The Economics of Hubs: The Case of Monopoly
Author(s): Ken Hendricks, Michele Piccione and Guofu Tan
Source: *The Review of Economic Studies*, Vol. 62, No. 1 (Jan., 1995), pp. 83-99
Published by: Oxford University Press
Stable URL: http://www.jstor.org/stable/2297842
Accessed: 14-04-2017 15:06 UTC

The Economics of Hubs:
The Case of Monopoly

KEN HENDRICKS, MICHELE PICCIONE, and GUOFU TAN
University of British Columbia

In this paper, we study the optimization problem of an unregulated air carrier which is given the exclusive right to satisfy demand for air travel between any pair of cities. It chooses a network of connections and a set of prices to maximize profits. Thus, both network design and prices are endogenous. We characterize the solution to this optimization problem when demands and costs are symmetric. Our main result is that, if there are economies of density in the number of individuals travelling between two directly connected cities, the optimal network is either a hub of size $n-1$ or one in which every pair of cities is connected directly.

1. INTRODUCTION

Prior to deregulation, the Civil Aeronautics Board (CAB) in the United States was empowered to supervise and control entry into the airline industry, the cities served, and the rates charged. In particular, if an air carrier wanted to supply air transport between cities, it first had to apply to the Board for a route certificate. The certificate entitled the air carrier to service a named set of cities arranged sequentially. Approval of the application was based primarily on "public convenience and necessity". In many instances, the certificate was a monopoly grant, since price regulation presumably made competition unnecessary. In fact, the CAB would often implicitly agree to protect future revenue of certificate holders from the effects of competition by not issuing more than one route certificate. This regulatory process resulted in networks comprised of several linear segments, pieced together over time (see Tretheway (1991) for details).

The Airline Deregulation Act of October, 1978 changed the circumstances under which air carriers competed with each other. It allowed air carriers to fly when and where they like and to set their own fares. In an excellent article, Levine (1987) reviews the subsequent changes in the industry and examines some of deregulation's unpredicted consequences: hub-spoke networks, complicated fare structures, frequent flyer programmes, the enhanced role of travel agencies and computer reservation systems, failure of new entrants, vertical integration, and mergers. He argues convincingly that neither a simple theory of perfect competition nor its more sophisticated version, contestability theory, is able to explain any of these phenomena. He concludes that they are the result of air carriers exploiting scale economies and strategic advantages that they could not pursue under regulation.

This paper focuses on the effect of scale economies on the structure of networks. Given the freedom to choose its own route structure and prices, air carriers transformed their networks into hub-spoke networks. Many analysts believe that the main reason for this transformation is economies of density in route costs (e.g., Bailey and Panzar (1981), Keeler (1972), and Caves, Christianson, and Tretheway (1984)). This occurs when costs

83

per passenger on a route declines with the number of passengers travelling on that route. The hub-spoke network possesses higher traffic densities than larger networks with more direct connections. The distance travelled per passenger is usually longer, since most travellers have to travel two connections to reach their destination instead of one. But, if economies of density are sufficiently large, the total costs of satisfying a given set of travel demands may be lower in the hub-spoke network than in a point-to-point network.

Our objective in this paper is to formalize the above argument in the context of a monopoly. One firm is given the exclusive right to satisfy the demand for air travel between any pair of n cities. Its problem is to choose prices for travelling between city pairs that are connected, a network to connect city pairs, and an allocation of travellers to the paths which connect city pairs. Thus, network design, flows, and prices are endogenous. In particular, we do not restrict the set of networks to hub-spoke or point-to-point networks, but permit the monopolist to choose any pattern of connections among the cities. We then investigate necessary and sufficient conditions for hub-spoke networks to be a solution. Our main result is that, if there are economies of density in the total number of individuals travelling between a pair of directly connected cities, the optimal network when demands and costs are symmetric is either a hub of size $n-1$ or a point-to-point network.

Our approach differs from that of operations research. This literature is primarily interested in finding an algorithm which determines, for any exogenously specified set of flows and unit costs, a network that minimizes total costs subject to the constraint that the flow demands for each city-pair are satisfied. By contrast, we endogenize the quantity flows through prices, and seek conditions on demands and costs under which the optimal network has a simple geometric structure, such as the hub-spoke network.

In the economics literature, Berechman and Shy (1991) study a three-city example in which they compare the profitability of a hub network to a point-to-point network, and Brueckner and Spiller (1991a, b) study optimal pricing in a given network. The more substantive work is by Starr and Stinchcombe (1992), who have been working independently from us. They are also interested in identifying conditions under which hub-spoke networks are optimal for a monopolist. Their model differs from ours in several respects. Following the tradition in operations research, they implicitly assume that prices are exogenous by assuming that quantity flows between city-pairs are fixed. More importantly, they restrict their analysis to linear cost functions in which either fixed costs or variable costs are small. We consider a more general class of cost functions. Finally, they consider a different type of fixed cost, which can lead to different results.

There is an extensive empirical literature on the airline industry which tries to measure the effect of network characteristics and number of competitors on prices. Examples of this work inlcude Bornstein (1989, 1990), Morrison and Winston (1990), Berry (1992), Brueckner, Dyer, and Spiller (1990) and Brueckner and Spiller (1991a, b). A recent survey of this literature can be found in Bornstein (1992).

The paper is organized as follows. The model is introduced in the next section. In Section 3, we characterize the solution to the monopoly problem when there are economies of density and discuss the two main assumptions, the aggregation of directional flows and symmetric demands. Section 4 consists of two examples. Concluding remarks follow in Section 5. Formal proofs of our results are presented in the appendix.

2. THE OPTIMIZATION PROBLEM

In this section we formalize the optimization problem of the monopolist. The basic components of the model are the network, the demands, network flows, and costs. We introduce and discuss each component in turn.

2.1. *Networks*

There is a set $N = \{1, 2, \ldots, n\}$ of $n \geq 3$ distinct cities, and individuals living in each city who wish to travel one-way to other cities in N. The assumption of one-way travel is to simplify notation and interpretation. The results in this paper also hold for two-way travel. In what follows, subscripts g, h, g', h' shall be used to index cities.

In choosing to connect a pair of cities, the monopolist has to decide whether to establish a *direct* connection between the pair of cities or to connect them *indirectly* through a sequence of direct connections. A direct connection involves the acquisition of airport space and associated facilities at each endpoint city which permits the possibility of travel between them, from city g to city h and vice versa. A *network* is a set of direct connections. Specifically, it is a function $X: N \times N \to \{0, 1\}$ such that $X(g, h) = X(h, g)$, and

$$X(g, h) = \begin{cases} 1 & \text{if there is a direct connection between cities } g \text{ and } h, g \neq h, \\ 0 & \text{otherwise.} \end{cases}$$

Note that establishing a direct connection between cities g and h does not imply that the number of travellers from g to h is the same as the number from h to g. As we shall see later, this depends critically upon the structure of costs. The total number of direct connections associated with X is given by $m(X)$ where $m(X) = \sum_h \sum_{g>h} X(g, h)$. For notational simplicity, we will often refer to $m(X)$ as m.

A network induces a set of connections between city pairs. A sequence $a = (n_1, n_2, \ldots, n_{z+1})$ of distinct cities such that $n_1 = g, n_{z+1} = h$ and $X(n_t, n_{t+1}) = 1$ for $t = 1, \ldots, z$, is called a *path* between g and h. The length of the path is z, the number of connections. We denote by a^{-1} the path $(n_{z+1}, n_z, \ldots, n_1)$. Of course, there may be more than one path between cities g and h. The set of all such paths is denoted by L_{gh}.

Definition 1.   A pair of cities (g, h) is *connected* if there exists a path between g and h.

One class of networks that is worth singling out is the set of networks in which passengers can travel from any city g to any other city h.

Definition 2.   A network of X is *connected* if every pair of cities is connected.

Note that a connected network does not imply that every city is directly connected to every other city.

In what follows, two particular types of networks will receive special attention.

Definition 3.   A network of X is called a *point-to-point* network if $X(g, h) = 1$ for any $g, h \in N, g \neq h$.

Definition 4.   A network X is called a *hub-spoke* network of size $m(X) \leq n - 1$ if $\exists h \in N$ such that $\sum_g X(g, h) = m(X)$.

Definition 3 implies that in a point-to-point network every city is directly connected to every other city. Definition 4 implies that in a hub-spoke network city h is connected directly to m other cities, and that these cities are in turn connected to each other only through city h. Clearly, a hub which is connected has to be of size $n - 1$.

2.2. *Demand*

To travel in the network, an individual must purchase a ticket from the monopolist. A ticket specifies a travel path, and its price depends upon the origin and destination cities.[1] Thus, a *price configuration* P for a network X is a mapping that assigns for each city-pair (g, h), $g \neq h$, a non-negative number P_{gh}. If there is no path connecting cities g and h, then P_{gh} is defined to be infinite.

We shall assume that tickets can be used only by the traveller to whom it is issued. It cannot be transferred in whole or in part to other travellers. This is in accord with current airline practices. For a moment, we shall also assume that consumers cannot arbitrage the difference between the price of a route and the prices of its sub-routes by purchasing separate tickets for each sub-route. Later we show that, under mild conditions on the revenue function, the arbitrage constraint is not binding at the set of optimal choices. Together, these two assumptions imply that the monopolist can discriminate among consumers based upon their origin and destination.

A price configuration P determines the number of individuals who wish to travel between each pair of cities in the network. Each set of travellers is indexed by their origin and destination cities. Thus, the set of $g - h$ traveller refers to individuals who begin their trip in city g and end it in city h. We let q_{gh} denote their number. Demand in the $g - h$ market is given by $D(p_{gh})$, where p_{gh} is the price of a ticket to travel from city g to city h. Demand is continuous and strictly decreasing on an interval $[0, \bar{p}]$ where $D(\bar{p}) = 0$, and zero at higher prices. The inverse demand function is denoted by $P(q_{gh})$. Revenue in the $g - h$ market can then be written as $R(q_{gh}) = P(q_{gh})q_{gh}$.

This specification of demands abstracts from reality in several ways. Demand in each city-pair market is assumed to be independent of prices in other city-pair markets. The idea here is that customers who wish to travel from city g to city h have no desire to travel anywhere else. We have also ignored the possibility of price discrimination within a market. In practise, of course, air carriers can and do discriminate among groups of travellers in a city-pair market using fare restrictions, offering lower price to those who are willing to pay in advance, stay a weekend, and risk not being reimbursed if they miss their flight.[2] Finally, we have assumed that customers care only about reaching their destination and the price of the trip, not how this destination is reached. In particularly, they are indiffferent to the number and distance of the connections travelled.[3] This assumption can be relaxed to admit preferences which depend upon the number of stops in the obvious way.

The above assumptions are made primarily to focus the analysis on economies of density and inter-market price discrimination. A more substantive assumption is that the demand functions in the city-pair markets are identical. This is essentially equivalent to assuming that the cities are similar in size. In general, the structure of optimal networks is determined by the pattern of demands across city-pair markets. As we shall see later in Section 3.4 if demands are allowed to vary arbitrarily across city-pairs, very little can be said.

---

1. We do not allow the monopolist to charge different prices for different paths of travel between pairs of cities.

2. Business people typically want to minimize time spent flying and waiting in airports and are more likely to experience changes in their schedules. The leisure traveller is far less sensitive to the time factor, but more sensitive about price. For example, in their study of U.S. travel demand, Morrison and Whinston (1986) found that doubling the frequency of flights would increase demand by business travellers by 21%. For leisure travellers, the increase would be only 5%. Price elasticities for the U.S. business and leisure travellers were estimated by Gillen, Oum and Noble (1986) to be 1·15 and 1·5. respectively.

3. Tretheway (1991) uses estimates from Kanafani and Ghobrial (1985) and De Vany (1974) to argue that the average traveller is willing to pay between $20–$30 more for a non-stop flight than a one-stop flight.

2.3. *Flows*

In hub networks, there is a unique path connecting each pair of cities. Consequently, everyone travelling from city g to city h must have the same itinerary. However, for other networks, there may be multiple paths connecting a pair of cities. In these instances, there is an additional dimension of choice: which path should travellers follow? We shall initially assume that the monopolist has the power to choose the paths that individuals travel. This assumption appears quite strong. However, we shall subsequently show that, in the optimal network, no one has an incentive to choose a path different from the one assigned to her by the monopolist.

The problem of allocating flows does not arise if transport costs are proportional to the number of travellers. In that case, network costs are independent of how flows are allocated across paths. But, in determining the conditions under which hub-spoke networks are likely to arise, it is important not to restrict the analysis a priori to linear cost functions. With non-linear costs, the choice of path affects network costs, so it is necessary to formalize this choice. This requires some additional notation.

For a connected pair of cities (g, h), let Δ_{gh} denote a probability distribution over L_{gh}, and let $\Delta_{gh}(a)$ be the proportion of $g-h$ travellers that the monopolist allocates to path $a \in L_{gh}$. A *flow configuration* Δ for a network X is a mapping that assigns for each connected city-pair (g, h), $g \neq h$, a probability distribution Δ_{gh}.

Using this notation, we can now define the flows on each direct connection. We distinguish flows by direction. For each (g, h) such that $X(g, h) = 1$, let $\gamma_{g'h'}(g, h)$ be the proportion $g'-h'$ travellers who use (g, h) associated with $\Delta_{g'h'}$. The set of origin-destination pairs for travellers that use (g, h) is then given by $D_{gh} = \{(g', h') \in N \times N | \gamma_{g'h'}(g, h) > 0\}$. Given a price configuration P and a flow configuration Δ, the total flow from city g to city h is defined as

$$Q_{gh}(P, \Delta) = \sum_{(g',h') \in D_{gh}} \gamma_{g'h'}(g, h)D(p_{g'h'}).$$

Note that there may not be any relation between q_{gh} and Q_{gh}. The set of $g-h$ travellers may follow a path that does not involve the direct connection between these two cities.

2.4. *Costs*

There are basically two kinds of costs involved in establishing a direct connection between two cities. Station and ground site costs, ticketing and promotion, and administration costs are more or less independent of the total volume of traffic on a particular connection. We represent such costs by F. According to industry reports, these costs represent approximately 40–50% of air carrier's total costs. Note that, given our definition of a network, the payment of F establishes the possibility of travelling in both directions on a connection.

The remaining costs involve aircraft leasing and operating costs. These typically depend upon the number and sizes of the aircraft required to service the number of travellers travelling in either direction on the connection. (Note that a connection does not imply one flight in each direction!) In general, these costs may depend on the composition of the flow travelling each way on the connection. However, we shall assume that all sets of travellers travelling in the same direction on a direct connection can be aggregated, regardless of destination or origin. We also ignore any differences between city-pairs by

assuming that costs are identical across connections.[4] Variable costs for a direct connection between cities g and h can then be expressed as $V(Q_{gh}, Q_{hg})$.[5]

Tretheway (1991) argues that there are substantial economies of scale in aircraft size. Both leasing prices and fuel costs are strictly concave in size, at least for jet aircraft. Unit labour cost are clearly declining in size, since there is only one pilot and co-pilot for every aircraft. This suggests that V is likely to be strictly concave for connections with traffic densities that require only one airplane.[6] For connections with larger densities, the indivisibilities associated with the number of aircraft are likely to be less important, so marginal costs in these instances may be approximately constant.

We denote total costs of a connection as the sum of fixed costs and variable costs,

$$C(Q, Q') = F + V(Q, Q').$$

We assume that $V(\cdot, \cdot)$ is symmetric with $V(0, 0) = 0$. In addition, we impose the regularity conditions that it is continuous and strictly increasing in (Q, Q').

Definition 5. A network exhibits *economies of density* if $C(\theta Q, \theta Q') < \theta C(Q, Q')$ for all $\theta > 1$ and for all $(Q, Q') \neq (0, 0)$.

The definition of economies of density is equivalent to the condition that average costs of a direct connection decreases with proportionate increases in both flows on that connection. It also implies that $C(Q, Q')/Q$ is decreasing in Q for all Q'. Finally, it is straightforward to show that concavity of C implies economies of density. Note that the economies of density can arise from two sources: spreading fixed costs over a larger volume of passengers or declining marginal costs.

Following the literature, we shall assume that airline networks exhibit economies of density.

2.5. *The optimization problem*

We can now formally state the monopolist's optimization problem. It consists of choosing a triple (X, P, Δ) to maximize

$$\sum_{g=1}^{n} \sum_{h \neq g} R(D(p_{gh})) - \sum_{g=1}^{n} \sum_{h > g} V(Q_{gh}(P, \Delta), Q_{hg}(P, \Delta)) - Fm(X). \quad (1)$$

The first term simply sums up the total revenues which the monopolist can collect from each market. The second and the third terms represent the total costs of connecting up city pairs. We denote the maximized value of (1) by $\Pi(F)$, which we assume is positive for some positive F. This rules out the null network in which the monopolist does not service any of the markets.

4. This is more reasonable than might first appear, since approximately 60% of the fuel costs of a flight are incurred in take-off and landing. Consequently, distance is not as important a factor in costs as it is in demand.

5. Implicit in this form of aggregation is the assumption that scheduling problems can be ignored. For example, consider a point-to-point network with three cities in which travellers always move in the same (clockwise or counterclockwise) direction. Then, on each outgoing flight, there are three sets of customers. However, it is clearly impossible to stagger the times of the flights so that every plane is indeed carrying the three sets of travellers.

6. The Boeing 747 has a seating capacity of 500, and is the largest commercial aircraft currently available.

3. OPTIMAL NETWORKS

In this section we characterize the solution to the optimization problem when there are economies of density. We first present the main theorems and the steps involved in proving them. The analysis assumes that the monopolist can discriminate perfectly across city-pair markets. We then discuss the plausibility of this assumption, that is, whether travellers are willing to identify themselves in the optimal network by purchasing the appropriate city-pair ticket. Finally, we discuss the key assumptions.

3.1. *The main results*

We begin by examining a polar case in which fixed costs are positive but variable costs are zero. Our first result is on the number of connections required to service all of the markets.

Lemma 1. *Suppose X is a network such that $m(X) \leq n-1$. Then the maximum number of connected city pairs is $m(m+1)$.*

The effect of fixed costs on the size of the networks captured by the following theorem, which can easily be established as an implication of Lemma 1. To state the theorem, let $\bar{F} = \sup\{F: \Pi(F) \geq 0\}$ be the highest fixed cost that allows the firm to attain non-negative profits, and let $P(X)$ and $\Delta(X)$ denote an optimal price and flow configuration for network X.

Theorem 1. *Suppose $V(Q, Q')=0$ for all (Q, Q'). If $F \in (0, \bar{F})$, $(X, P(X), \Delta(X))$ maximizes* (1) *if and only if X is a connected network with $m(X)=n-1$.*

The intuition is as follows. If variable costs are zero, profits in each market are independent of prices set in the other markets. Consequently, the optimal pricing policy is to charge the revenue-maximizing price in every market serviced by the network. Given this configuration, the monopolist wants to service all $n(n-1)$ markets as long as fixed costs are not too high. It then follows from Lemma 1 that the smallest network which achieves this objective consists of $n-1$ connections.

Economies of density give the monopolist an incentive to concentrate the flow of traffic on a small number of connections. Theorm 1 formalizes this intuition for a special form of concavity. However, it fails to justify the hub network, since the optimal configuration is not unique. A hub of size $n-1$ is optimal, but so is the linear or "snake" configuration in which cities g and $g+1$ are directly connected. There are many other optimal configurations including multiple hubs.

The indeterminacy in network structure arises for two reasons: variable costs are zero and travellers are indifferent to the particular path of travel between their origin and destination cities. If travellers dislike stops, then a connected hub is the uniqe optimal network. This is established as an implication of the following lemma.

Lemma 2. *Suppose that X is a connected network and that $m(X)=n-1$. Then no path in X has more than two direct connections if and only if X is a hub.*

Lemma 2 also implies that, if variable costs are positive, then the hub of size $n-1$ is optimal among the set of networks with exactly $n-1$ connections.[7] However, in general, it may no longer be optimal for the monopolist to choose a network of size $n-1$.[8] To obtain stronger results, it is necessary to assume that V exhibits economies of density. As we shall see, however, it is not sufficient. We shall need to make additional restrictions, namely, that variable costs depend upon the *sum* of the directional forms. This additvity assumption is quite strong. We explore ways of relaxing this restriction later.

The theorem below characterizes the solution to the monopolist's optimization problem under these cost conditions. To state the theorem, we require some additional notation. Given a real number A, define $F_A = \{F \in [0, \bar{F}): F < A\}$ and $F^A = \{F \in [0, \bar{F}): F > A\}$. Notice that, for $A \leq 0$, F_A is empty, and for $A \geq F$, F^A is empty.

Theorem 2. *Suppose $V(Q, Q') = \phi(Q + Q')$, where ϕ is concave and strictly increasing. Then there exists a constant A such that:*

(i) *For $F \in F_A$, $(X, P(X), \Delta(X))$ is an optimal configuration if and only if X is a point-to-point network;*

(ii) *For $\in F^A$, $(X, P(X), \Delta(X))$ is an optimal configuration if and only if X is a hub network of size $n-1$.*

The value of A is defined in the proof of the theorem. When A is positive, it represents the level of fixed costs at which the hub and point-to-point networks are equally profitable. However, A can be negative when marginal costs are small at relatively low denstiies. In this case, the hub network is optimal even though F is zero.

The proof of the theorem requires a number of steps. We first determine the optimal flow configuration in a network.

Lemma 3. *Suppose $V(Q, Q') = \phi(Q + Q')$ and ϕ is concave. Then, given any X and P there is a cost-minimizing allocation of flows Δ such that, for each pair $\{g, h\}$ for which $L_{gh} \neq \phi$:*

(i) *$\Delta_{gh}(a) = 1$ for some $a \in L_{gh}$;*

(ii) *if $\Delta_{gh}(a) = 1$, then $\Delta_{hg}(a^{-1}) = 1$;*

(iii) *if $\Delta_{gh}(a) = 1$, where $a = \{n_1, n_2, .., n_z\}$, then $\Delta_{n_t n_v}(\{n_t, n_{t+1}, .., n_v\}) = 1$ for $1 \leq t < v \leq z$.*

Part (i) of Lemma 3 states that all $g-h$ travellers use the same path. Part (ii) states $h-g$ travellers use the inverse path of $g-h$ travellers. Part (iii) states that, if $g-h$ travellers stop in another city h' en route to city h, then individuals who live in city h' and who wish to travel to another city g' on the path a, must do so using the same sequence of connections as the $g-h$ travellers. Note that it implies that everyone who travels from city g to city h, regardless of origin and destination, uses the same path as the $g-h$ customers. This in turn means that, in an optimal network, if cities g and h are directly connected, then the $g-h$ and $h-g$ customers travel on the direct connection. (Otherwise, the connection can be dropped at a saving of F.)

7. Starr and Stinchcombe (1992) prove a similar result. They show that if fixed costs are positive and variable costs are arbitrarily small, then the optimal network is a hub of size $n-1$.

8. In our working paper, we prove that, if F is zero and V is convex, the optimal network is the point-to-point network. We also provide examples which establish that, if F is positive and $V(Q, Q')$ is convex, then very little can be said about the size and structure of the optimal network.

The proof is based upon concavity of ϕ. It implies that the cost of transporting a fixed number of individuals between two cities g and h is lowest when the $g-h$ travellers all use the same path, and that path is the inverse of the one used by the $h-g$ travellers. In other words, all travellers between two cities traverse the same sequence of direct connections.

The next step consists of showing that any network of size m, where m is an integer between $n-1$ and $n(n-1)/2$, cannot be optimal. To understand why this is true, suppose marginal costs are constant at c. In this case, the costs of servicing a city-pair market depends only upon the number of travellers and the number of connections that they travel. In particular, it does not depend upon the volume of traffic from the city-pair markets that may be using the route. Consequently, given any network X, network profits are equal to

$$\sum_{g=1}^{n} \sum_{h \neq g} R(q_{gh}) - ck_{gh}(X)q_{gh} - Fm(X),$$

where $k_{gh}(X)$ represents the number of connections travelled by $g-h$ travellers. It is then easy to show that, given any $m(X)$ between $n-1$ and $n(n-1)/2$, network gross profits can be bounded from above by

$$\bar{\Pi}_m = 2m\pi(c) + (n(n-1) - 2m)\pi(2c), \tag{2}$$

where $\pi(kc) = \max_q [R(q) - kcq]$. Furthermore, the bound is achievable by starting with a hub of size $n-1$ and then directly connecting $m-(n-1)$ non-hub city pairs. The monopolist collects $\pi(c)$ from each of the $2m$ markets served by direct connections, and $\pi(2c)$ from each of the $n(n-1)-2m$ markets served indirectly. The gain from adding a direct connection is $2(\pi(c) - \pi(2c))$. If it exceeds F, the optimal network is the point-to-point network; if it is less than F, then the hub network of size $n-1$ yields higher profits than *any* larger network.

For the concave case, we use piece-wise linear, concave functions. Given a collection of positive real numbers $\{c_i\}_{i \in I}$ and a collection of non-negative real numbers $\{f_i\}_{i \in I}$, where I is a finite index set, define a piecewise-linear cost function by

$$\phi_L(Q) = \min_{i \in I} \{c_i Q + f_i\}.$$

Clearly, $\phi_L(Q)$ is a concave function.

The next lemma establishes the validity of approximating concave functions with piecewise-linear functions. For any network X, define

$$\Pi_\phi(X) = \max_{P,\Delta} \sum_{g=1}^{n} \sum_{h \neq g} R(D(p_{gh})) - \sum_{g=1}^{n} \sum_{h > g} \phi(Q_{hg}(P, \Delta) + Q_{hg}(P, \Delta)). \tag{3}$$

If the network X posseses direct connections on which flows are zero, then it can be improved to a network $\hat{X}$, the network defined by dropping the unused connections.

Lemma 4.   *For any given X and any strictly increasing, concave cost function $\phi(\cdot)$, there exists a piecewise-linear function ϕ_L where $I = \{1, \ldots, m(\hat{X})\}$ such that $\phi_L(Q) \geqq \phi(Q)$ for any Q and $\Pi_\phi(X) = \Pi_{\phi_L}(X)$.*

The proof of Lemma 4 is a standard application of the separating hyperplane theorem.

To state the next result, we require some additional notation. Let H denote a hub of size $n-1$, and T denote the point-to-point network. For $\phi_L(Q) = \min_{i \in I} \{c_i Q + f_i\}$, define

$$\Pi_i^H = (n-1)(2\pi(c_i) - f_i + (n-2)\pi(2c_i)),$$

$$\Pi_i^T = n(n-1)(2\pi(c_i) - f_i)/2.$$

Notice that $\pi(c_i)$ is convex in c_i. It is clear that $\Pi_\phi(H) \geq \max_{i \in I} \Pi_i^H$ and $\Pi_\phi(T) \geq \max_{i \in I} \Pi_i^T$. Finally, for any m between $n-1$ and $n(n-1)/2$, define a linear combination of $\Pi_\phi(H)$ and $\Pi_\phi(T)$ by

$$\bar{\Pi}_m = \Pi_\phi(H) + 2(m-n+1)(n-1)^{-1}(n-2)^{-1}(\Pi_\phi(T) - \Pi_\phi(H)).$$

We can now state the lemma.

Lemma 5. *Let X be a network such that $m(\hat{X}) = m$ where $n-1 < m < n(n-1)/2$. Then, $\Pi_\phi(X) \leq \bar{\Pi}_m$.*

The proof of Lemma 5 deserves a remark. Fixing the size of a network and solving for the optimal network within that class of networks does not yield "simple" networks. (Non-uniqueness is also a problem.) Moreover, the structure of the optimal network changes dramatically from one value of m to another. Hence, it does not seem possible to characterize the profit frontier for values of m in the range between $n-1$ and $n(n-1)/2$. Lemma 5 avoids this difficulty by establishing that the gross profits of *every* network in this size range are dominated by the profits of either the hub network of size $n-1$ or the directly connected network.

We find this result somewhat surprising. In the concave case, marginal costs are decreasing in traffic densities. One might, therefore, expect the gain from an additional connection to decline with the number of connections, and the optimal size to depend upon the magnitude of the economies of density. Lemma 5 indicates that this reasoning is not correct. If it is optimal to add one more direct connection to the hub network, it is optimal to go all the way and connect every pair of cities directly.

In Lemma 6, we establish that the gross profits of a hub of size m, where $m \leq n-1$, exceeds the profits of any other network of size m.

Lemma 6. *Let (X, P, Δ) be an optimal configuration such that $m(X) = m$, where $n-1 \geq m \geq 1$ and $\Pi_\phi(X) - mF > 0$. Then, X is a hub of size $n-1$.*

Lemma 6 permits a characterization of the profit frontier in the size range $m \leq n-1$. It is easy to show that the frontier is strictly convex. Adding another connection to a network of size $m(m < n-1)$ means the monopolist can service an additional $2(m+1)$ markets and *lower* average costs per traveller everywhere in the network. Thus, it cannot be optimal not to connect the network.

Together, Lemma 5 and 6 imply that the optimal network is either a hub of size $n-1$ or a point-to-point network. Theorem 2 then provides a characterization of the conditions under which each network is optimal.

3.2. *Extensions*

The scope of Theorem 2 may appear somewhat limited due to the restrictive form of the cost function. One might plausibly argue that costs are related to the maximum flow in either direction on a connection rather than individual flows. The following proposition extends the results of Theorem 2 to a more general class of cost functions which includes the above specification.

Proposition 1. *Suppose* $V(Q, Q') = \psi(f(Q, Q'))$, *where f is convex and* $\psi(f(Q, Q))$ *is concave in Q. Then there exists a function* $\phi: \Re \to \Re$ *such that*:

  (i) $V(Q, Q') \geq \phi(Q + Q')$, *with equality holding if* $Q = Q'$, *and*
  (ii) ϕ *is positive, increasing, and concave.*

Convexity of the aggregator function f ensures that transport costs under V are at least as large off the diagonal as they are under ϕ. Theorem 2 can then be extended to the class of cost functions identified in Proposition 1 by noting first, that the bound constructed in Lemmas 5 and 6 is attainable under V and second, that it is an upper bound. The function $f = \max [Q, Q']$ is convex so, given the above proposition, the results of Theorem 2 still apply.

The restriction that variable costs depend upon the sum of the directional flows or, more generally, a convex aggregator of these flows, is essential. Suppose $V(Q, Q') = \phi(Q) + \phi(Q')$, and $\phi(Q)$ is approximately constant over the relevant range of Q (assuming it is positive). Starr and Stinchcombe (1992) consider this type of cost structure. The optimal network in this case is often a "circle", with everyone travelling in the same direction around the circle. Some individuals travel more connections than necessary, but this is offset by having fewer directional flows—only one direction is used on each direct connection. In fact, the circle network minimizes the number of directional flows subject to the constraint that every pair of cities is connected. It is a commonly observed network for delivery runs, and may be appropriate for networks in which demands in the city-pair markets are low enough and the number of cities small enough that one flight in each direction on a direct connection is sufficient.

In the context of Theorem 2, circle networks provide examples in which condition (ii) of Lemma 3 does not hold. That is, individuals in cities that are directly connected do not always use that connection to travel to the other city. Instead, they may use a more roundabout route. Hence, to extend the results of Theorem 2 to a more general class of cost functions, additional structure needs to be imposed.[9] Unfortunately, it is difficult to find conditions on V which are sufficient to ensure that the optimal flow allocation satisfies Lemma 3 (ii).

3.3. *Arbitrage*

Theorem 2 implicitly assumes that the monopolist can perfectly discriminate across city-pair markets. The plausibility of this assumption depends upon the willingness of travellers to identify themselves correctly by purchasing a ticket in the appropriate market. Is this the case in the optimal networks?

Consider the price configuration in the hub network. It is not difficult to show that there exist optimal prices for the direct connections which are symmetric, and that this is also true of optimal prices for the indirect connections. Therefore, let p_D denote the optimal price of a flight between a non-hub city and the hub city and let p_I denote the optimal price of a trip between two non-hub cities.

Proposition 2. *Suppose* $F \in F^4$ *and R is concave and differentiable. Then,* (i) $p_D < p_I$ *and* (ii) $p_I \leq 2p_D$ *if price elasticity is non-increasing.*

9. For example, if $V(Q, Q')$ is sub-modular and concave on (Q, Q'), then it is possible to prove that the characterization given in Theorem 2 holds, *provided* it is optimal for individuals to use the direct connection whenever it is available (see our working paper for details).

Part (i) states that the price of direct flights is less than the price of a ticket between non-hub cities. Travellers between non-hub cities are charged more because the cost of transporting them is twice as high as the cost of transporting travellers between a non-hub city and the hub city. Given this result, anyone travelling to the hub city has no incentive to pretend otherwise by buying a ticket to a non-hub city and de-planing at the hub city. The assumption of symmetric demands is clearly crucial.

Part (ii) states that, given a mild regularity condition on demand, the price of the trip between non-hub cities does not exceed twice the price of a direct flight. Thus, travellers between non-hub cities do not have an incentive to fly into the hub city on one ticket and fly out of the hub city on another ticket. Once again, the assumption that demands are symmetric plays an important role.[10]

There is no scope for arbitrage in point-to-point networks. As in the hub network, prices are symmetric. The cost of travelling on an indirect path to one's destination is at least twice the cost of using the direct connection. Hence, everyone purchases the direct ticket.

3.4. Non-symmetric demands

In this section we discuss the importance of the assumption of symmetric demands. To obtain some insights on this issue, suppose marginal costs are constant, so the network profit function is separable across city-pair markets. Demand conditions can differ across city pairs but, for notational convenience, demand in the $g-h$ market is assumed to be the same as in the $h-g$ market. Then the gain from servicing a pair of cities with a direct rather than a one-stop connection is given by

$$A_{gh}(c) = 2[\pi_{gh}(c) - \pi_{gh}(2c)].$$

If this gain exceeds F, it is profitable for the monopolist to connect cities g and h directly. Thus, a lower bound on the number of direct connections in the optimal network is the number of city-pairs for which this condition holds. If it holds for all city-pairs, the optimal network is obviously a point-to-point network.

When demand conditions are the same in every city, $A_{gh}(c)$ is the same for all city-pairs. Consequently, either it exceeds F for every city-pair, in which case the solution is a point-to-point network, or for none, in which case the solution is a hub-spoke network. But, when demand conditions differ across cities, $A_{gh}(c)$ can be less than F for some city-pairs and exceed F for other city-pairs. The optimal number of direct connections in these cases will lie somewhere between $n-1$ and $n(n-1)/2$, and depend on the pattern of network externalities. Furthermore, even if $A_{gh}(c)$ is less than F for every city-pair, the optimal network is not necessarily a hub-spoke network.[11]

It seems that very little can be said about optimal networks if demand conditions are allowed to vary arbitrarily across city-pairs. However, in reality, the value of a direct connection is often related to the scale of the demand in the markets served, which in turn depends upon the sizes of the cities being connected. For example, suppose there are four cities and cities 1 and 2 are large, and cities 3 and 4 are small. The gains from servicing

---

10. The reason why p_I can be less than $2p_D$ is because it is currently not legal to resell a ticket or a portion thereof. The ticket can only be used by the person to whom it is issued.

11. Suppose there are four cities and the gains from directly connecting cities g and $g+1$ for $g = 1, 2, 3$ are large relative to those of other city-pairs. In that case, ti is not difficult to show that a linear or "snake" network in which cities g and $g+1$ are directly connected for $g = 1, 2, 3$ can be more profitable than a hub-spoke network.

the (1–3), (1–4), (2–3) or (2–4) markets with a direct rather than a one-stop connection could reasonably be assumed to be the same, and less than the gains from directly connecting cities 1 and 2, but greater than those obtained from directly connecting cities 3 and 4. An ordering of city-pair markets based on their size may enable one to establish more general theorems on the structure of optimal networks. We hope to investigate this issue in later work.

4. HUB VERSUS POINT-TO-POINT NETWORKS

Given a set of n cities, when is a hub more likely to occur than the point-to-point network? To gain some insight into this issue, it will be useful to consider a couple of examples.

Suppose $D(p) = kp^{-\alpha}$ and $\phi(Q + Q') = c(Q + Q')$, where $c, k > 0$ and $\alpha > 1$. Then optimal prices in the hub network are given by

$$p_D = \alpha c/(\alpha - 1), \qquad p_I = 2\alpha c/(\alpha - 1).$$

Indirect travellers pay twice as much as the direct travellers. Optimal prices in the point-to-point network are p_D, since marginal costs are constant. Profit on each set of direct travellers is

$$\pi(c) = k\alpha^{-\alpha}[c/(\alpha - 1)]^{1-\alpha},$$

and $\pi(2c)$ on each set of indirect travellers. The value of fixed costs at which the hub and point-to-point networks are equally profitable is given by

$$A(c) = 2k\alpha^{-\alpha}(c/(\alpha - 1))^{1-\alpha}(1 - 2^{1-\alpha}).$$

In a hub network, marginal costs in $(n-1)$ $(n-2)$ markets are twice as high as they are in a point-to-point network. As a result, the optimal price in these markets is higher, and revenues are lower. The magnitude of these revenue losses depend upon the location and shape of the marginal revenue curve in the range between c and $2c$. In the constant elasticity case, these losses are an increasing function of k, the sale of demand parameter, and a decreasing function of c. Hence, the hub is the preferred network when demand is low and marginal costs are high.

The latter property is not general, since it relies upon demand being unbounded as p approaches zero. If $\bar{p}$ is finite, as we assumed earlier, then $A(c)$ is equal to zero when marginal costs are zero. This implies that $A(c)$ is not monotonic. For example, suppose demand is linear, $D(p) = k - p$ for $0 \leq p \leq k$. Optimal prices in the hub network for $c < k/2$ are

$$p_D = (k + c)/2, \qquad p_I = (k + 2c)/2.$$

(If c exceeds k, no one travels, and if c lies between k and $k/2$, no one travels between non-hub cities.) In the point-to-point network, optimal prices are equal to p_D. Given these prices, each set of direct travellers generate profits of $(k - c)^2/4$, and each set of direct travellers in a hub network generate profits of $(k - 2c)^2/4$. The two networks are equally profitable when fixed costs are equal to

$$A(c) = c(k - (3/2)c).$$

Thus, for a fixed F, a hub may be optimal at low and high values of c, but not at intermediate values.

5. CONCLUSION

The conditions identified in this paper as sufficient for the optimality of hub-spoke networks appear to describe costs in the airline industry. To that extent, our analysis confirms the wide-spread belief that economies of density can explain the emergence of hub-spoke networks as the network of choice for U.S. air carriers in the post-regulation era. It is also important to note the strength of the result. Small perturbations of the symmetric model will not cause the optimal network to change dramatically. It suggests that, even when there is significant variation in sizes of cities or distances across city-pairs, the effects on the network may be local.[12] For example, cities whose locations involve too much "backtracking" or whose sizes are quite large may be directly connected. Small cities may be placed one stop away from the hub. But, most of the cities will be connected through a hub-spoke network.

We have not examined other reasons for hub-spoke networks. Some analysts argue that the high traffic densities of hub-spoke networks permits an airline to offer more frequent service between cities. This is in turn allows the airline to attract more business travellers, who desire scheduling flexibility and are willing to pay for it. For example, Morrison and Winston (1986) report that a doubling of the frequency of air service would lead to a 21% increase in the demand for air service by business travellers. By comparison, the increase in demand by pleasure travellers was only 5%. Other analysts argue that hub-spoke networks are a consequence of the strategic interacion between firms. Both of these explanations deserve to be studied.

APPENDIX

Proof of Lemma 1.   Notice that the bound is attained by a hub of size m. To prove that this is an upper bound we proceed by induction. The claim is obviously true when $m=1$. Now suppose the claim is true for $m=r-1$. Then, for the bound to be attained when $m=r$, we need first to show that all the cities in the network must be connected, that is, $X(g, h)=1$ and $X(g', h')=1$ implies that cities g, h, g', and h' are connected.

Decompose N into a collection of disjoint subsets, $\{N_s\}$, such that for any $h \in N_s$, $X(h, g)=0$ for any $g \notin N_s$, and let r_s denote the number of direct connections in N_s. Now suppose $r_s < r$ for every s. Then, since $\sum_s r_s = r$, $\sum_s r_s^2 + 2\sum_s \sum_{s' \neq s} r_s r_{s'} = r^2$, which implies $\sum_s r_s(r_s+1) + 2\sum_s \sum_{s' \neq s} r_s r_{s'} = r(r+1)$. Hence, $\sum_s r_s(r_s+1) < r(r+1)$, which is a contradiction.

Next we show that the maximum number of connected city pairs cannot be larger than $r(r+1)$. If it is, the number of connected cities is $v > r+1$. Take a pair of cities (g, h) such that $X(g, h)=1$ and consider a network X' which is obtained from X by setting $X'(g, h')=0$ for all $h' \in N$, and $X'(h, h')=1$ if $X(g, h')=1$, $h' \neq h$, and leaving all other connections unchanged. As $\sum_{h'} \sum_{g > h'} X'(g, h') \leq r-1$ and X' connects $v-1$ cities to one another, a contradiction is obtained.   ‖

Proof of Lemma 2.   Let X be a connected network such that $m(X)=n-1$. Sufficiency is obvious. To prove necessity, we will show that if X is not a hub, then some L_{gh} contains only paths consisting of at least three direct connections.

First, it is easy to show that since X is connected and $m(X)=n-1$, L_{gh} is a singleton set for all pairs (g, h).

Suppose now that X is not a hub. Then, there is a city h and a set $I=\{g \in N: X(g, h)=1\}$ such that $2 \leq |I| < n-1$. Since all city pairs are connected, there is a city $g \in I$, $g \neq h$, and a city $h' \in I$ such that $X(g, h')=1$. Consider a city $g' \in I/h'$. Then, $X(g', h)=X(h, h')=X(h', g)=1$. Since $L_{g'g}$ is unique, the claim follows.   ‖

Proof of Lemma 3.   First, notice that an optimal allocation of flows always exists. Now, suppose that the claim (i) is not true. Then, there exist two positive quantities $\hat{q}$ and $\bar{q}$ and two collections $\{\hat{q}_t\}_{t=1}^{i}$ and $\{\bar{q}_t\}_{t=1}^{j}$

---

12. In the language of graph theory, there is no "ripple" effect.

such that

$$\sum_{t=1}^{i} \phi(\hat{q}_t + \hat{q}) + \sum_{t=1}^{j} \phi(\bar{q}_t + \bar{q}) \leq \sum_{t=1}^{i} \phi(\hat{q}_t) + \sum_{t=1}^{j} \phi(\bar{q}_t + \bar{q} + \hat{q})$$

and

$$\sum_{t=1}^{i} \phi(\hat{q}_t + \hat{q}) + \sum_{t=1}^{j} \phi(\bar{q}_t + \bar{q}) \leq \sum_{t=1}^{i} \phi(\hat{q}_t + \bar{q} + \hat{q}) + \sum_{t=1}^{j} \phi(\bar{q}_t)$$

with at least one strict inequality. However, the concavity of ϕ implies that

$$\bar{q}(\bar{q} + \hat{q})^{-1} \phi(\hat{q}_t) + \hat{q}(\bar{q} + \hat{q})^{-1} \phi(\hat{q}_t + \bar{q} + \hat{q}) \leq \phi(\hat{q}_t + \hat{q})$$

and

$$\hat{q}(\bar{q} + \hat{q})^{-1} \phi(\bar{q}_t) + \bar{q}(\bar{q} + \hat{q})^{-1} \phi(\bar{q}_t + \bar{q} + \hat{q}) \leq \phi(\bar{q} + \bar{q})$$

for any t. A contradiction is then obtained by simple calculations. Claims (ii) and (iii) are proved in a similar fashion. ∥

Proof of Lemma 5. First, choose a solution (P, Δ) to (3) such that Δ satisfies Lemma 3. Now, consider a piecewise-linear function $\phi_L(Q)$ for which Lemma 4 holds. Now, construct a collection $\{\rho_{ij}\}_{i,j \in I}$, where $\rho_{ij} \in \{0, 1\}$, as follows. For any pair (g, h) for which $X(g, h) = 0$ and $L_{gh} \neq \phi$, choose the path $a \in L_{gh}$ such that $\Delta_{gh}(a) = 1$, where $a = \{n_1, n_2, \ldots, n_z\}$. Then, set $\rho_{ij} = 1$ if and only if the pair (g, h) is connected indirectly by a path for which the initial connection is indexed by i and final connection is indexed by j. Notice that (iii) in Lemma 3 implies that there is a one-to-one relationship between a pair of connections i and j for which $\rho_{ij} = 1$ and a city pair (g, h). All the remaining ρ_{ij} are set equal to zero.

We now construct an upper bound for $\Pi_{\phi_L}(X)$ by considering for each indirect connection only the cost incurred for the initial and final connection. Thus,

$$\Pi_{\phi_L}(X) \leq \sum_{i \in I} (2\pi(c_i) - f_i)) + \sum_{i,j \in I} \rho_{ij} \pi(c_i + c_j).$$

Since $\pi(c_i)$ is convex in c_i we have

$$\Pi_{\phi_L}(X) \leq \sum_{i \in I} (2\pi(c_i) - f_i) + (1/2) \sum_{i,j \in I} \rho_{ij} (\pi(2c_i) + \pi(2c_j)). \tag{A1}$$

Notice that since Δ satisfies Lemma 3, $\rho_{ij} = \rho_{ji}$, $\sum_{i,j \in I} \rho_{ij} \leq n(n-1) - 2m$ and $\sum_{j \in I} \rho_{ij} \leq n - 2$ for any $i, j \in I$. Now define $\alpha_i = \sum_{j \in I} \rho_{ij} / (n-2)$. Thus, (A1) can be rewritten as

$$\Pi_{\phi_L}(X) \leq \sum_{i \in I} \alpha_i (2\pi(c_i) - f_i + (n-2)\pi(2c_i)) + \sum_{i \in I} (1 - \alpha_i)(2\pi(c_i) - f_i).$$

Therefore,

$$\Pi_{\phi_L}(X) \leq (n-1)^{-1} \max_i \Pi_i^H \sum_{i \in I} \alpha_i + n^{-1}(n-1)^{-1} 2 \max_i \Pi_i^T \sum_{i \in I} (1 - \alpha_i).$$

Since $(n-1)^{-1} \max_i \Pi_i^H \geq n^{-1}(n-1)^{-1} 2 \max_i \Pi_i^T$ and $\sum_{i \in I} \alpha_i \leq (n(n-1) - 2m)(n-2)^{-1}$, then

$$\Pi_{\phi_L}(X) \leq (n-1)^{-1} \Pi_\phi(H) \beta + n^{-1}(n-1)^{-1} 2 \Pi_\phi(T)(m - \beta)$$

where $\beta = (n(n-1) - 2m)(n-2)^{-1}$. ∥

Proof of Lemma 6. First notice that, since (X, P, Δ) is an optimal configuration, $X = \hat{X}$. Now define ϕ_L as in Lemma 5 and consider a collection $\{\rho_{ij}\}_{i,j \in I}$ such that $\rho_{ij} = 0$ if $i = j$ and $\rho_{ij} = 1$ otherwise. By Lemmas 1, 3 and 4, it follows that $\Pi_\phi(X) \leq \sum_{i \in I} (2\pi(c_i) - f_i) + \sum_{i,j \in I} \rho_{ij} \pi(c_i + c_j)$. The convexity of $\pi(c_i)$ implies

$$\Pi_\phi(X) \leq \sum_{i \in I} ((2\pi(c_i) - f_i) + (m-1)\pi(2c_i)) \leq \Pi_\phi(H_m) \tag{A2}$$

where H_m is a hub of size m. It is easy to show that $\Pi_\phi(H_m) - mF > 0$ implies that $\Pi_\phi(H_m) - mF$ is strictly increasing in m for $m \leq n - 1$. Therefore, $m(X) = n - 1$.

To prove uniqueness, suppose X is not a hub and $m(X) = n - 1$. Then, (A2) implies that $\Pi_\phi(X) = \max_{i \in I} \Pi_i^H$ and $\Pi_\phi(X) = \Pi_i^H$ for all i. Since $\Pi_\phi(X) - mF > 0$, it follows that $\pi(2c_i) > 0$ for all i. Otherwise, the point-to-point network would attain strictly higher profits. Hence, $\pi(c_i + c_j) > 0$ for all i, j, which implies that $\pi(c_i + c_j) > \pi(c_i + c_j + c_z)$ for any i, j, z. Therefore, by Lemmas 2, 3 and 4 it follows that $\Pi_\phi(X) < \sum_{i \in I} (2\pi(c_i) - f_i) + \sum_{i,j \in I} \rho_{ij} \pi(c_i + c_j) \leq \Pi_\phi(H)$. A contradiction. ∥

Proof of Theorem 2. Define $A = 2(n-1)^{-1}(n-2)^{-1}(\Pi_\phi(T) - \Pi_\phi(H))$ as the level of fixed costs at which the net profits of the hub and point-to-point networks are equal. If $F \in F_A$, it follows from Lemma 5 that, for $n - 1 < m(X) < n(n-1)/2$

$$\Pi_\phi(X) - mF < \Pi_\phi(T) - n(n-1)F/2.$$

Since $\Pi(F) > 0$, it follows by Lemma 6 that if X is an optimal network for which $m(X) \leqq n - 1$, $X = H$. Since

$$\Pi_\phi(T) - n(n-1)F/2 > \Pi_\phi(H) - (n-1)F$$

part (i) of the theorem follows. Part (ii) is proved by a similar argument. ‖

Proof of Proposition 1. Using symmetry of V and convexity of f, we obtain the relation

$$V(Q, Q') \geqq \psi(f((Q + Q')/2, (Q + Q')/2)).$$

Define $\phi(Q) \equiv V(Q/2, Q/2)$. Then Part (i) follows immediately. Part (ii) follows directly from the properties of $V(\cdot, \cdot)$. ‖

Proof of Proposition 2. The first-order conditions to the profit maximization for the hub yields the equation $R'(q_I) = 2R'(q_D) > 0$, where $q_I = D(p_I)$ and $q_D = D(p_D)$. Concavity of R implies $q_I < q_D$, and hence, that $p_I > p_D$.

To prove part (ii), recall that $R'(q) = P(q)(1 + \varepsilon(q))$, where $\varepsilon(q) = qP'(q)/P(q)$ is the price elasticity. Rewriting the first-order condition yields

$$p_I(1 + \varepsilon(q_I)) = 2p_D(1 + \varepsilon(q_D)).$$

Since $\varepsilon(q)$ is non-increasing, this implies $p_I \geqq 2p_D$. ‖

REFERENCES

BAILEY, E. and PANZAR J. (1981), "The Contestability of Airline Markets During the Transition to Deregulation", *Law and Contemporary Problems*, 44, 125–45.

BERECHMAN, J. and SHY, O. (1991), "Airline Deregulation and the Choice of Networks" (Working Paper No. 2-91, The Sackler Institute of Economic Studies, Tel-Aviv University, Israel).

BERRY, S. (1990), "Airport Presence as Product Differentiation", *American Economic Review, Papers and Proceedings*, 80, 394–399.

BERRY, S. (1992), "Estimation of a Model of Entry in the Airline Industry", *Econometrica*, 60, 889–917.

BORNSTEIN, S. (1992), "The Evolution of U.S. Airline Competition", *Journal of Economic Perspectives*, 6, 45–73.

BORNSTEIN, S. (1989), "Hubs and High Fares: Dominance and Market Power in the U.S. Airline Industry", *Rand Journal of Economics*, 20, 344–365.

BORNSTEIN, S. (1990), "Airline Mergers, Airport Dominance, and Market Power", *American Economic Review, Papers and Proceedings*, 80, 400–404

BRUECKNER, J. K. and SPILLER, P. T. (1991a), "Competition and Mergers in Airline Networks", *International Journal of Industrial Organization*, 9, 323–342.

BRUECKNER, J. K. and SPILLER, P. T. (1991b), "Fares and Network Feed": Estimating Economics of Traffic Density in Airline Hub-and-Spoke Systems" (BEBR Working Paper No. 91–0180, University of Illinois at Urbana-Champaign).

BRUECKNER, J. K., DYER, N J. and SPILLER, P. T. (1990), "Fare Determination in Airline Hub-and-Spoke Networks" (BEBR Working Paper No. 90–1665, University of Illinois at Urbana-Champaign).

CAVES, D., CHRISTENSEN, L. and TRETHEWAY, M. (1984), "Economies of Density Versus Economies of Scale: Why Trunk and Local Service Airline Costs Differ", *RAND Journal of Economics*, 15, 471–89.

DE VANY, (1974), "The Revealed Value of Time in Air Travel", *Review of Economics and Statistics*, 56, 77–82.

GILLEN, D., OUM, T. and TRETHEWAY, M. (1990), "Airline Cost Structure and Policy Implications", *Journal of Transport Economics and Policy*, 24, 9–34.

HENDRICKS, K., PICCIONE, M. and TAN, G. (1992), "The Economics of Hubs: The Case of Monopoly" (UBC Discussion Paper 92–09).

KANAFANI, A. and GHOBRIAL, A. (1985), "Airline Hubbing—Some Implications for Airport Economics", *Transportation Research*, 19, 15–27

KEELER, T. (1972), "Airline Regulation and Market Performance", *Bell Journal of Economics*, 3, 399–424.

LEVINE, M. (1987), "Airline Competition in Deregulated Markets: Theory, Firm Strategy and Public Policy", *Yale Journal on Regulation*, 4, 393–393.

MORRISON, S. and WINSTON, C. (1990), "The Dynamics of Airline Pricing and Competition", *American Economic Review*, **80**, 389–393.

MORRISON, S. and WINSTON, C. (1986), *The Economic Effects of Airline Deregulation* (Washington D. C.: Brookings Institution).

OUM, T., GILLEN, D. and NOBLE, S. (1986), "Demands for Fare Classes and Pricing and Airline Markets", *Logistics and Transportation Review*, **22**, 195–222.

REISS, P. and SPILLER, P. (1989), "Competition and Entry in Small Airline Markets", *Journal of Law and Economics*, **32**, S179–202.

SPILLER, P. (1989), "A Note on Pricing of Hub-and-Spoke Networks", *Economics Letters*, **30**, 165–169.

TRETHEWAY, M. (1991), "The Characteristics of Modern Post-Deregulation Air Transport" (UBC Transportation and Logistics).

STARR, R., and STINCHCOMBE, M. (1992), "An Economic Analysis of the Hub and Spoke System" (mimeo, University of California, San Diego).