Unanticipated Price Changes

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Outline

Background and Roadmap

- So far inflation in our OLG model is perfectly anticipated.
- Effect of monetary surprises (changes in money supply on output)?
- Legacy of Bill Phillips: Empirical Regularities or Irregularities?
- An Island Model with Signal Extraction Problems: dangerously Endogenous Phillips curve
- Pitfalls of Keynesian policy based on WYSIWYG modeling?

Alban William Housego Phillips

- born at Te Rehunga, near Dannevirke, New Zealand
- studied electrical engineering
- outbreak of World War II, Phillips joined the Royal Air Force and was sent to Singapore
- spent three and a half years interned in a prisoner of war camp in Indonesia; learned Chinese from other prisoners, repaired and miniaturised a secret radio
- in 1958 Phillips published the relationship between inflation and unemployment: Phillips curve
- went to Australia in 1967 at Australian National University; died in Auckland on 4 March 1975

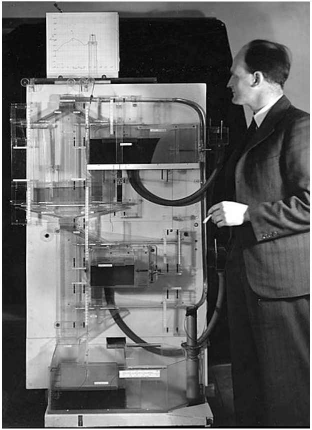


Figure: Phillips and the MONIAC. Source: Wikipedia

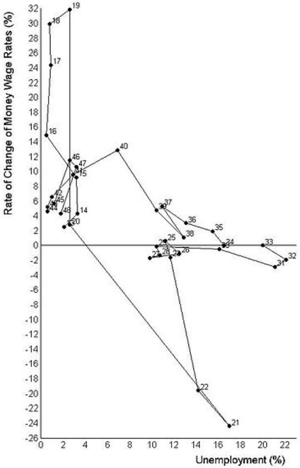


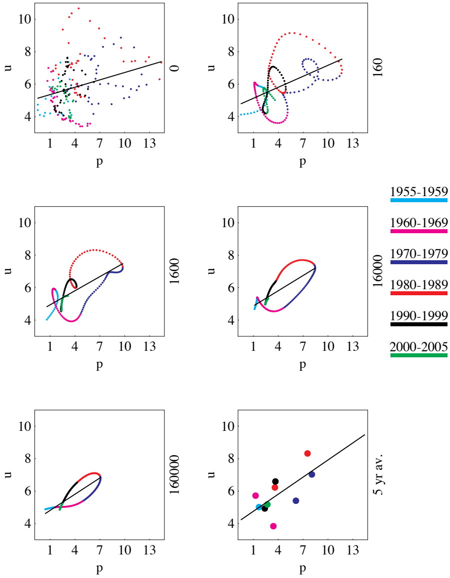
Figure: Original Phillips data for the U.K., 1913-1948.

Empirical Regularities?

- Before 1970's stable inverse relationship between inflation and unemployment;
- Or positive relationship between inflation and output.
- This is often referred to as a Phillips curve.
- Empirical support for Keynesian government policy – improve employment/output by trading off inflation.
- Since 1970's "Stagflation", this stable trade-off disappeared: Lucas (1973, AER); Berentsen-Menzio-Wright (2011, AER) [next figure]

More recently documented:

- Berentsen-Menzio-Wright (2011, AER) [next figure]:
- Panel (1,1): raw data; Panel (2,1): business cycle frequency; Panel (2,2) to (3,2) low frequency data (long run).
- if one "filters" out the long run and focus on business cycle frequencies of the data, appears only decade 1960-1969 that corroborates the Phillips curve tradeoff.
- When one filters out high-frequencies and focus on low frequencies, long-run data suggests a positive relationship!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2000-2005

Lucas (1972)

- Lucas (1972, JET): *Expectations and the Neutrality of Money*.
- Key mechanism:
  - OLG and spatial (island) separations;
  - Information friction.
- Here we present simplified story as in Champ and Freeman (2001) and Wallace (1980).
- Islands: parable for spatial separation of traders with localized information imperfection.

Lucas (1972)

- Model accounts for Phillips curve correlation between inflation and output/employment.
- Only under imperfect information about money supply and location/market specific price.
- Attempts to stimulate economy in a Keynesian way will invert "Phillips" correlation.
- Warning for reduced-form policy modeling and analysis: Lucas critique.

A Lucas-Island-type Model

Assumptions

- Two islands: $i \in \{A, B\}$.
- Population on both islands constant over time. Total young population: N.
- Independent of location when young, the current old are randomly and equally distributed across the islands.
- Unequal distribution of young agents on $\{A, B\}$ – w.l.o.g. assume distribution is $(q, 1-q) = (\frac{1}{3}, \frac{2}{3})$.
- Each period, equiprobability each island has fraction $2/3$ of young agents.
- Lump-sum transfer of new money to old agents each period.

Assumptions (con't) and reinterpretation

- y is time endowment when young
- Now c_{1t}^{i} is nonmarket good (e.g. leisure)
- p_t^i is island-i price of non-storable output y_t^i. Observed only by island-i individuals.
- Publicly observed aggregate price P_t
- $l_t^i = L(p_t^i)$ is labor supply by island-i young.
- On-the-spot production technology: $y_t^i = l_t^i$.
- Aggregate money supply growth rule:

$$M_{t+1} = \gamma_t M_t.$$

Tracking individuals

- Young's (1) consumption on island $i \in \{A, B\}$ at time t :

c^{i}_{1t} (Nonmarket consumption)

- Old (2) (born on island i) consumption on ex-ante random island $j \in \{A, B\}$ at time $t+1$:

$c^{i,j}_{2t+1}$ (Market consumption)

Individuals' budget constraints

- Island-i young's budget constraint:

$$c_{1t}^i + l_t^i = y \qquad \text{(Home production + Tradable production)}$$

Since no storage, tradable output sold at location-i price p_t^i in exchange for money:

$$l_t^i := L(p_t^i) = y_t^i = \frac{m_t^i}{p_t^i},$$

- When old (island-i born), face possible constraints:

$$c_{2t+1}^{i,j} = \frac{m_t^i}{p_t^i}\frac{p_t^i}{p_{t+1}^j} + \frac{T_{t+1}}{P_{t+1}}, \qquad \forall j \in \{A, B\},$$

where nominal lump-sum transfer is $T_{t+1} = \left(1 - \frac{1}{\gamma_t}\right)\frac{M_{t+1}}{N}$.

Note:

- Second-period (old-age) budget constraint is random from a period-one perspective.
- Given i as birthplace, random reassignment to another island j next period.

Decision Problem

Island i's young agent at time t solves:

$$\max_{c^i_{1t},c^{i,j}_{2t+1}} U(c^i_{1t}) + \beta \mathbb{E}\left[U(c^{i,j}_{2t+1})\right]$$

such that

$$c^i_{1t} + l^i_t = y,$$

and,

$$c^{i,j}_{2t+1} = \frac{m^i_t}{p^i_t}\frac{p^i_t}{p^j_{t+1}} + \frac{T_{t+1}}{P_{t+1}}, \qquad \forall j \in \{A, B\}.$$

Note:

- Location j in $t+1$ is a random variable (with distribution $\Pr\{j = A\} = 1/2$) for young agent at i in period t.
- Implies $c_{2t+1}^{i,j}$ also a random variable.
- Hence $U(c_{2t+1}^{i,j})$ also a random variable.
- $\mathbb{E}[\cdot]$ is mathematical expectations operator.

Decision Problem (cont'd)

Island i's young agent at time t solves equivalent unconstrained problem of:

$$\max_{l_t^i} U\left(y - l_t^i\right) + \beta \mathbb{E}\left[U\left(l_t^i \frac{p_t^i}{p_{t+1}^j} + \frac{T_{t+1}}{P_{t+1}}\right)\right].$$

First-order condition w.r.t. l_t^i:

$$U_c\left(y - l_t^i\right) = \beta \mathbb{E}\left\{\left[U_c\left(l_t^i \frac{p_t^i}{p_{t+1}^j} + \frac{T_{t+1}}{P_{t+1}}\right)\right] \frac{p_t^i}{p_{t+1}^j}\right\}.$$

where $U_c(c) := \partial U(c)/\partial c$.

If $U_c : \mathbb{R}_+ \to \mathbb{R}$ is a bijection (i.e. one-to-one and onto function), then the FOC:

$$U_c\left(y - l_t^i\right) = \beta \mathbb{E}\left\{\left[U_c\left(l_t^i \frac{p_t^i}{p_{t+1}^j} + \frac{T_{t+1}}{P_{t+1}}\right)\right]\frac{p_t^i}{p_{t+1}^j}\right\}. \qquad (\star)$$

implies an optimal supply of labor (equivalently demand for money):

$$l_t^i = L\left(p_t^i, y\right) := L\left(p_t^i\right).$$

Example ($U(c) = \ln(c)$ for $c > 0$)

Given distribution of old next period is independently $(1/2, 1/2)$ on the set $\{A, B\}$, we can calculate the FOC as

$$\left(\frac{1}{y - l_t^i}\right)\frac{1}{p_t^i} = \frac{1}{2}\beta\left\{\left[\frac{1}{l_t^i \frac{p_t^i}{p_{t+1}^A} + \frac{T_{t+1}}{P_{t+1}}}\right]\frac{1}{p_{t+1}^A}\right\} + \frac{1}{2}\beta\left\{\left[\frac{1}{l_t^i \frac{p_t^i}{p_{t+1}^B} + \frac{T_{t+1}}{P_{t+1}}}\right]\frac{1}{p_{t+1}^B}\right\}, \quad i \in \{A, B\}.$$

This says: Marginal utility value of money today = P.V. of expected marginal utility value of money tomorrow.

Discussion

- Implicit in the general FOC ($\star$) is the optimal supply of labor effort L by the young in each island i.
- It is also symmetrically, the optimal demand for real money balances L.
- Why? Recall assumption that production of i-goods are on the spot. The medium of exchange for these goods is money.
- Problem: We cannot explicitly solve for $l_t^i = L\left(p_t^i\right)$.

Discussion

- Recall consumer theory and effect of a relative price change: Slutsky decomposition – wealth/income vs. substitution effect.
- Assume preferences are such that the substitution effect dominates the income effect from changes in relative prices p_t^i / p_{t+1}^j.
- That is, a higher p_t^i, *ceteris paribus*, implies a higher supply of labor (demand for money) $L(p_t^i)$.
- Lucas (1972) provides assumptions on U and general characterizations of L.
- We can proceed by working with a general $L(p_t^i)$ that is an increasing function of p_t^i.

Constant Money Supply Growth

- Suppose $\gamma_t = \gamma$ for all t and observed.
- Agents know this.
- Market clearing on Island $i \in \{A, B\}$ with $N^i \in \{\frac{2}{3}N, \frac{1}{3}N\}$ young people:

$$N^i L(p_t^i) = \frac{1}{2}\frac{M_t}{p_t^i}$$

  - Total demand for real money balances by young on Island i is $N^i L(p_t^i)$; and
  - Distribution of old is: $(\frac{1}{2}, \frac{1}{2})$ on $\{A, B\}$. Total supply of nominal money stock on Island i is $\frac{1}{2}M_t$.

- Rearranging, we have:

$$p_t^i = \frac{\frac{1}{2} M_t}{N^i L(p_t^i)}.$$

- Note: p_t^i is a function of random variable N^i.
- Since we assume only agents on i observe island price p_t^i, then i-agents can infer own population of young, N^i.

Analysis

- Suppose current distribution of the N young agents on the set $\{A, B\}$ is $(1/3, 2/3)$, then we have:

$$p_t^A = \frac{\frac{1}{2}M_t}{\left(\frac{1}{3}N\right)L(p_t^A)}.$$

  and

$$p_t^B = \frac{\frac{1}{2}M_t}{\left(\frac{2}{3}N\right)L(p_t^B)}.$$

  Recall this event occurs with ex-ante probability of $1/2$.
- Since L is increasing in p_t^i, we can deduce that $p_t^A > p_t^B$.

Proposition

With constant money supply growth, the price of island i's good is higher when it has the smaller population of young agents.

Proof.

This can be easily proved by contradiction. Suppose not: $p_t^A \leq p_t^B$ and $N^A < N^B$. Since L is increasing in p_t^i we can derive a contradiction. □

Analysis (cont'd)

- The one-period rate of return between state i and j, is p_t^i / p_{t+1}^j.
- Since p_{t+1}^j will be independent of p_t^i, $i, j \in \{A, B\}$, then the greater p_t^i implies a greater rate of return to producing good y_t^i.
- So the RHS of the FOC tends to increase:

$$U_c\left(y - l_t^i\right) = \beta \mathbb{E}\left\{\left[U_c\left(l_t^i \frac{p_t^i}{p_{t+1}^j} + \frac{T_{t+1}}{P_{t+1}}\right)\right] \frac{p_t^i}{p_{t+1}^j}\right\}. \quad (\star)$$

Since L increasing in p_t^i, and U_c decreasing in c, from the marginal utility terms on LHS (increase) and on RHS (decrease) with $L(p_t^i)$ to maintain the equality of the FOC.

Example ($U(c) = \ln(c)$)

The FOC on labor supply or money demand is

$$\left(\frac{1}{y - L(p_t^i)}\right) = \frac{1}{2}\beta\left\{\left[\frac{1}{L(p_t^i)\frac{p_t^i}{p_{t+1}^A} + \frac{T_{t+1}}{P_{t+1}}}\right]\frac{p_t^i}{p_{t+1}^A}\right\}$$

$$+\frac{1}{2}\beta\left\{\left[\frac{1}{L(p_t^i)\frac{p_t^i}{p_{t+1}^B} + \frac{T_{t+1}}{P_{t+1}}}\right]\frac{p_t^i}{p_{t+1}^B}\right\}, \qquad i \in \{A, B\}.$$

In words

- On island with too many [few] producers (young) available to sell to the consuming old, relative price of that island-good is lower [higher].
- So rate of return on working is lower [higher].
- Optimal to supply less [more] labor.
- Demand for real balances lower [higher] for own old age consumption, given fixed transfers from government.
- Without randomness in monetary policy, i.e. $\gamma_t = \gamma$, prices here reveal true signal of the state N^i of the individual island economies.
- These prices support the allocation of resources (i.e. labor, real balances and thus output) consistent with individual utility maximization.

Two more observations

Proposition

Money is neutral in this economy

Note:

$$\frac{p_t^i}{p_{t+1}^j} = \frac{N^j L(p_{t+1}^j) M_t}{N^i L(p_t^i) M_{t+1}}$$

Increasing M_t and M_{t+1} by the same portion does not affect one-period, across-state, relative prices of goods.

Proposition

Money is not superneutral in this economy.

- Anticipated gross inflation is $M_{t+1}/M_t = \gamma$.
- An increase in γ lowers M_t/M_{t+1}, so rate of return to working is lowered.
- This is an anticipated inflation tax on real money balances. So labor effort falls and output falls.
- Implies a negative inflation-output relationship as empirically studied by Lucas (1973).

Figure: Inverted-Phillips curve when inflation tax is anticipated.

Random Money Supply Growth

- Now suppose for all t:

$$\gamma_t = \begin{cases} 1 & \text{w.p. } \theta \in (0,1) \\ 2 & \text{w.p. } 1-\theta \end{cases}.$$

  Money growth shocks are identically and independently distributed.

- Imperfect information: Suppose agents do not observe realization of γ_t until all decisions at t are made.
- So agents only learn about M_t at the end of period t.

Signal Extraction Problem

- Now recall, p_t^i depends on knowing realization of random variables N^i and M_t. Recall market clearing condition:

$$p_t^i = \frac{\frac{1}{2} M_t}{N^i L(p_t^i)}.$$

- Young agents are assumed to observe p_t^i, but not N^i and M_t.
- A signal extraction problem:
  - Cannot directly infer "signal" N^i from observed p_t^i now.
  - M_t as "noise".
  - A high Island-i price p_t^i now, may be due to either a small population of sellers (young) or a higher fiat money stock, or both.
  - Why does this matter?

Signal Extraction Problem

Why does this matter?

- If M_t were observed and if a high price were due to a high M_t, then there is no reason to work harder.
- Since γ_t is i.i.d. random variable, observing higher M_t does not affect anticipated rates of return on money.
- If a high price were due to a higher N^i, then there is reason to work harder, as they anticipate a higher return on holding money.

Describing the signal extraction problem here is simple since each of the r.v.'s have finite states, so product state space is finite and isomorphic with set $\{a, b, c, d\}$.

Table: Possible states

	$N^i = \frac{2}{3}N$	$N^i = \frac{1}{3}N$
$\gamma_{t-1} = 1$	a	b
$\gamma_{t-1} = 2$	c	d

Relation from $S = \{a, b, c, d\}$ to $\{p_t^{i,s} | s \in S\}$ is tabulated as:

Table: Possible state-island-prices

	$N^i = \frac{2}{3}N$	$N^i = \frac{1}{3}N$
$\gamma_{t-1} = 1$	$p_t^{i,a} = \frac{\frac{1}{2}M_{t-1}}{\frac{2}{3}NL(p_t^{i,a})}$	$p_t^{i,b} = \frac{\frac{1}{2}M_{t-1}}{\frac{1}{3}NL(p_t^{i,b})}$
$\gamma_{t-1} = 2$	$p_t^{i,c} = \frac{M_{t-1}}{\frac{2}{3}NL(p_t^{i,c})}$	$p_t^{i,d} = \frac{M_{t-1}}{\frac{1}{3}NL(p_t^{i,d})}$

Discussion

- Note order: $p_t^{i,a} < p_t^{i,b} = p_t^{i,c} < p_t^{i,d}$.
- Only $p_t^{i,a}$ and $p_t^{i,d}$ are unique.
- $p_t^{i,a}$ is detectable as consistent with the state $(\gamma_{t-1}, N^i) = (1, \frac{2}{3}N)$: Work little $l_t^{i,a}$ to maximize expected utility.
- $p_t^{i,a}$ is detectable as consistent with the state $(\gamma_{t-1}, N^i) = (2, \frac{1}{3}N)$: Work harder $l_t^{i,d}$ to maximize expected utility.
- Problem when observing $p_t^{i,b}$ or $p_t^{i,c}$, since $p_t^{i,b} = p_t^{i,c}$: Cannot infer which market (island) they are in.

1. Problem when observing $p_t^{i,b}$ or $p_t^{i,c}$, since $p_t^{i,b} = p_t^{i,c}$: Cannot infer which market (island) they are in.
2. Denote $l_t^{i,b}$ and $l_t^{i,c}$ denote equilibrium labor decision/allocation with perfect information about M_t.
3. Now since states $b \equiv (2, 1N/3)$ and $c \equiv (1, 2N/3)$ are indistinguishable, sellers (young) will optimally produce l^* with corresponding price p^*:

$$l_t^{i,c} < l^* < l_t^{i,b} \Rightarrow p_t^{i,a} < p^* < p_t^{i,d}.$$

Figure: If current policy state is $\gamma_{t-1} = 1$.

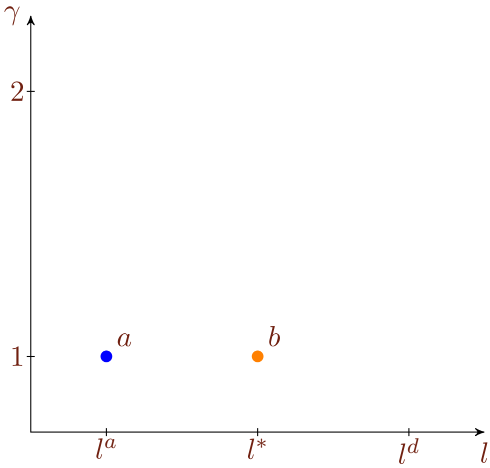


Figure: If current policy state is $\gamma_{t-1} = 2$.

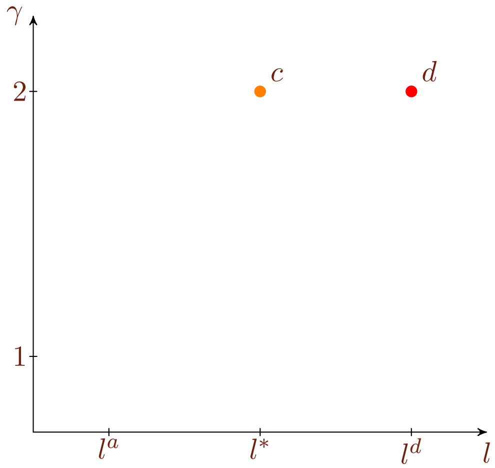


Figure: Phillips curve across islands with imperfect information.

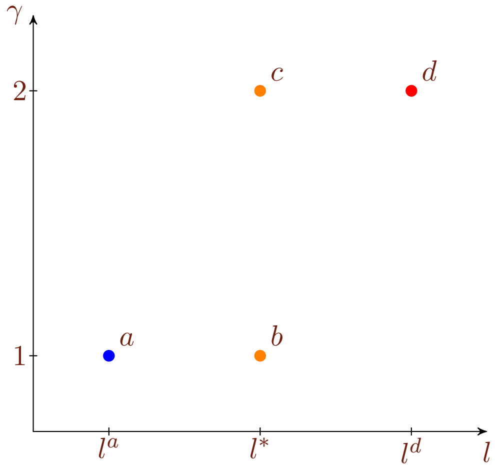
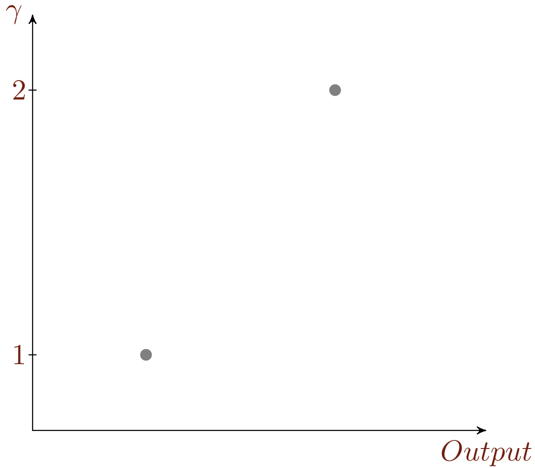


Figure: Aggregate Phillips curve across islands with imperfect information.



The Phillips Curve in the Island Model

Remark:

- So imperfect information regarding the aggregate state M_t and island-specific state N^i results in a relationship between inflation and output that resembles the Phillips curve in the same space.
- Output is the Lucas island economy with a signal extraction problem can rationalize positive correlation between output (employment) and inflation.
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Pitfalls of Keynesian Policy

Remark:

- Suppose policymaker observes the relationship over a long time that $corr(Inflation, Output) > 0$.
- A Keynesian policymaker would be tempted to exploit this Phillips curve: Increase money supply to stimulate output growth.
- What happens if this is done persistently: suppose inflate at constant rate γ?
- We know that results in equilibrium reaction of economy to produce inverted Phillips curve [See non-superneutrality proposition].
- What if policy inflates almost always? Will not work either. Why?

Figure: Interpreting History: Before 1970.

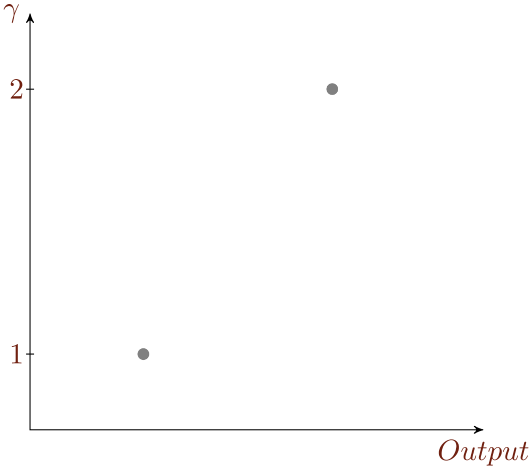

Figure: After 1970.

Lucas Critique and Policy Analysis

Lesson from this parable:

- Observed correlation (e.g. inflation-output) in the data is likely to be an equilibrium outcome from best-responses of agents to prices and policy.
- A change in policy may change these best responses, and equilibrium relationship may change altogether.
- Pitfalls of making policy conclusions using Old-Keynesian macro models that econometrically assume a reduced-form (and fixed) relationship capturing these historical data correlations.
- Ideal: Any model-based policy analysis must start from policy invariant description of primitives: tastes, technology, trading environments.

Lucas Critique and Policy Analysis

- Lucas (1972, JET): provided a microfoundation for the possibility of a Phillips curve relationship.
- More importantly, this paper changed how economists thought about macroeconomic policy analysis and modeling.
- Making policy conclusions on ad-hoc estimated reduced form relationships may lead to counterproductive policy outcomes.
- This was demonstrated by the policy stance in the 1970's stagflation that was prescribed by ad-hoc Keynesian models.