Supplementary Material for "Hybrid and Oriented Harmonic Potentials for Safe Task Execution in Unknown Environment"

Shuaikang Wang and Meng Guo[1]

Department of Mechanics and Engineering Science,
College of Engineering, Peking University, Beijing 100871, China.

Abstract

This document contains the additional definitions, proofs, algorithmic and experiment details that are omitted in the main manuscript of "Hybrid and Oriented Harmonic Potentials for Safe Task Execution in Unknown Environment", due to limited space. References to the main manuscript including equations, definitions, lemmas, theorems are marked by "P[X]".

1. Definition of Contraction-like Transformation

The transformation $\Phi_{\mathcal{M}\to\tilde{\mathcal{P}}}(q)$ form a sphere world $\mathcal{M}$ to a bounded point world in P[1] is given by $\tilde{\mathcal{P}} = O_0\backslash\{\tilde{P}_1, \cdots, \tilde{P}_M\}$.

$$\begin{aligned} &\Phi_{\mathcal{M}\to\mathcal{P}}(q) \triangleq \psi \circ \Phi_{\mathcal{M}\to\tilde{\mathcal{P}}}(q),\\ &\Phi_{\mathcal{M}\to\tilde{\mathcal{P}}}(q) \triangleq \mathrm{id}(q) + \sum_{i=1}^{M} \big(1 - s_\delta(q, O_i)\big)(q_i - q),\\ &\psi(\tilde{q}) \triangleq \frac{\rho_0}{\rho_0 - ||\tilde{q} - q_0||}(\tilde{q} - q_0) + q_0, \end{aligned} \tag{1}$$

where $s_\delta(q, O_i)$ is the contraction-like transformation for obstacle O_i, which is composed by $\eta_\delta(x) \circ \sigma(x) \circ b_i(x)$ as the switch function, smoothing function and distance function, respectively. The specifical definitions is introduced in Loizou & Rimon (2022,2021) and defined as follows.

$$\eta_\delta(x) \triangleq \frac{\sigma(x)}{\sigma(x) + \sigma(\delta - x)};$$

$$\sigma(x) \triangleq \begin{cases} e^{-1/x}, & x > 0\\ 0, & x \leqslant 0 \end{cases};$$

and

$$b_i(x) \triangleq \|x - P_i\| - \rho_i,$$

where $P_i \in \mathbb{R}^2$ is the position of the point obstacle $\mathcal{O}_i$ and ρ_i is the radius of the sphere obstacle O_i.

2. Online Computation of Harmonic Potentials

2.1. Independent Stars

Lemma 5. *Each time an independent obstacle $\mathcal{O}_{k+1}$ is added to the workspace, the following holds:*

(i) the online omitted product for $\mathcal{O}_0$ is given by

$$\overline{\beta}_0^{k+1}(q) = \overline{\beta}_0^{k}(q)\,\beta_{k+1}(q);$$

(ii) the online analytic switch for $\mathcal{O}_0$ is given by

$$s_0^{k+1}(q) = \frac{s_0^k(q)}{\alpha^k(q)\,(1 - s_0^k(q)) + s_0^k(q)},$$

where $\alpha^k(q) = \lambda_{k+1}/\big(\lambda_k\,\beta_{k+1}(q)\big)$.

Proof. (i) By Definition P[4],

$$\overline{\beta}_0^{k}(q) = \prod_{j=0, j\neq i}^{k} \beta_j(q)$$

at step-k and

$$\overline{\beta}_0^{k+1}(q) = \prod_{j=0, j\neq i}^{k+1} \beta_j(q)$$

at step-$(k+1)$. Comparing these two equations, it follows that

$$\overline{\beta}_0^{k+1}(q) = \overline{\beta}_0^{k}(q)\,\beta_{k+1}(q).$$

(ii) By Definition P[5],

$$s_0^k(q) = \frac{\gamma_G(q)\,\overline{\beta}_0^k(q)}{\lambda_k\,\beta_0(q) + \gamma_G(q)\,\overline{\beta}_0^k(q)},$$

which can be rearranged as

$$\overline{\beta}_0^k(q) = \frac{\lambda_k\, s_0^k(q)\,\beta_0(q)}{(1 - s_0^k(q))\,\gamma_G(q)}.$$

Further by Definition P[5] and (i),

$$\begin{aligned} s_0^{k+1}(q) &= \frac{\gamma_G(q)\,\overline{\beta}_0^{k+1}(q)}{\lambda_{k+1}\,\beta_0(q) + \gamma_G(q)\,\overline{\beta}_0^{k+1}(q)}\\ &= \frac{\gamma_G(q)\,\beta_{k+1}(q)\,\overline{\beta}_0^k(q)}{\lambda_{k+1}\,\beta_0(q) + \gamma_G(q)\,\beta_{k+1}(q)\,\overline{\beta}_0^k(q)}. \end{aligned}$$

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Substituting the expression for $\overline{\beta}_0^k(q)$ derived earlier, it holds that

$$s_0^{k+1}(q) = \frac{\gamma_G(q)\,\beta_{k+1}(q)\,\frac{\lambda_k\, s_0^k(q)\,\beta_0(q)}{(1-s_0^k(q))\,\gamma_G(q)}}{\lambda_{k+1}\,\beta_0(q) + \gamma_G(q)\,\beta_{k+1}(q)\,\frac{\lambda_k\, s_0^k(q)\,\beta_0(q)}{(1-s_0^k(q))\,\gamma_G(q)}} = \frac{\lambda_k\, s_0^k(q)\,\beta_{k+1}(q)}{\lambda_{k+1}\,[1-s_0^k(q)] + \lambda_k\, s_0^k(q)\,\beta_{k+1}(q)}.$$

Finally, considering

$$\alpha^k(q) = \lambda_{k+1}/[\lambda_k\,\beta_{k+1}(q)],$$

it follows

$$s_0^{k+1}(q) = \frac{s_0^k(q)}{\alpha^k(q)\,[1-s_0^k(q)] + s_0^k(q)}.$$

□

Lemma 6. *Each time an independent obstacle $\mathcal{O}_{k+1}$ is added to the workspace, the following holds:*

(i) the online omitted product for $\mathcal{O}_{i+1}$ is given by

$$\overline{\beta}_{i+1}^{k+1}(q) = \overline{\beta}_i^{k+1}(q)\,\frac{\beta_i(q)}{\beta_{i+1}(q)};$$

(ii) the online analytic switch for $\mathcal{O}_{i+1}$ is given by

$$s_{i+1}^{k+1}(q) = \frac{s_i^{k+1}(q)}{\alpha_i(q)\,(1-s_i^{k+1}(q)) + s_i^{k+1}(q)},$$

where

$$\alpha_i(q) = (\beta_{i+1}(q))^2/(\beta_i(q))^2.$$

Proof. (i) By Definition P[4],

$$\overline{\beta}_i^{k+1}(q) = \prod_{j=0, j\neq i}^{k+1} \beta_j(q)$$

and

$$\overline{\beta}_{i+1}^{k+1}(q) = \prod_{j=0, j\neq i+1}^{k+1} \beta_j(q).$$

Comparing these two equations, it follows that

$$\overline{\beta}_{i+1}^{k+1}(q) = \overline{\beta}_i^{k+1}(q)\,\frac{\beta_i(q)}{\beta_{i+1}(q)}.$$

(ii) By Definition P[5],

$$s_i^{k+1}(q) = \frac{\gamma_G(q)\,\overline{\beta}_i^{k+1}(q)}{\lambda_{k+1}\,\beta_i(q) + \gamma_G(q)\,\overline{\beta}_i^{k+1}(q)},$$

which can be rearranged as

$$\overline{\beta}_i^{k+1}(q) = \frac{\lambda_{k+1}\, s_i^{k+1}(q)\,\beta_i(q)}{(1-s_i^{k+1}(q))\,\gamma_G(q)}.$$

Further by Definition P[5] and (i),

$$s_{i+1}^{k+1}(q) = \frac{\gamma_G(q)\,\overline{\beta}_{i+1}^{k+1}(q)}{\lambda_{k+1}\,\beta_{i+1}(q) + \gamma_G(q)\,\overline{\beta}_{i+1}^{k+1}(q)} = \frac{\gamma_G(q)\,(\beta_i(q))^2\,\overline{\beta}_i^{k+1}(q)}{\lambda_{k+1}\,[\beta_{i+1}(q)]^2 + \gamma_G(q)\,(\beta_i(q))^2\,\overline{\beta}_i^{k+1}(q)}.$$

Substituting the expression for $\overline{\beta}_0^k(q)$ derived earlier, it holds that

$$s_{i+1}^{k+1}(q) = \frac{\gamma_G(q)\,(\beta_i(q))^2\,\frac{\lambda_{k+1}\, s_i^{k+1}(q)\,\beta_i(q)}{(1-s_i^{k+1}(q))\,\gamma_G(q)}}{\lambda_{k+1}\,(\beta_{i+1}(q))^2 + \gamma_G(q)\,(\beta_i(q))^2\,\frac{\lambda_{k+1}\, s_i^{k+1}(q)\,\beta_i(q)}{(1-s_i^{k+1}(q))\,\gamma_G(q)}} = \frac{s_i^{k+1}(q)\,(\beta_i(q))^2}{(1-s_i^{k+1}(q))\,(\beta_{i+1}(q))^2 + s_i^{k+1}(q)\,(\beta_i(q))^2}.$$

Finally, considering

$$\alpha_i(q) = (\beta_{i+1}(q))^2/(\beta_i(q))^2,$$

it follows

$$s_{i+1}^{k+1}(q) = \frac{s_i^{k+1}(q)}{\alpha_i(q)\,(1-s_i^{k+1}(q)) + s_i^{k+1}(q)}.$$

□

2.2. Overlapping Stars

Lemma 7. *Each time an obstacle $\mathcal{O}_{k+1}$ is added to the workspace and overlapping with an existing obstacle, the following holds:*

(i) the online omitted product for $\mathcal{O}_{k+1}$ is given by

$$\overline{\beta}_{k+1}(q) = \overline{\beta}_k(q)\,\frac{(\beta_k(q))^2\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)}{\tilde{\beta}_k(q)\,\beta_{(k+1)^\star}(q)};$$

(ii) the online analytic switch for $\mathcal{O}_{k+1}$ is given by:

$$\sigma_{k+1}(q) = \frac{\sigma_k(q)}{\alpha_k(q)(1-\sigma_k(q)) + \sigma_k(q)},$$

where

$$\alpha_k(q) = \frac{\xi_{k+1}\,\beta_{k+1}(q)\,\tilde{\beta}_k(q)\,\beta_{(k+1)^\star}(q)}{\xi_k\,(\beta_k(q))^3\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)}.$$

Proof. (i) By Definition P[6],

$$\overline{\beta}_{k+1}(q) \triangleq \Big(\prod_{j=0, j\neq(k+1)^\star}^{k} \beta_j(q)\Big)\Big(\prod_{j\in\mathcal{L}\setminus\{k+1\}} \beta_j(q)\Big)\tilde{\beta}_{k+1}(q),$$

and

$$\overline{\beta}_k(q) \triangleq \Big(\prod_{j=0, j\neq k^\star}^{k-1} \beta_j(q)\Big)\Big(\prod_{j\in\mathcal{L}\setminus\{k\}} \beta_j(q)\Big)\tilde{\beta}_k(q).$$

Comparing these two equations, it follows that

$$\overline{\beta}_{k+1}(q) = \overline{\beta}_k(q)\,\frac{(\beta_k(q))^2\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)}{\tilde{\beta}_k(q)\,\beta_{(k+1)^\star}(q)}.$$

(ii) By Definition P[7],

$$\sigma_k(q) = \frac{\gamma_G(q)\,\overline{\beta}_k(q)}{\xi_k\,\beta_k(q) + \gamma_G(q)\,\overline{\beta}_k(q)},$$

which can be rearranged as

$$\overline{\beta}_k(q) = \frac{\xi_k\,\sigma_k(q)\,\beta_k(q)}{(1-\sigma_k(q))\,\gamma_G(q)}.$$

Further by Definition P[7] and (i),

$$\begin{aligned}\sigma_{k+1}(q) &= \frac{\gamma_G(q)\,\overline{\beta}_{k+1}(q)}{\xi_{k+1}\,\beta_{k+1}(q) + \gamma_G(q)\,\overline{\beta}_{k+1}(q)}\\ &= \frac{\gamma_G(q)\,\xi_k\,[\beta_k(q)]^3\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)\,\overline{\beta}_k(q)}{\xi_{k+1}\,\beta_{k+1}(q)\,\tilde{\beta}_k(q)\,\beta_{(k+1)^\star}(q) + \gamma_G(q)\,\xi_k\,[\beta_k(q)]^3\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)\,\overline{\beta}_k(q)}.\end{aligned}$$

Substituting $\overline{\beta}_k(q)$ results that

$$\sigma_{k+1}(q) = \frac{\sigma_k(q)}{\alpha_k(q)\,[1-\sigma_k(q)] + \sigma_k(q)}$$

with

$$\alpha_k(q) = \frac{\xi_{k+1}\,\beta_{k+1}(q)\,\tilde{\beta}_k(q)\,\beta_{(k+1)^\star}(q)}{\xi_k\,[\beta_k(q)]^3\,\tilde{\beta}_{k+1}(q)\,\beta_{k^\star}(q)}.$$

$\square$