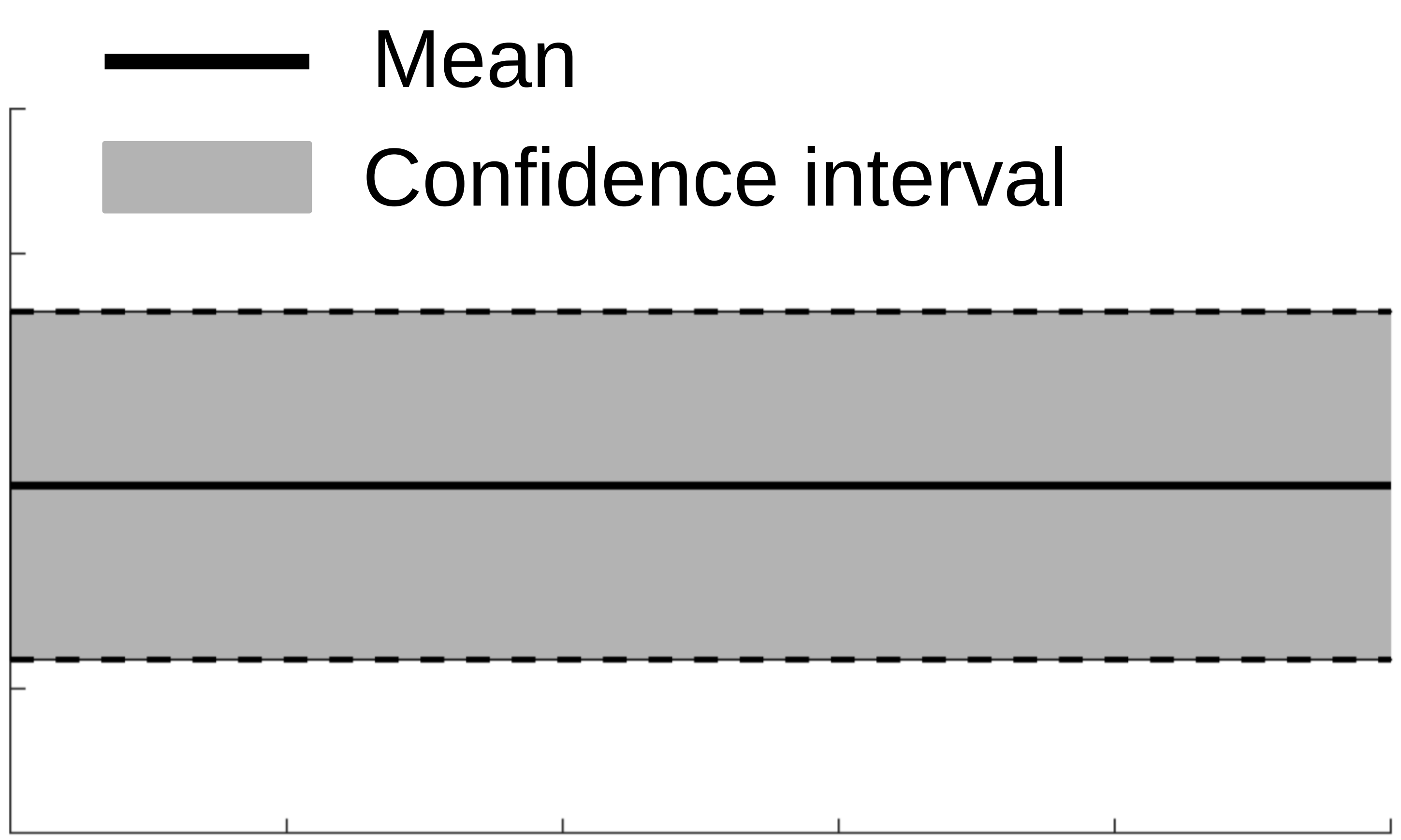
Gaussian Process uses unknown constant as its global trend



Prior

$$f(x) \sim \mathcal{GP}(\beta, k(x, x'))$$

Gaussian Process uses unknown constant as its global trend



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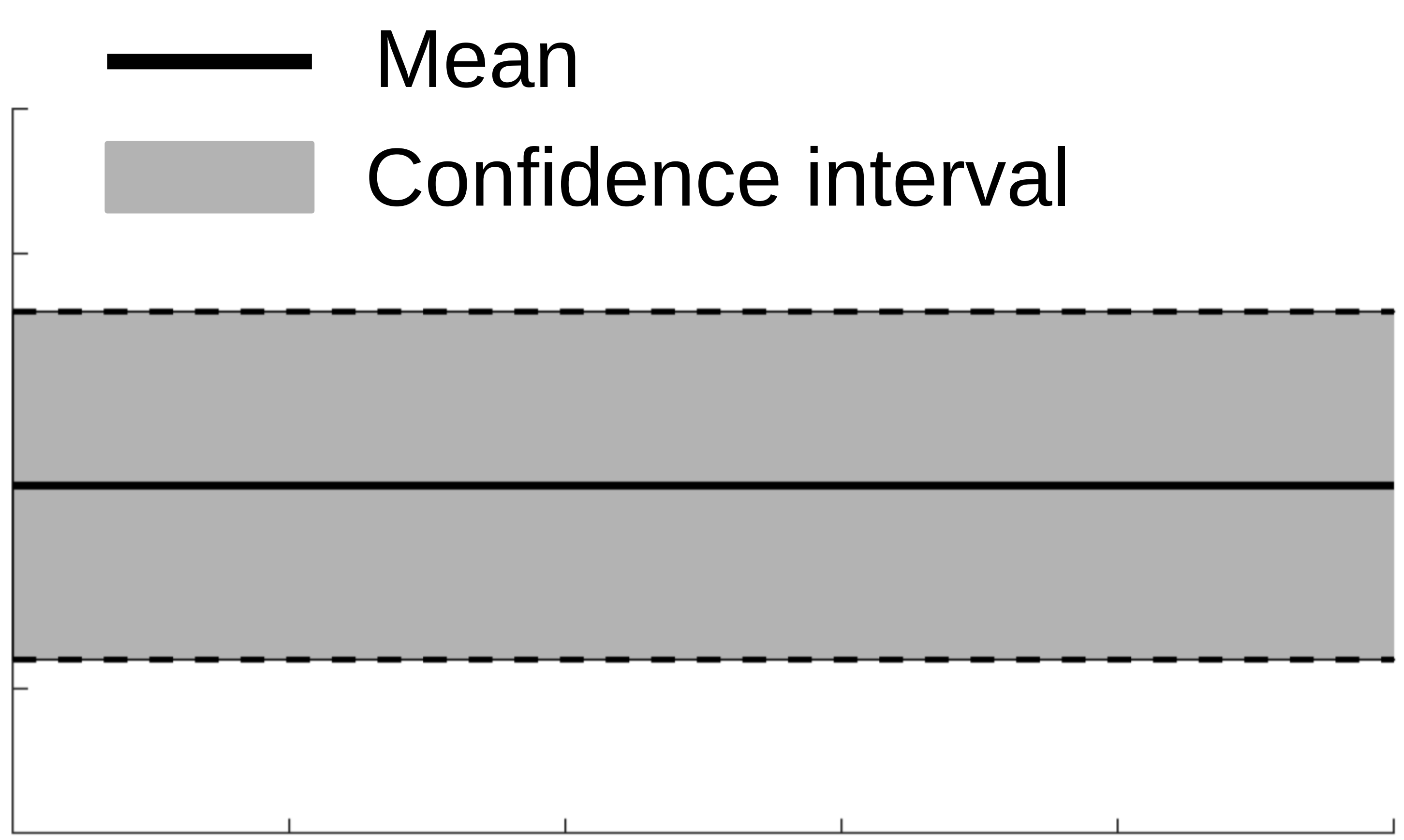

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$$f(x) \sim \mathcal{GP}(\beta, k(x, x'))$$

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$k(x, x') = \sigma^2 \text{exp}(-\theta |x - x'|^2)$: Kernel

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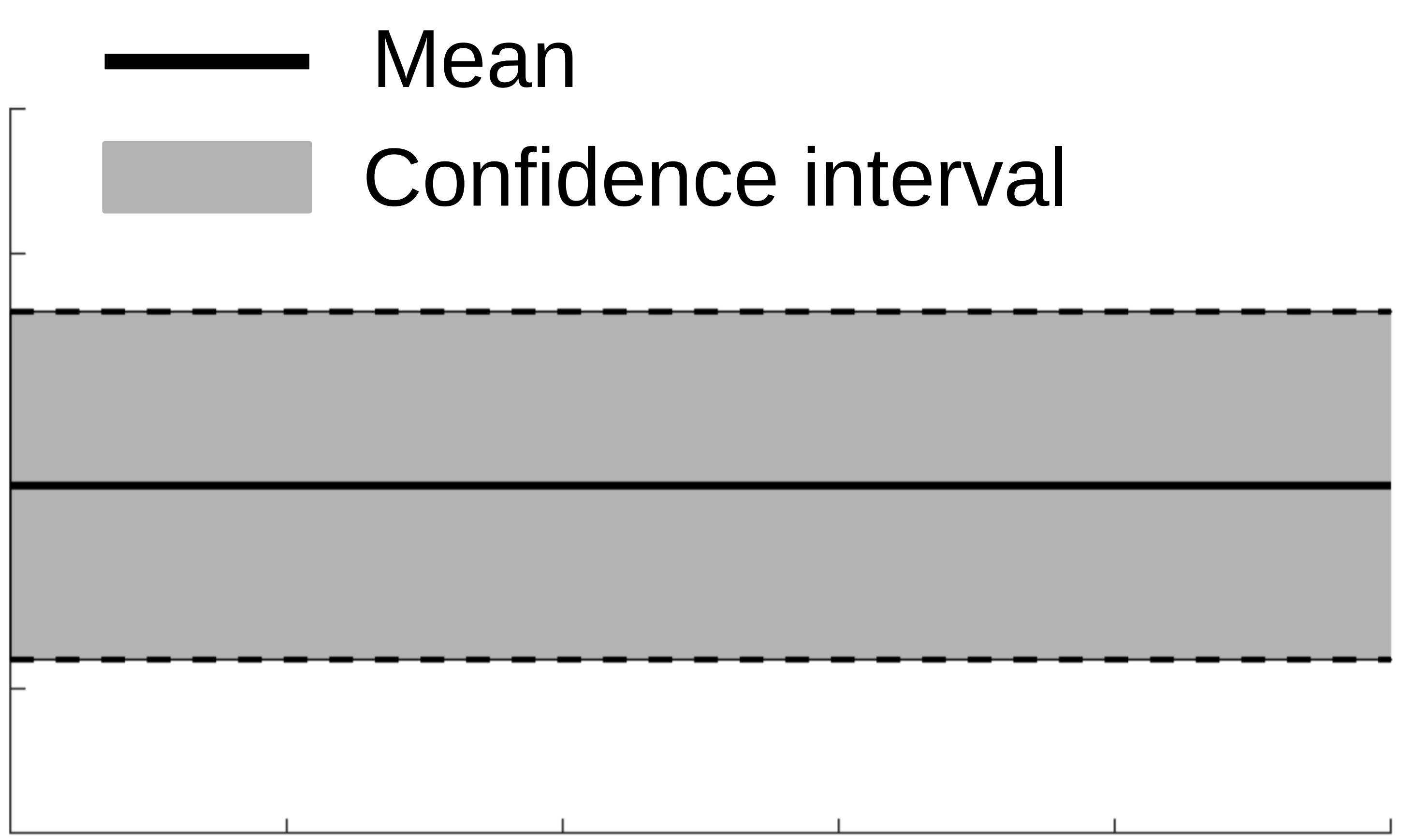

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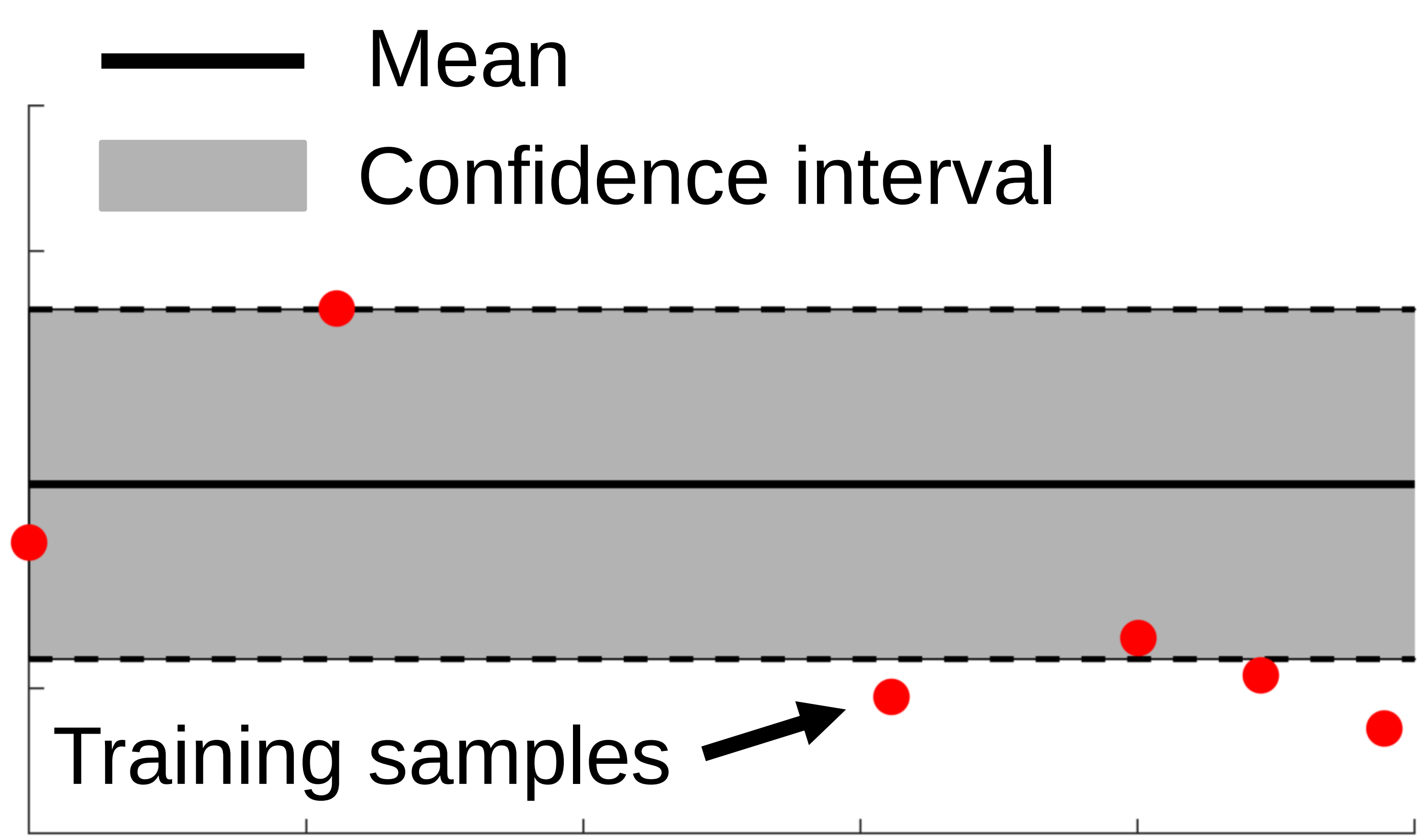

Data

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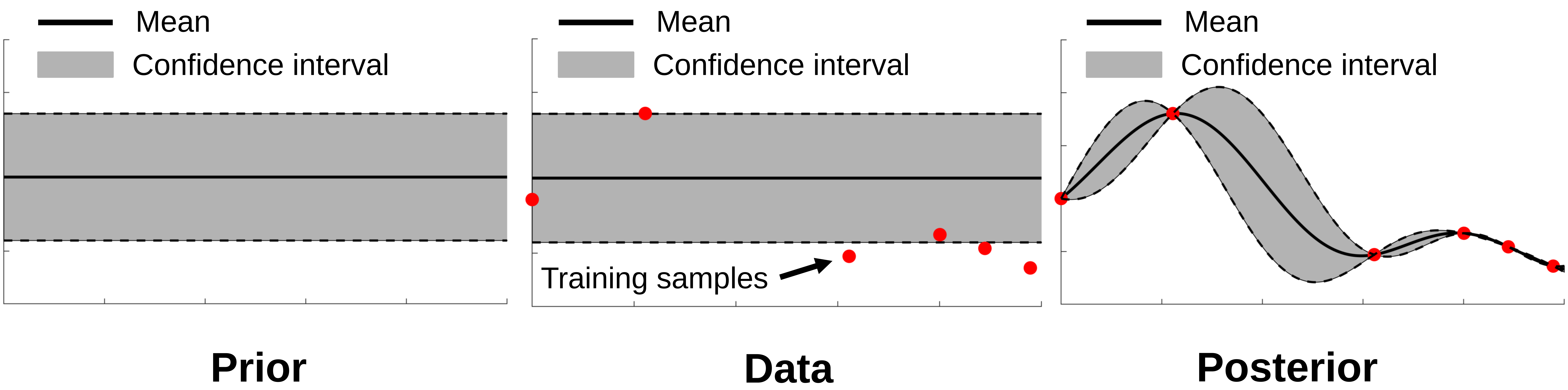

$$f(x) \sim \mathcal{GP}(\beta, k(x, x')) \quad \Longrightarrow \quad f^*(x) \sim \mathcal{GP}(m^*(x), k^*(x, x'))$$

β : Constant

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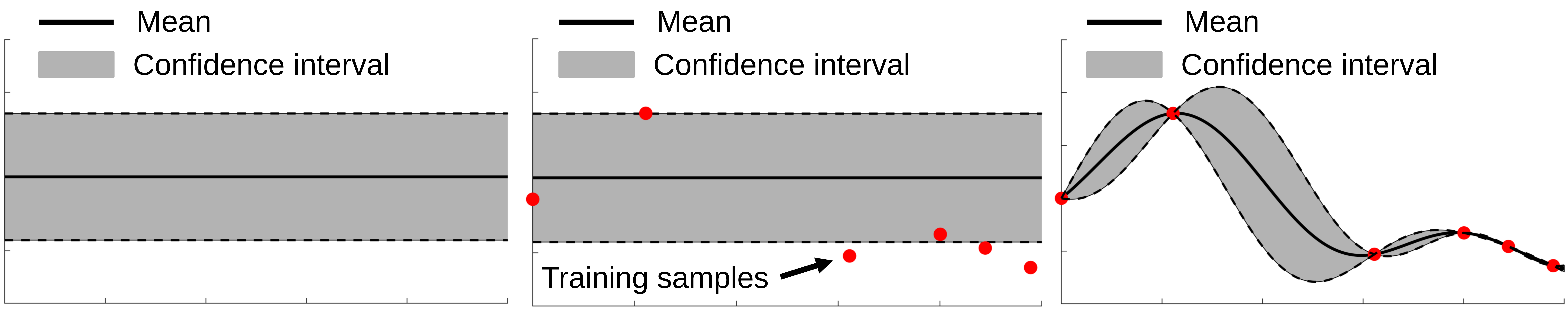
Multi-fidelity Gaussian Process uses low-fidelity results as its global trend



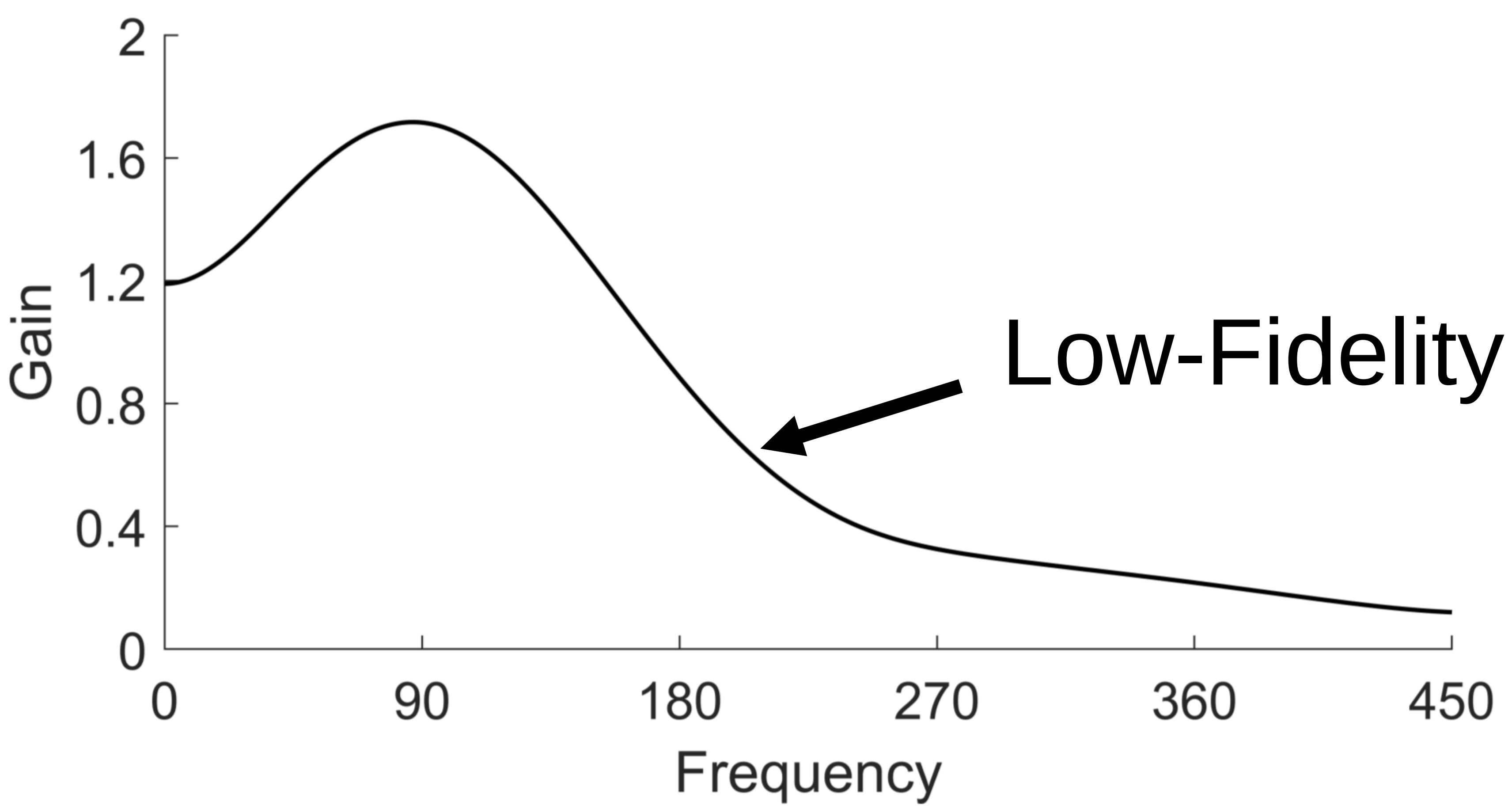
Prior **Data** **Posterior**

$$f(x) \sim \mathcal{GP}(\text{LoFi}(x), k(x, x')) \quad \Longrightarrow \quad f^*(x) \sim \mathcal{GP}(m^*(x), k^*(x, x'))$$

$$k(x, x') = \sigma^2 \text{exp}(-\theta |x - x'|^2)$$: Kernel

[3] Z. H. Han, S. Gortz, 2012, AIAA Journal.

Multi-fidelity Gaussian Process is employed to fuse results from both fidelities



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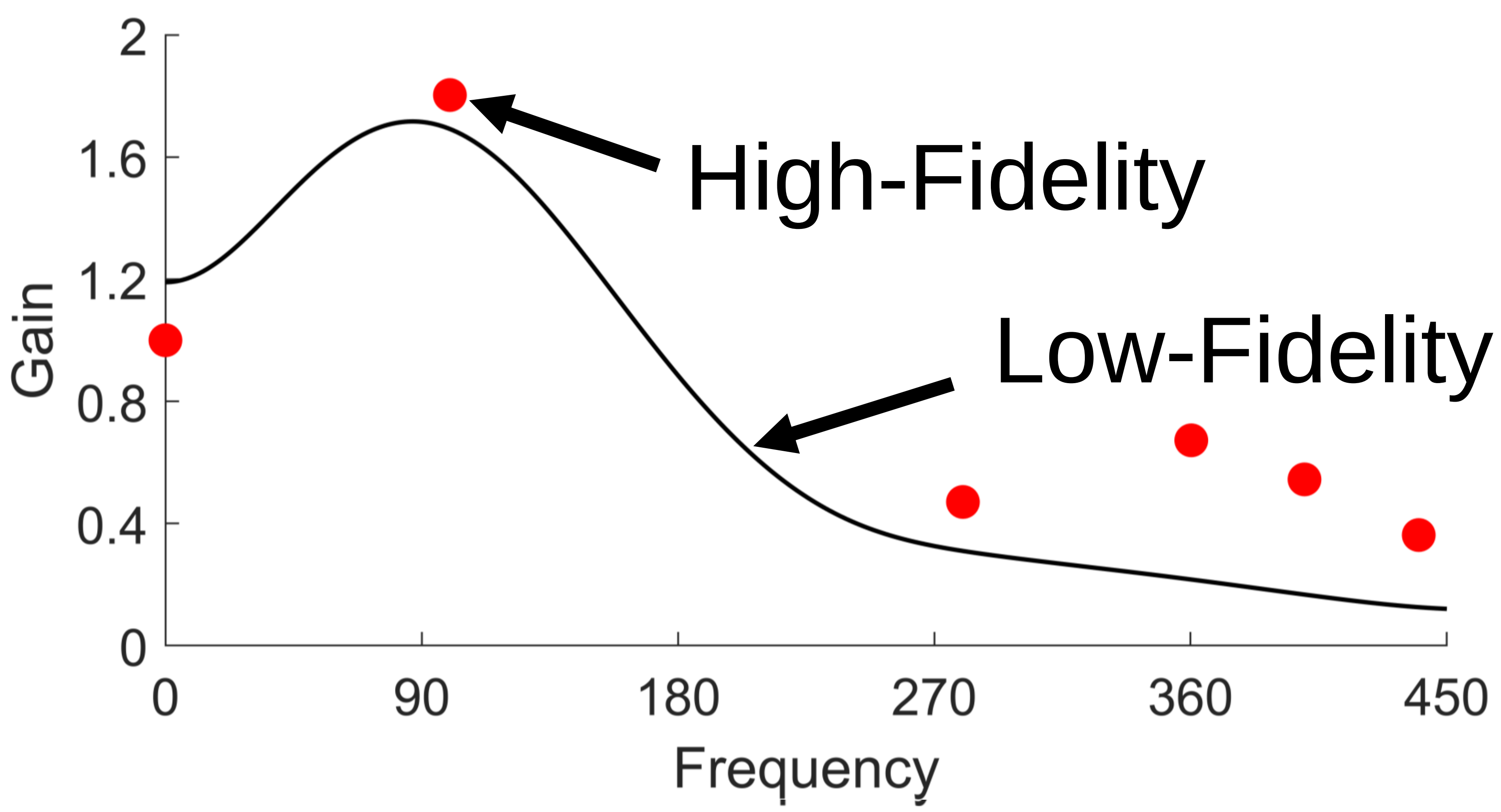

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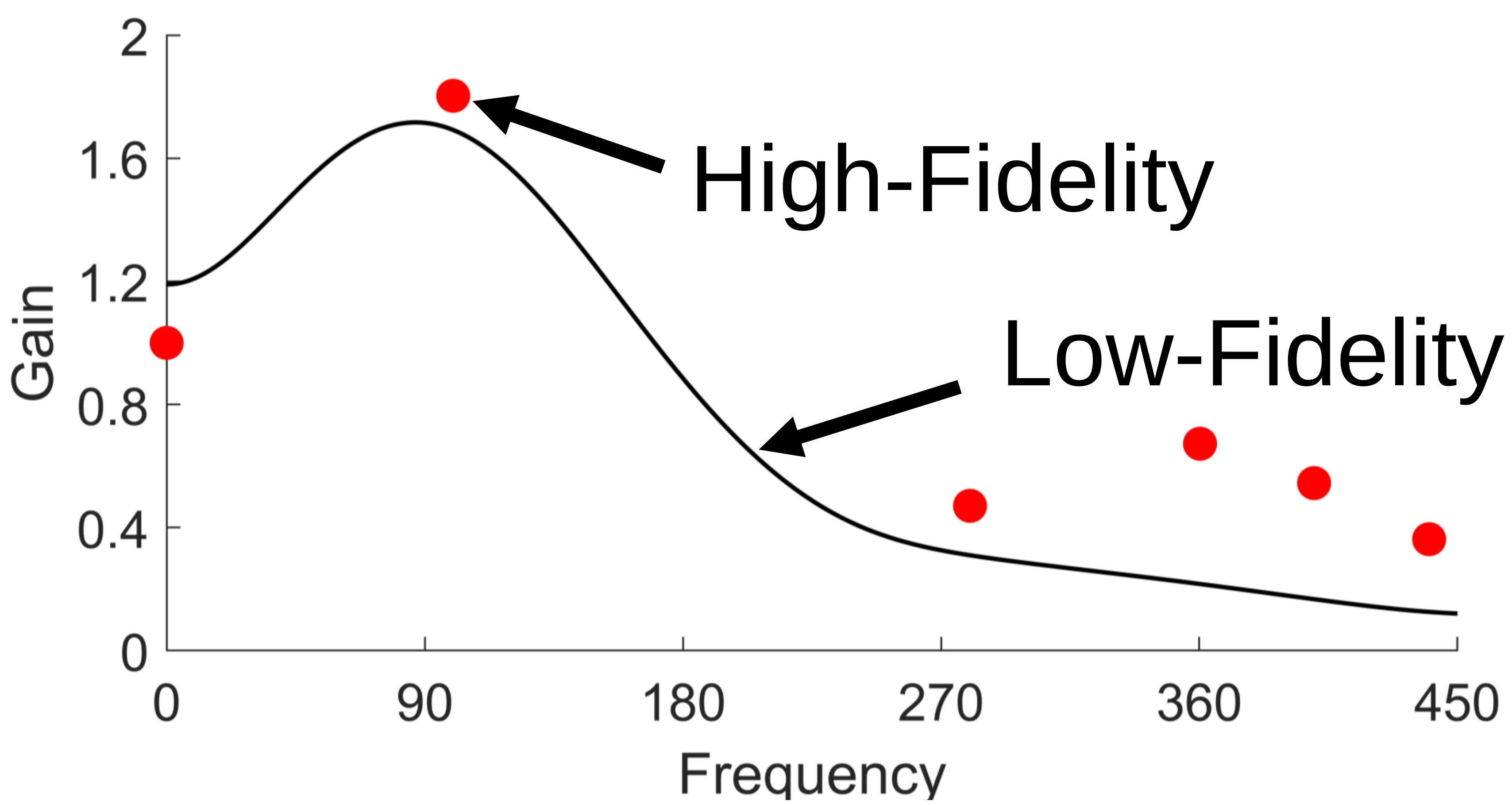

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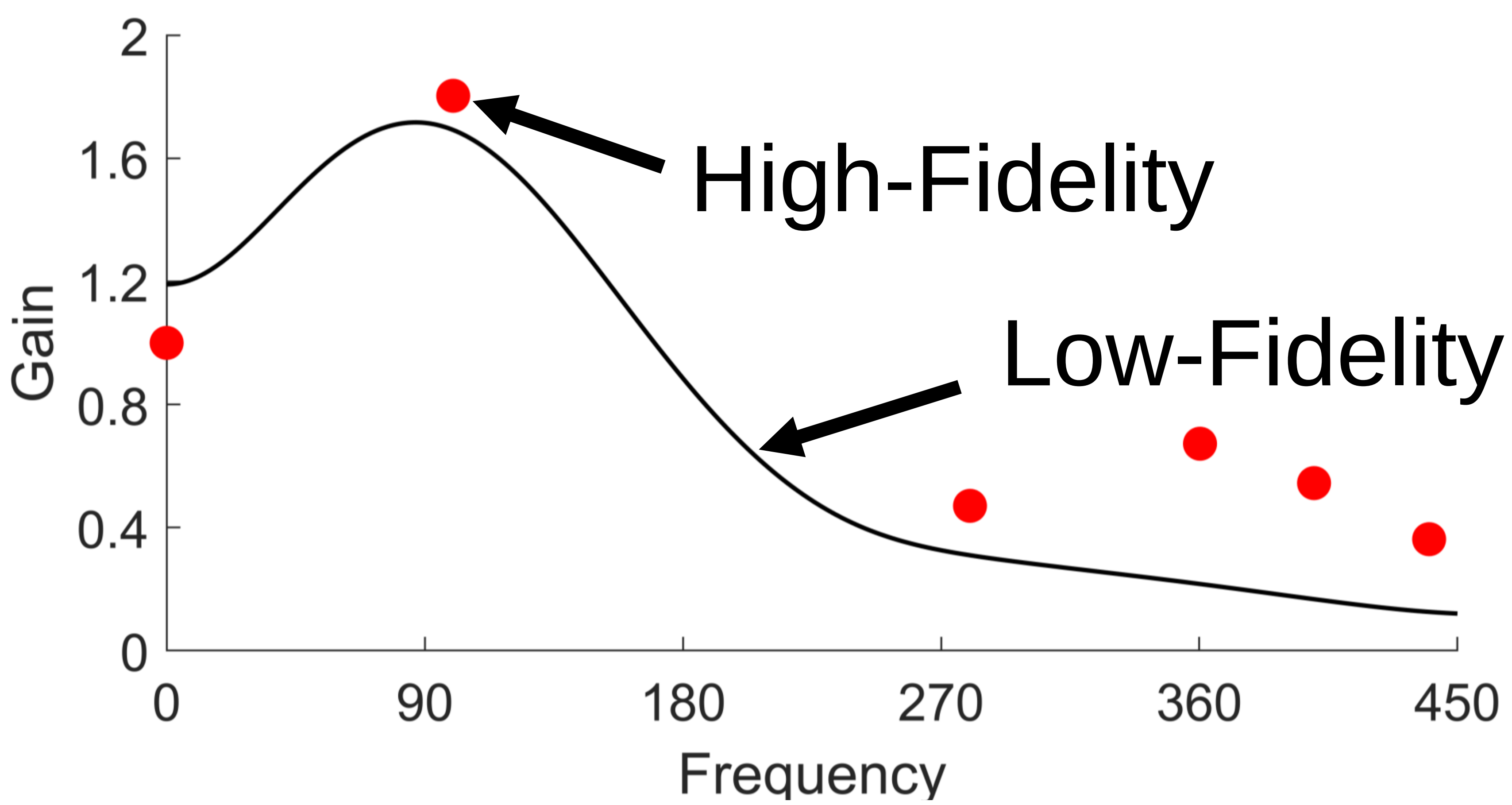
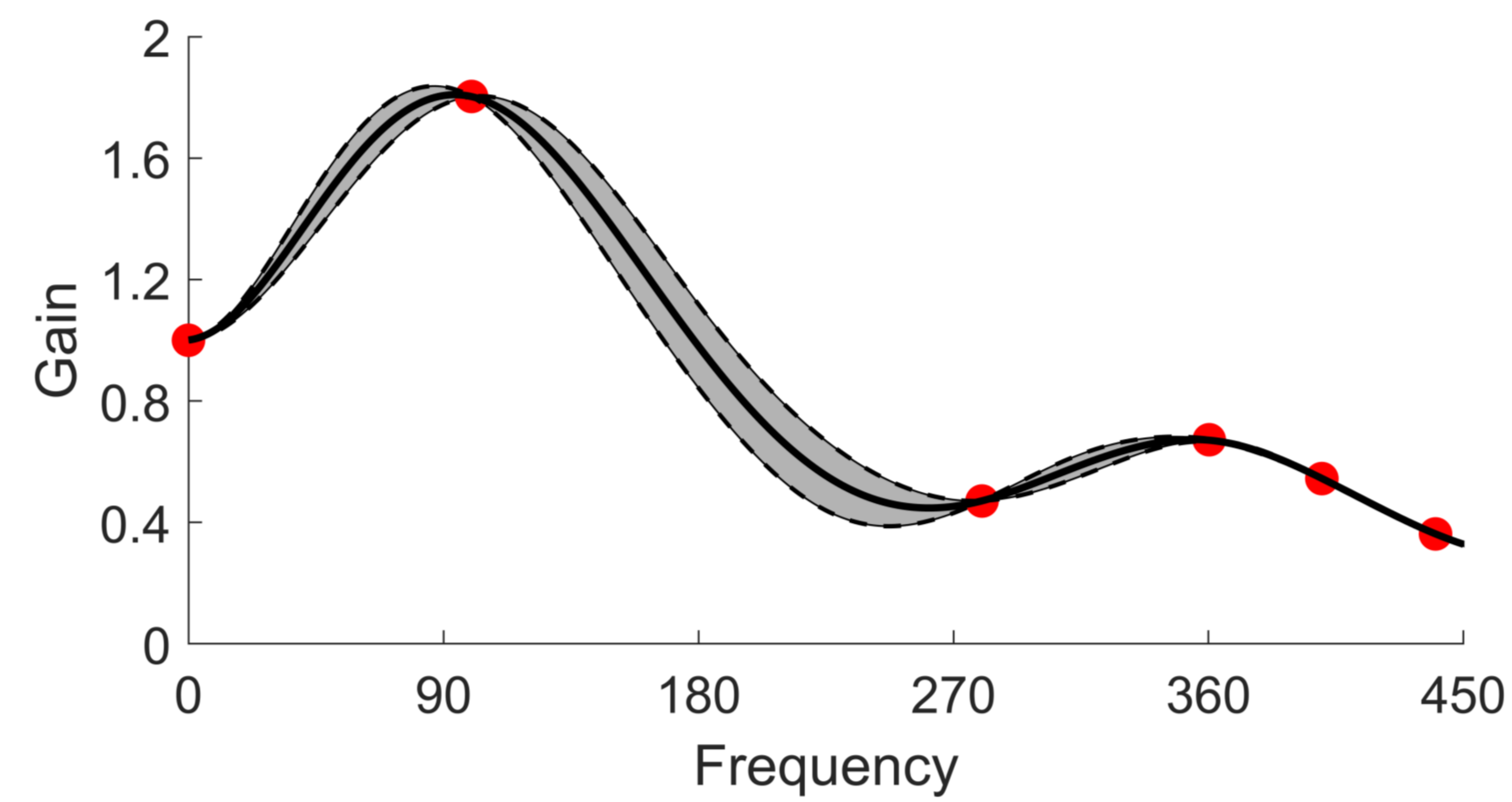
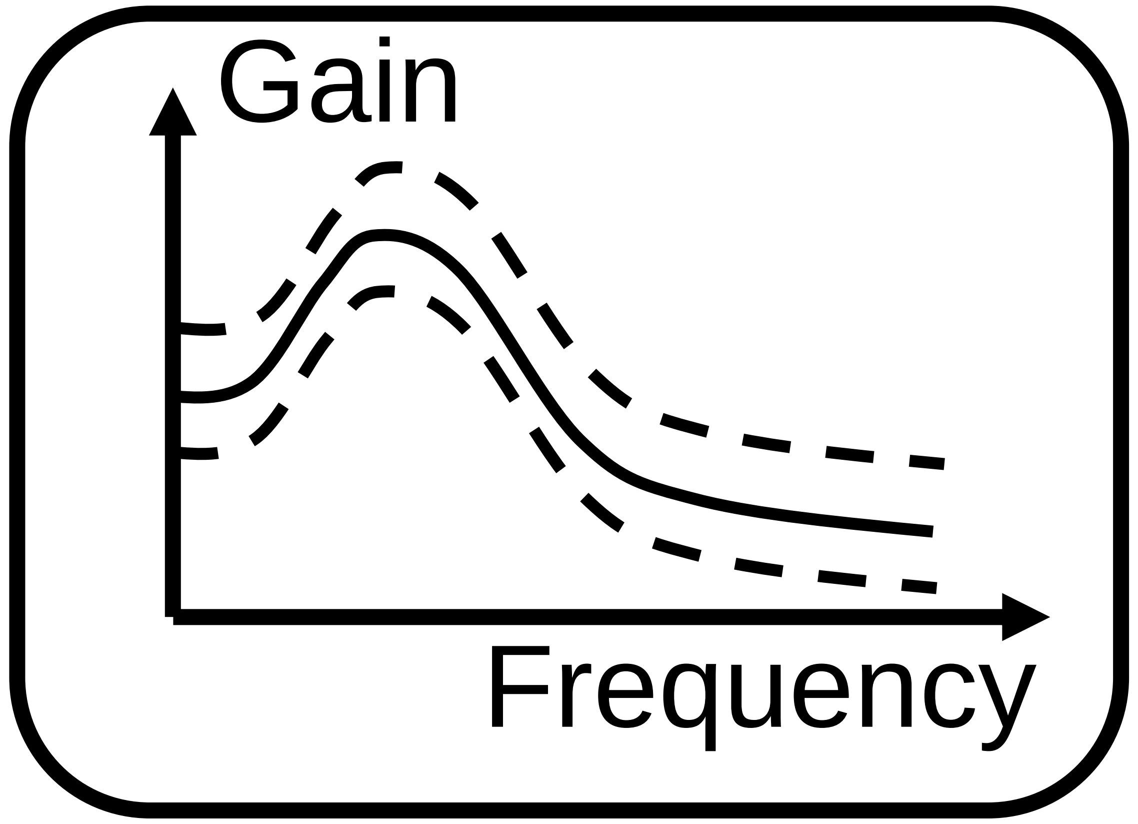




$$f(x) \sim \mathcal{GP}(\text{LoFi}(x), k(x, x')) \quad \Longrightarrow \quad f^*(x) \sim \mathcal{GP}(m^*(x), k^*(x, x'))$$

We propose a bootstrapping procedure to account for uncertainties from both fidelities



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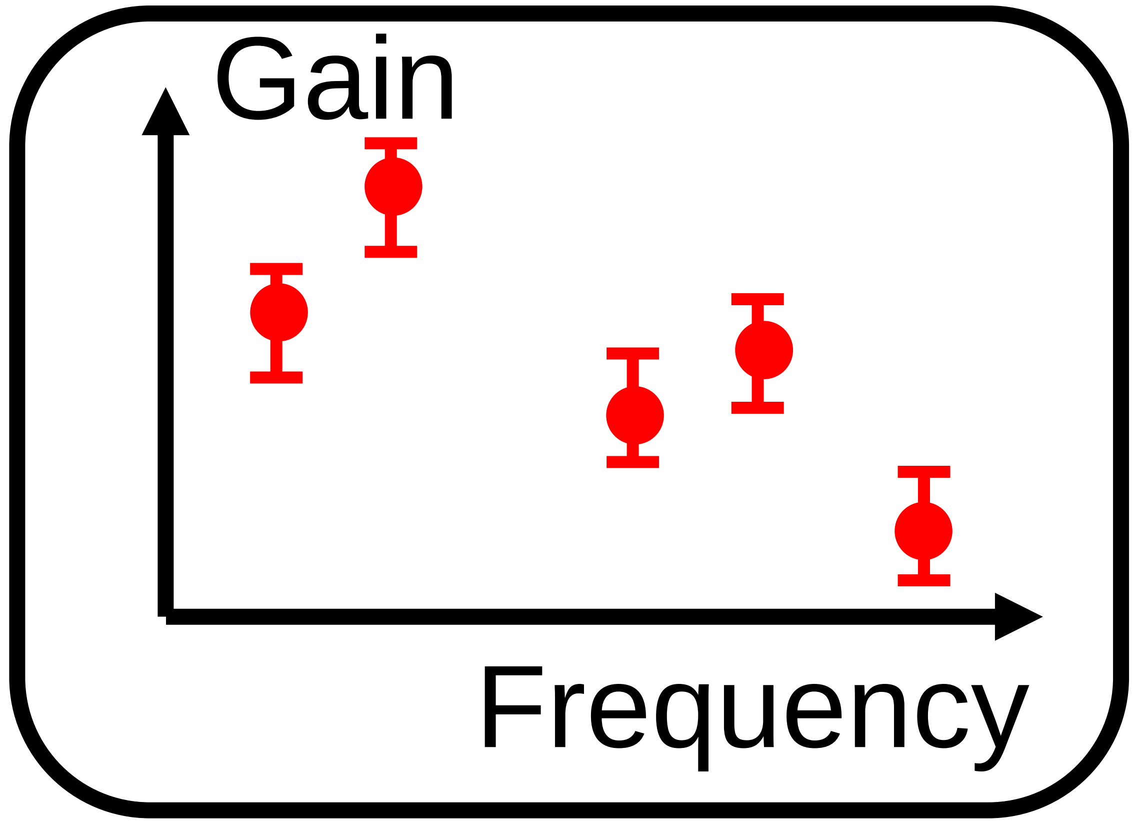

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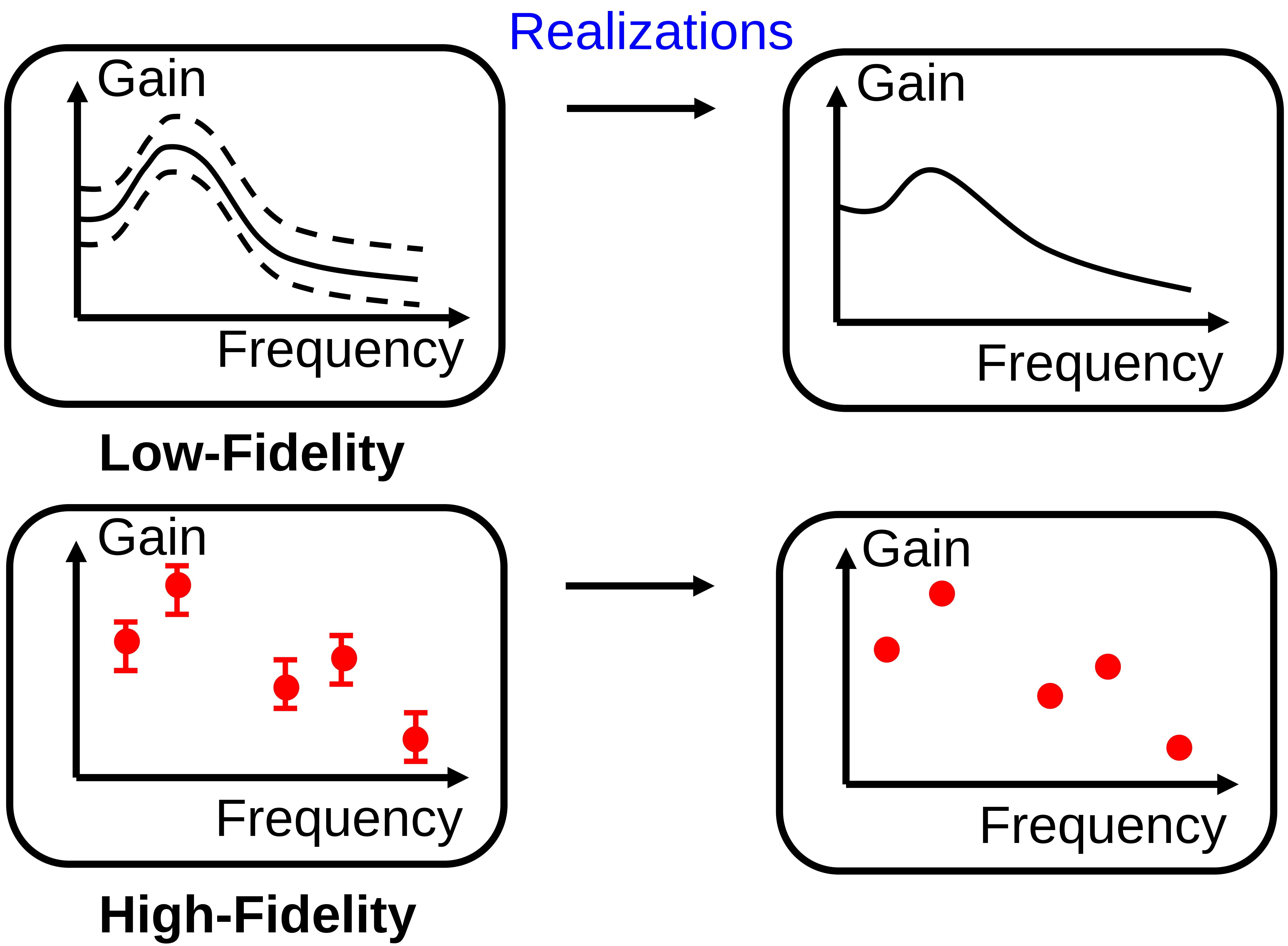

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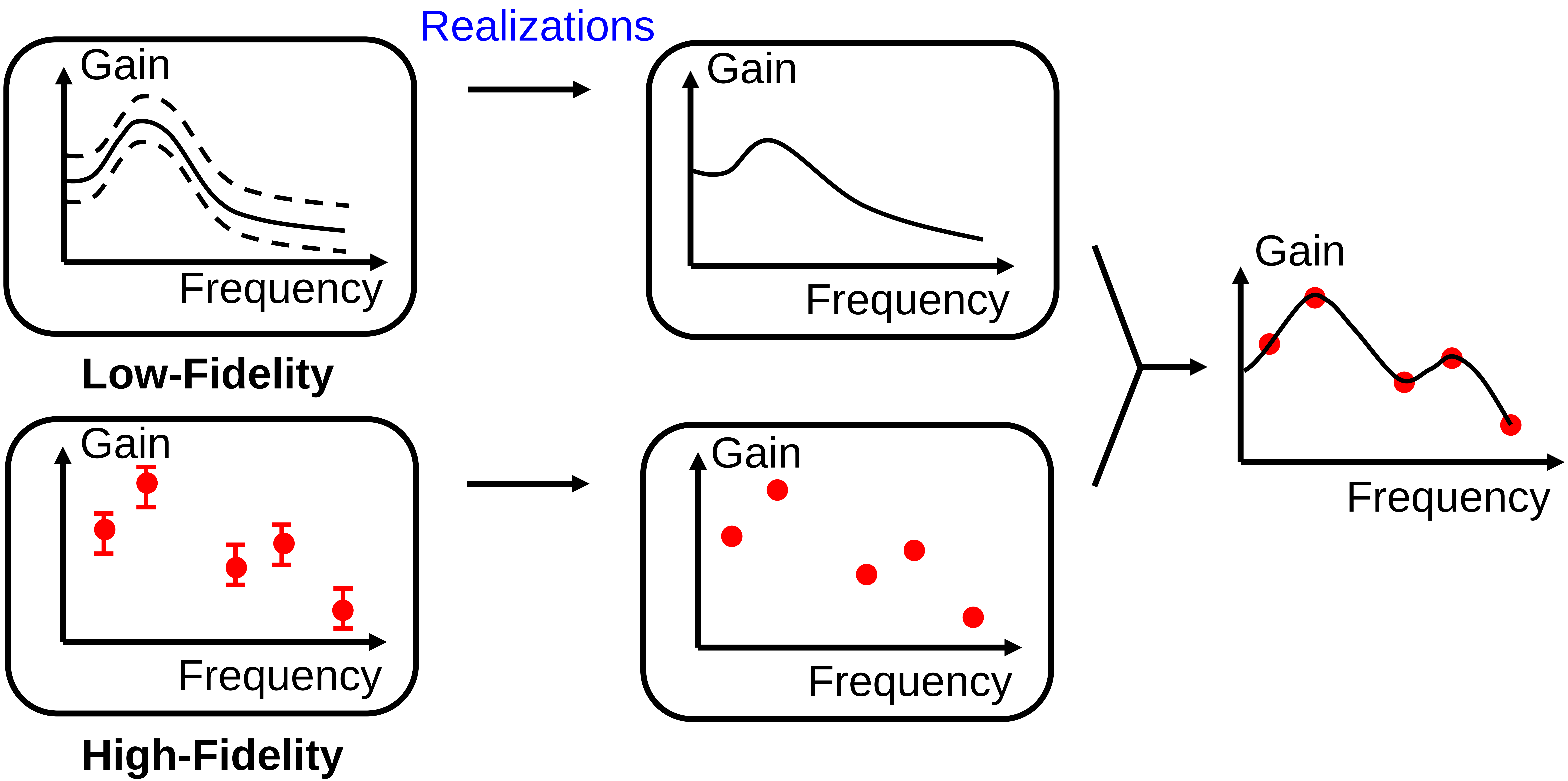

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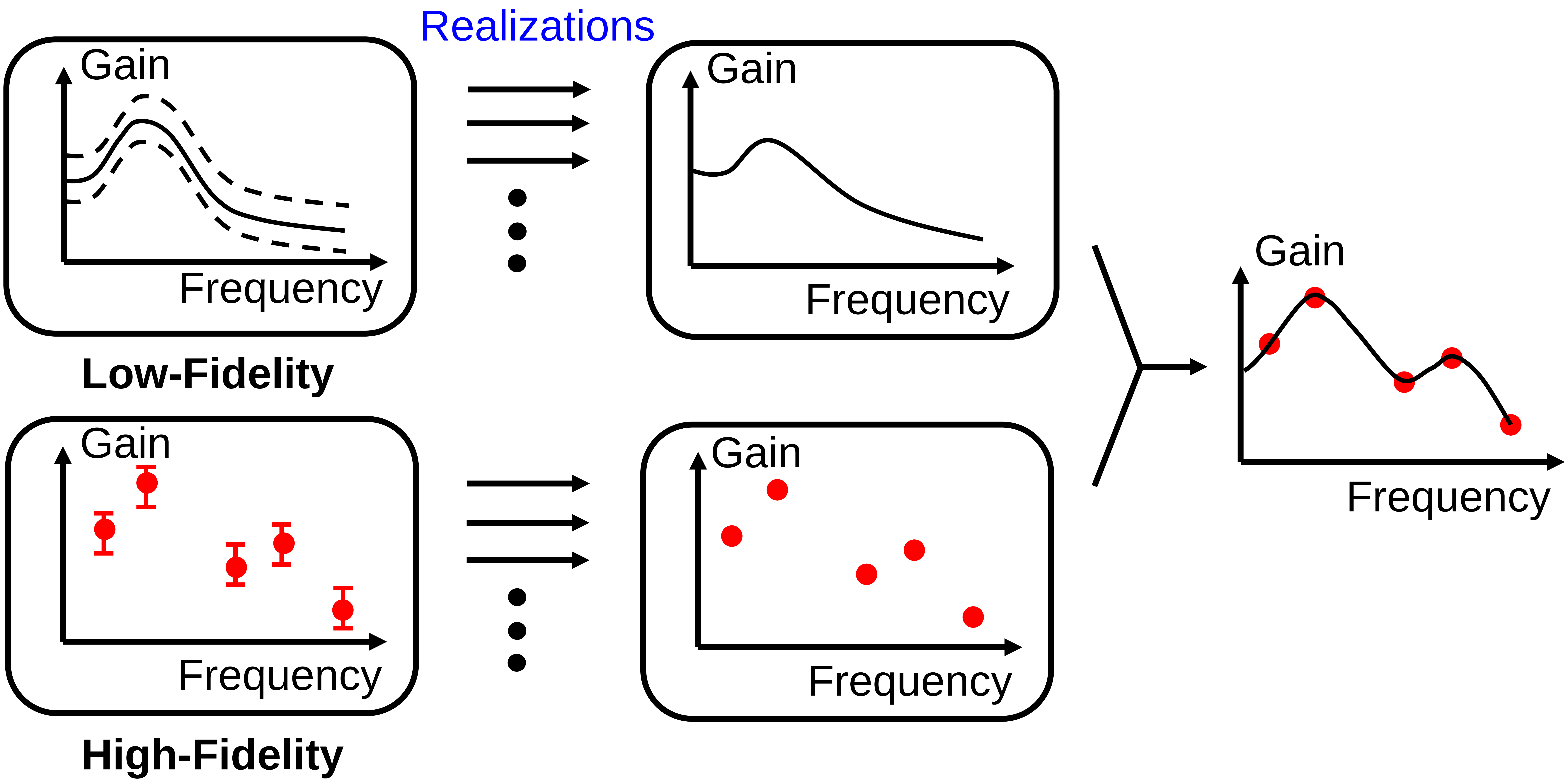

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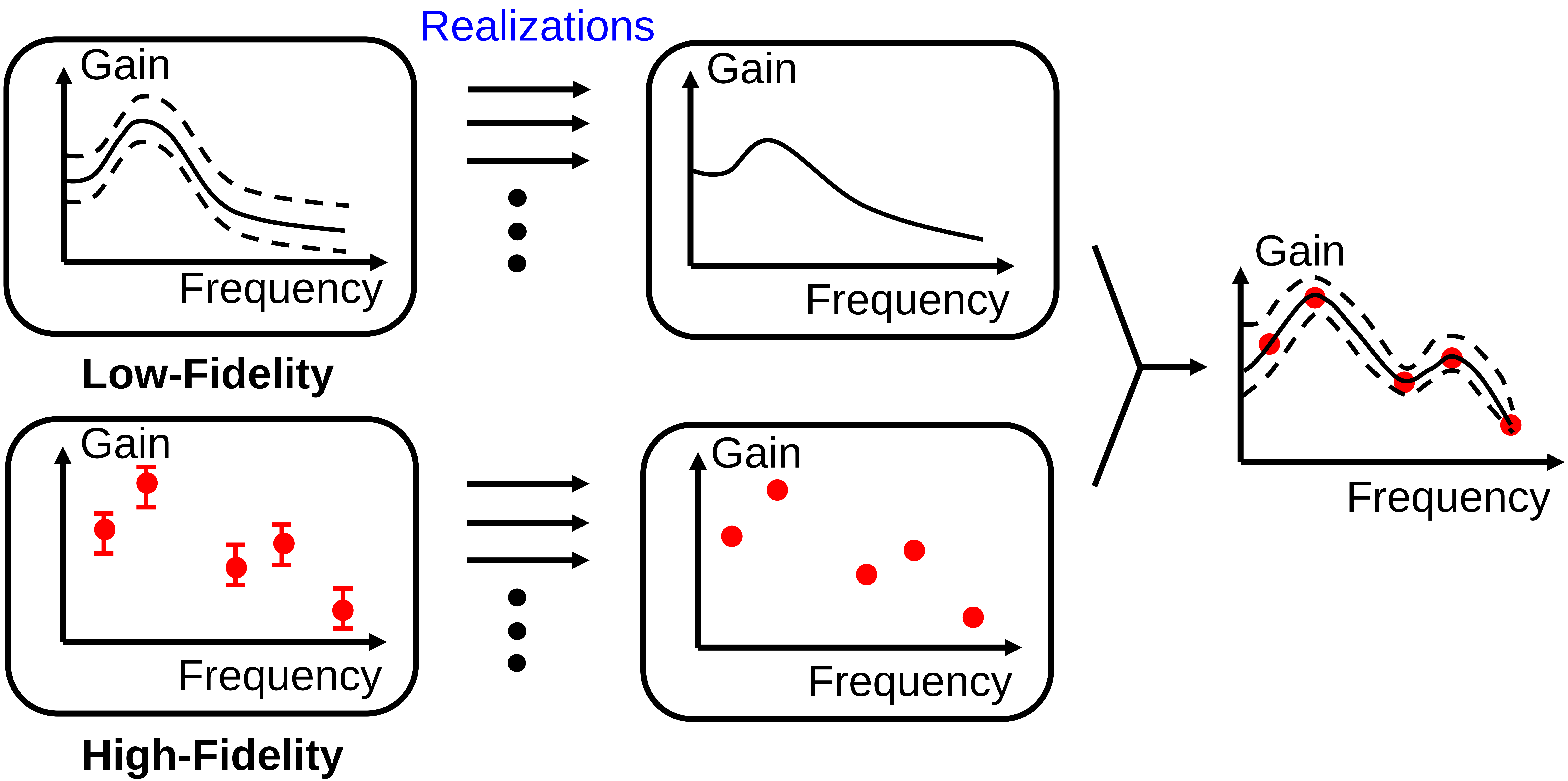
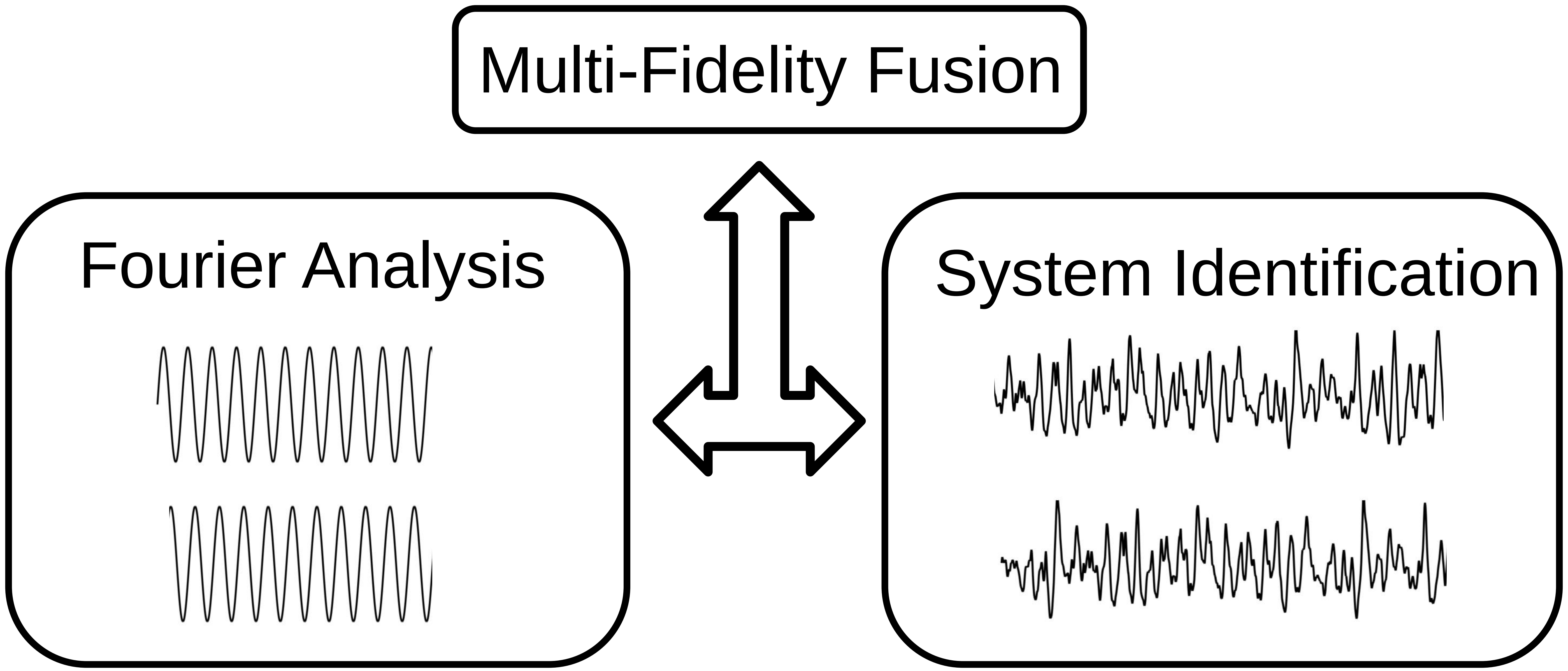


The costs of multi-fidelity analysis is negligible compared with CFD simulations



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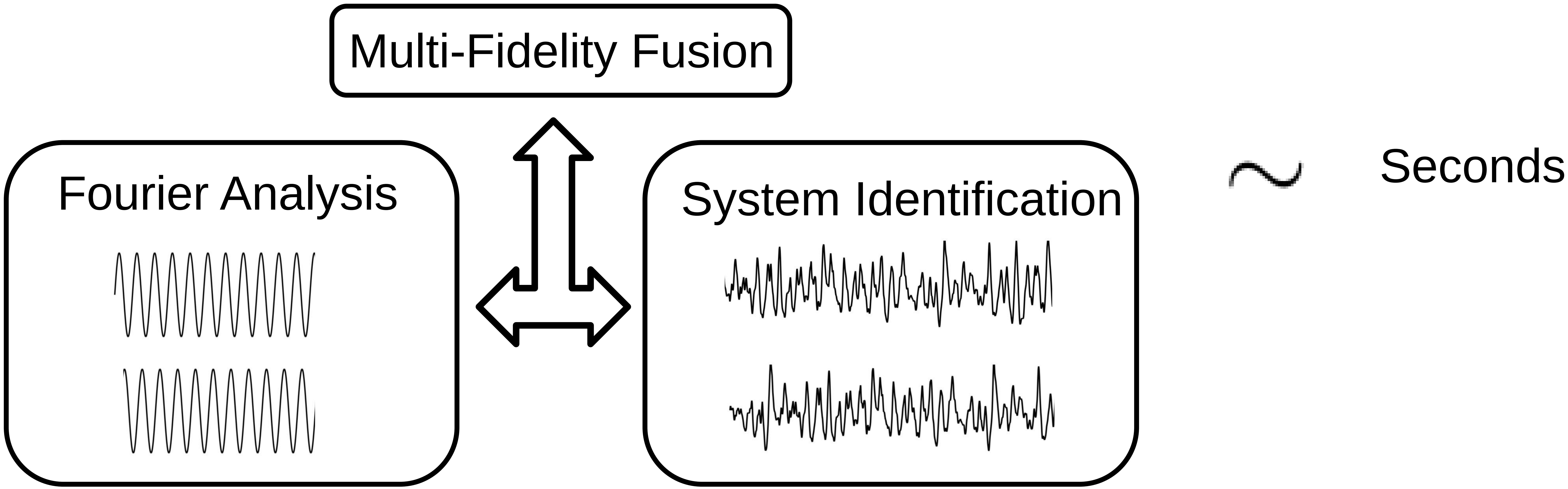

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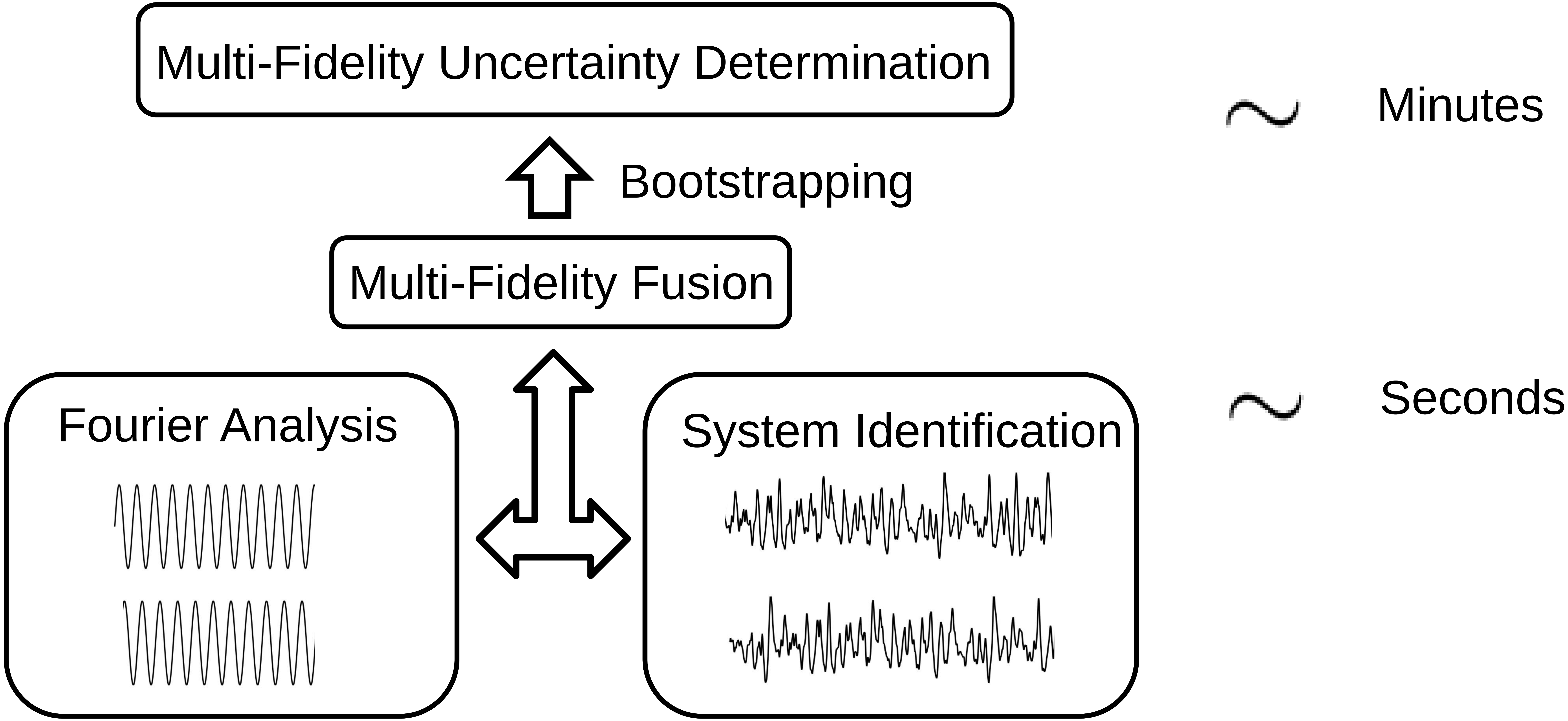

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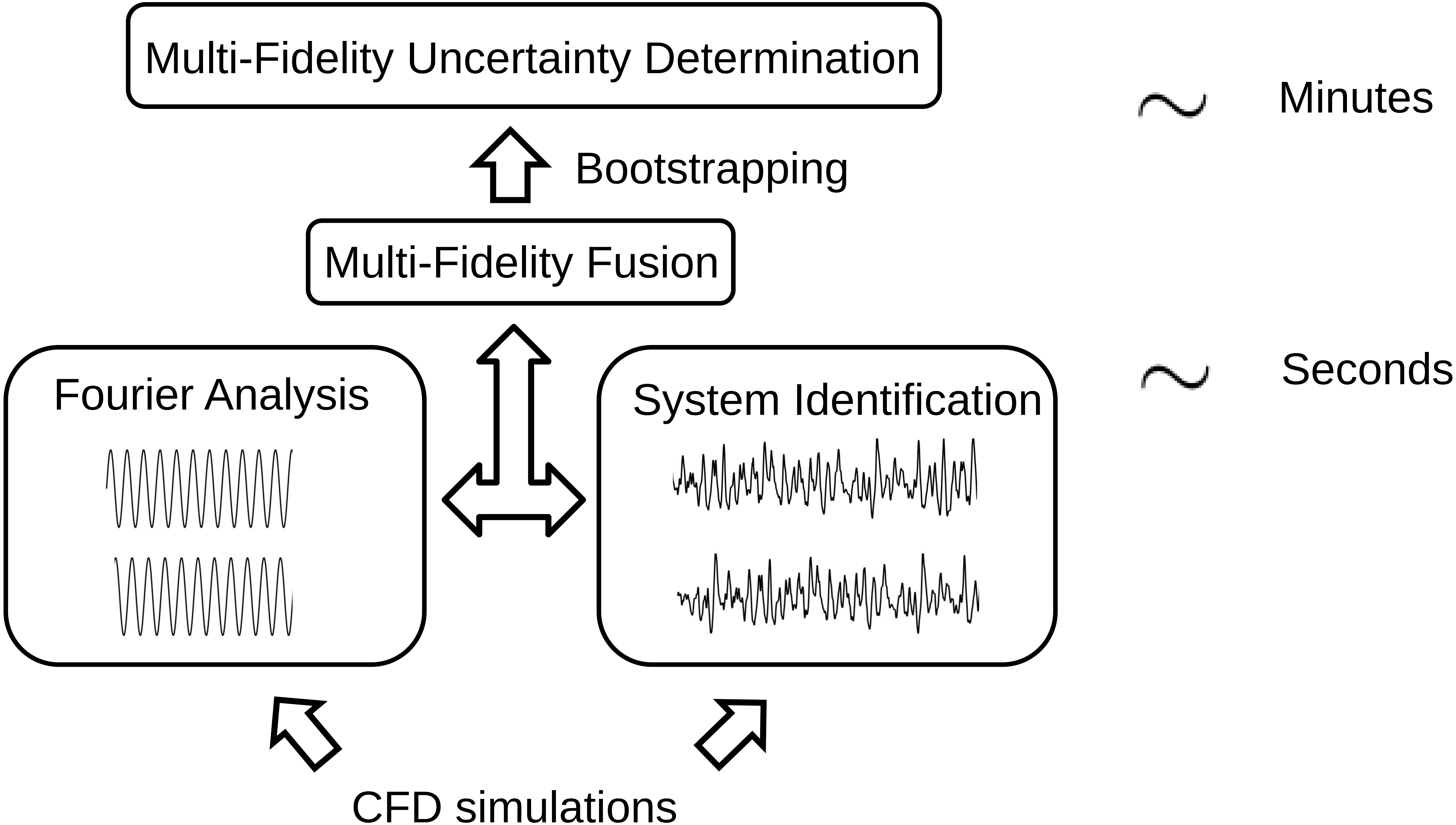

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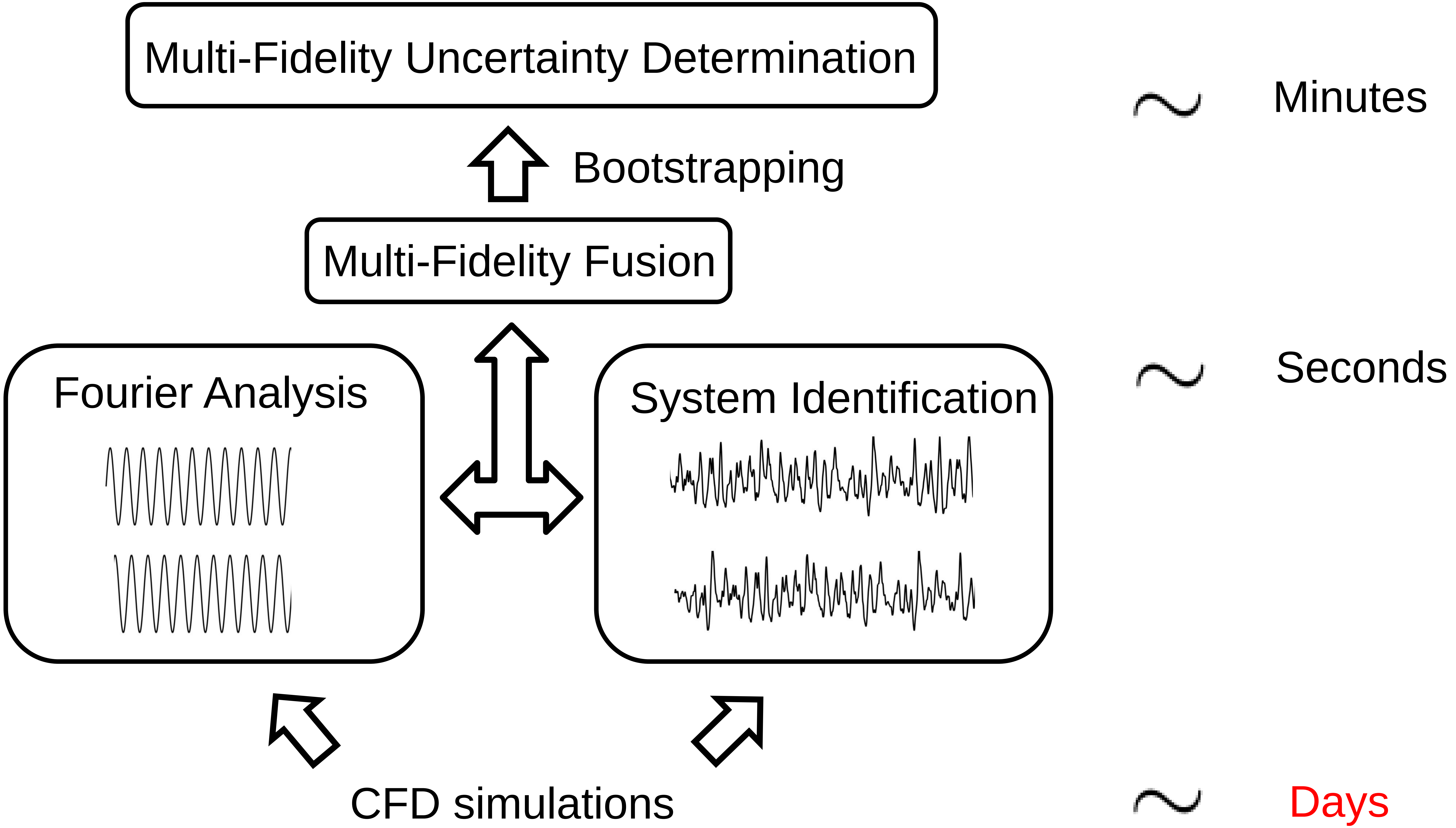
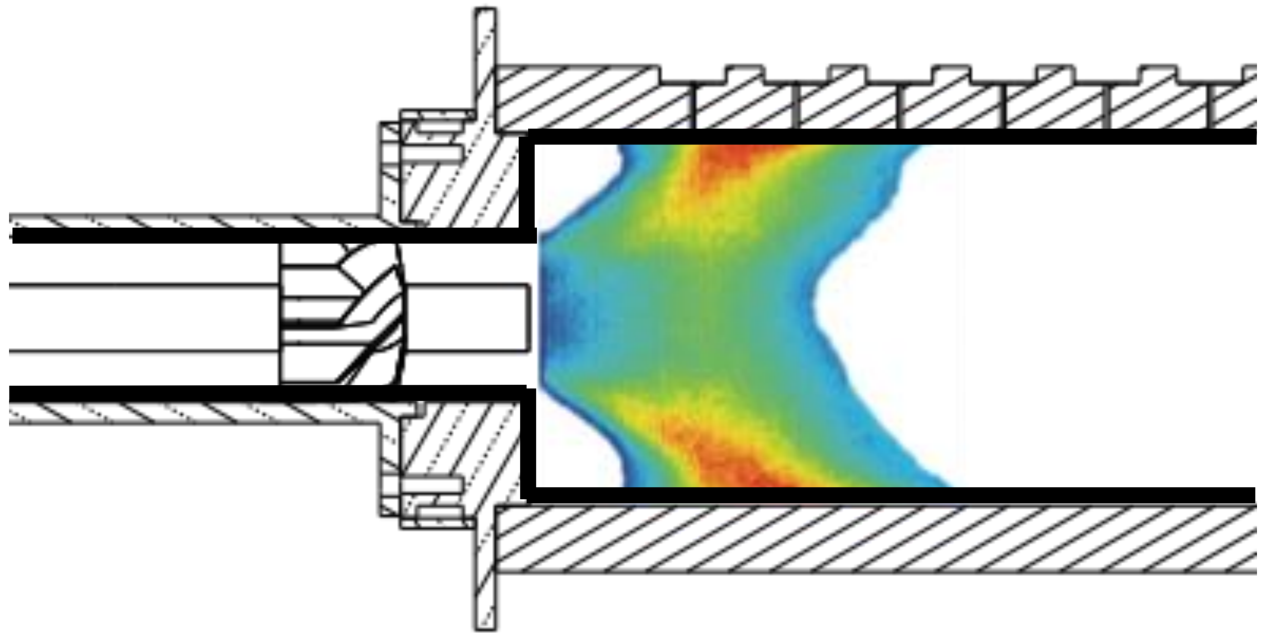


Presentation Overview

- Motivation
- Multi-fidelity Gaussian Process
  - → How to aggregate different fidelities?
  - → How to combine uncertainties from individual fidelities?
- Case study
  - → Set-up
  - → Results & Discussions
- Conclusion & Outlook

We force a combustor network model to identify flame frequency response



Premixed swirl burner

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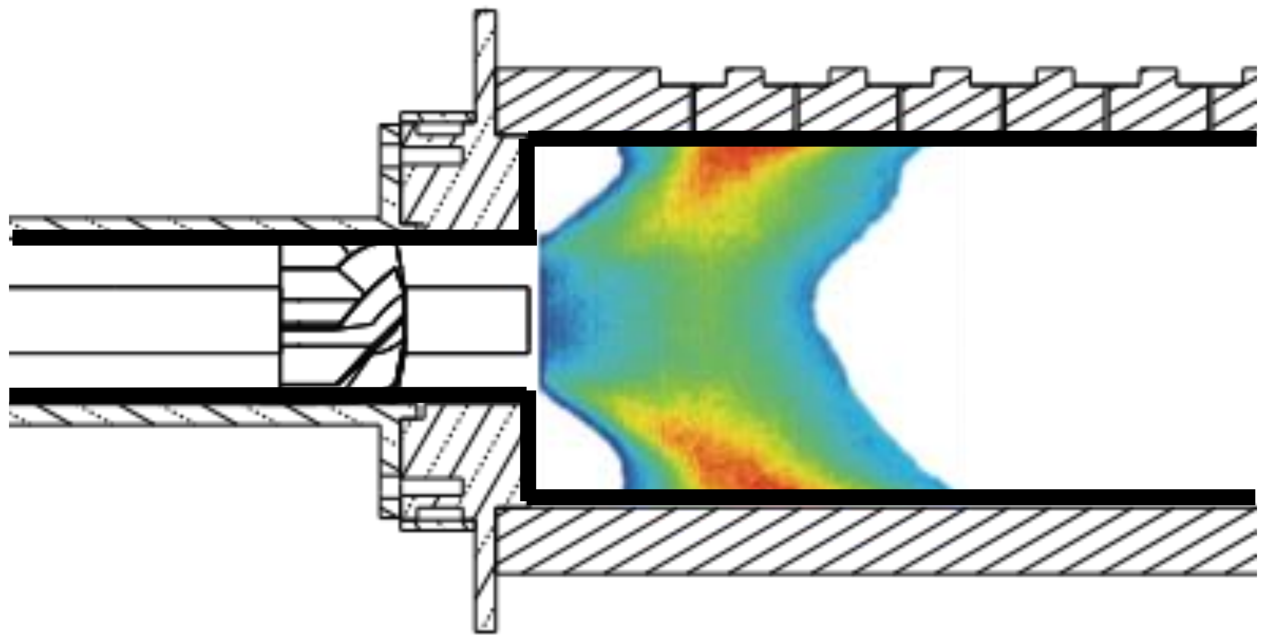
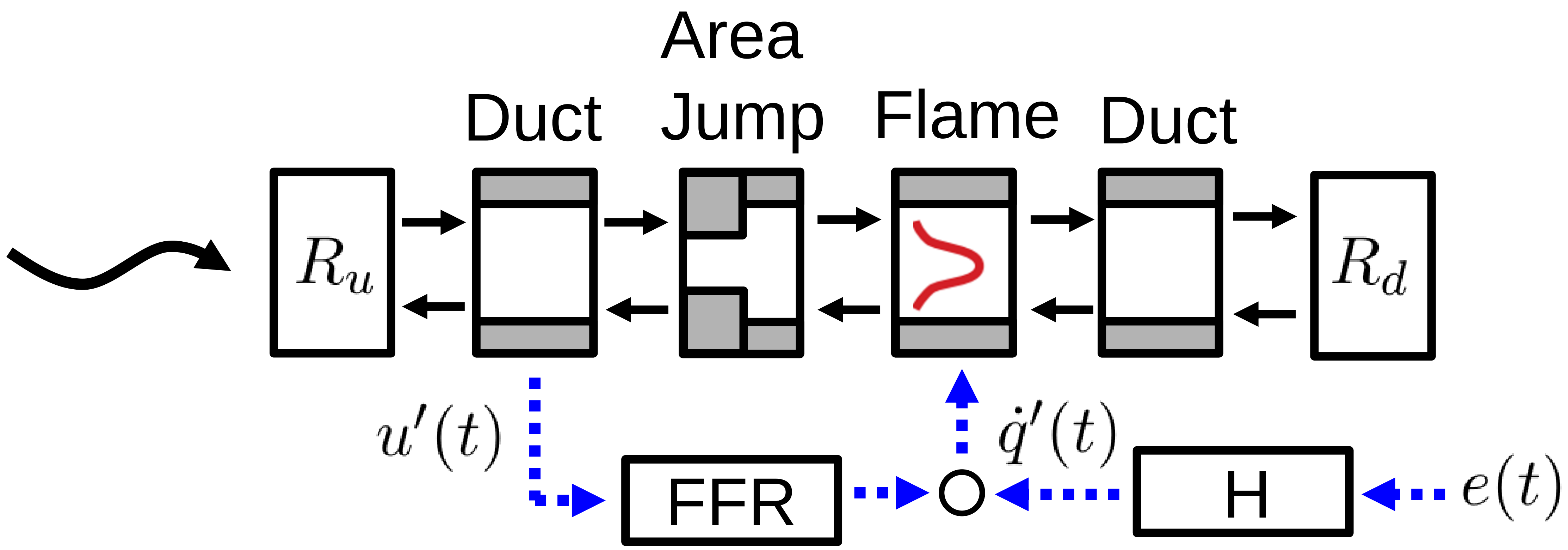
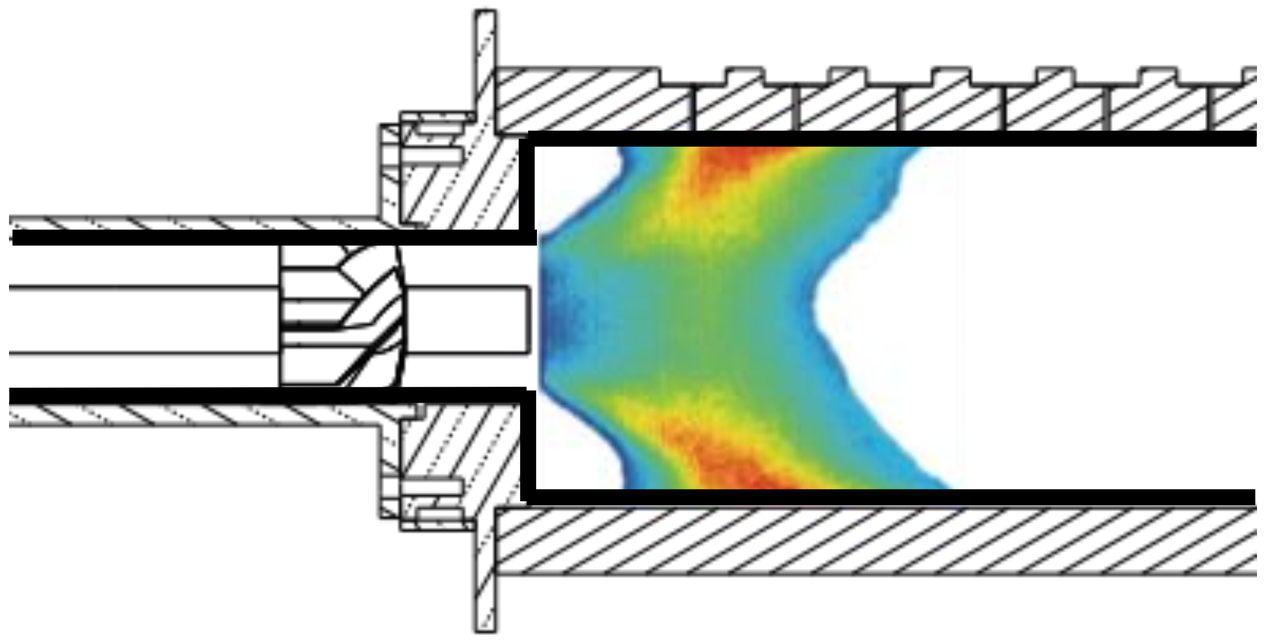


Premixed swirl burner



Network model

We force a combustor network model to identify flame frequency response



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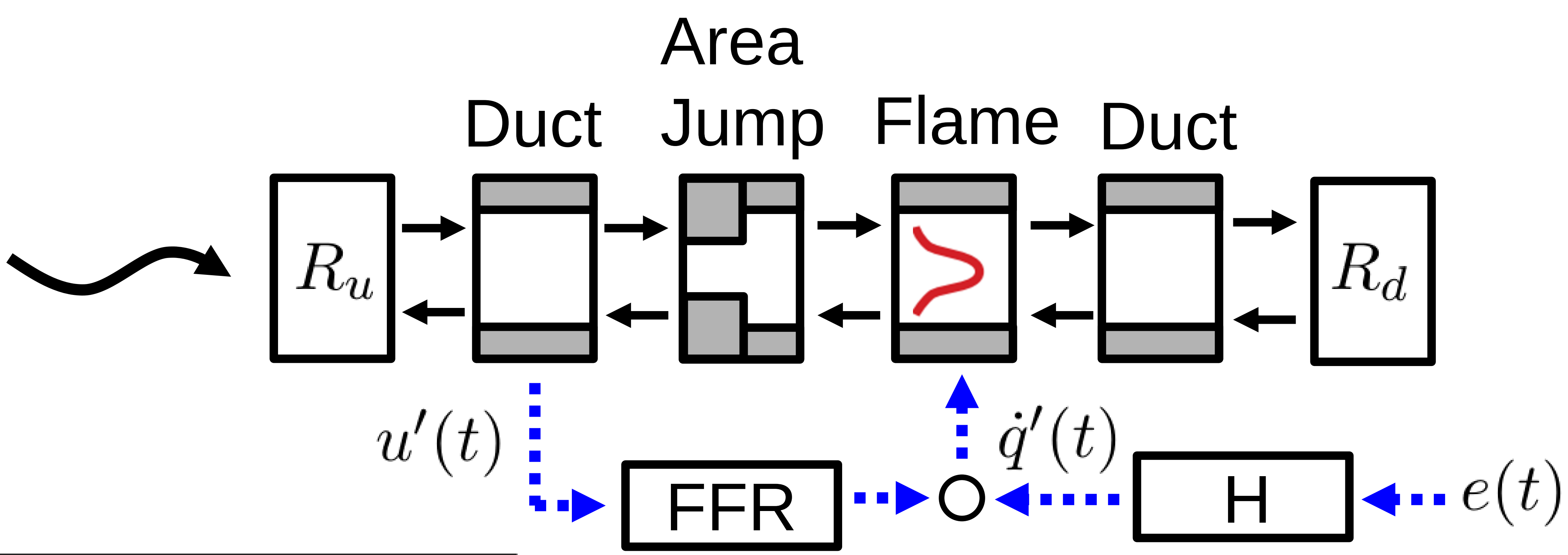
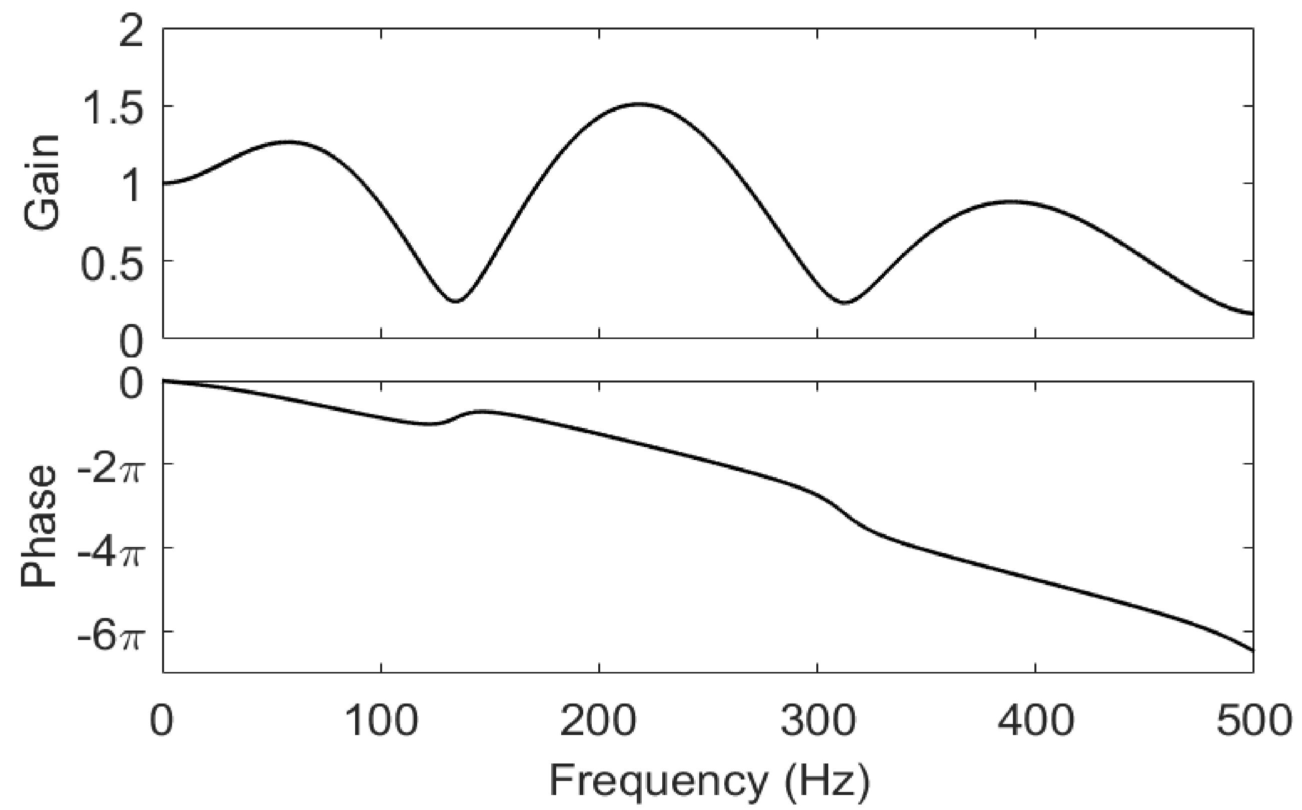


Network model



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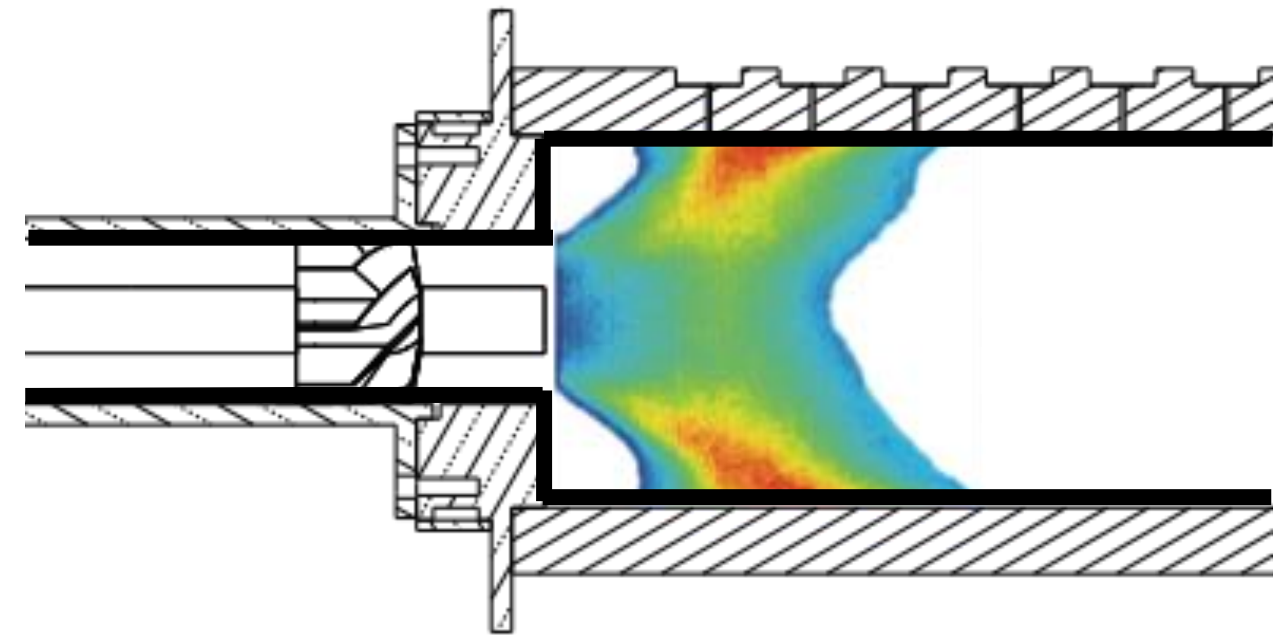
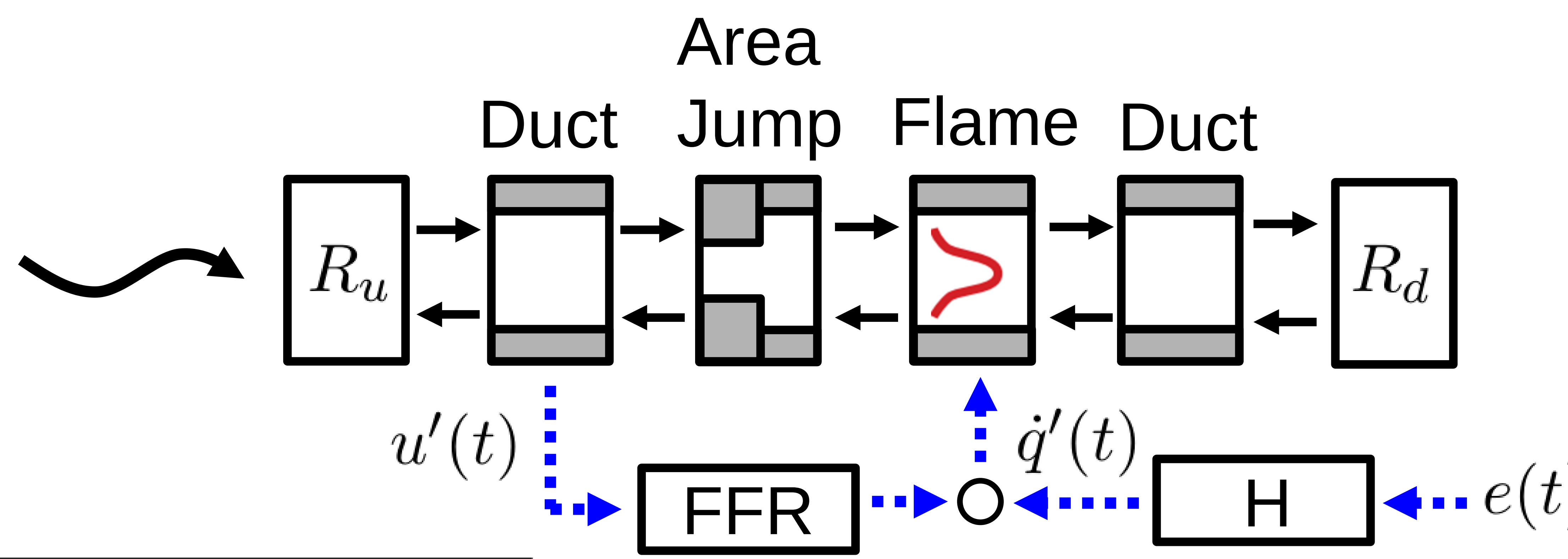
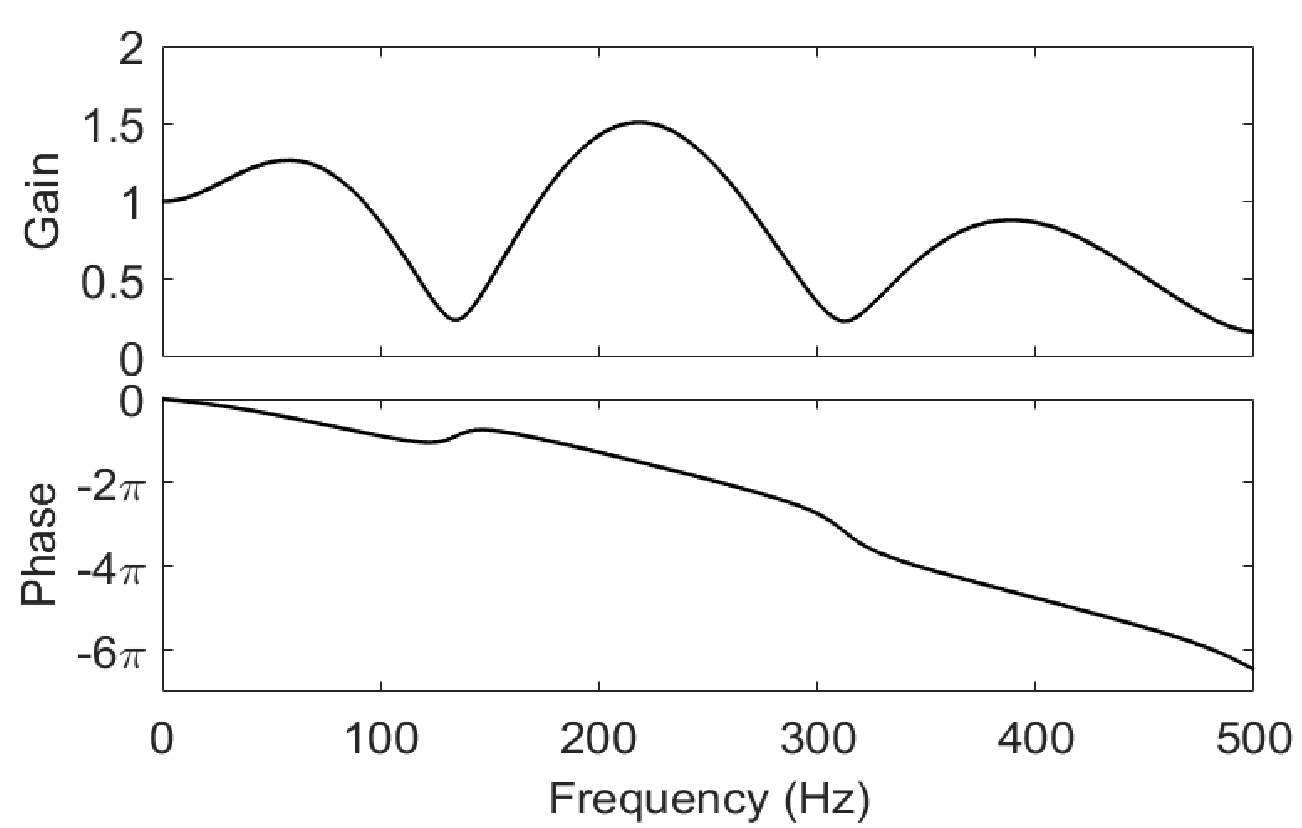
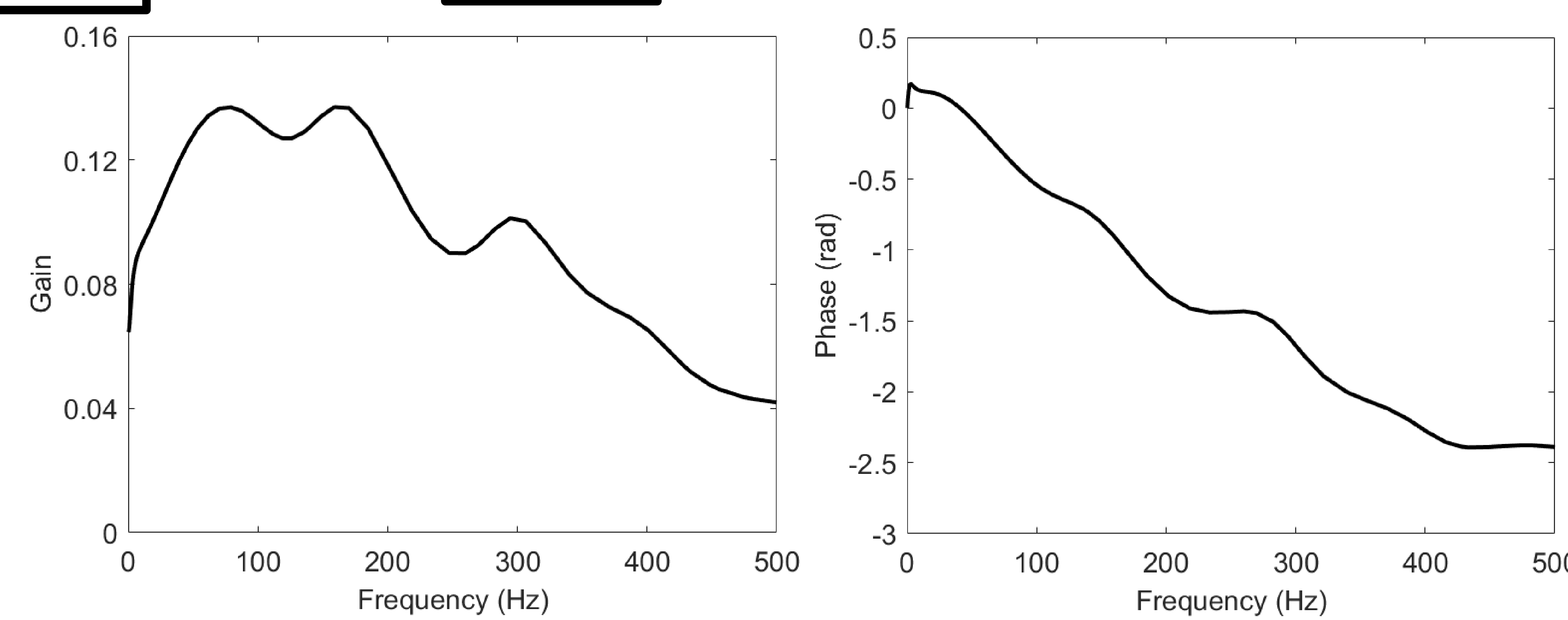


Premixed swirl burner



Network model





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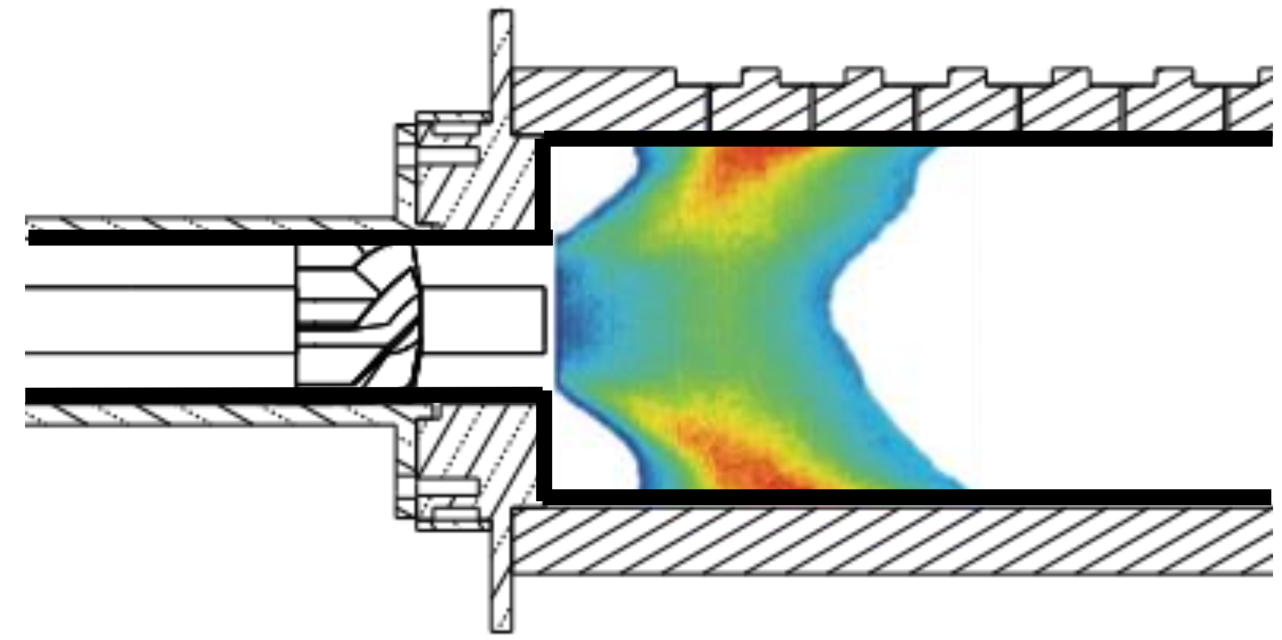
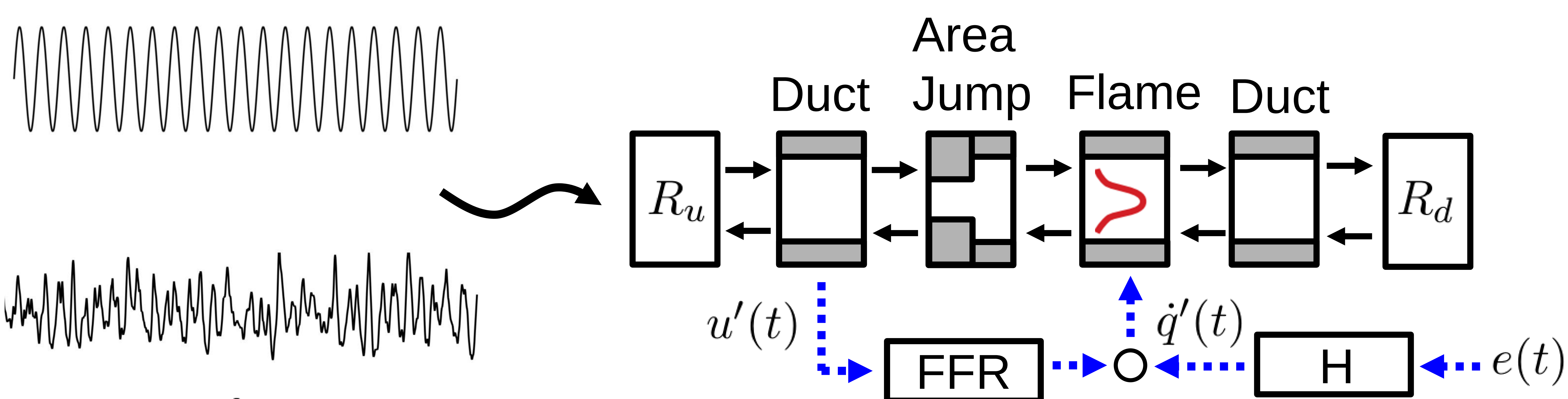


Premixed swirl burner



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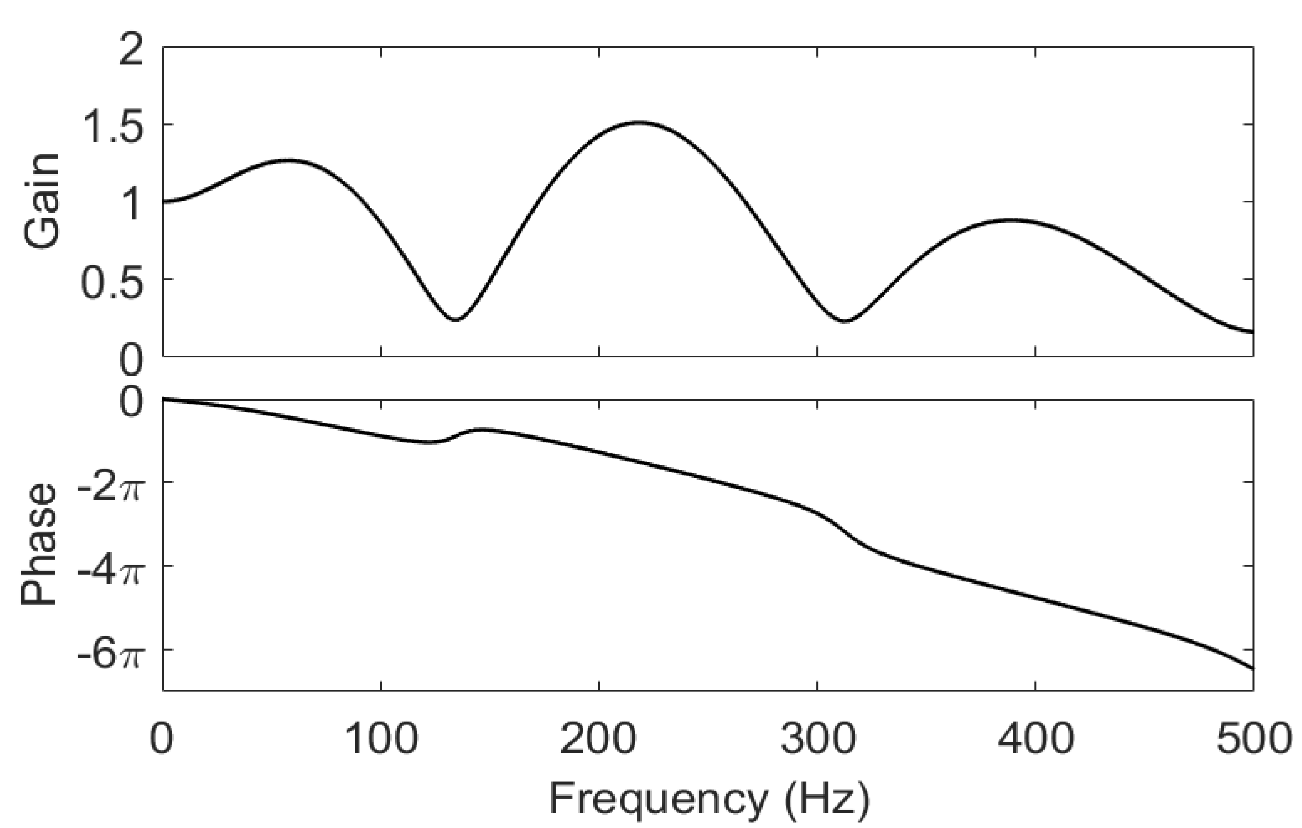
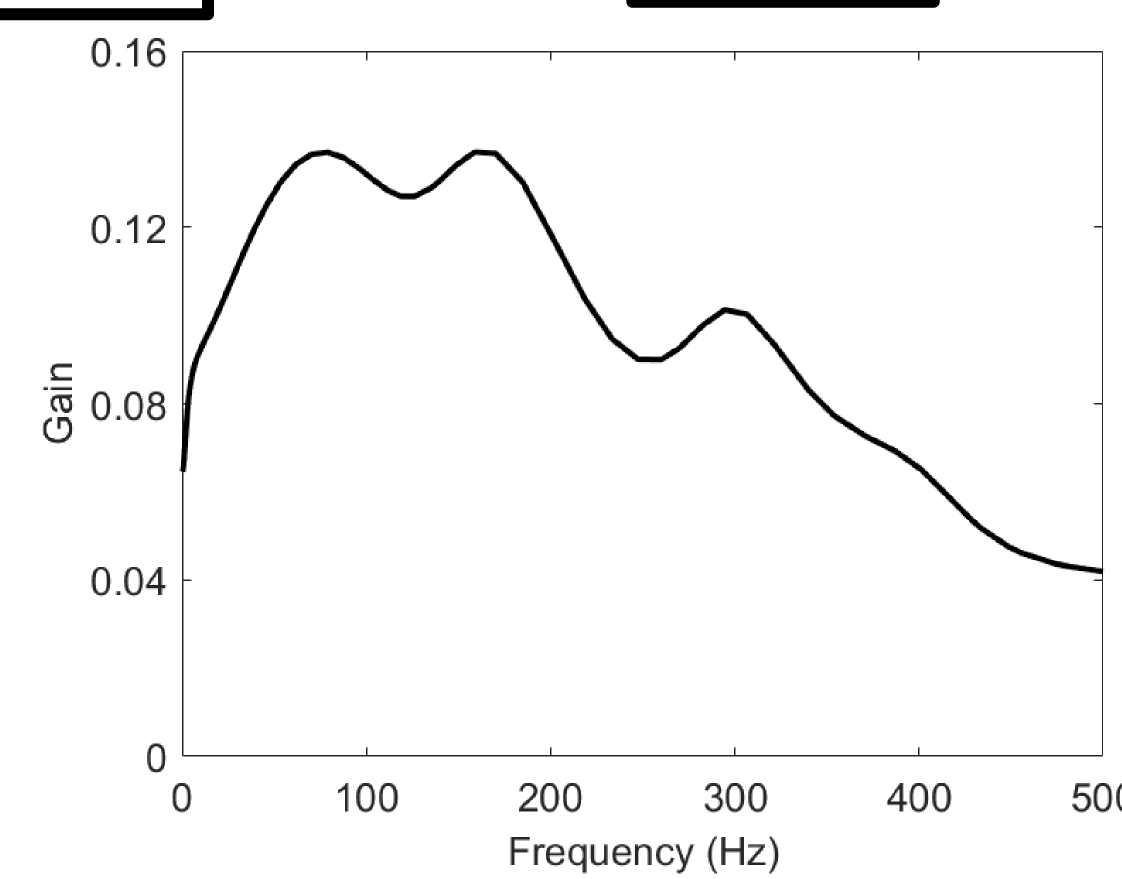
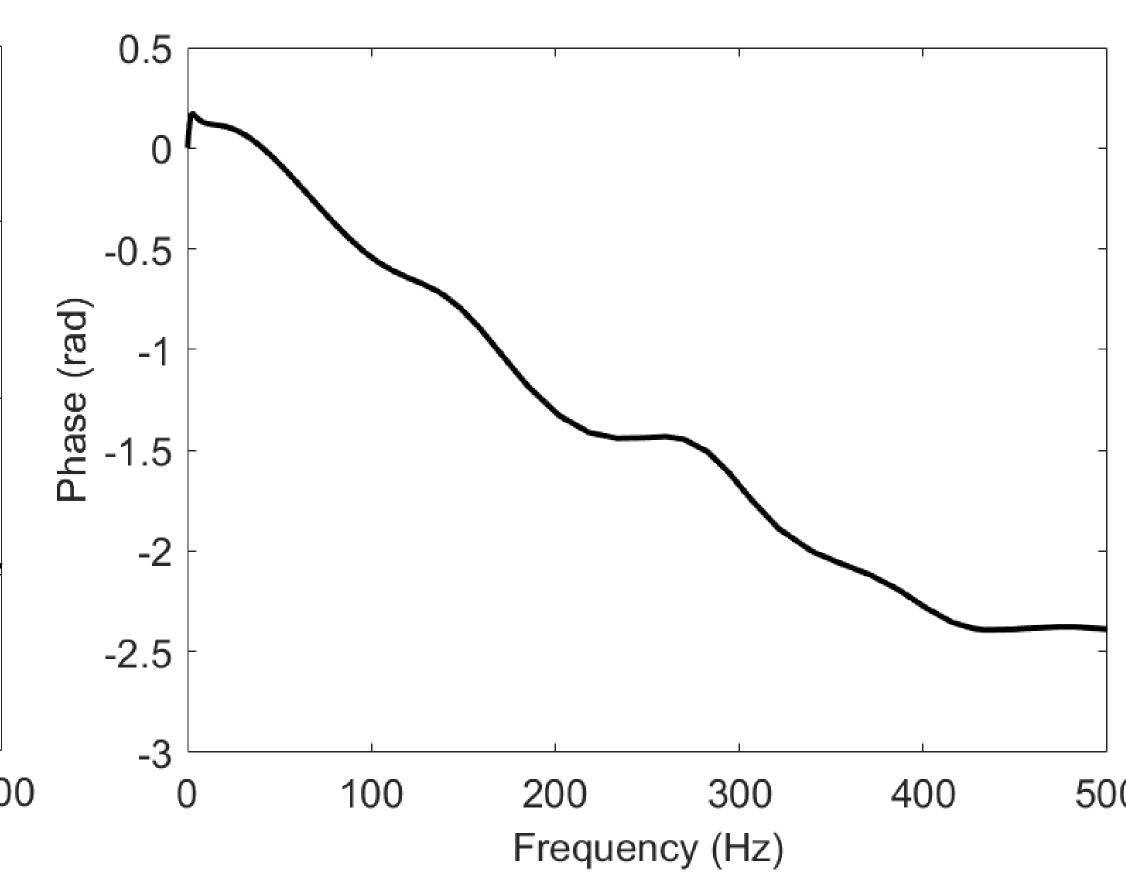


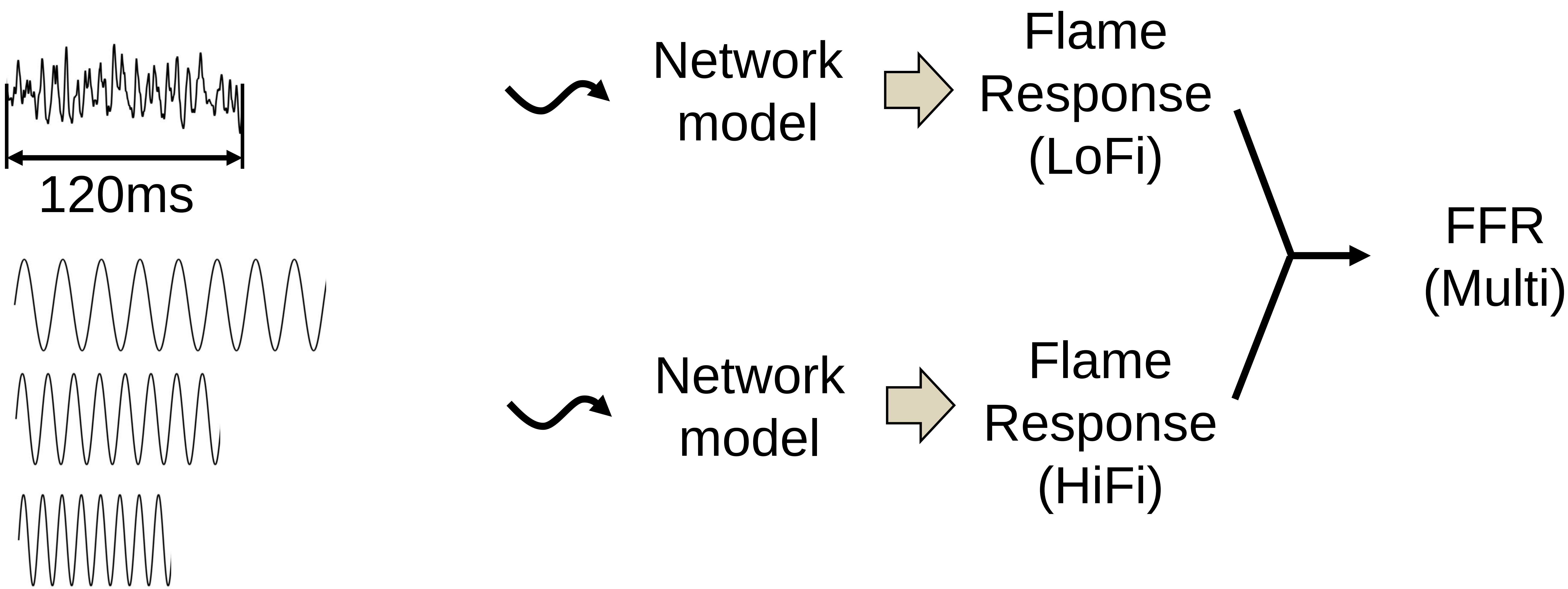




We force a combustor network model to identify flame frequency response

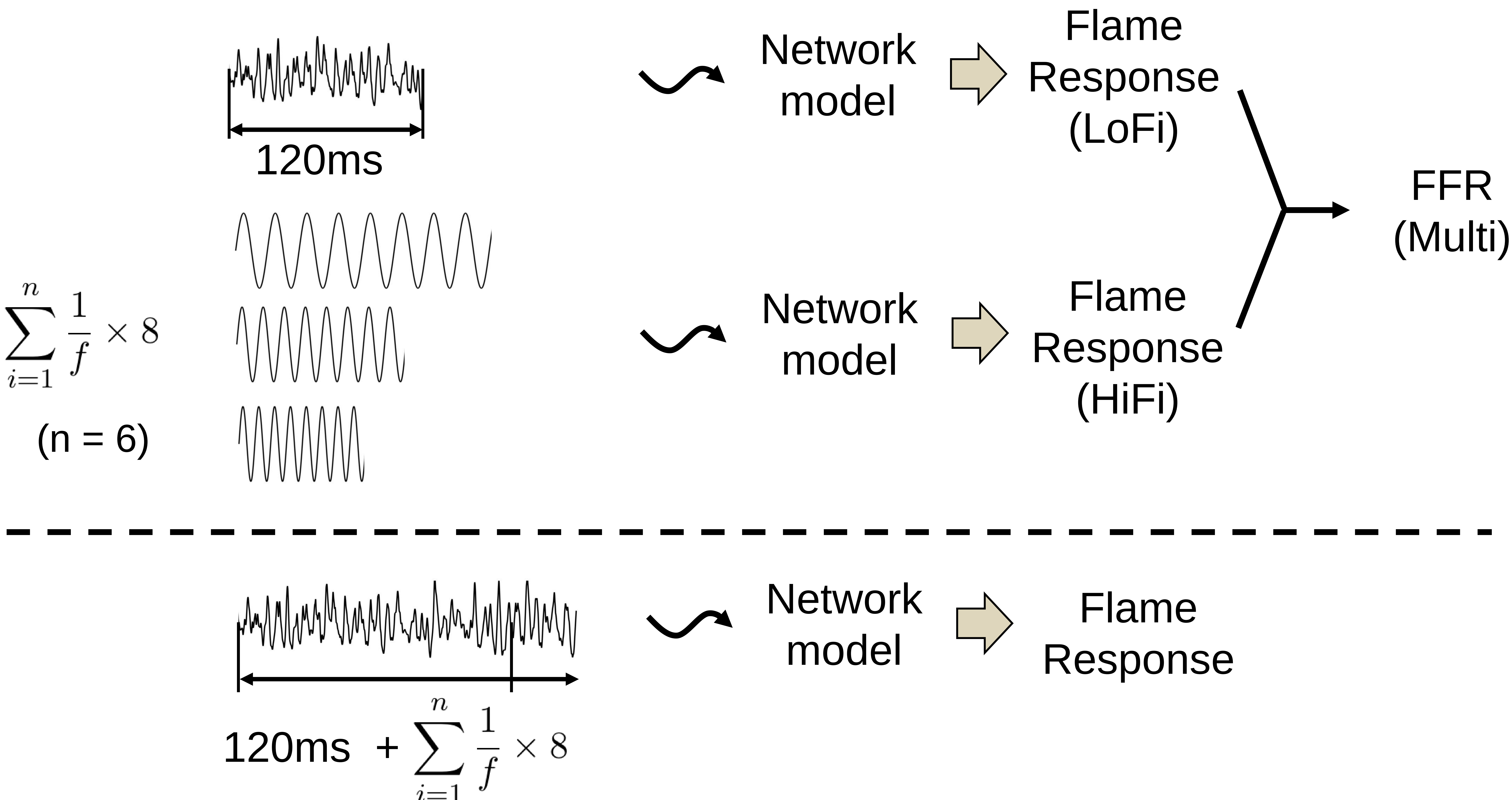
$$\sum_{i=1}^{n} \frac{1}{f} \times 8$$

(n = 6)



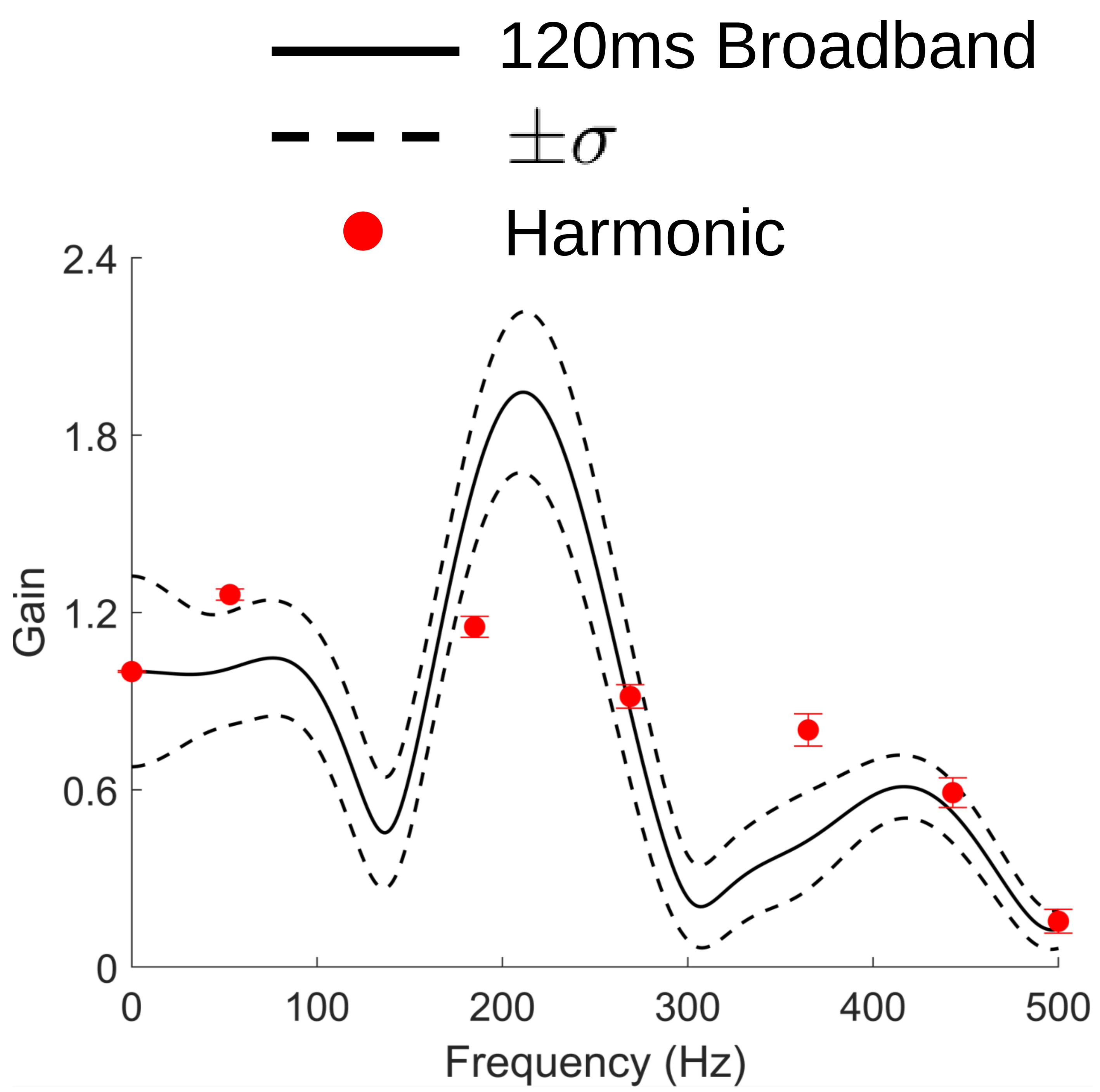
[4] A. K. Tangirala, 2014, Principles of System Identification.

We force a combustor network model to identify flame frequency response

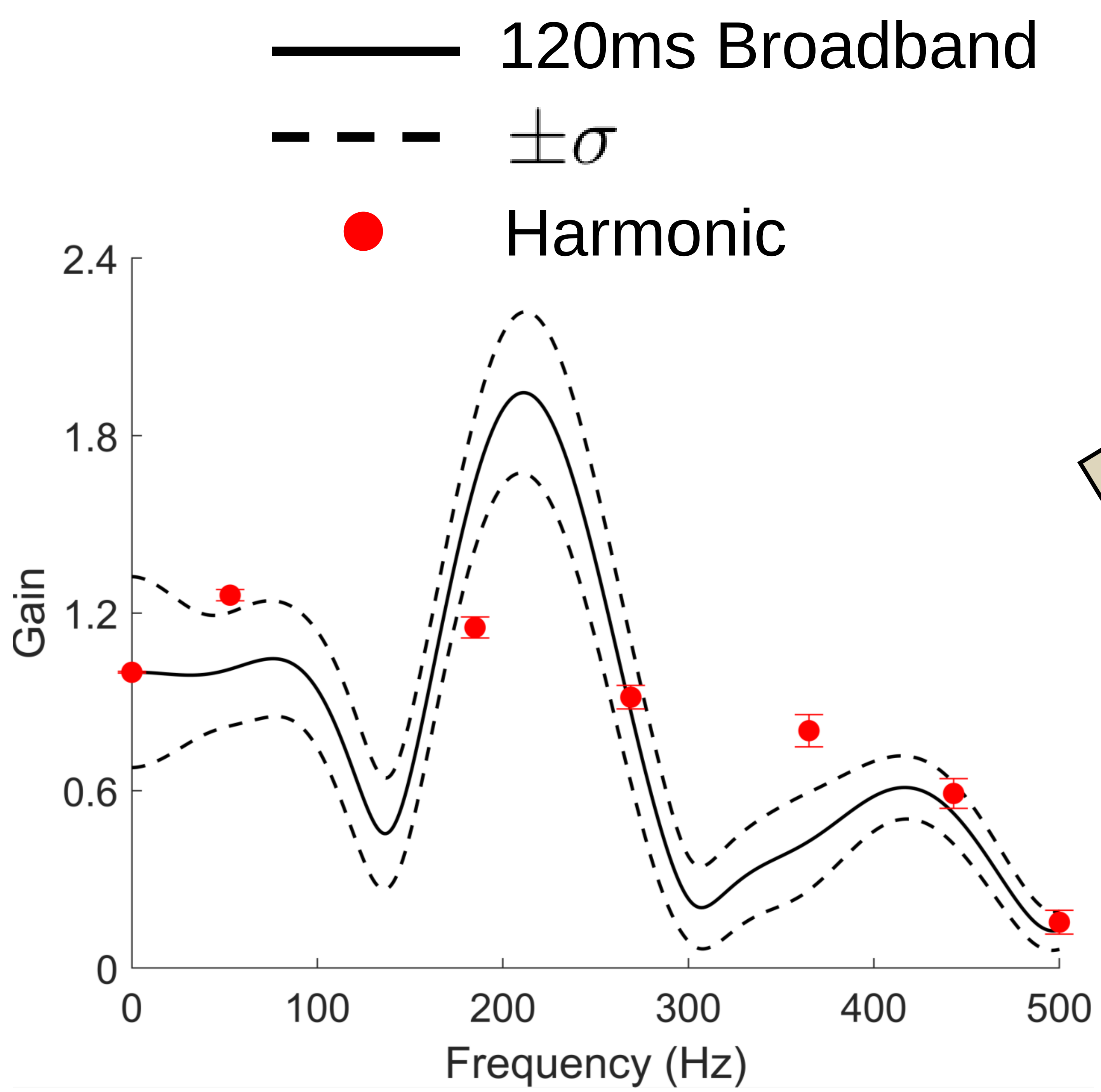


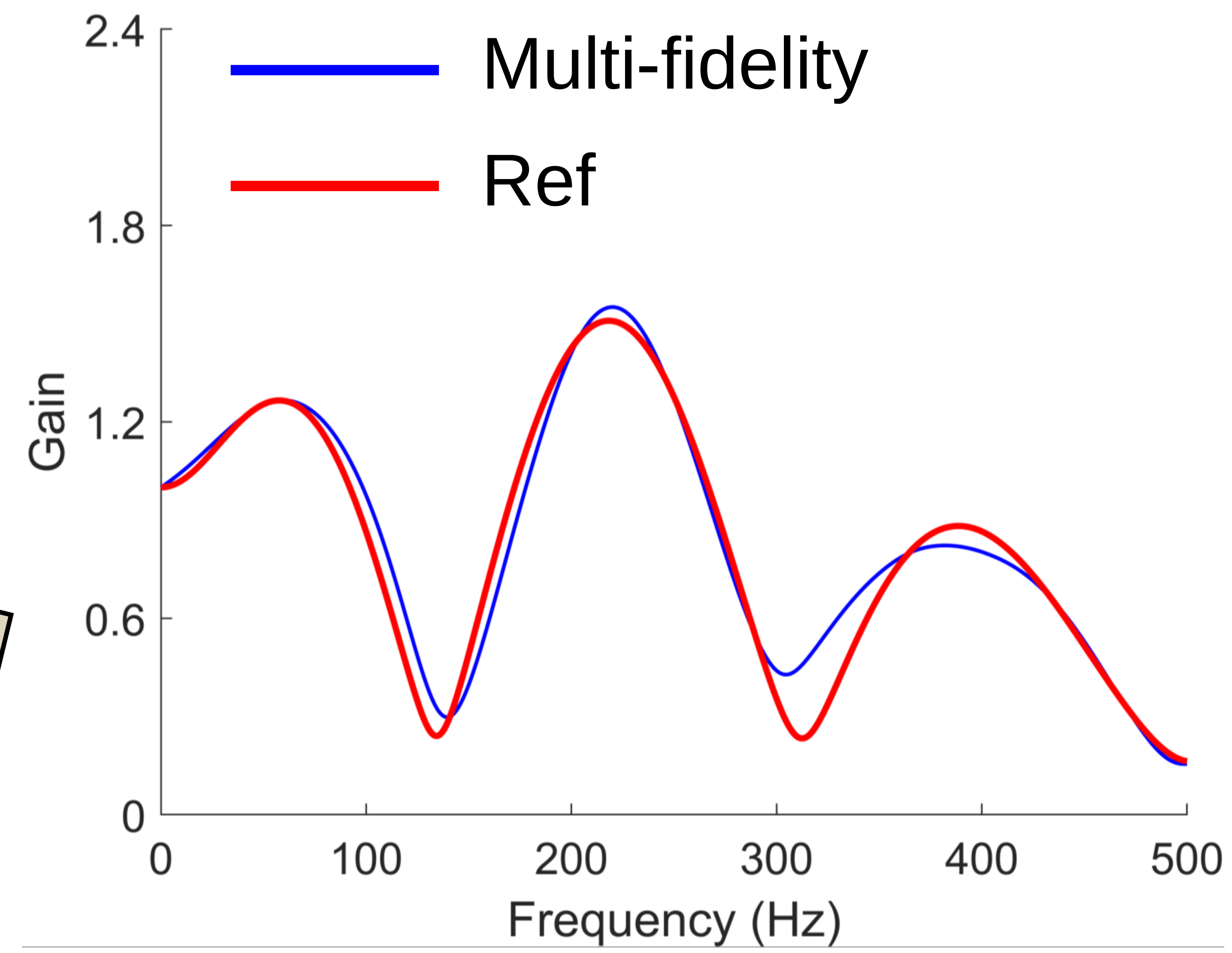
[4] A. K. Tangirala, 2014, Principles of System Identification.

We start with a short broadband results and harmonic results at several frequencies



For gain, multi-fidelity approach yields a more globally accurate prediction





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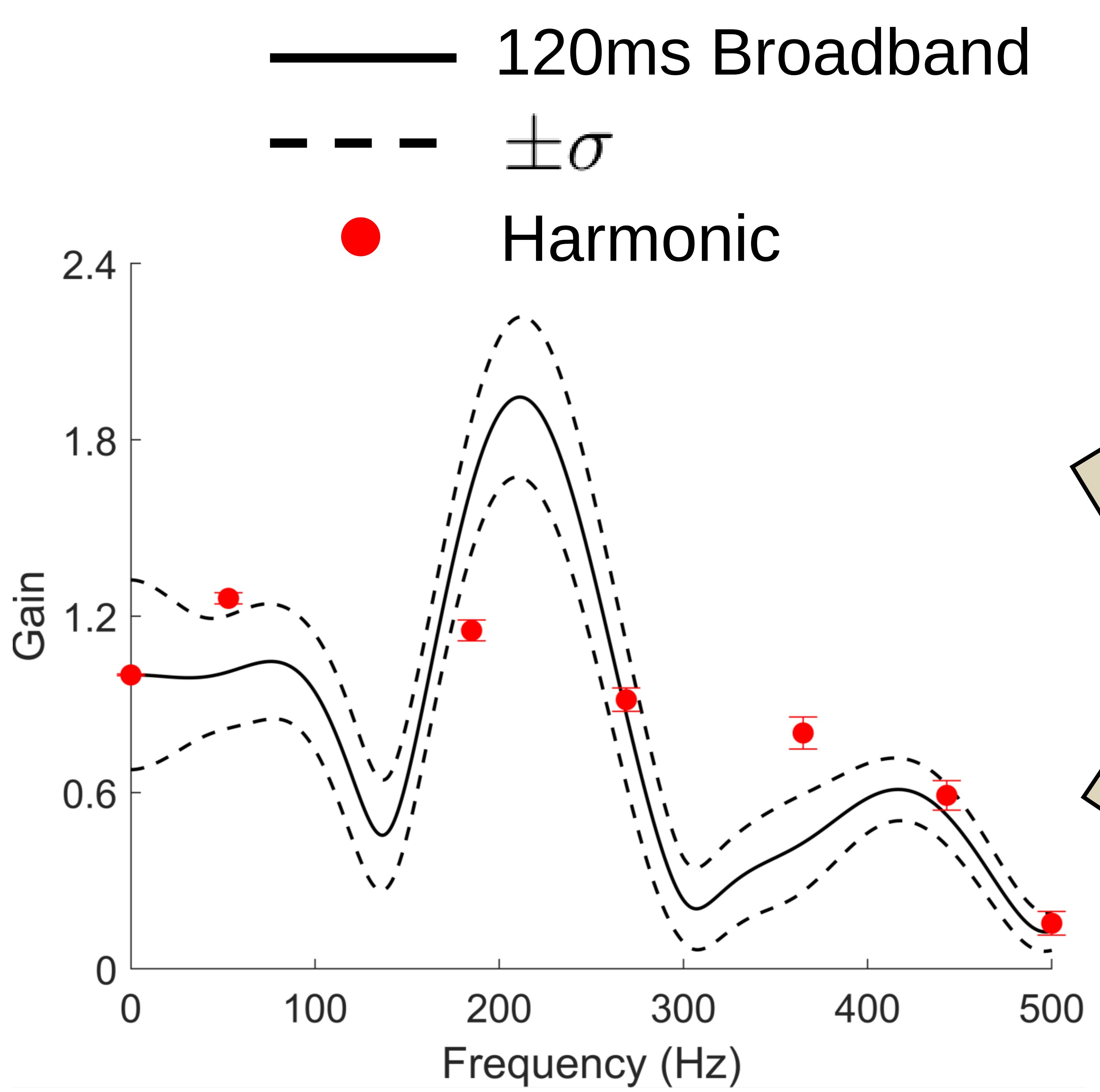
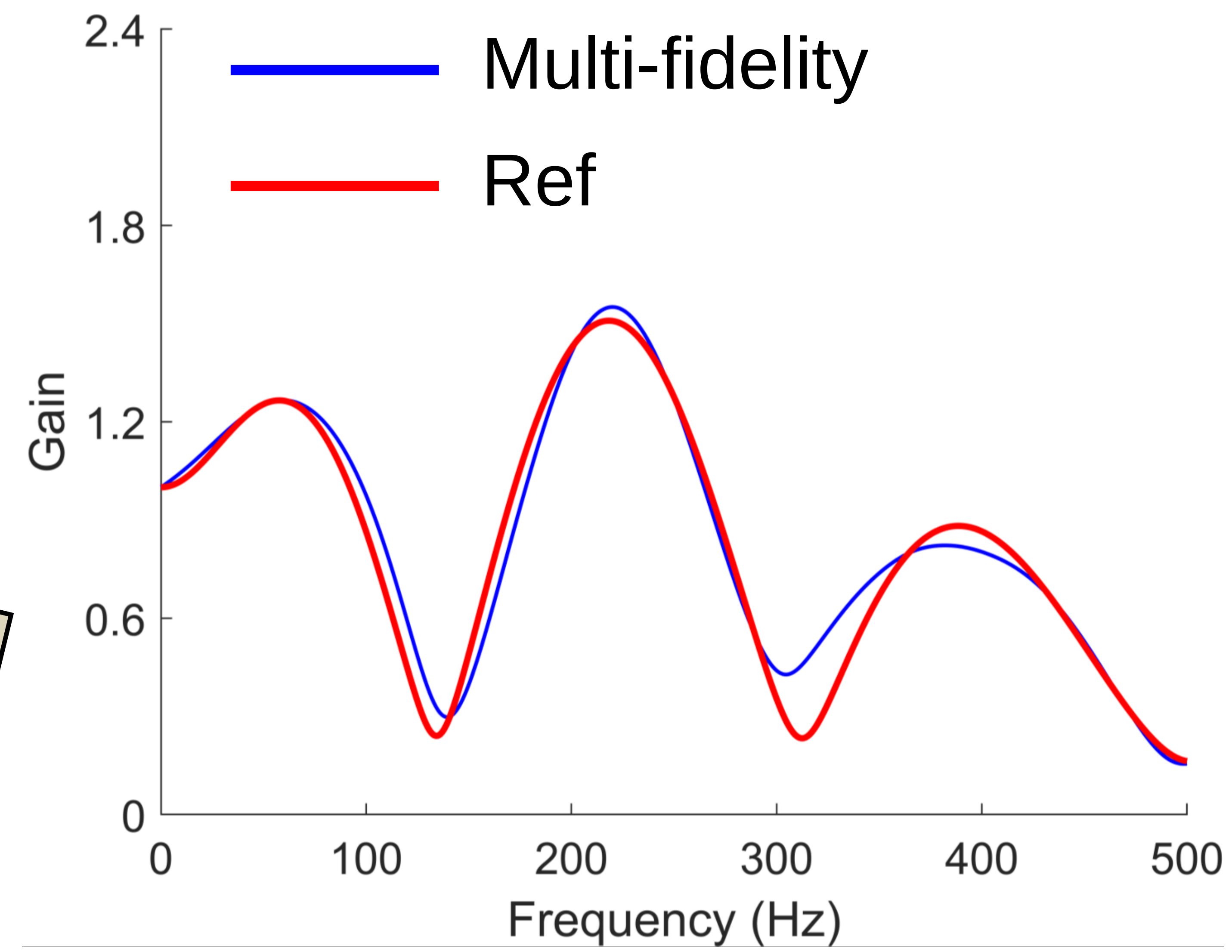
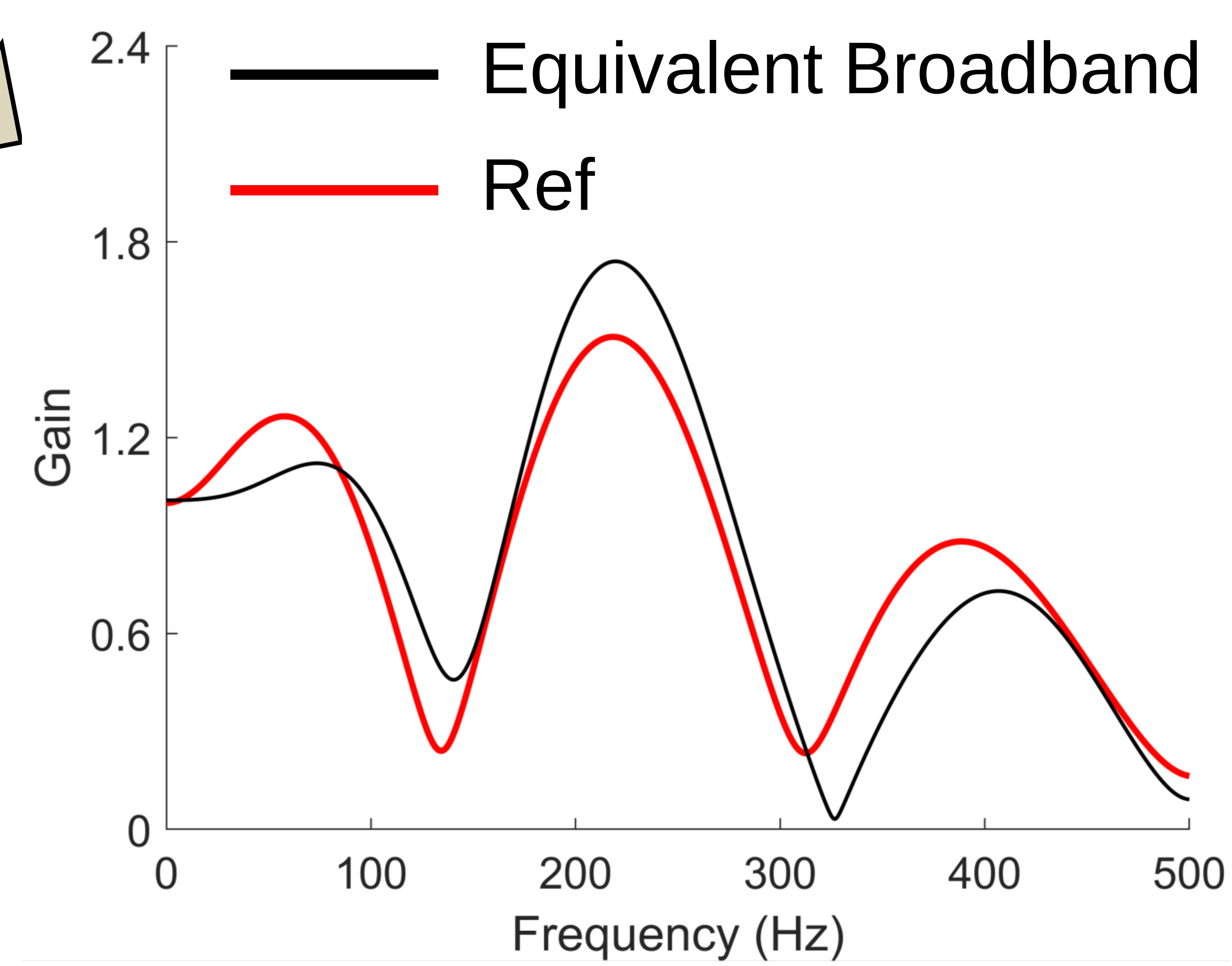
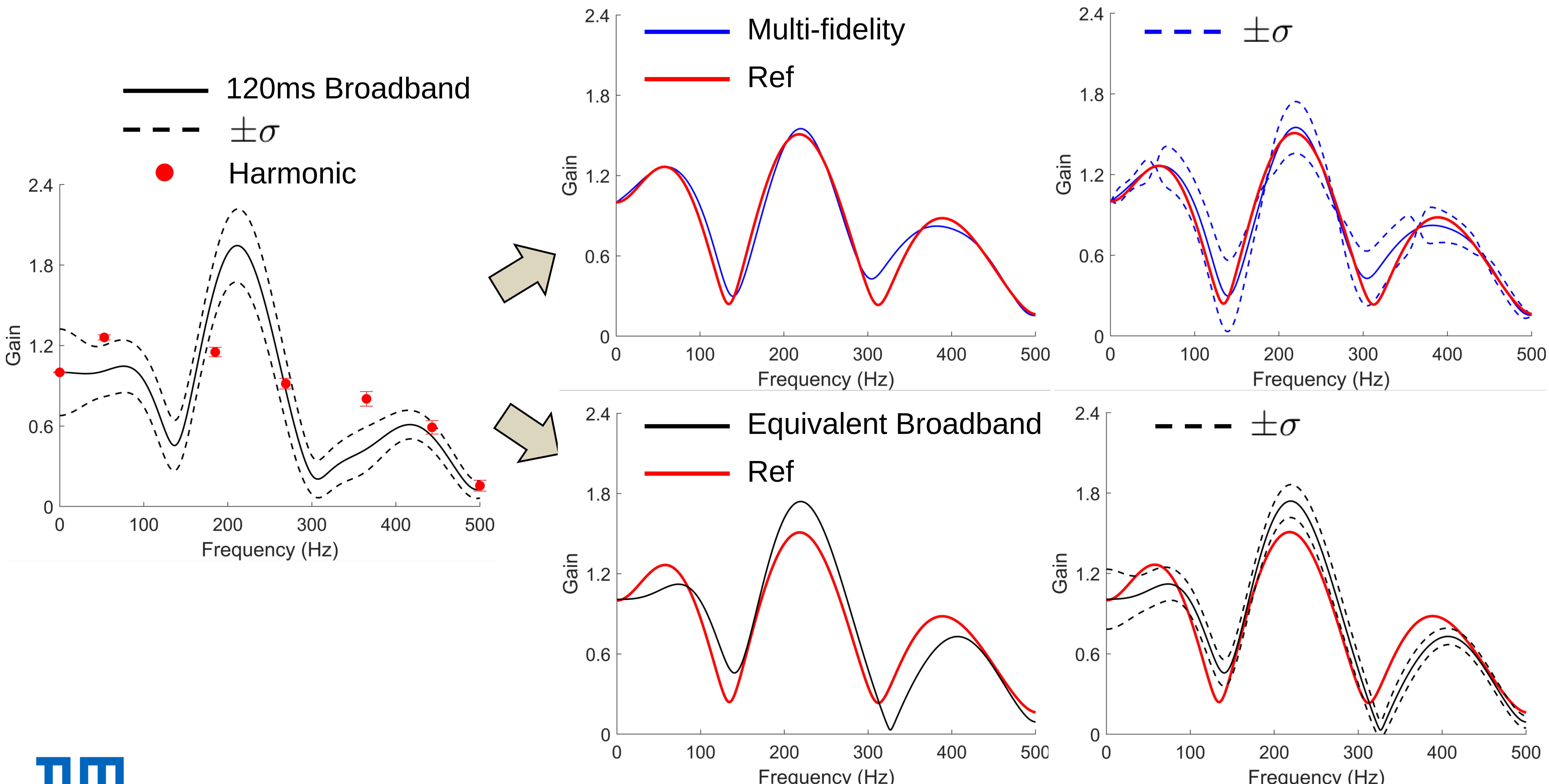


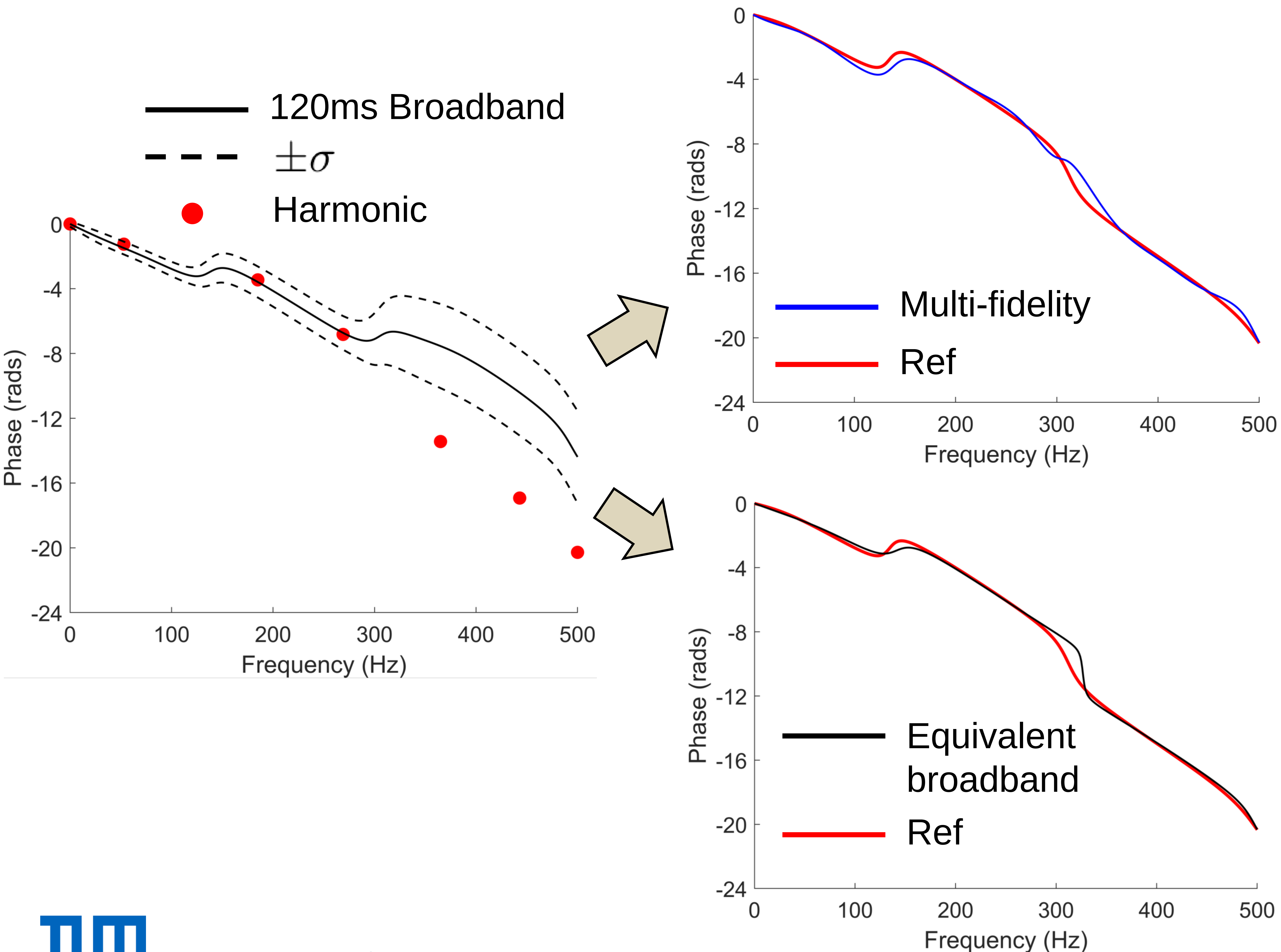




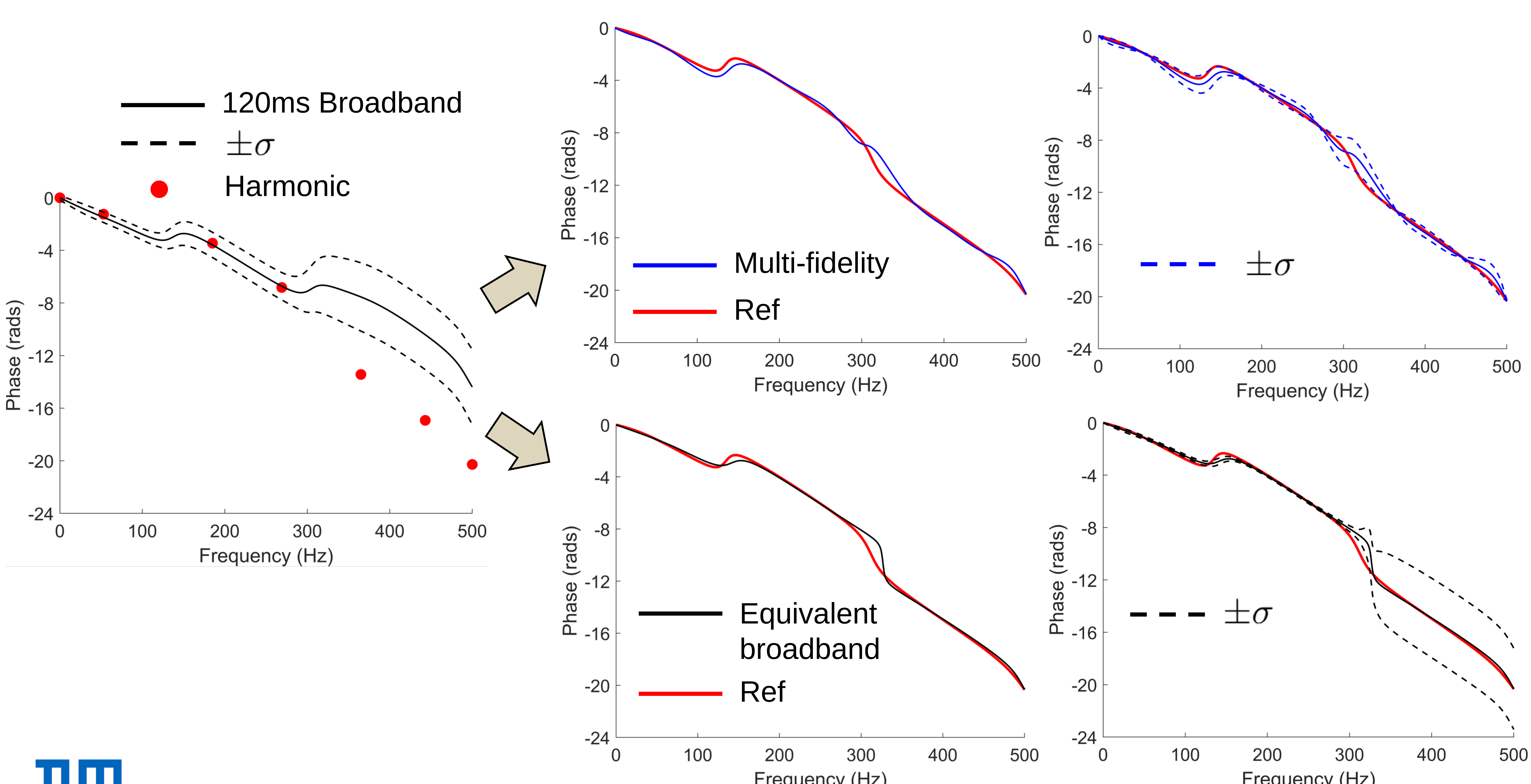
For gain, multi-fidelity approach yields more robust uncertainty estimation



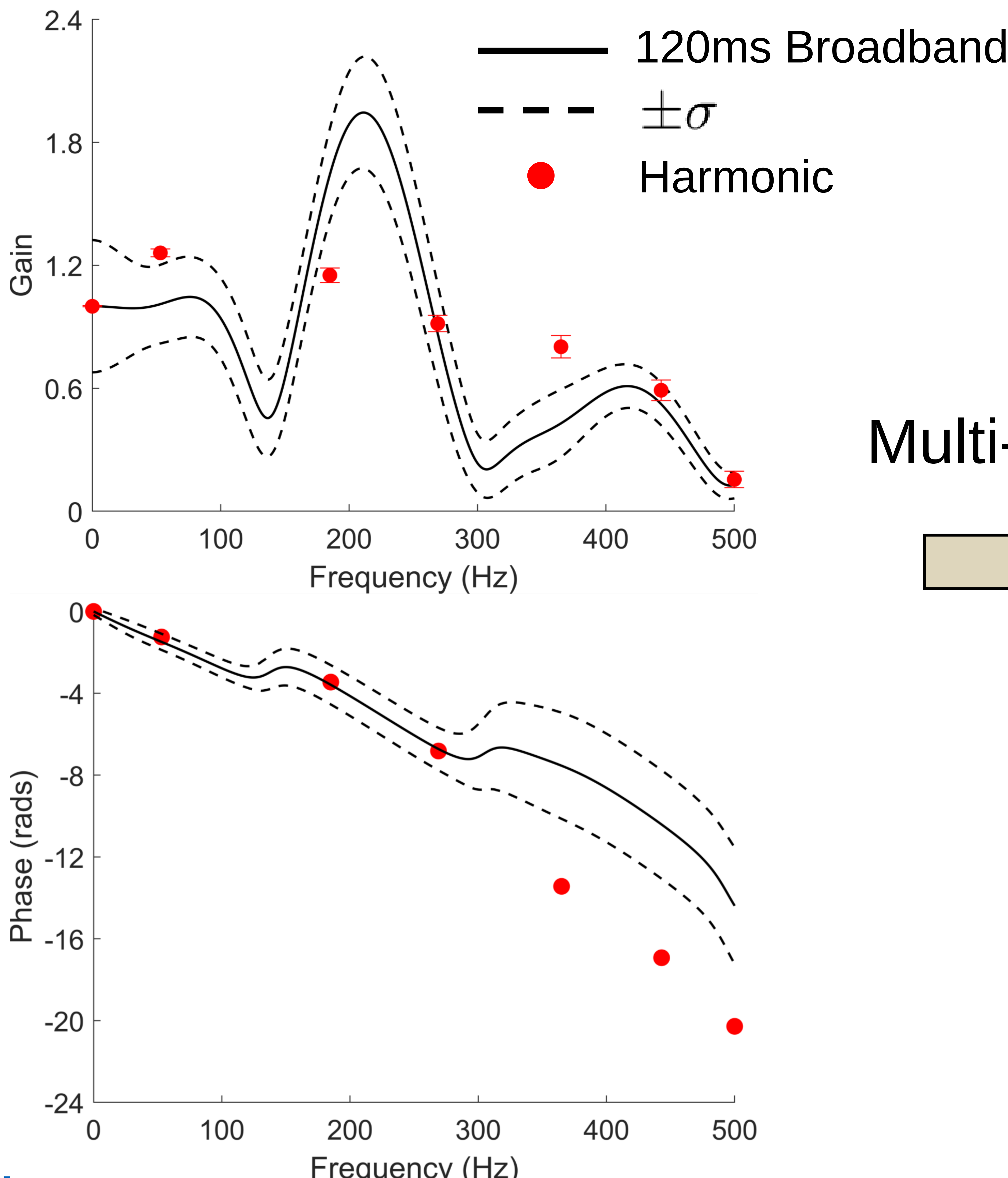
For phase, predictions made by both methods have similar accuracy



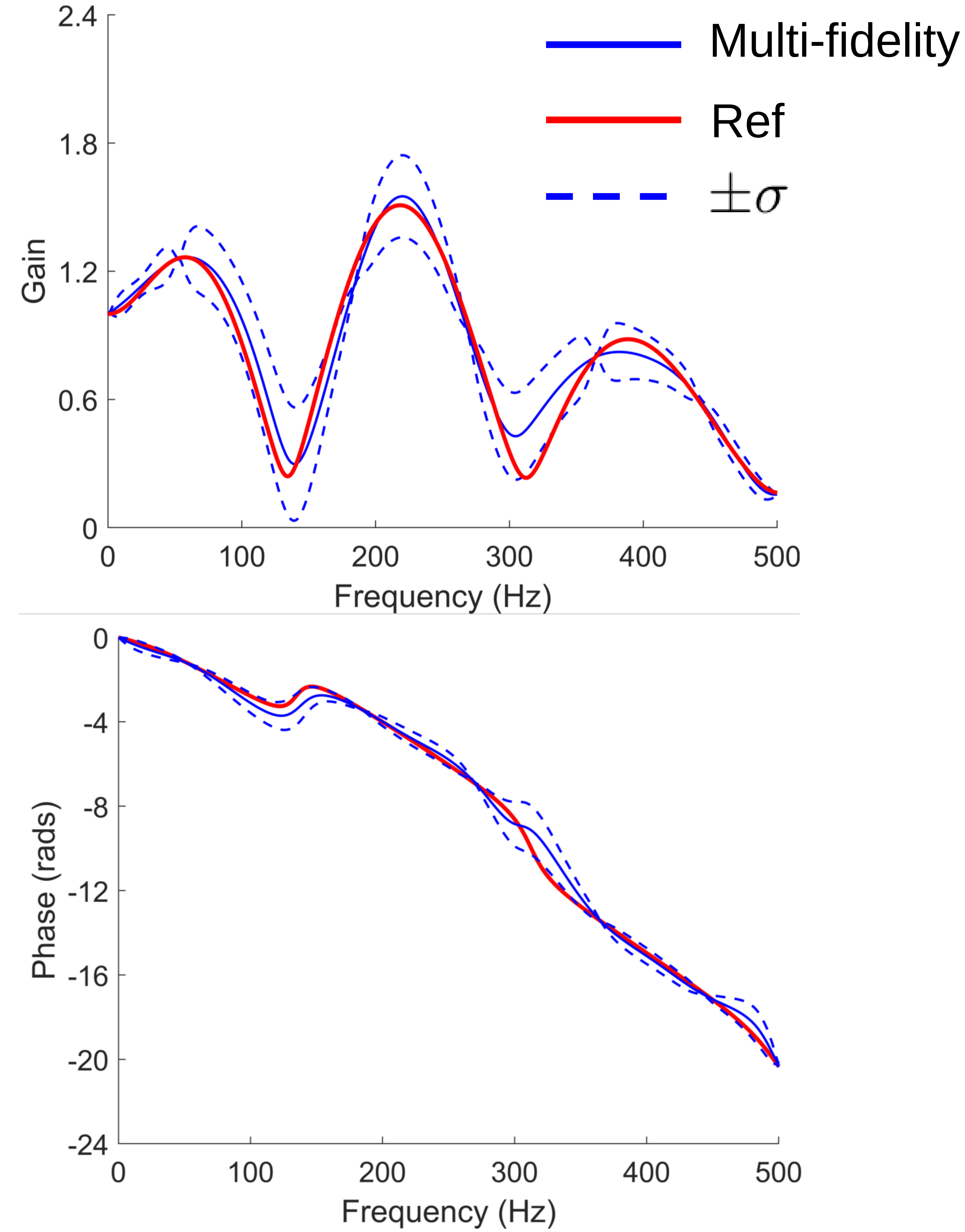
For phase, predictions made by multi-fidelity approach has less uncertainty



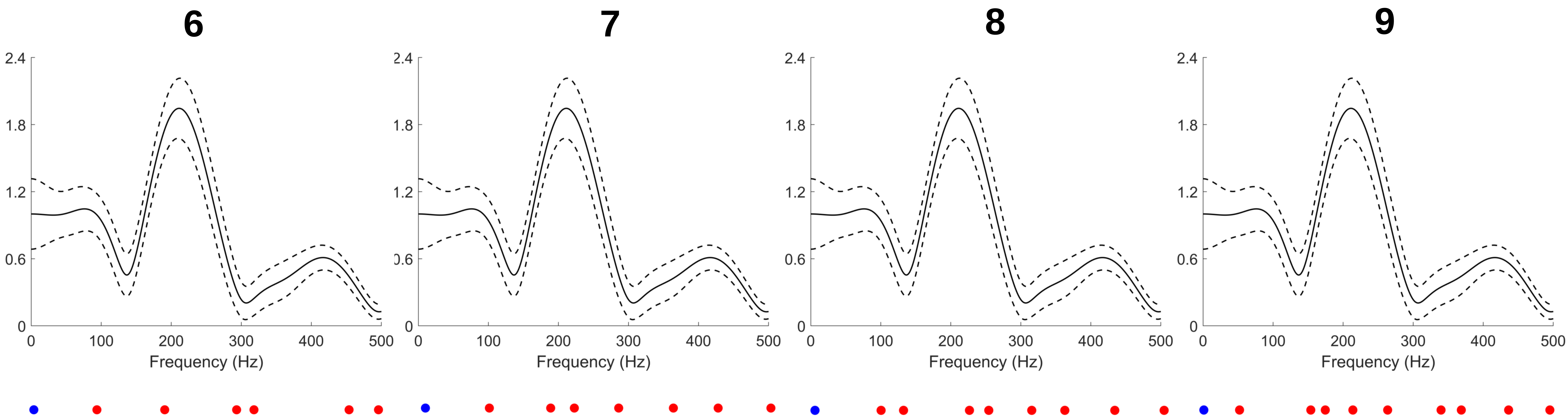
Overall, multi-fidelity approach yields globally more accurate and robust flame frequency response identification



Multi-fidelity



We investigated the impact of harmonic sample numbers and locations on multi-fidelity performance



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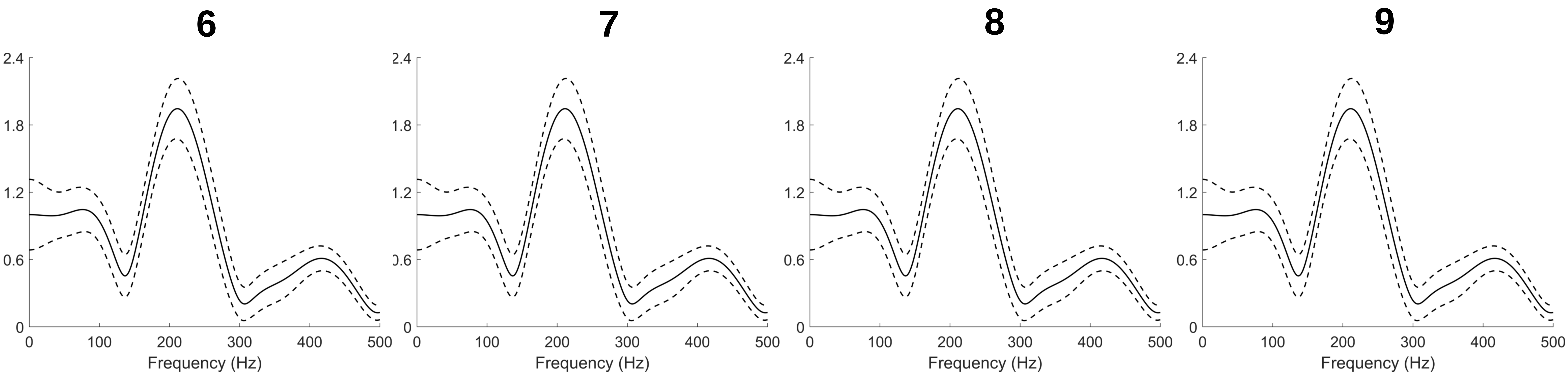

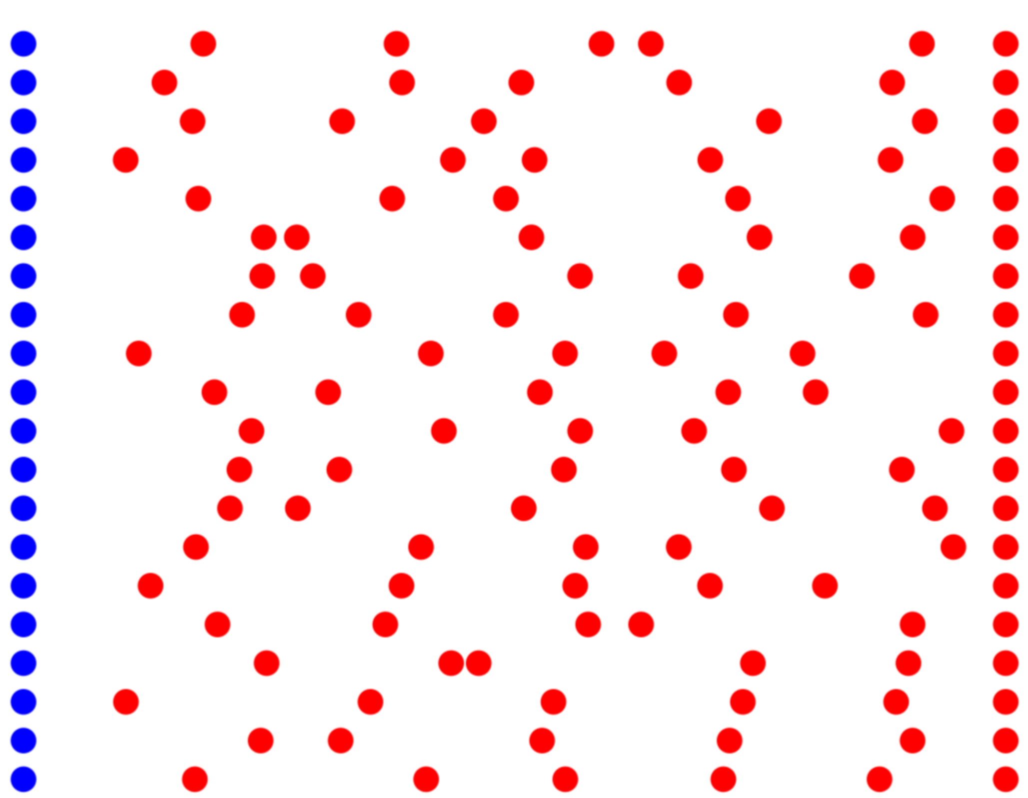

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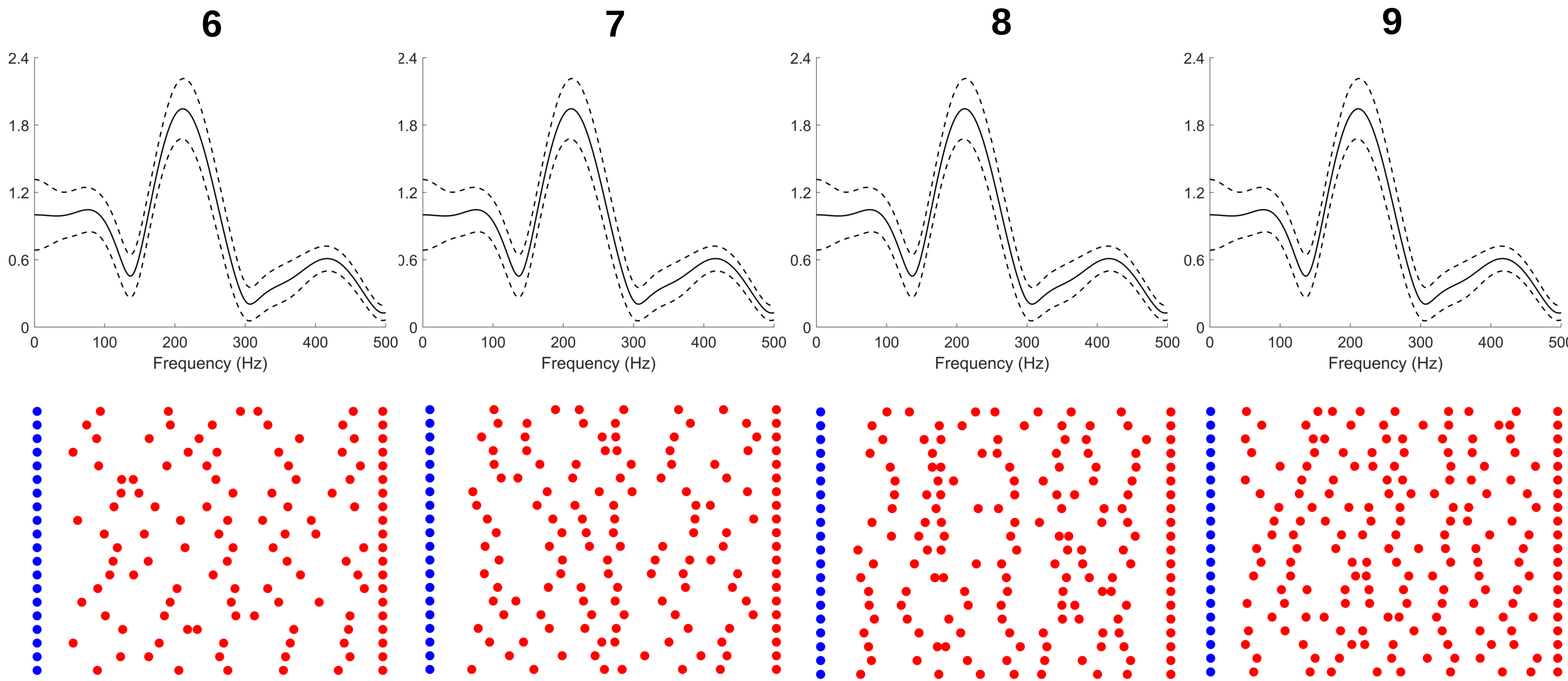

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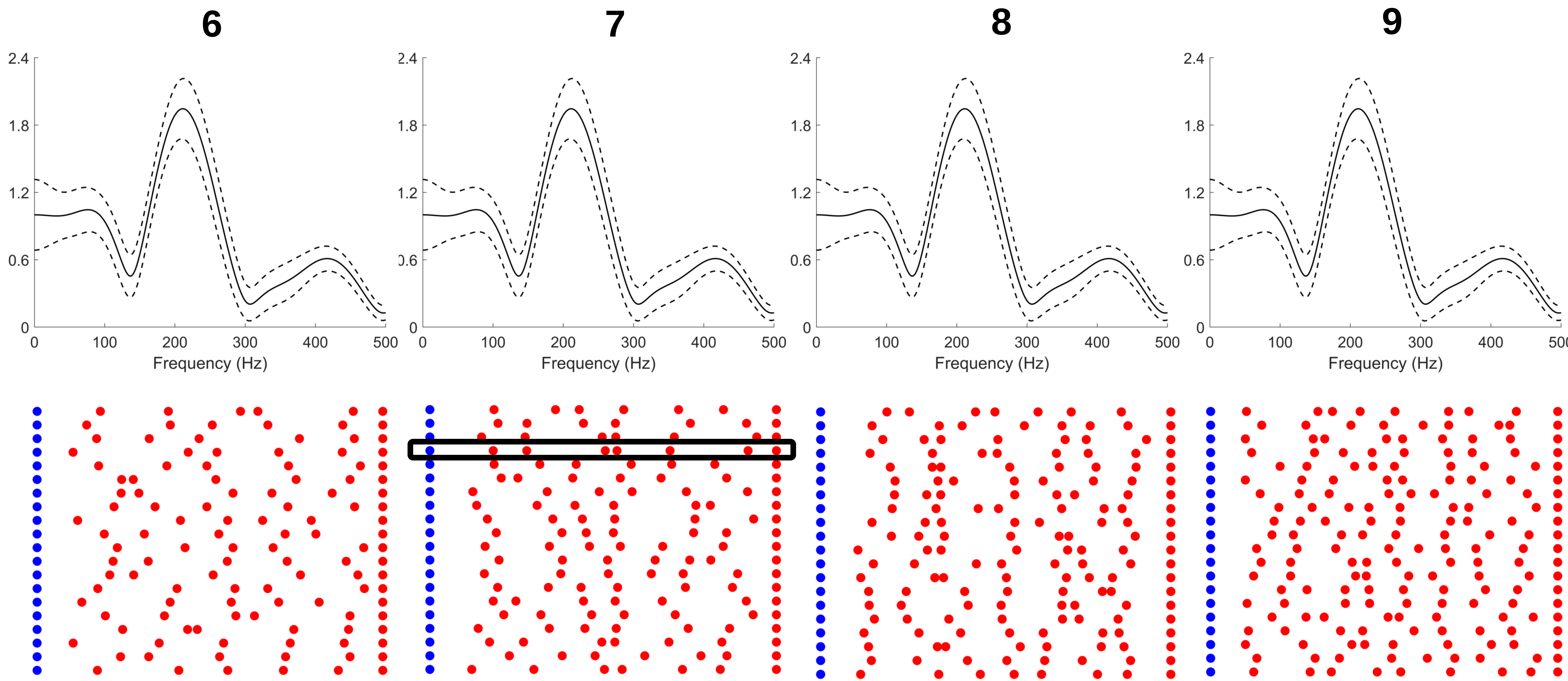
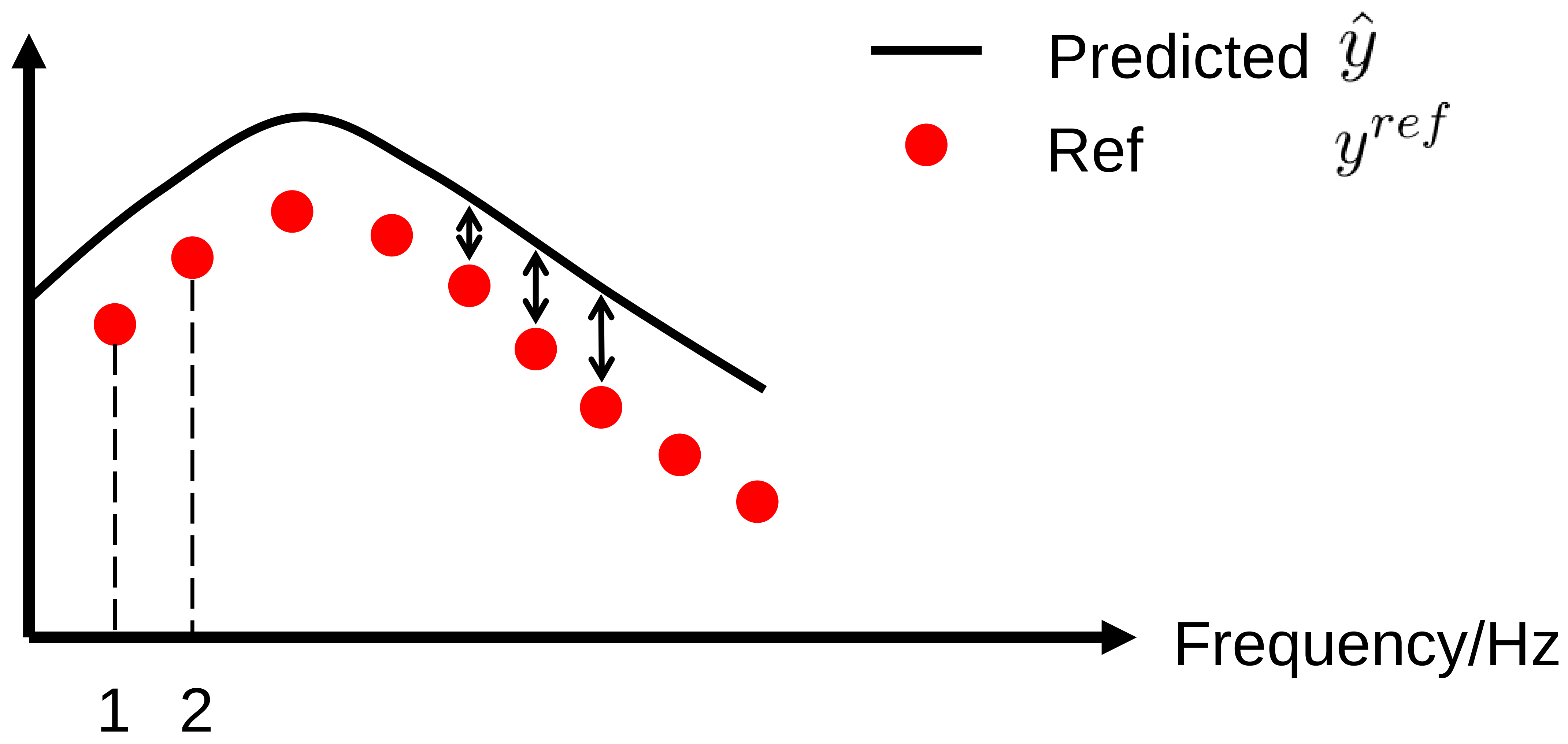
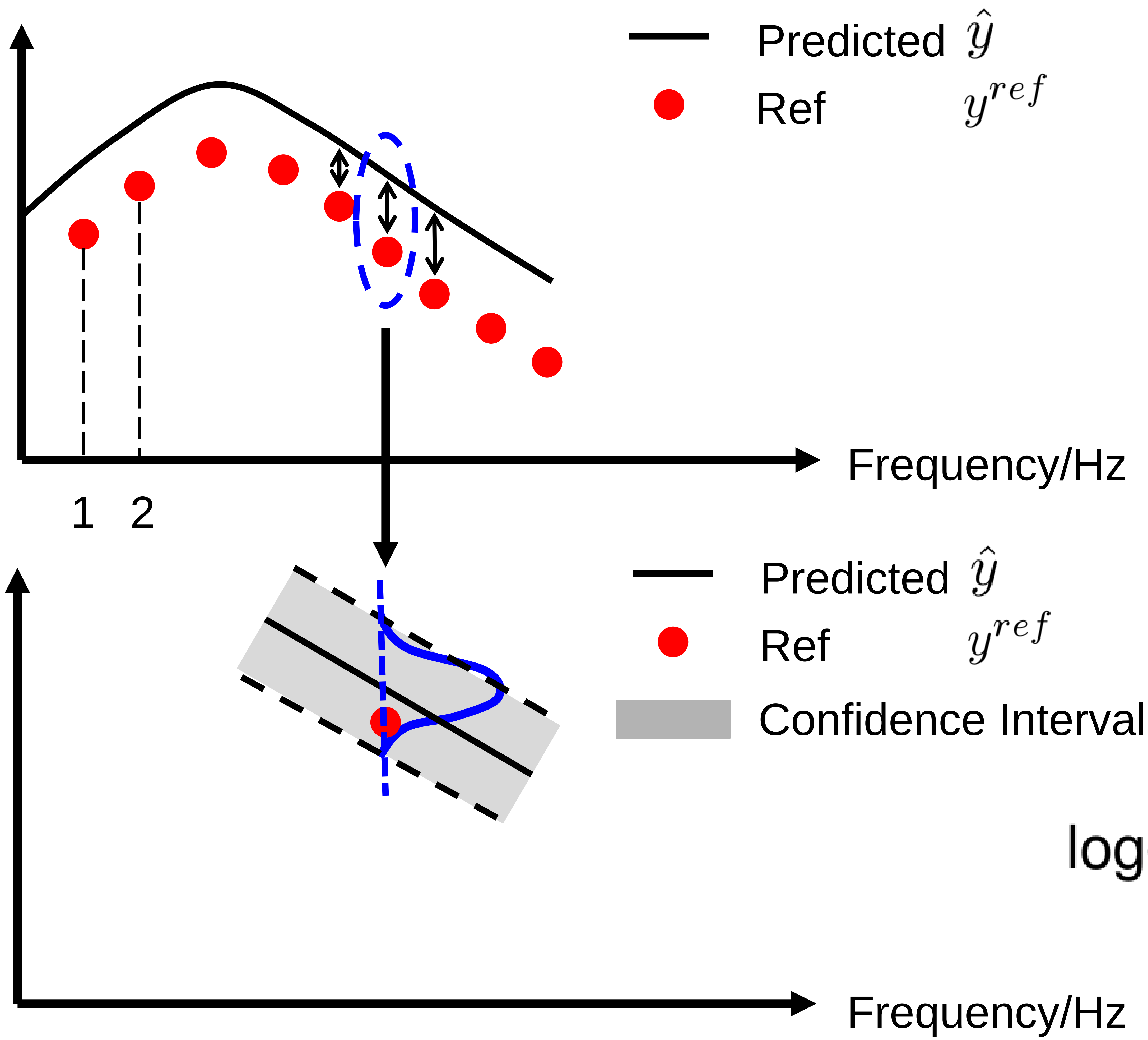


Root mean square error is used to assess the prediction accuracy



$$\text{RMSE} = \frac{\sqrt{\frac{1}{500}\sum_{i=1}^{500}(\hat{y}_i - y_i^{ref})}}{\text{range}(y^{ref})}$$

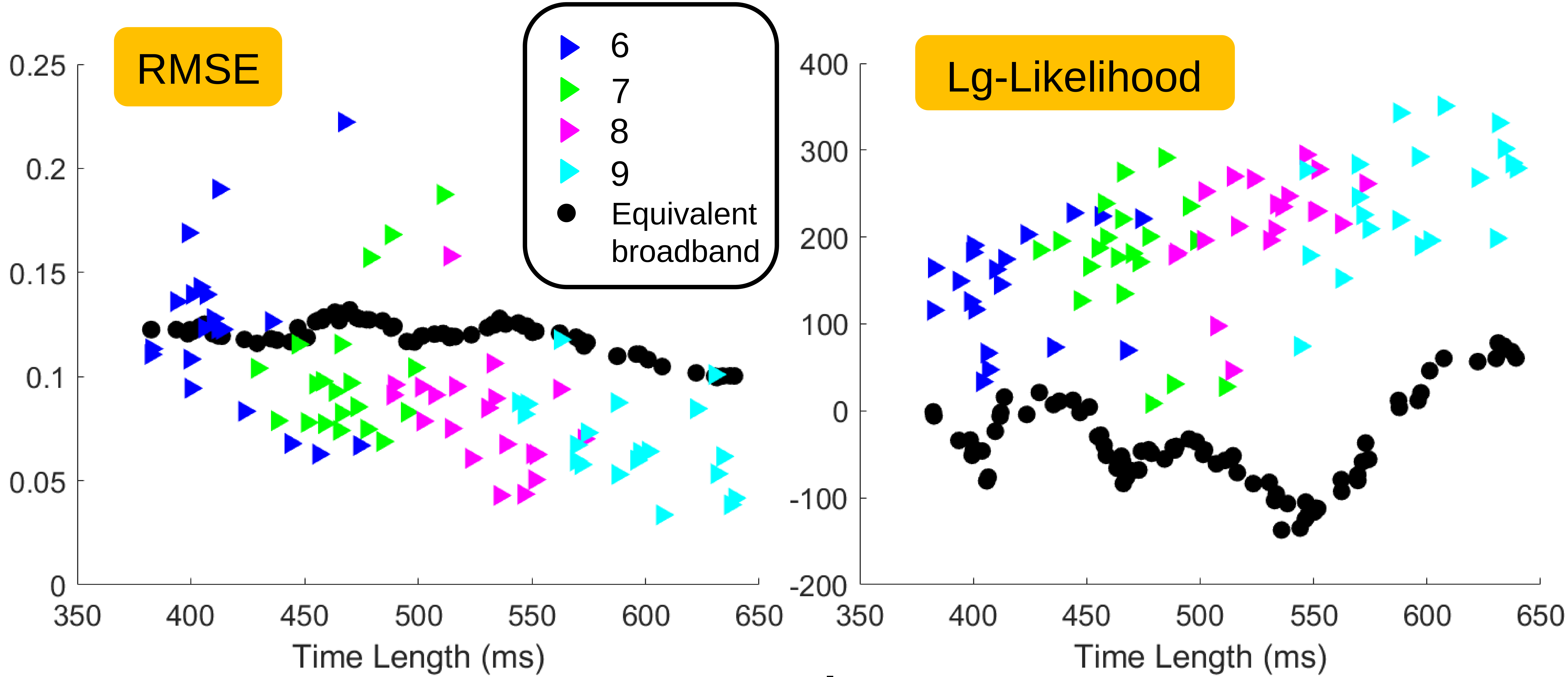
Lg-Likelihood is used to assess the prediction robustness



$$\text{RMSE} = \frac{\sqrt{\frac{1}{500}\sum_{i=1}^{500}(\hat{y}_i - y_i^{ref})}}{\text{range}(y^{ref})}$$

$$\text{log-likelihood} = \sum_{i=1}^{500} log_{10} \frac{1}{\sqrt{2\pi}\sigma_i} e^{\frac{-(\hat{y}_i - y_i^{ref})^2}{2\sigma_i^2}}$$

Number and location of harmonic excitations have direct impact on the performance of multi-fidelity approach

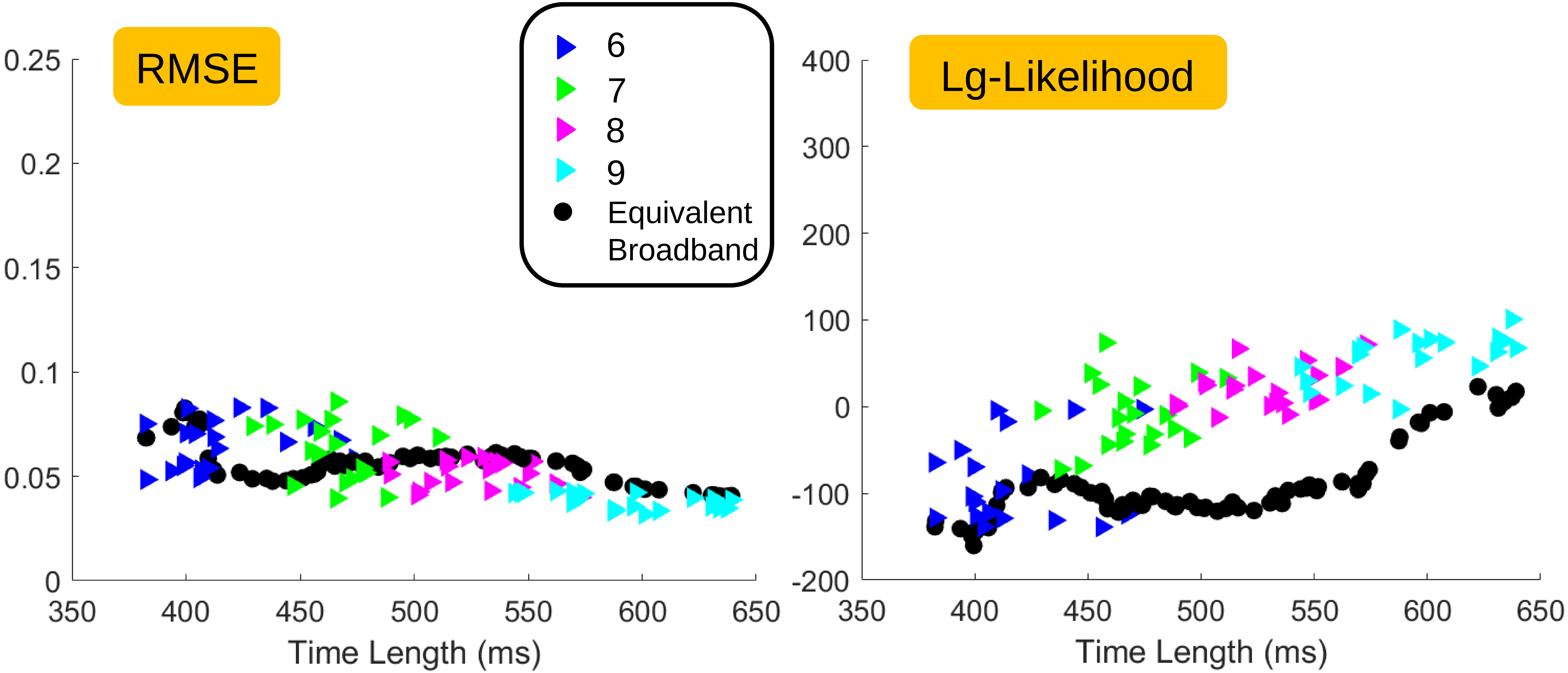


Gain

$$\text{RMSE} = \frac{\sqrt{\frac{1}{500}\sum_{i=1}^{500}(\hat{y}_i - y_i^{ref})}}{\text{range}(y^{ref})}$$

$$\text{Lg-Likelihood} = \sum_{i=1}^{500} log_{10}\frac{1}{\sqrt{2\pi}\sigma_i}e^{\frac{-(\hat{y}_i - y_i^{ref})^2}{2\sigma_i^2}}$$

Number and location of harmonic excitations have direct impact on the performance of multi-fidelity approach



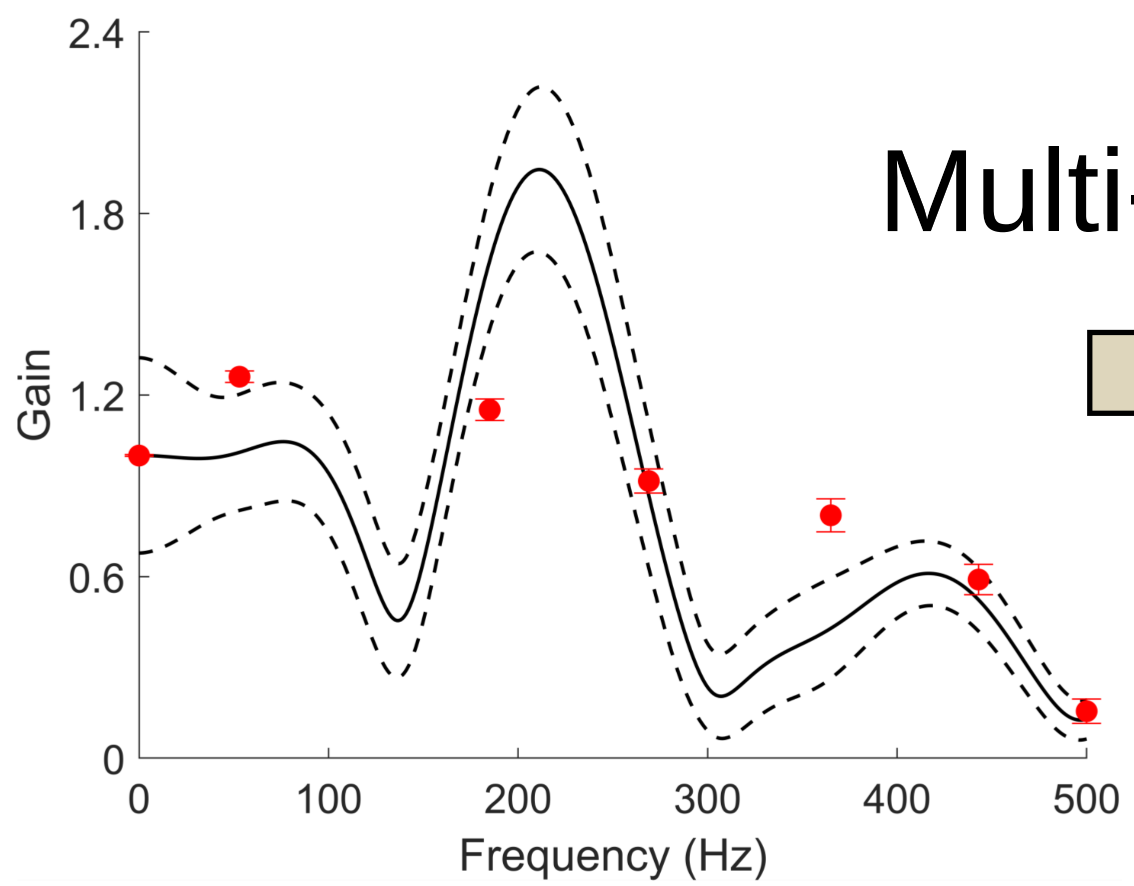
Phase

$$\text{RMSE} = \frac{\sqrt{\frac{1}{500}\sum_{i=1}^{500}(\hat{y}_i - y_i^{ref})}}{\text{range}(y^{ref})}$$

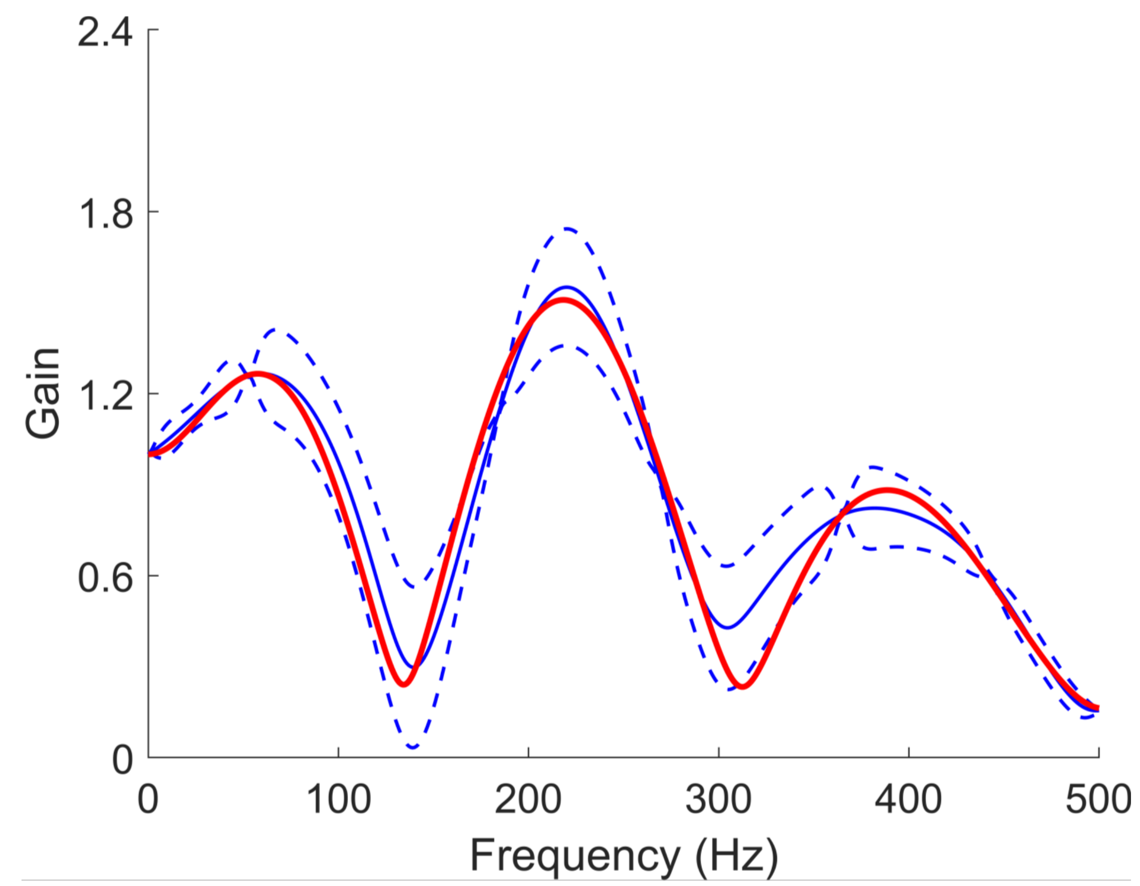
$$\text{Lg-Likelihood} = \sum_{i=1}^{500} log_{10} \frac{1}{\sqrt{2\pi}\sigma_i} e^{\frac{-(\hat{y}_i - y_i^{ref})^2}{2\sigma_i^2}}$$

Conclusion & Outlook

1

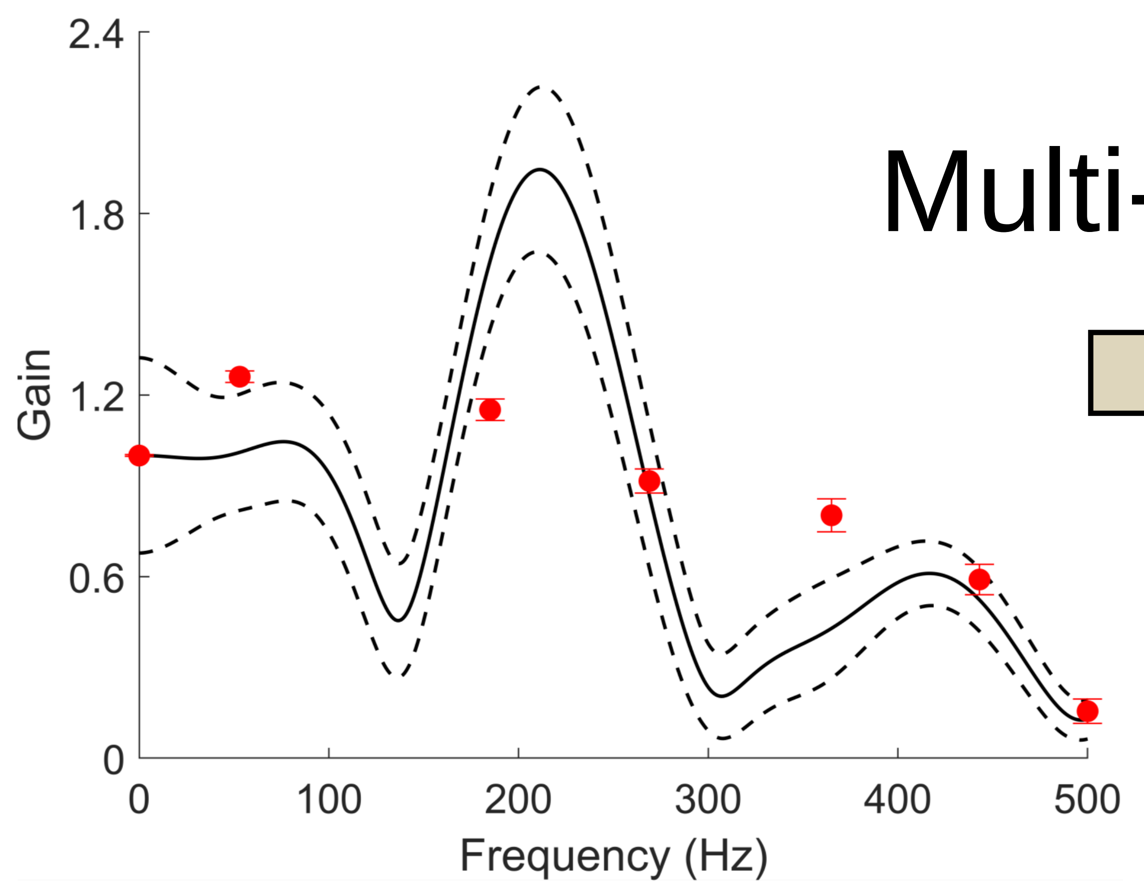


Multi-fidelity

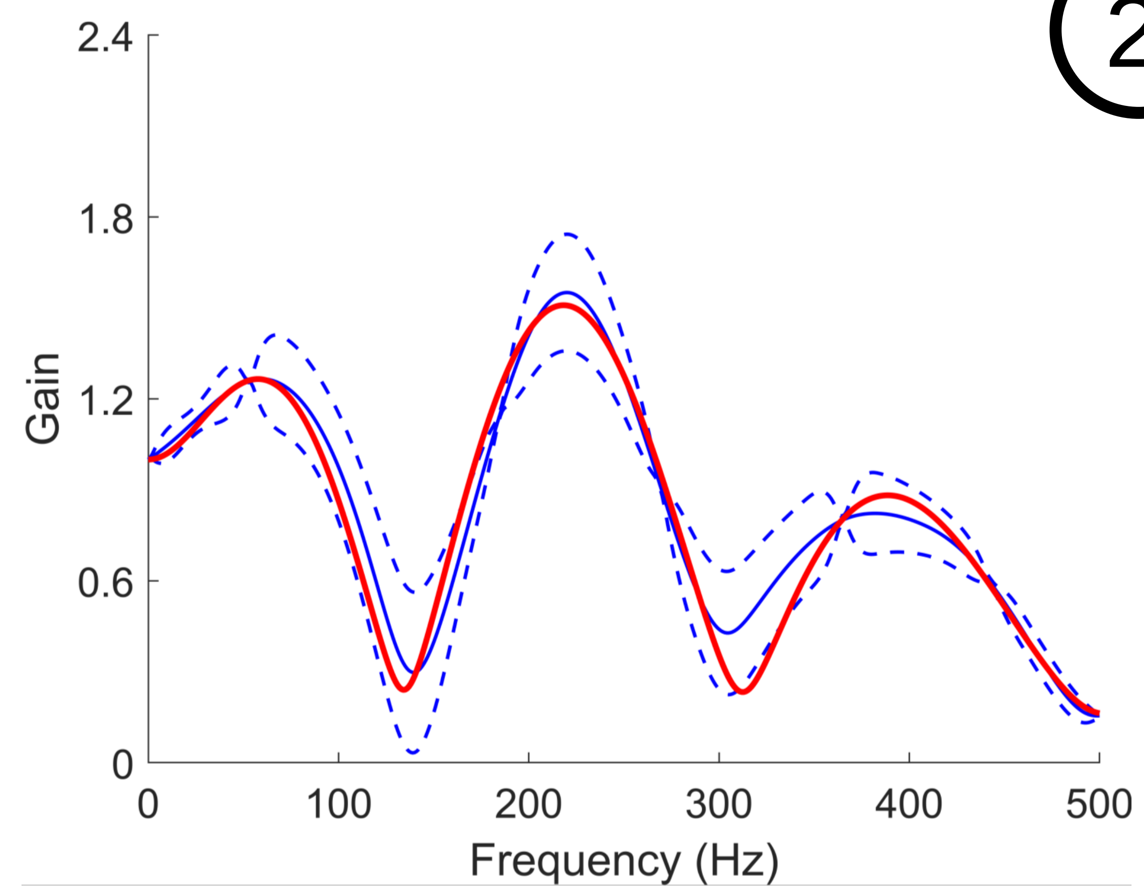


Conclusion & Outlook

1



Multi-fidelity



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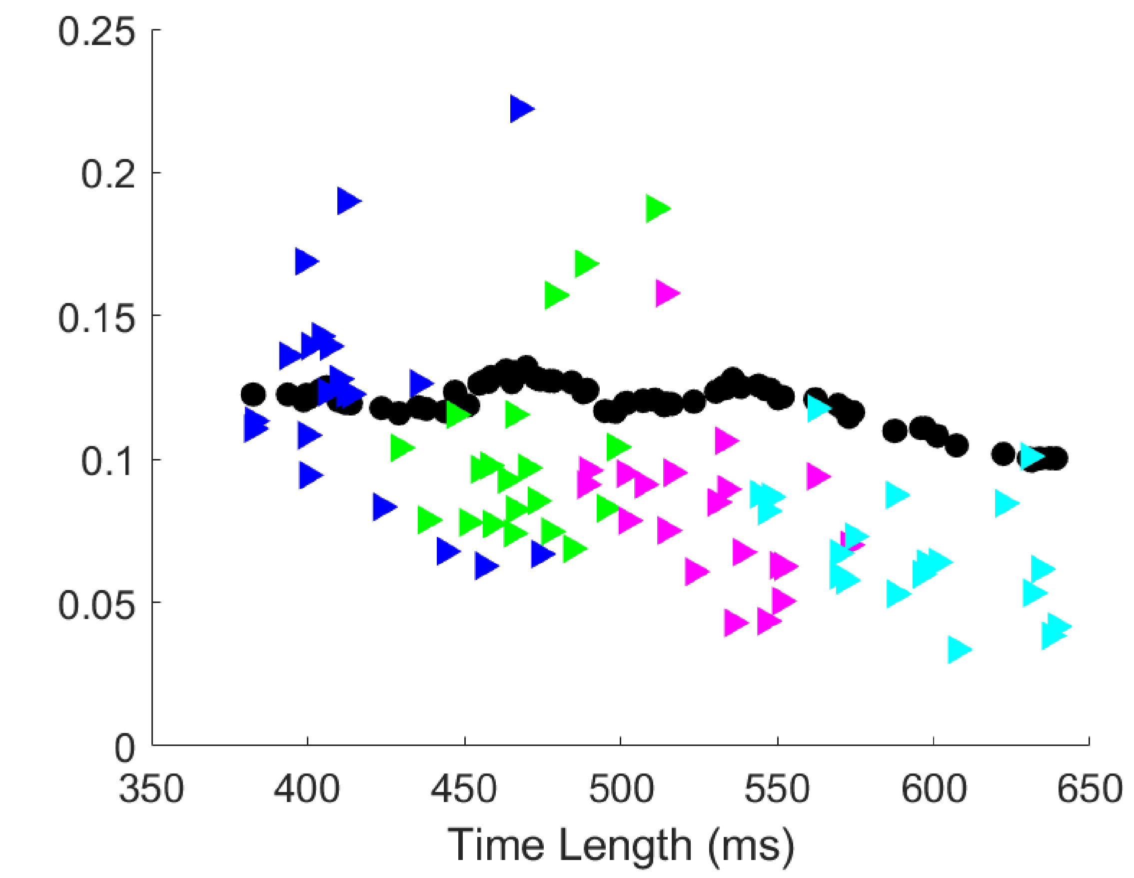
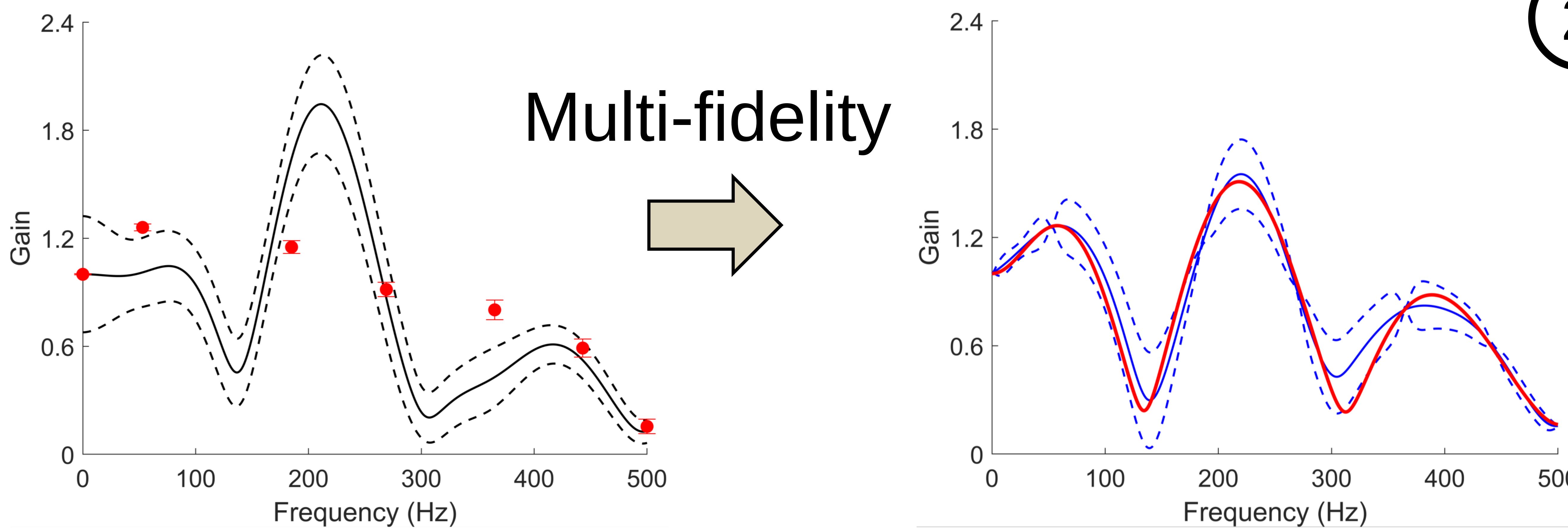


Conclusion & Outlook

1



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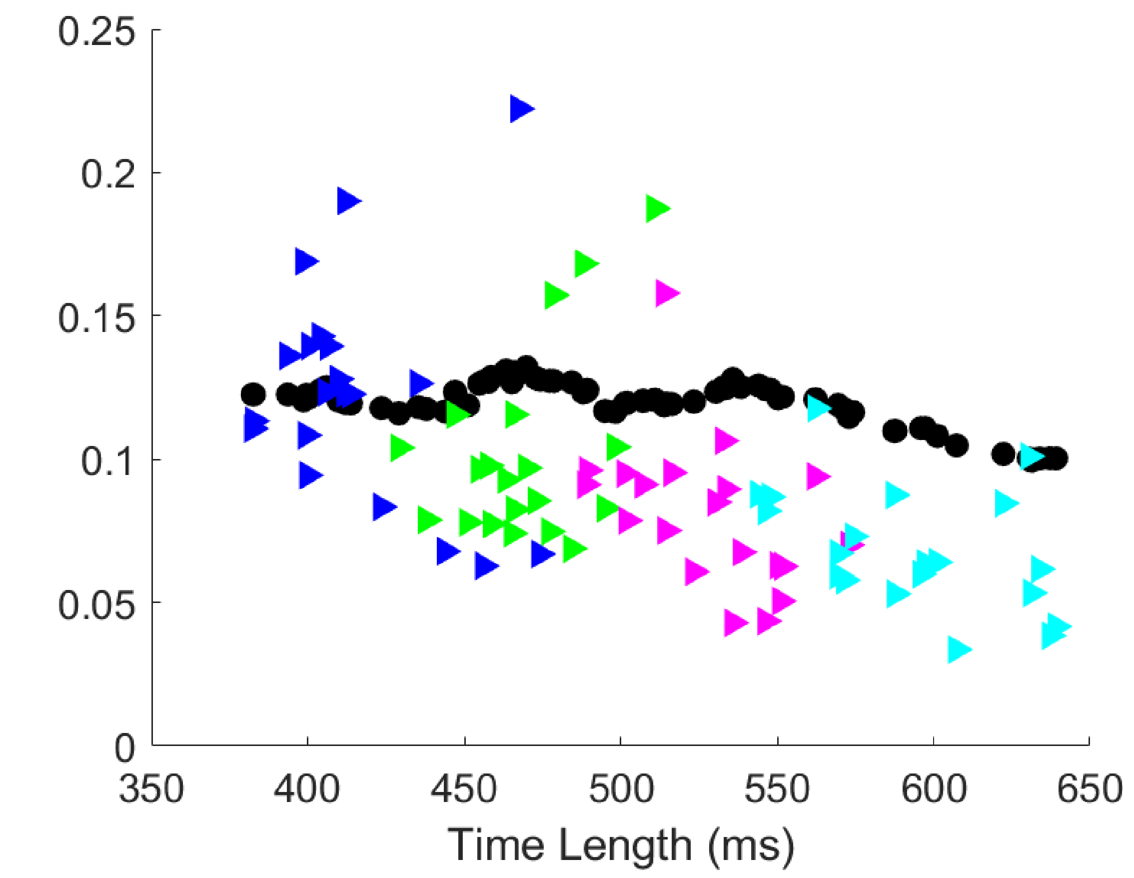
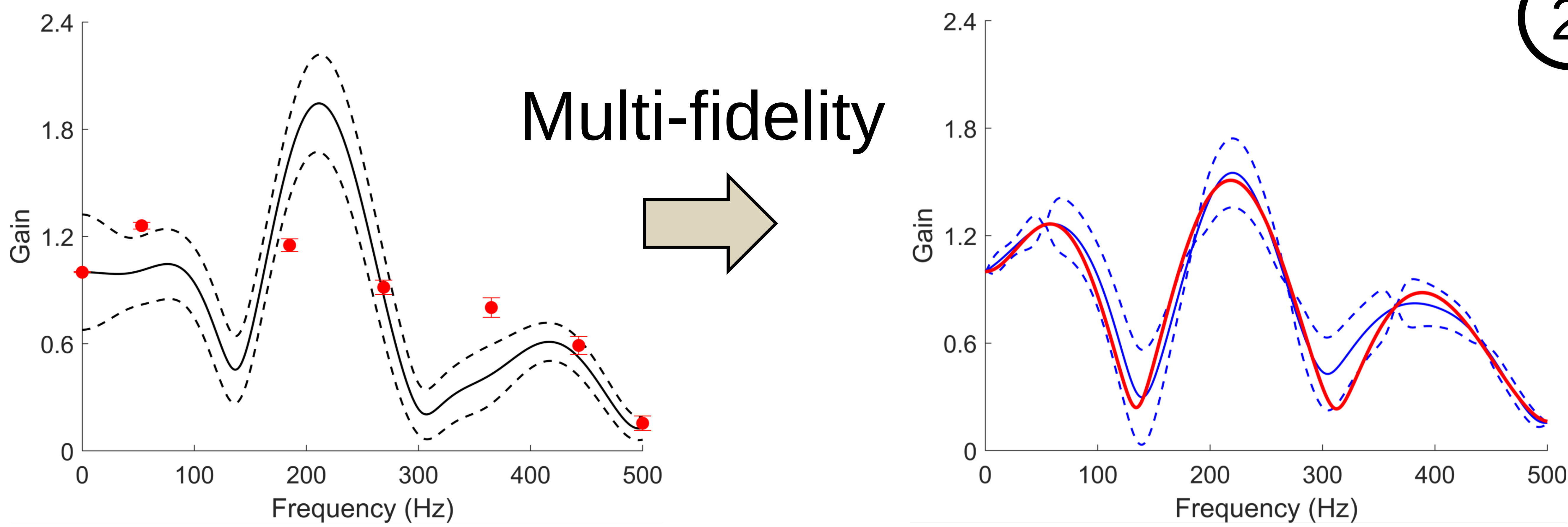


In future study:

a. Impact of noise level?

1



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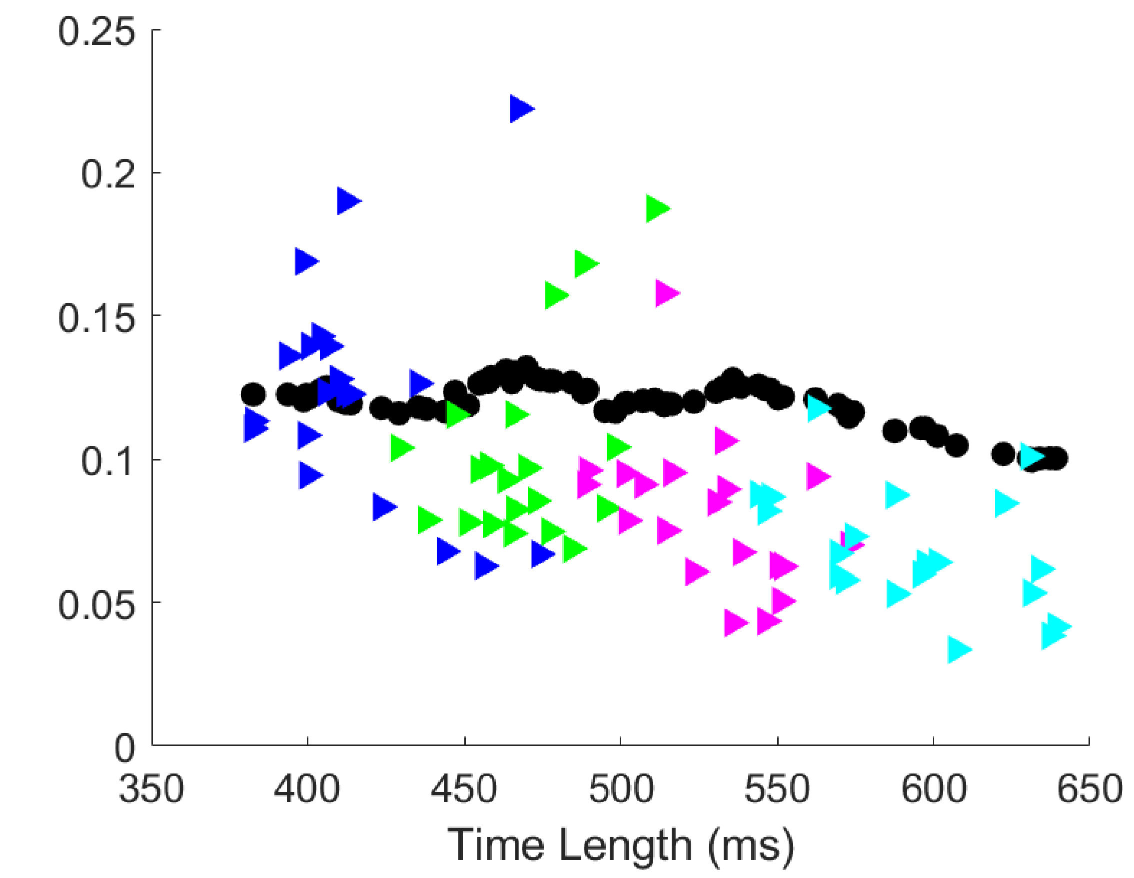


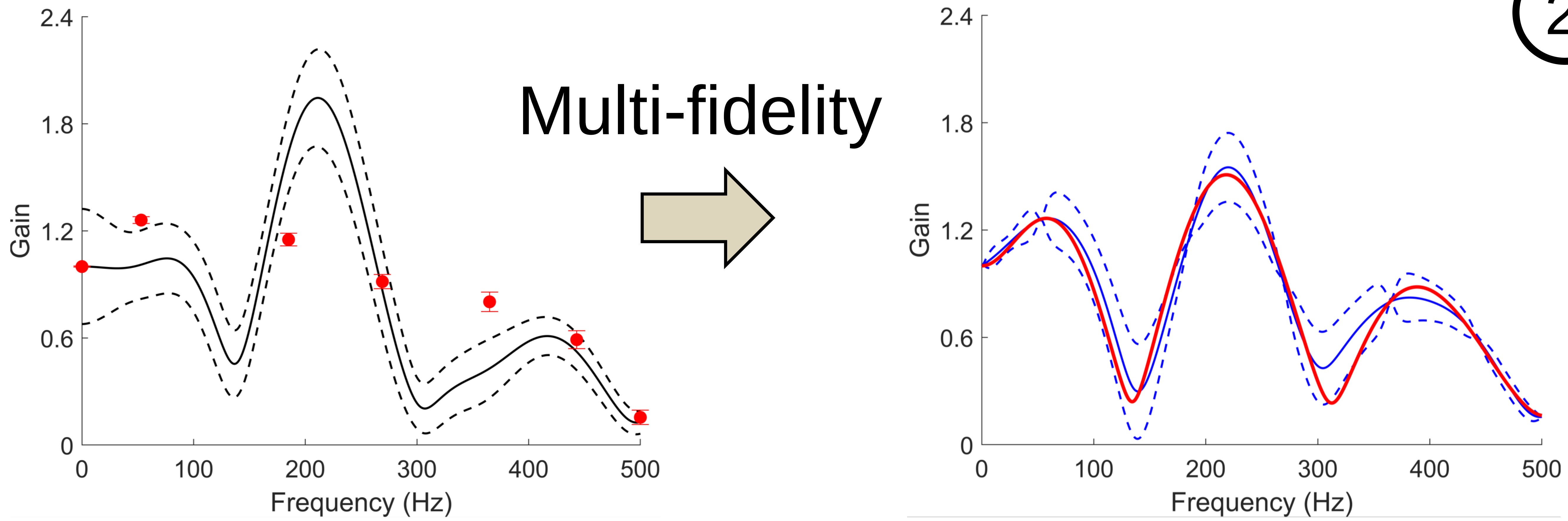
In future study:

a. Impact of noise level?

b. Intelligent frequency selection for harmonic excitations?

Conclusion & Outlook

1



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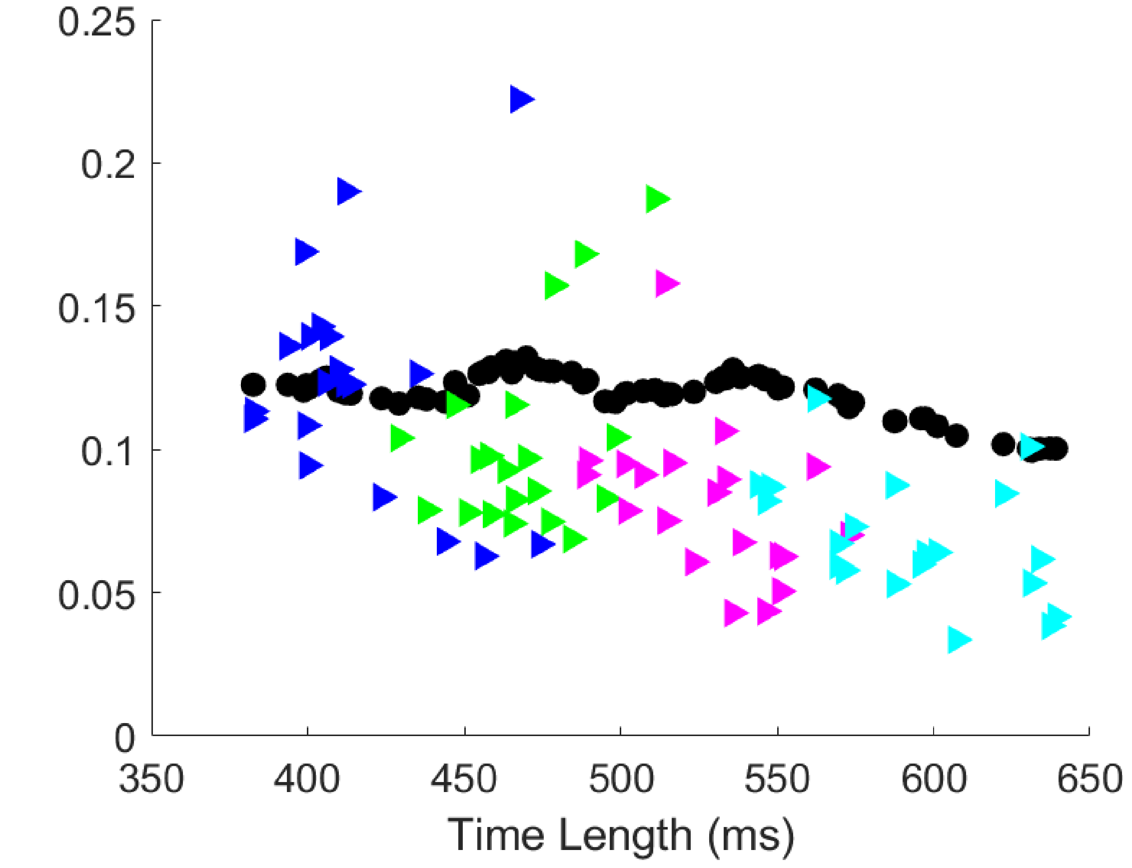

In future study:

a. Impact of noise level?

b. Intelligent frequency selection for harmonic excitations?

c. Extend to identify frequency response of other dynamic systems?