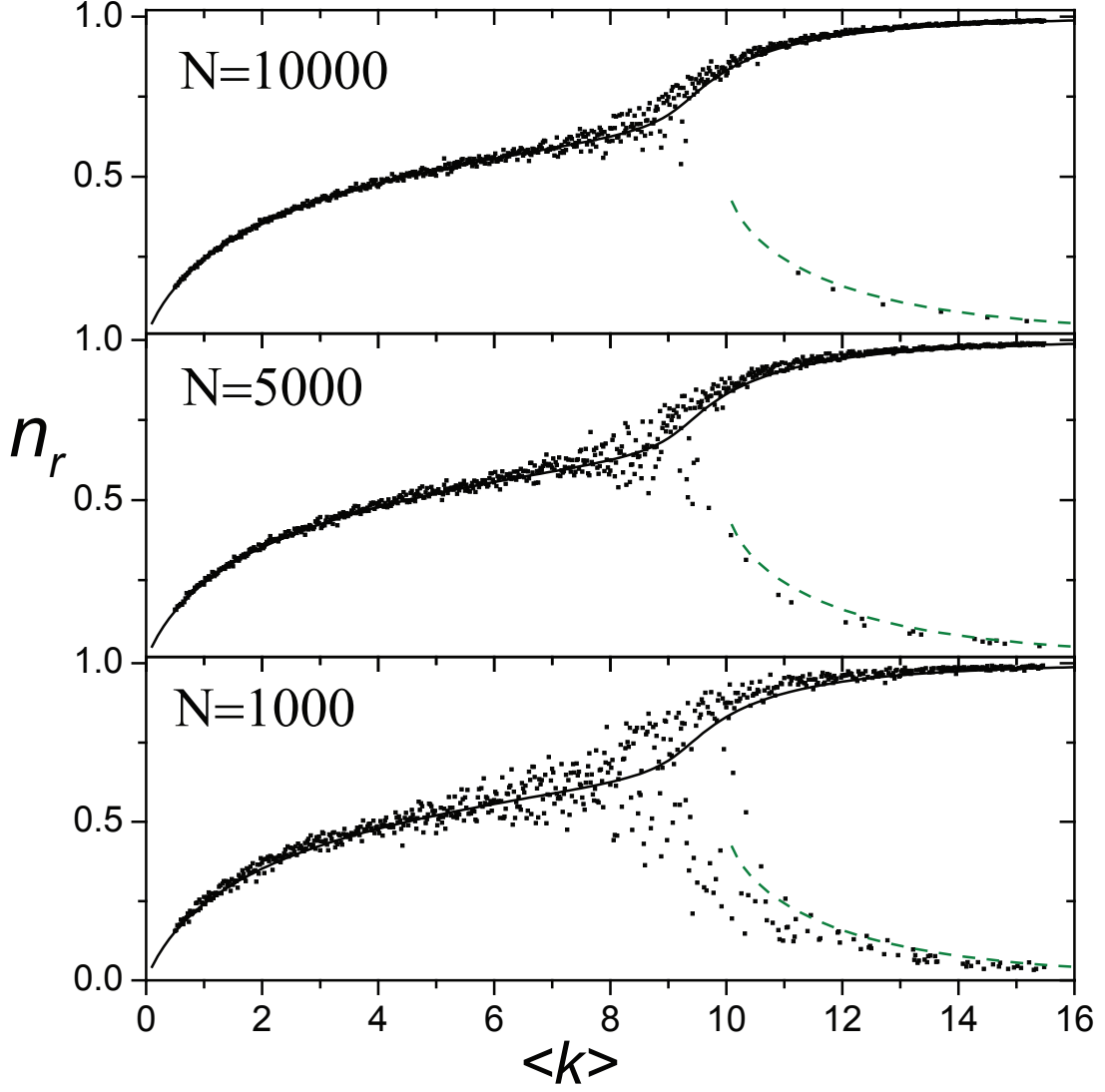
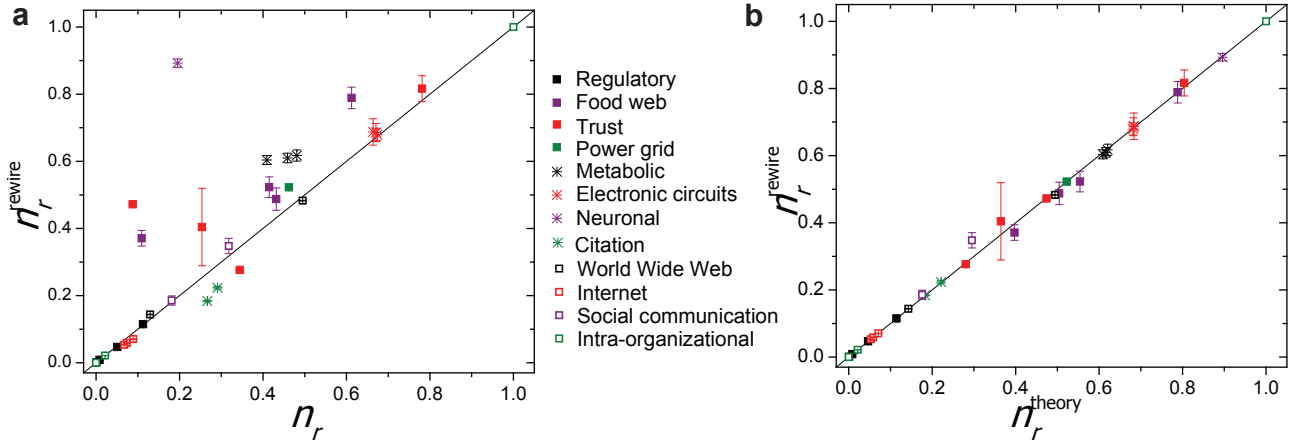


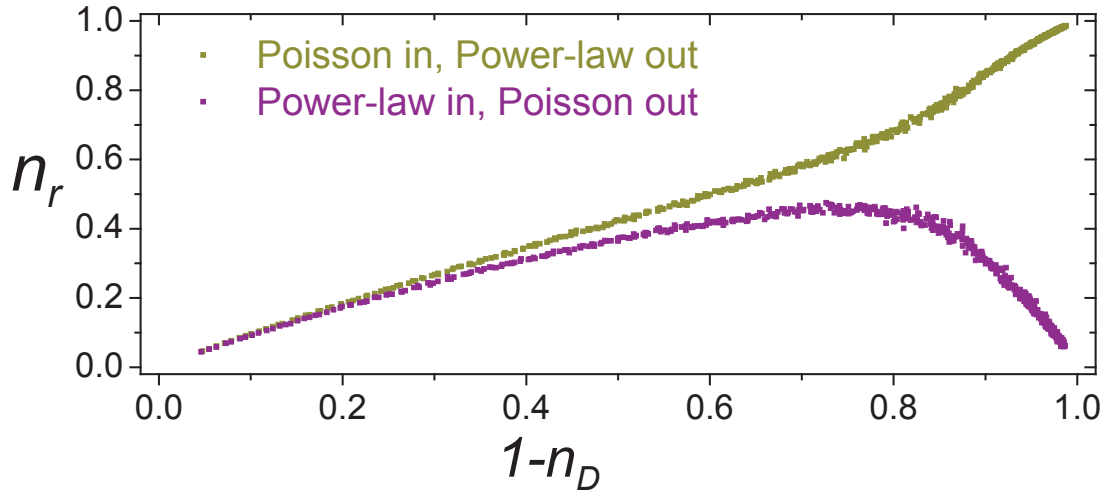
**Supplementary Figure S1: Bimodality and the size dependence under certain degree asymmetry.** Distribution of  $n_r$  in scale-free networks ( $\gamma_{\text{out}} = 3$ ,  $\gamma_{\text{in}} = 2.85$  and  $\langle k \rangle = 9.8$ ) with different sizes. When the system size is small, the distribution of  $n_r$  is bimodal, corresponding to the observed bifurcation. As system size increases,  $P(n_r)$  eventually develops a single peak. This indicates that the two modes can coexist in small systems with a modest degree asymmetry, with one mode statistically dominating the other. However infinite systems only follow one control mode when the in- and out-degrees are asymmetric.



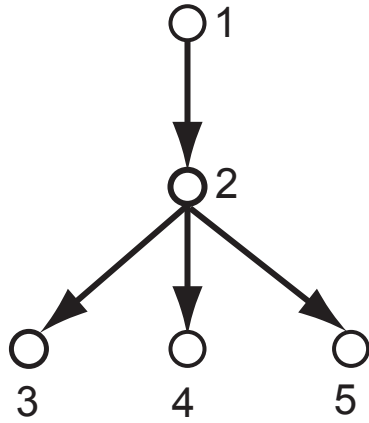
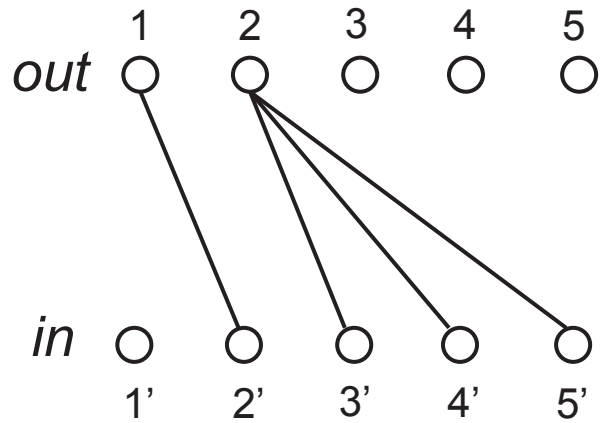
**Supplementary Figure S2: Bifurcation and the size dependence under certain degree asymmetry.** Different realizations of scale-free networks ( $\gamma_{\text{out}} = 3$  and  $\gamma_{\text{in}} = 2.85$ ) with different  $\langle k \rangle$  and network size. When the system size is small, the bifurcation is observed. However, as system size increases, fewer networks will be on the lower branch. The solid line is the  $n_r$  based on the continuous solution of equations (1) and (2) and the dashed line corresponds to the  $n_r$  found by the discontinuous solution. The minorities that are able to jump over the gap are in line with the prediction given by the discontinuous solution.



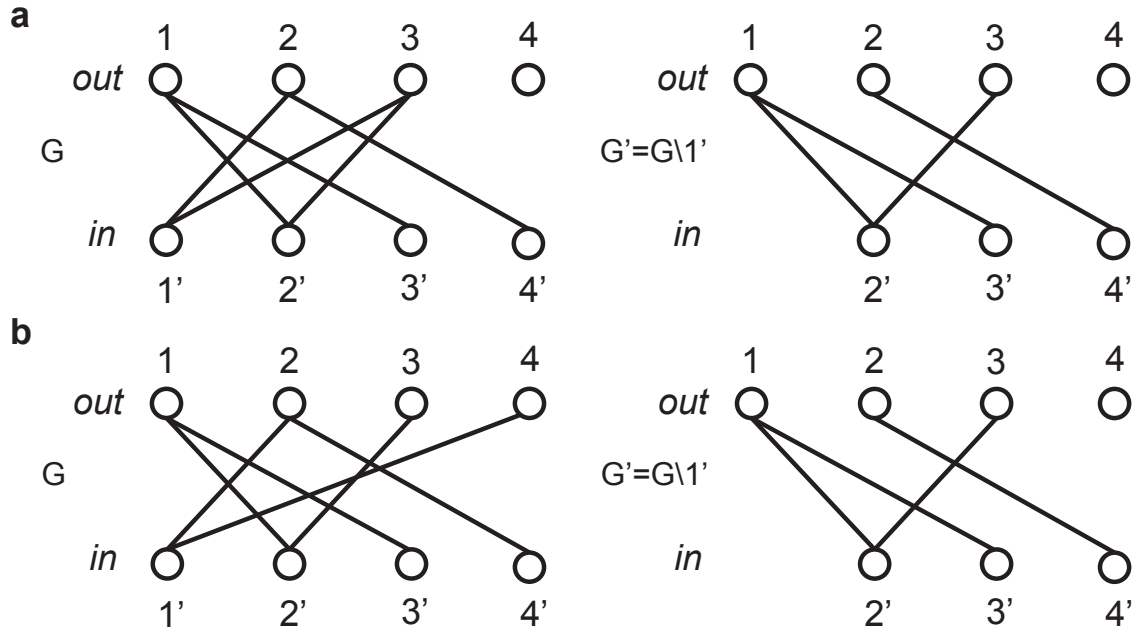
**Supplementary Figure S3: Comparison between the analytical predication  $n_r^{\text{theory}}$ ,  $n_r^{\text{rewire}}$  and the actual  $n_r$ .** (a)  $n_r$  in real networks and their degree-preserving randomized counterparts. For most real networks,  $n_r$  does not change much after rewiring the links, indicating that the degree distribution is the main factor determining the number of redundant nodes. (b)  $n_r$  calculated based on degree distributions of real networks agrees with  $n_r$  of networks obtained after degree-preserving randomization. The error bars in both figures correspond to the variance in 100 different rewiring realizations. The result implies that equations (1) and (2) work best for uncorrelated networks.



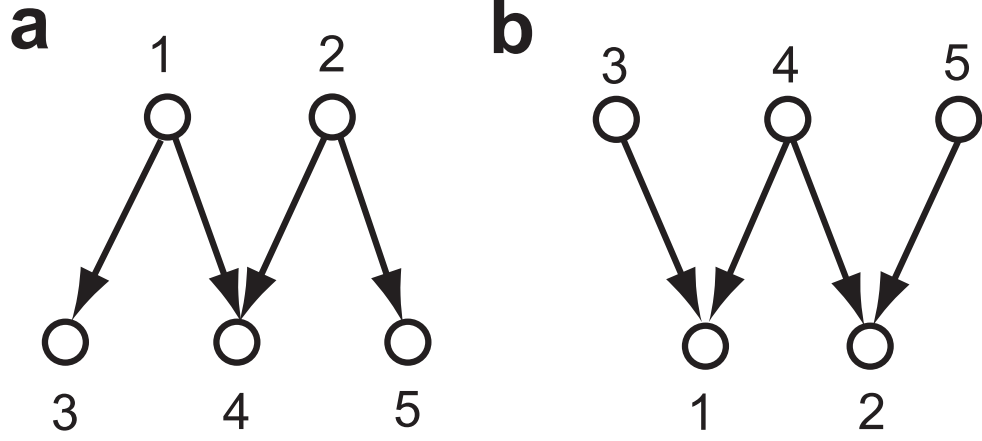
**Supplementary Figure S4:  $n_r$  vs  $1 - n_D$  for two model networks.** Upper branch is for  $P_{\text{in}}(k) \sim k^{-3}$  and  $P_{\text{out}}(k)$  follows a Poisson distribution, lower branch is for  $P_{\text{out}}(k) \sim k^{-3}$  and  $P_{\text{in}}(k)$  follows a Poisson distribution.

**a****b**

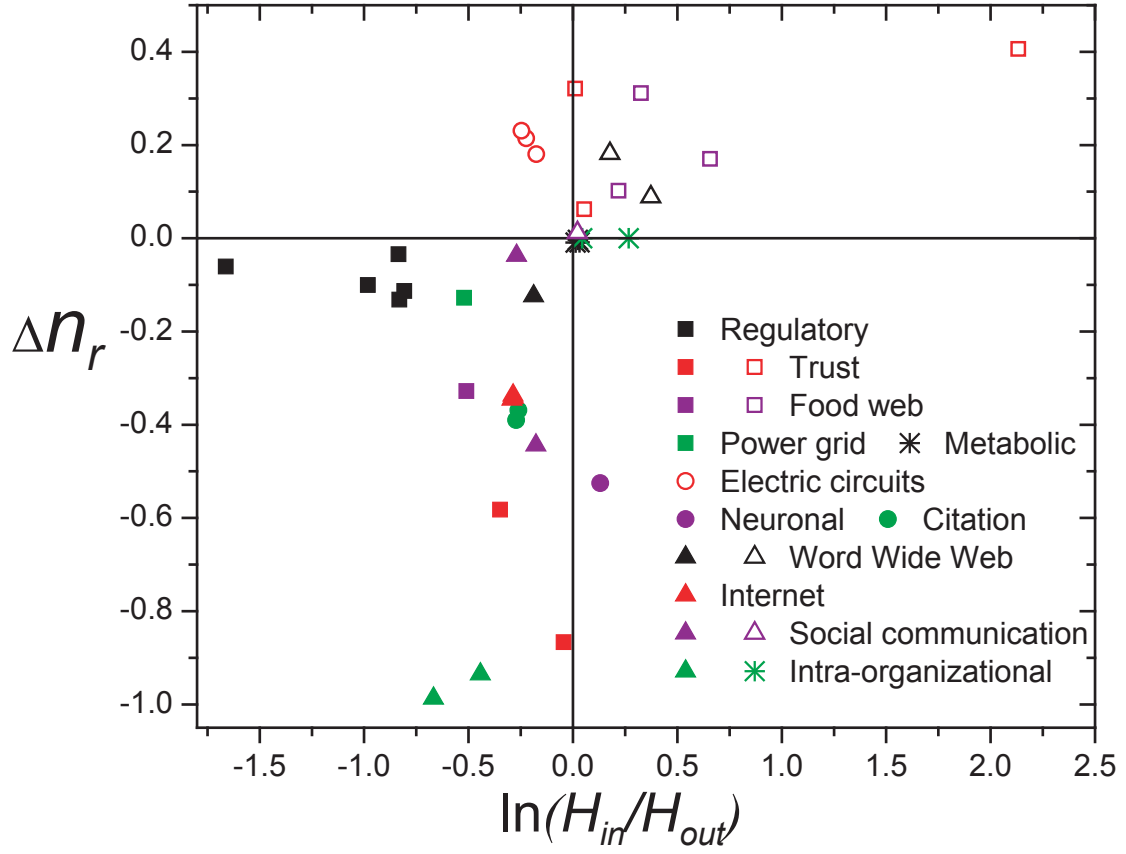
**Supplementary Figure S5: Conversion between a directed network and the corresponding bipartite representation.** (a) An example of a directed network (b) The bipartite representation of the directed network in (a). To distinguish, the *out* nodes are labeled by numbers and *in* nodes are labeled by numbers with superscript  $\prime$ .



**Supplementary Figure S6: Examples of bipartite graph  $G$  and the subgraph  $G'$ .  $G' = G \setminus 1'$  when taking off node  $1'$  and all its links. (a) Node  $1'$  is not always matched in  $G$  as both neighbors (node 2 and 3) are always matched in subgraph  $G'$ . (b) Node  $1'$  is always matched in  $G$  because node 4 (one of its neighbors) can not be matched (thus not always matched) in subgraph  $G'$ .**



**Supplementary Figure S7: An example of the control mode and the degree divergence dependence.** (a) A network with  $N_D = 3$  and  $N_r = 0$ . To control the system, we need to control node 1 and 2, and control one node among 3, 4 and 5 (b) The inverse of (a) which has  $N_D = 3$  and  $N_r = 2$ . There is only one MDS: controlling nodes 3, 4 and 5. This suggests that control mode is correlated with the degree divergence.



**Supplementary Figure S8: Control mode can be predicted by degree heterogeneity difference.** The value of  $\Delta n_r$  and  $\ln(H_{in}/H_{out})$  for real networks. Most points fall in regions  $(x \geq 0, y \geq 0)$  and  $(x \leq 0, y \leq 0)$ , indicating that the difference between heterogeneities of the in- and out-degree distribution determines the mode of  $n_r$ . Hollow symbols represent networks with  $\Delta n_r > 0$  (centralized control mode), solid symbols correspond to  $\Delta n_r < 0$  and star symbols are for  $\Delta n_r \approx 0$ .



|                      | Name                      | Description  |
|----------------------|---------------------------|--|
| Regulatory           | TRN-Yeast-1 [31]          | Transcriptional regulatory network of <i>S. cerevisiae</i> |
|                      | TRN-Yeast-2 [30]          | Same as above (compiled by different group).               |
|                      | TRN-EC-1 [32]             | Transcriptional regulatory network of <i>E. coli</i>       |
|                      | TRN-EC-2 [30]             | Same as above (compiled by different group).               |
|                      | Ownership-USCorp [61]     | Ownership network of US corporations.                      |
| Trust                | College student [52,53]   | Social networks of positive sentiment (college students).  |
|                      | Prison inmate [52,53]     | Same as above (prison inmates).                            |
|                      | Slashdot [39]             | Social network (friend/foe) of Slashdot users.             |
|                      | WikiVote [39]             | Who-vote-whom network of Wikipedia users.                  |
|                      | Epinions [62]             | Who-trust-whom network of Epinions.com users.              |
| Food Web             | Ythan [63]                | Food Web in Ythan Estuary.                                 |
|                      | Little Rock [54]          | Food Web in Little Rock lake.                              |
|                      | Grassland [63]            | Food Web in Grassland.                                     |
|                      | Seagrass [64]             | Food Web in St. Marks Seagrass.                            |
| Power Grid           | TexasPowerGrid [65]       | Power grid in Texas.                                       |
| Metabolic            | <i>E. coli</i> [45]       | Metabolic network of <i>E. coli</i> .                      |
|                      | <i>S. cerevisiae</i> [45] | Metabolic network of <i>S. cerevisiae</i> .                |
|                      | <i>C. elegans</i> [45]    | Metabolic network of <i>C. elegans</i> .                   |
| Electronic Circuits  | s838 [30]                 | Electronic sequential logic circuit.                       |
|                      | s420 [30]                 | Same as above.   |
|                      | s208 [30]                 | Same as above.   |
| Neuronal             | <i>C. elegans</i> [66]    | Neural network of <i>C. elegans</i> .                      |
| Citation             | ArXiv-HepTh [35]          | Citation networks in HEP-TH category of Arxiv.             |
|                      | ArXiv-HepPh [35]          | Citation networks in HEP-PH category of Arxiv.             |
| WWW                  | nd.edu [37]               | WWW from nd.edu domain.                                    |
|                      | stanford.edu [39]         | WWW from stanford.edu domain.                              |
|                      | Political blogs [67]      | Hyperlinks between weblogs on US politics.                 |
| Internet             | p2p-1 [68]                | Gnutella peer-to-peer file sharing network.                |
|                      | p2p-2 [68]                | Same as above (at different time).                         |
|                      | p2p-3 [68]                | Same as above (at different time).                         |
| Social Communication | UCIonline [69]            | Online message network of students at UC, Irvine.          |
|                      | Email-epoch [70]          | Email network in a university.                             |
|                      | Cellphone [49]            | Call network of cell phone users.                          |
| Intra-organizational | Freemans-2 [71]           | Social network of network researchers.                     |
|                      | Freemans-1 [71]           | Same as above (at different time).                         |
|                      | Manufacturing [51]        | Social network from a manufacturing company.               |
|                      | Consulting [51]           | Social network from a consulting company.                  |

**Supplementary Table S1:** For each network, we show its type, name, reference and brief description.

|                      | Name                 | $N$     | $L$       | $n_D$ | $n_r$              | $n_r^T$ | $n_r^{\text{theory}}$ | Mode |
|----------------------|----------------------|---------|-----------|-------|--------------------|---------|-----------------------|------|
| Regulatory           | TRN-Yeast-1          | 4,441   | 12,873    | 0.965 | $9 \times 10^{-4}$ | 0.035   | $9 \times 10^{-4}$    | D    |
|                      | TRN-Yeast-2          | 688     | 1,079     | 0.821 | 0.051              | 0.164   | 0.046                 | D    |
|                      | TRN-EC-1             | 1,550   | 3,340     | 0.891 | 0.008              | 0.109   | 0.008                 | D    |
|                      | TRN-EC-2             | 418     | 519       | 0.751 | 0.112              | 0.244   | 0.114                 | D    |
|                      | Ownership-USCorp     | 7,253   | 6,726     | 0.820 | 0.112              | 0.173   | 0.114                 | D    |
| Trust                | College student      | 32      | 96        | 0.188 | 0.781              | 0.375   | 0.804                 | C    |
|                      | Prison inmate        | 67      | 182       | 0.134 | 0.254              | 0.836   | 0.365                 | D    |
|                      | Slashdot             | 82,168  | 948,464   | 0.045 | 0.088              | 0.954   | 0.474                 | D    |
|                      | WikiVote             | 7,115   | 103,689   | 0.666 | 0.334              | 0.013   | 0.333                 | C    |
|                      | Epinions             | 75,888  | 508,837   | 0.549 | 0.344              | 0.282   | 0.280                 | C    |
| Food Web             | Ythan                | 135     | 601       | 0.511 | 0.415              | 0.104   | 0.554                 | C    |
|                      | Little Rock          | 183     | 2,494     | 0.541 | 0.109              | 0.437   | 0.397                 | D    |
|                      | Grassland            | 88      | 137       | 0.523 | 0.432              | 0.261   | 0.504                 | C    |
|                      | Seagrass             | 49      | 226       | 0.265 | 0.612              | 0.510   | 0.788                 | C    |
| Power Grid           | TexasPowerGrid       | 4,889   | 5,855     | 0.325 | 0.462              | 0.590   | 0.522                 | D    |
| Metabolic            | <i>E. coli</i>       | 2,275   | 5,763     | 0.382 | 0.409              | 0.418   | 0.609                 | N/A  |
|                      | <i>S. cerevisiae</i> | 1,511   | 3,833     | 0.329 | 0.481              | 0.467   | 0.620                 | N/A  |
|                      | <i>C. elegans</i>    | 1,173   | 2,864     | 0.302 | 0.481              | 0.484   | 0.620                 | N/A  |
| Electronic Circuits  | s838                 | 512     | 819       | 0.232 | 0.673              | 0.443   | 0.681                 | C    |
|                      | s420                 | 252     | 399       | 0.234 | 0.670              | 0.456   | 0.682                 | C    |
|                      | s208                 | 122     | 189       | 0.238 | 0.663              | 0.484   | 0.683                 | C    |
| Neuronal             | <i>C. elegans</i>    | 297     | 2,345     | 0.165 | 0.195              | 0.721   | 0.896                 | D    |
| Citation             | ArXiv-HepTh          | 27,770  | 352,807   | 0.216 | 0.291              | 0.660   | 0.221                 | D    |
|                      | ArXiv-HepPh          | 34,546  | 421,578   | 0.232 | 0.266              | 0.657   | 0.183                 | D    |
| WWW                  | nd.edu               | 325,729 | 1,497,134 | 0.677 | 0.130              | 0.253   | 0.143                 | D    |
|                      | stanford.edu         | 281,903 | 2,312,497 | 0.317 | 0.495              | 0.406   | 0.494                 | C    |
|                      | Political blogs      | 1,224   | 19,025    | 0.356 | 0.493              | 0.311   | 0.618                 | C    |
| Internet             | p2p-1                | 10,876  | 39,994    | 0.552 | 0.071              | 0.430   | 0.071                 | D    |
|                      | p2p-2                | 8,846   | 31,839    | 0.578 | 0.059              | 0.410   | 0.059                 | D    |
|                      | p2p-3                | 8,717   | 31,525    | 0.577 | 0.053              | 0.413   | 0.053                 | D    |
| Social Communication | UCIonline            | 1,899   | 20,296    | 0.323 | 0.181              | 0.625   | 0.176                 | D    |
|                      | Email-epoch          | 3,188   | 39,256    | 0.426 | 0.318              | 0.355   | 0.295                 | D    |
|                      | Cellphone            | 36,595  | 91,826    | 0.204 | 0.694              | 0.682   | 0.576                 | C    |
| Intra-organizational | Freemans-2           | 34      | 830       | 0.029 | 0.971              | 0.971   | 1                     | N/A  |
|                      | Freemans-1           | 34      | 695       | 0.029 | 0.971              | 0.971   | 1                     | N/A  |
|                      | Manufacturing        | 77      | 2,228     | 0.013 | 0.02               | 0.987   | 0.022                 | D    |
|                      | Consulting           | 46      | 879       | 0.043 | 0.022              | 0.957   | 0.022                 | D    |

**Supplementary Table S2:** For each network, we show its type, name; number of nodes ( $N$ ) and edges ( $L$ ); fraction of redundant nodes in the original ( $n_r$ ), transpose network ( $n_r^T$ ) and theoretical prediction based on degree distribution ( $n_r^{\text{theory}}$ ); and the control mode (D for the distributed mode  $\Delta n_r < 0$ , C for the centralized mode  $\Delta n_r > 0$  and N/A for networks with  $\Delta n_r \approx 0$ )

### Supplementary Note 1

*Theorem 1:* A node is critical if and only if it has no incoming link.

Theorem 1 is equivalent to the following statement: node  $n$  in the *in* set of the bipartite graph can never be matched if and only if there is no link to  $n$ .

*Proof:* We first convert the maximum matching to an integer programming problem. Assume that the adjacency matrix of a directed network is  $A = \{a_{i,j}\}$  where  $a_{i,j} = 1$  if there is a link from node  $i$  to  $j$  and  $a_{i,j} = 0$  otherwise. Correspondingly in the bipartite representation  $a_{i,j} = 1$  is when there is one link between node  $i$  in the *out* set and node  $j$  in the *in* set. To find the number of maximumly matched nodes in the *in* set, we use integer programming as

$$\begin{aligned} \text{Max : } & \sum_j x_j \\ \text{Subject to : } & 0 \leq y_{i,j} \leq a_{i,j} \quad \forall i, j \quad (1) \\ & \sum_i y_{i,j} \leq 1 \quad (2) \\ & \sum_j y_{i,j} \leq 1 \quad (3) \\ & \sum_i y_{i,j} = x_j \quad (4) \\ & y_{i,j} \in \{0, 1\} \quad \forall i, j. \quad (5) \end{aligned}$$

Here the variable  $y_{i,j} = 1$  if the link  $i \rightarrow j$  is matched and  $x_j = 1$  if the node  $j$  in the *in* set is matched. The objective function is to maximize the number of matched nodes in the *in* set. Constraint (1) limits  $y_{i,j} = 0$  if there is no link  $i \rightarrow j$ . Constraint (2) and (3) represent the fact that a node can only be matched once.

We first prove the *necessary* condition: node  $n$  in the *in* set of the bipartite graph can never be matched  $\Leftarrow$  it has no link. Node  $n$  in the *in* set has no link gives  $a_{i,n} = 0$ . With constraint (1) and (4),  $x_{i,n} = 0$  and correspondingly  $y_n = 0$ , indicating that node  $n$  can not be matched.

Now we prove the *sufficient* condition: node  $n$  in the *in* set of the bipartite graph can never be matched  $\Rightarrow$  it has no link. Assuming this statement is *not* true, then there is the situation that node  $n$  has at least one link (suppose  $a_{m,n} = 1$ ) and the node can never be matched ( $x_n$  can not be 1).  $x_n$  can not be 1 if and only if (i)  $x_n = 1$  is infeasible or (ii)  $x_n = 1$  is not optimal. Because  $a_{m,n} = 1$  allows  $y_{m,n} = 1$  according to constraint (1). Hence  $x_n = 1$  is feasible and situation (i) is not true. For situation (ii), assume there is one optimal solution in which  $x_n = 0$ . By making  $y_{m,n} = 1$  we have  $x_n = 1$ . To satisfy constraint (3) we need to put some  $y$  value (assuming  $y_{m,l}$ ) to

be 0. Yet, the objective value remained unchanged. Therefore situation (ii) is not true. In sum, there is no condition that  $a_{m,n} = 1$  and  $x_n$  can not be 1. This leads to a contradiction. Hence the sufficient condition holds.

## Supplementary Note 2

### Erdős-Rényi network

For directed Erdős-Rényi random networks, both  $P_{\text{in}}(k)$  and  $P_{\text{out}}(k)$  follow a Poisson distribution, i.e.  $P_{\text{in}}(k) = P_{\text{out}}(k) = e^{-c}c^k/k!$  with  $c = \langle k_{\text{in}} \rangle = \langle k_{\text{out}} \rangle = \langle k \rangle/2$ . The generating function  $G(x)$  and  $H(x)$  are given by

$$G_{\text{in,out}}(z) = H_{\text{in,out}}(z) = e^{-c(1-z)}. \quad (\text{S1})$$

The parameter  $\theta_{\text{out}}$  can be obtained by solving the equation

$$1 - \theta_{\text{out}} = \exp(-ce^{-c(1-\theta_{\text{out}})}). \quad (\text{S2})$$

By defining  $\theta'$  as the largest root of equation (S2) (the one that is closest to 1 if there are two solutions), the two branches of the  $n_r$  curve can be written as

$$n_r = \begin{cases} n_r^u = \theta' \\ n_r^l = 1 - e^{-c(1-\theta')}, \end{cases} \quad (\text{S3})$$

where  $n_r^h$  and  $n_r^l$  correspond to the upper and lower branch of bifurcation diagram of  $n_r$  vs  $\langle k \rangle$ .

### Scale-free network

The other model network used in the paper is the static scale-free networks. We start from  $N$  disconnected nodes indexed by the integers  $i$  ( $i = 1, \dots, N$ ). We assign a weight or expected degree  $w_i^{\text{out,in}} = i^{-\alpha_{\text{out,in}}}$  to each node in the *out* and the *in* set, where  $\alpha_{\text{out,in}}$  is a real number in the range  $[0, 1)$ . Nodes  $i$  and  $j$  are randomly selected from the set of  $N$  nodes, with probability proportional to  $w_i$  and  $w_j$ , respectively. If they have not been connected, we connect them. Otherwise randomly choose another pair. This process is repeated until  $c = \langle k_{\text{in}} \rangle = \langle k_{\text{out}} \rangle = \langle k \rangle/2$  links are created. Note that in case  $\alpha = 0$ , this model is equivalent to the Erdős-Rényi random graph. The degree distribution is given by  $P_{\text{out,in}}(k) = \frac{[m(1-\alpha_{\text{out,in}})]^{1/\alpha_{\text{out,in}}}}{\alpha_{\text{out,in}}} \frac{\Gamma(k-1/\alpha_{\text{out,in}}, m[1-\alpha_{\text{out,in}}])}{\Gamma(k+1)}$  with  $\Gamma(s)$  the gamma function and  $\Gamma(s, x)$  the upper incomplete gamma function. In the large  $k$  limit,  $P_{\text{out,in}}(k) \sim k^{-(1+\frac{1}{\alpha_{\text{out,in}}})} = k^{-\gamma_{\text{out,in}}}$  where  $\gamma_{\text{out,in}} = 1 + \frac{1}{\alpha_{\text{out,in}}}$ . The parameter  $\alpha_{\text{out,in}}$  can be tuned independently to create networks with different asymmetries.

The generating functions used in our analytical framework are then given by

$$G_{\text{out,in}}(x) = \frac{1}{\alpha_{\text{out,in}}} E_{1+\frac{1}{\alpha_{\text{out,in}}}}[(1-x)c(1-\alpha_{\text{out,in}})] \quad (\text{S4})$$

$$H_{\text{out,in}}(x) = \frac{1-\alpha_{\text{out,in}}}{\alpha_{\text{out,in}}} E_{\frac{1}{\alpha}}[(1-x)c(1-\alpha_{\text{out,in}})]. \quad (\text{S5})$$

Here  $E_n(x) = \int_1^\infty dy e^{-xy} y^{-n}$ .

### Supplementary Note 3

By this definition, the control mode is undetermined when  $\Delta n_r = 0$ . Typically  $\Delta n_r = 0$  corresponds to two situations. (1) for symmetric in- and out-degrees and  $\langle k \rangle \leq k_c$ , both the original and the transpose network have the same  $n_r$  (*e.g.*, Metabolic networks in Supplementary Tables S1 and S2). In this case the control mode is unknown. (2) when  $\langle k \rangle$  is very high and there is a perfect matching. By our definition, a perfect matching gives  $n_r = 1 - 1/N$  and the network is in the centralized mode. However,  $\Delta n_r = 0$  under perfect matching and can not be used to identify the control mode (*e.g.*, Social network of network researchers in the Supplementary Tables S1 and S2).

### Supplementary Note 4

The control mode is correlated with the degree divergence: if the out-degree is more divergent than in-degree, the network will have fewer redundant nodes and in the distributed mode (Supplementary Fig. S7a), and vice versa (Supplementary Fig. S7b). The simplest quantity capturing of the divergence is the variance. Indeed, degree variance gives good predication in uncorrelated networks. In Supplementary Fig. S7a, we have  $\sigma_{\text{kout}}^2 = 8$ ,  $\sigma_{\text{kin}}^2 = 6$  and  $\sigma_{\text{kout}}^2 > \sigma_{\text{kin}}^2$ . The network is with low  $n_r$  and in the distributed mode. In our observation in model networks (Fig 2e of the main text), we have  $\gamma_{\text{out}} = 2.67$ ,  $\gamma_{\text{in}} = 3$  with  $\sigma_{\text{kout}}^2 > \sigma_{\text{kin}}^2$  and the network is in the distributed mode (lower branch of the bifurcation diagram). Therefor a simple prediction can be made based on degree variance: we expect  $\sigma_{\text{kout}}^2 > \sigma_{\text{kin}}^2$  for distributed networks and  $\sigma_{\text{kout}}^2 < \sigma_{\text{kin}}^2$  for centralized networks.

Yet, due to the degree correlation, this prediction fails in real systems. We find, however, that the heterogeneity is less sensitive to the degree correlation and can be applied to quantify the degree asymmetry in both real and model networks. The heterogeneity of a degree distribution can be introduced via  $H = \sum_i \sum_j |i - j| P(i) P(j) / \langle k \rangle$ . We predict that  $H_{\text{out}} > H_{\text{in}}$  yields distributed networks and  $H_{\text{out}} < H_{\text{in}}$  generates centralized networks where  $H_{\text{in}}$  and  $H_{\text{out}}$  are heterogeneities of the in- and out-degree distribution. For example in Supplementary Fig. S7a,  $H_{\text{out}} = 0.96$ ,  $H_{\text{in}} = 0.8$  and the network is distributed.

To further test this prediction, we plot  $\Delta n_r$  as function of  $\ln(H_{\text{in}}/H_{\text{out}})$ . If our prediction holds all data points will fall into the first and third quadrant. Indeed we observe that most real networks analyzed are in line with our expectation (Supplementary Fig. S8). This result allows us to predict the control mode of a network starting only from its degree distribution. It also helps us choose the continuous solution of equation (2) when two solution exist.



### Supplementary Note 5

When there is a perfect matching that all nodes in the *in* set are always matched, the number of driver nodes is 1, as controlling any one node is sufficient to control the whole system. In this situation our definition on redundant nodes encounters a controversy. On one hand, by the mapping that always matched nodes in the *in* set are redundant, all nodes are redundant in this case therefore  $N_r = N$  where  $N$  is the network size. On the other hand, the number of redundant nodes needs to be less than the network size as we always need to control at least one node to control. Finally, in terms of the participation in MDS's, all nodes are intermittent because controlling any of these nodes yields the control. To avoid the inconsistency, we define that in a perfect matching,  $N_r = N - 1$ .

### Supplementary Note 6

Denote with  $G$  a bipartite graph and  $G' = G \setminus i$  the subgraph of  $G$  by removing node  $i$  and all its links (Supplementary Figure S6).

*Theorem 2:* node  $i$  is not always matched in  $G$  if and only if all its neighboring nodes are always matched in the subgraph  $G'$ .

*Proof:*

*Necessity:* If the statement is *not* true, there exist a situation that node  $i$  is not always matched in  $G$  and one of its neighboring nodes (denoted by node  $j$ ) is not always matched in  $G'$ . Then there is at least one matching configuration of  $G'$  that node  $j$  is not matched. When adding back all links of node  $i$  and converting  $G'$  to  $G$ ,  $j$  need to match  $i$ . In this case we can see that  $i$  should always be matched otherwise matching in  $G$  is not maximum. This leads to a contradiction. Therefore, if node  $i$  is not always matched in  $G$ , all its neighboring nodes are always matched in  $G'$ .

*Sufficiency:* If the neighboring nodes are always matched in  $G'$ , in all matching configurations of  $G'$  these nodes are matched. Under such matching configurations, the neighboring nodes will not match  $i$  when adding back the links between them (going back to  $G$  from  $G'$ ). Therefore there exist at least one matching configuration of  $G$  that  $i$  is not matched. Hence if all neighboring nodes of node  $i$  are always matched in  $G'$ , node  $i$  is not always matched in  $G$ .

## Supplementary References

- [61] Norlen, K., Lucas, G., Gebbie, M. & Chuang, J. Eva: Extraction, visualization and analysis of the telecommunications and media ownership network. *Proceedings of International Telecommunications Society 14th Biennial Conference, Seoul Korea* (2002).
- [62] Richardson, M., Agrawal, R. & Domingos, P. Trust management for the semantic web. In *Proceedings of the second international semantic web conference*, 351–368 (2003).
- [63] Dunne, J. A., Williams, R. J. & Martinez, N. D. Food-web structure and network theory: The role of connectance and size. *Proceedings of the National Academy of Sciences* **99**, 12917–12922 (2002).
- [64] Christian, R. R. & Luczkovich, J. J. Organizing and understanding a winter’s seagrass foodweb network through effective trophic levels. *Ecological Modelling* **117**, 99–124 (1999).
- [65] Bianconi, G., Gulbahce, N. & Motter, A. E. Local structure of directed networks. *Physical Review Letters* **100**, 118701 (2008).
- [66] Watts, D. J. & Strogatz, S. H. Collective dynamics of ‘small-world’ networks. *Nature* **393**, 440–442 (1998).
- [67] Adamic, L. A. & Glance, N. The political blogosphere and the 2004 us election. *Proceedings of the 3rd international workshop on Link discovery*, 36–43 (2005).
- [68] J. Leskovec, J. K. & Faloutsos, C. Graph evolution: Densification and shrinking diameters. *ACM Transactions on Knowledge Discovery from Data (ACM TKDD)* **1**, 2 (2007).
- [69] Opsahl, T. & Panzarasa, P. Clustering in weighted networks. *Social Networks* **31**, 155–163 (2009).
- [70] Eckmann, J.-P., Moses, E. & Sergi, D. Entropy of dialogues creates coherent structures in e-mail traffic. *Proceedings of the National Academy of Sciences* **101**, 14333–14337 (2004).
- [71] Freeman, S. & Freeman, L. The networkers network: A study of the impact of a new communications medium on sociometric structure. *Social Science Research Reports* **46**. University of California, Irvine, CA. (1979).