Question 1:

1. ① $MP_K = \frac{\partial zF(K,N)}{\partial K} = z\frac{\partial F(K,N)}{\partial K} = z\frac{\partial F(\frac{K}{N},1)}{\partial \frac{K}{N}} = z\frac{\partial f(k)}{\partial k} = zf'(k)$

② $y = zf(k) = zF(\frac{K}{N},1) = MP_N + MP_K \cdot k$

$= MP_N + zf'(k)\cdot k$

$MPN = zf(k) - zf'(k)\cdot k = z[f(k) - f'(k)\cdot k]$

2.

Assume that k^* is the capital in the steady state.

$r_K = \frac{k'}{k} - 1$

$k' = (1-\delta)k + sf(k,1) = (1-\delta)k + s\cdot zf(k,1)$

$r_K = \frac{(1-\delta)k + s\cdot zF(k,N)}{k} - 1 = (1-\delta) + s\cdot zf(1,\frac{1}{k}) - 1$

$= -\delta + s\cdot zF(1,\frac{1}{k})$

$r'_K = \frac{\partial r_K}{\partial k} = -\frac{1}{k^2}\cdot szf(1,\frac{1}{k}) < 0$

Then r_K is a decreasing function of k. Also the production function is an increasing function of k.

Thus as a country has the higher competitive equilibrium growth rate, it is further to its initial steady state.

3. $Y = zF(K,N) = zK^{\alpha}N^{1-\alpha}$ where $\alpha \in (0,1)$

① $MP_K = \frac{\partial zF(K,N)}{\partial K} = \alpha \cdot z K^{\alpha-1} \cdot N^{1-\alpha} = \alpha \cdot z \cdot \frac{K^{\alpha-1}}{N^{\alpha-1}} = \alpha \cdot z \cdot k^{\alpha-1}$

$f(k) = zF(\frac{K}{N},1) = z(\frac{K}{N})^{\alpha} = z k^{\alpha}$ $\quad y'(k) = \alpha \cdot z k^{\alpha-1} = MP_K$.

② $y = zf(k) = zF(\frac{k}{N},1) = MP_N + MP_K \cdot k$

$\frac{zK^{\alpha} \cdot N^{1-\alpha}}{N} = z(\frac{K}{N})^{\alpha} = zk^{\alpha} = MP_N + \alpha \cdot z k^{\alpha-1}$

$MP_N = zk^{\alpha} - \alpha \cdot z k^{\alpha-1} = z(k^{\alpha} - \alpha k^{\alpha-1})$

$= z[f(k) - f'(k) \cdot k]$

③ $r_K = \frac{k'}{k} - 1$ $\quad f(k,n) = \frac{Y}{N} = \frac{zK^{\alpha}N^{1-\alpha}}{N} = zk^{\alpha}$

$k' = (1-s)k + s \cdot z k^{\alpha}$

$r_K = \frac{(1-s)k + s \cdot z k^{\alpha}}{k} - 1$

$= (1-s) + s \cdot z k^{\alpha-1} - 1 = -s + s \cdot z \cdot k^{\alpha-1}$

$r'_K = (\alpha-1) \cdot s \cdot z k^{\alpha-2}$ Because $\alpha \in (0,1)$, thus $r'_K < 0$.

Then r_K is a decreasing funcetion of k. Also the production function is an increasing function of k.

It satisfis the previous findings.

Question 2

1. ① $\frac{x_{t+1}}{x_t} = 1+g$ $\rightarrow$ Geometric Segence : $x_t = x_0(1+g)^t$

If g (the growth rate) keeps unchanged, x_{t+1} is $1+g$ times of x_t.

② $\ln x_{t+1} = \ln(1+g)x_t$

$\ln x_{t+1} = \ln(1+g) + \ln x_t$

$\ln x_{t+1} - \ln x_t = \ln(1+g)$ $\rightarrow$ Arithmetic progression :

$x_t = \ln x_0 + t\ln(1+g) = \ln(x_0(1+g)^t)$

x_{t+1} is always $\ln(1+g)$ more than x_t.

2. $\frac{Y_t}{N_t} = z F(\frac{k_t}{N_t}, x_t)$

$y_t = z f(k_t, x_t)$

$k_{t+1} = (1-\delta)k_t + s \cdot z f(k_t, x_t)$

$\frac{k_{t+1}}{k_t} = (1-\delta) + s \cdot z f(1, \frac{x_t}{k_t})$

From question 2.1, we know x_t grows as a geometric sequence, so $\frac{k_{t+1}}{k_t}$ will not be constant and k_t also grows over time.

y_t^* is the function of k_t and x_t, and k_t、x_t both are not constant, so y_t^* is not constant and it grows over time.

3. $\tilde{k}_t = \frac{K_t}{X_t} N_t = \frac{k_t}{X_t}$

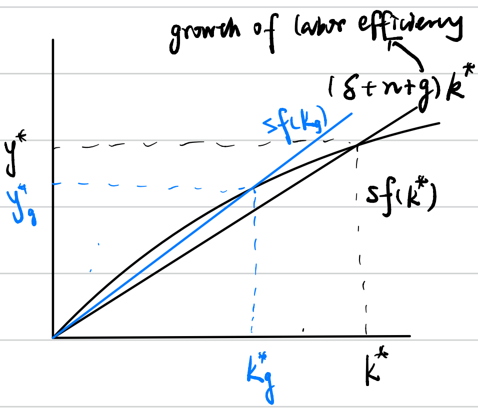
$$\frac{\tilde{k}_{t+1}}{\tilde{k}_t} = \frac{\frac{k_{t+1}}{X_{t+1}}}{\frac{k_t}{X_t}} = \frac{k_{t+1}}{k_t} \cdot \frac{X_{t+1}}{X_t} = \frac{k_{t+1}}{k_t} \cdot (1+g)$$

$$\frac{k_{t+1}}{k_t} = (1-\delta) + s \cdot z f(1, \frac{X_t}{k_t}) = (1-\delta) + s \cdot z f(1, \frac{1}{\tilde{k}_t})$$

$$\frac{\tilde{k}_{t+1}}{\tilde{k}_t} = (1-\delta) + s z f(1, \frac{1}{\tilde{k}_t}) \cdot (1+g)$$

Thus we know $\tilde{k}^*$ is unchanging over time in steady state, Since output y depends on $\tilde{k}^*$ through the production function, it is also unchanging.

4.



If g (the growth rate) increased permanently, we have g_1.

$$\frac{X_{t+1}}{X_t} = 1+g_1 > 1+g_0.$$

$$y = z f(k_t, X_t)$$

In this case, $sf(k)$ will be steeper, and the steady point moves left.

①

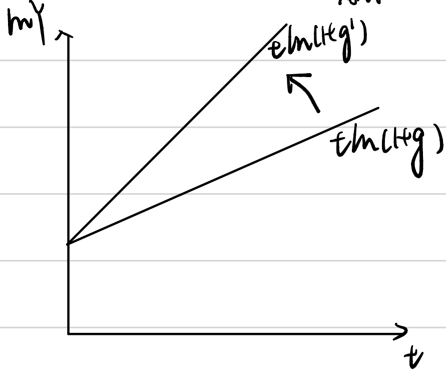
3. $\ln Y = \ln z + \ln F(k_t, X_t N_t)$

$= \ln z + \ln F(\frac{k_t}{X_t N_t}, 1) \cdot (X_t N_t)$

$= \ln z + \ln F(\frac{k_t}{X_t N_t}, 1) + \ln(X_t N_t)$

$= \ln z + \ln(\frac{k_t}{X_t N_t}, 1) + \ln X_t + \ln N_t$

$= \ln z + \ln(\frac{k_t}{X_t N_t}, 1) + t\ln(1+g) + \ln X_0 + \ln N_t$

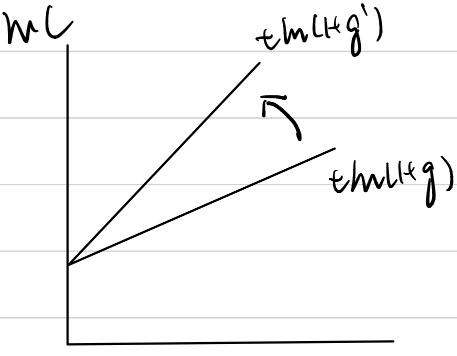


As the graph shown, with the increase of g, the line is getting steeper.

② $C = (1-s)Y \qquad s \in (0,1)$

$\ln C = \ln(1-s) + \ln Y$

$= \ln(1-s) + \ln z + \ln(\frac{k_t}{X_t N_t}, 1) + t\ln(1+g) + \ln X_0 + \ln N_t$.



Same as above.