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INDIVIDUAL HETEROGENEITY AND AVERAGE WELFARE

BY JERRY A. HAUSMAN AND WHITNEY K. NEWEY[1]

Individual heterogeneity is an important source of variation in demand. Allowing for general heterogeneity is needed for correct welfare comparisons. We consider general heterogeneous demand where preferences and linear budget sets are statistically independent. Only the marginal distribution of demand for each price and income is identified from cross-section data where only one price and income is observed for each individual. Thus, objects that depend on varying price and/or income for an individual are not generally identified, including average exact consumer surplus. We use bounds on income effects to derive relatively simple bounds on the average surplus, including for discrete/continuous choice. We also sketch an approach to bounding surplus that does not use income effect bounds. We apply the results to gasoline demand. We find tight bounds for average surplus in this application, but wider bounds for average deadweight loss.

KEYWORDS: Consumer surplus, deadweight loss, identification, quantiles.

1. INTRODUCTION

UNOBSERVED INDIVIDUAL HETEROGENEITY is thought to be a large source of variation in empirical demand equations. Often r-squareds are found to be low in cross-section and panel data applications, suggesting that much variation in demand is due to unobserved heterogeneity. The magnitude of heterogeneity in applications makes it important to account correctly for heterogeneity.

Demand functions could vary across individuals in general ways. For example, it seems reasonable to suppose that price and income effects are not confined to a one-dimensional curve as they vary across individuals, meaning that heterogeneity is multi-dimensional. Demand might also arise from combined discrete and continuous choice, where heterogeneity has different effects on discrete and continuous choices. For these reasons, it seems important to allow for general heterogeneity in demand analysis. In this paper, we do so.

Exact consumer surplus quantifies the welfare effect of price changes, including the deadweight loss of taxes. The average surplus over individuals is a common welfare measure. We show that for continuous demand, average surplus is generally not identified from the distribution of demand for a given price and income. Nonidentification motivates a bounds approach. We use bounds on income effects to derive bounds on average surplus. Surplus bounds are constructed from the average of quantity demanded across consumers.

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In the Supplemental Material (Hausman and Newey (2016)), we extend these bounds to combined discrete and continuous choice. We also sketch how surplus bounds can be obtained from the distribution of quantity demanded without knowing bounds on income effects.

Empirical application of these bounds is based on independence of preferences and budget sets, possibly conditioned on covariates and control functions. Under independence, average demand is the conditional expectation of quantity, which can be estimated by nonparametric, semiparametric, or parametric methods in cross-section data. The distribution of demand can be also estimated in analogous ways.

We apply average surplus bounds to gasoline demand, using data from the 2001 U.S. National Household Transportation Survey. We find that average surplus bounds based on income effects are quite tight in the application, but that bounds on deadweight loss are wider. We give confidence intervals for an identified set with income effect bounds. We find that bounds are substantially wider when we just impose utility maximization.

In this paper, bounds on derivatives lead to useful bounds for objects of interest, as have restrictions like monotonicity and concavity in other settings. The focus here on derivatives is driven by economics, where income effects play a pivotal role in bounding average surplus.

Choice models with general heterogeneity have previously been considered. In their analysis of nonlinear taxes, Burtless and Hausman (1978) allowed the income effect to vary over individuals. McFadden and Richter (1991) and McFadden (2005) allowed for general heterogeneity in a revealed preference framework. Our paper specializes the revealed preference work in imposing single valued, smooth demands to facilitate estimation. Lewbel (2001) provided conditions for the average demand to satisfy the restrictions of utility maximization. Hoderlein and Stoye (2014) showed how to impose the weak axiom of revealed preference. Dette, Hoderlein, and Neumeyer (2011) proposed tests of downward sloping compensated demands. Kitamura and Stoye (2012) gave tests of the revealed preference hypothesis. Blomquist and Newey (2002) derived the form of average demand with nonlinear budget sets; see also Blomquist, Kumar, Liang, and Newey (2015). Recently, Bhattacharya (2015) has derived average surplus for discrete demand and general heterogeneity.

Blundell, Horowitz, and Parey (2012), Hoderlein and Vanhems (2010), and Blundell, Kristensen, and Matzkin (2011) have considered identification and estimation of welfare measures when demand depends continuously on a single unobserved variable. Blundell, Kristensen, and Matzkin (2011) imposed revealed preference restrictions on demand functions in that setting. Recently, Lewbel and Pendakur (2013) have considered restricted multivariate heterogeneity. Here we go beyond these specifications and consider general heterogeneity.

The results of this paper build on Hausman and Newey (1995). This paper is about heterogeneity in demand, which was largely ignored in the previous paper, but we do make use of asymptotic estimation theory from the previous paper. In this paper, we find that bounds should be computed from the nonparametric regression of quantity on price and income, and not log quantity or some other function of quantity, because the conditional expectation of quantity averages across individuals in the desired way. The results of Hausman and Newey (1995) can then be applied for inference in large samples.

2. DEMAND FUNCTIONS WITH GENERAL HETEROGENEITY

We consider a demand model where the form of heterogeneity is unrestricted. To describe the model, let q denote the quantity of a vector of goods, a the quantity of a numeraire good, p the price vector for q relative to a, and y the individual income level relative to the numeraire price. Also let $x = (p^T, y)^T$, where throughout we adopt the notational convention that vectors are column vectors. The unobserved heterogeneity will be represented by a vector η of unobserved disturbances of unknown dimension. We think of each value of η as corresponding to a consumer, but do allow η to be continuously distributed.

For each consumer η, the demand function $q(x, \eta)$ will be obtained by maximizing a utility function $U(q, a, \eta)$ that is monotonic increasing in q and a, subject to the budget constraint, with

$$q(x, \eta) = \arg \max_{q \geq 0, a \geq 0} U(q, a, \eta) \quad \text{s.t.} \quad p^T q + a \leq y. \tag{2.1}$$

Here we assume that demand is single valued and not a correspondence. This assumption is essentially equivalent to strict quasi-concavity of the utility function. We impose no form on the way η enters the utility function U, and hence the form of heterogeneity in $q(x, \eta)$ is also unrestricted. Demand functions are allowed to vary across individuals in general ways.

Utility maximization imposes restrictions on the demand functions as a function of prices and income. For continuously differentiable demands and positive prices and income, these restrictions are summarized in the following condition.

ASSUMPTION 1: *For each η, $p^T q(x, \eta) + a(x, \eta) = y$, $q(x, \eta)$ is continuously differentiable in x at all x with strictly positive prices and income, and $\partial q(x, \eta)/\partial p + q(x, \eta)[\partial q(x, \eta)/\partial y]^T$ is symmetric and negative semi-definite for all $x \in \chi$.*

By Hurwicz and Uzawa (1971), this condition is also sufficient for existence of a utility function, with $q(x, \eta)$ maximizing the utility function subject to the budget constraint. In this sense, formulating a model with demand functions satisfying Assumption 1 is equivalent to formulating a model based on utility maximization. In what follows, we take as primitive demand functions satisfying Assumption 1. We also need technical conditions in order to make probability statements using these demand functions. For convenience, we reserve these technical conditions to Assumption A1 of the Supplemental Material.

Let r denote a possible value of quantity demanded, G the distribution of η, and $F(r|x, q, G)$ the CDF of quantity r when prices and income equal x for all individuals,

$$F(r|x, q, G) = \int 1\big(q(x, \eta) \leq r\big) G(d\eta). \tag{2.2}$$

The model we consider is one with a CDF for this form for $q(x, \eta)$ satisfying Assumption 1 and a distribution G of η.

This model is a random utility model (RUM) of the kind considered by McFadden (2005; see also McFadden and Richter (1991)). The model here specializes the RUM to single valued demands that are smooth in prices and income. Single valued, smooth demand specifications are often used in applications. In particular, smoothness has often proven useful in applications of nonparametric models and we expect it will here. Estimation under a RUM with general preference variation is not often done in applications. We do so here for average surplus.

Much of the revealed stochastic preference literature is concerned with deriving restrictions on $F(r|x, q, G)$ as a function of r and x that are necessary and sufficient for a RUM. McFadden (2005) provided a set of inequalities that are necessary and sufficient for the RUM with continuous demands. With two goods and single valued, smooth demand, there is a simple, alternative characterization in terms of quantiles that is useful in the identification analysis to follow. The characterization is that each quantile is a demand function. Let $Q(\tau|x) = \inf\{r : F(r|x, q, G) \geq \tau\}$ denote the τth conditional quantile corresponding to $F(r|x, q, G)$, where we drop dependence of Q on q and G for notational convenience. The following result holds under technical conditions that are given in Assumption A2 of the Supplemental Material.

THEOREM 1: *Suppose that there are two goods, so that $q(x, \eta)$ and p are scalars. If Assumptions 1 and A2 are satisfied then $Q(\tau|x)$ is a demand function for all $0 < \tau < 1$ and $p, y > 0$. If $Q(\tau|x)$ satisfies Assumption 1 for each $0 < \tau < 1$ then for $\tilde{\eta}$ distributed as $U(0, 1)$ with CDF $\tilde{G}$ such that Assumption 1 is satisfied for $\tilde{q}(x, \tilde{\eta}) = Q(\tilde{\eta}|x)$ and $F(r|x, q, G) = F(r|x, \tilde{q}, \tilde{G})$.*

Dette, Hoderlein, and Neumeyer (2011) showed that the quantile function is a demand function under conditions similar to those of Assumption A2. Theorem 1 also shows that if the quantile is a demand function, then a demand model with a one-dimensional uniformly distributed η gives the distribution of demand. Together, these results show that, under Assumption A2, the CDF of quantity takes the form $F(r|x, q, G)$ for some q satisfying Assumption 1 if and only if each quantile is a demand function. In this sense, for two goods and single valued smooth demands, the revealed, stochastic preference conditions are that each quantile is a demand function. This result will be used in the identification analysis to follow and is of interest in its own right.

3. EXACT CONSUMER SURPLUS

We focus on equivalent variation, though a similar analysis could be carried out for compensating variation. Let $e(p, u, \eta) = \min_{q\geq 0, a\geq 0}\{p^T q + a \text{ s.t. } U(q, a) \geq u\}$ be the expenditure function and $S(\eta) = \bar{y} - e(p^0, u^1, \eta)$ be the equivalent variation for individual η for a price change from p^0 to p^1 with income $\bar{y}$ and u^1 the utility at price p^1. The corresponding deadweight loss is $D(\eta) = S(\eta) - q(p^1, \bar{y}, \eta)^T \Delta p$, where $\Delta p = p^1 - p^0$.

It is helpful to express surplus and deadweight loss in terms of demand. Let $\{p(t)\}_{t=0}^1$ be a continuously differentiable price path with $p(0) = p^0$ and $p(1) = p^1$. As discussed in Hausman and Newey (1995), Shephard's Lemma implies that, for a scalar t, the equivalent variation $S(\eta)$ is the solution $s(0, \eta)$ at $t = 0$ to

$$\frac{ds(t, \eta)}{dt} = -q\big(p(t), \bar{y} - s(t, \eta), \eta\big)^T \frac{dp(t)}{dt}, \quad s(1, \eta) = 0, \tag{3.1}$$

where $s(1, \eta) = 0$ pins down the constant of integration in this ordinary differential equation. $S(\eta)$ does not depend on the price path as long as the demand function $q(x, \eta)$ satisfies Assumption 1 and $(p(t)^T, \bar{y} - s(t, \eta))^T$ remains in χ.

A change in the price of a single good, say the first one, is a common example. In that case, $p^0 = (p_1^0, \bar{p}_2^T)^T$ and $p^1 = (p_1^1, \bar{p}_2^T)^T$ for some fixed $\bar{p}_2$. A natural choice of price path is $p(t) = tp^1 + (1 - t)p^0 = (p_1^0 + t\Delta p_1, \bar{p}_2^T)^T$, where $\Delta p_1 = p_1^1 - p_1^0$. In this case, equation (3.1) becomes

$$\frac{ds(t, \eta)}{dt} = -q_1\big(p_1^0 + t\Delta p_1, \bar{p}_2, \bar{y} - s(t, \eta), \eta\big)\Delta p_1, \quad s(1, \eta) = 0.$$

Thus, with multiple goods, the exact consumer surplus for a price change for a single good can be computed from the demand function for that good by varying its price and varying income to keep utility constant, as shown by Hausman (1981).

The objects we will focus on and that are of common interest are the average surplus $\bar{S}$ and deadweight loss $\bar{D}$ across individuals, given by

$$\bar{S} = \int S(\eta)G(d\eta), \quad \bar{D} = \int D(\eta)G(d\eta).$$

As is known from Hicks (1939), when $\bar{S}$ is positive, it is possible to redistribute income so that individuals are better off under p^0 than under p^1. Also, weighted averages of $\bar{S}$ over different $\bar{y}$ values can provide measures of social welfare when income and heterogeneity are independent in the population. For these reasons, we will focus on average surplus and deadweight loss in this paper. Some results could be extended to other interesting objects, like the distribution of surplus. We leave such extensions to future work.

Average surplus depends only on average demand $\bar{q}(p, y) = \int q(x, \eta) \times G(d\eta)$ when the income effect is constant. Suppose that $\partial q_1(p_1, \bar{p}_2, y, \eta)/\partial y = b$ over $p_1 \in [p_1^0, p_1^1]$, $y \in [\bar{y} - S(\eta), \bar{y}]$, and η. Then $S(\eta)$ is the solution at $t = 0$ to

$$\frac{ds(t, \eta)}{dt} = -\big[q_1\big(p_1^0 + t\Delta p_1, \bar{p}_2, \bar{y}, \eta\big) - bs(t, \eta)\big]\Delta p_1, \quad s(1, \eta) = 0. \tag{3.2}$$

This is a linear differential equation with explicit solution

$$\begin{aligned} S(\eta) &= \Delta p_1 \int_0^1 q_1\big(p_1^0 + t\Delta p_1, \bar{p}_2, \bar{y}, \eta\big) \exp(-tb\Delta p_1)\, dt \\ &= \int_{p_1^0}^{p_1^1} q_1(p_1, \bar{p}_2, \bar{y}, \eta) \exp\big(-b\big(p_1 - p_1^0\big)\big)\, dp_1. \end{aligned}$$

Taking expectations under the integral gives

$$\bar{S} = \int_{p_1^0}^{p_1^1} \bar{q}_1(p_1, \bar{p}_2, \bar{y}) \exp\big(-b\big(p_1 - p_1^0\big)\big)\, dp_1.$$

This can also be represented as the solution at $t = 0$ to

$$\frac{d\bar{s}(t)}{dt} = -\bar{q}\big(p(t), \bar{y} - \bar{s}(t)\big)^T \frac{dp(t)}{dt}, \quad \bar{s}(1) = 0. \tag{3.3}$$

Comparing equation (3.3) with (3.1), we see that, with one price changing and income effect constant for that good, average surplus solves the same differential equation as individual surplus, with average demand replacing individual demand. This result generalizes to multiple price changes where the income effects are constant for all goods with changing prices.

Obtaining average surplus from average demand is consistent with the well-known aggregation results of Gorman (1961), who showed that constant income effects are necessary and sufficient for demand aggregation. The preceding discussion is a demonstration of a partial dual result, that when the price of one good is changing and the income effect is constant for that good, then exact surplus for average demand is the average of exact surplus. McFadden (2004) derived and used this result in the case where income effects are constant for all goods.

Marshallian surplus solves equation (3.1) while replacing $s(t, \eta)$ on the right-hand side with zero, that is, while not compensating income to remain on the same indifference curve. Average Marshallian surplus $\bar{S}_M$ is given by

$$\bar{S}_M = \int \left\{ \int_0^1 q\big(p(t), \bar{y}, \eta\big)^T \big[dp(t)/dt\big]\, dt \right\} G(d\eta)$$
$$= \int_0^1 \bar{q}\big(p(t), \bar{y}\big)^T \big[dp(t)/dt\big]\, dt.$$

This surplus measure is a function of average demand in general, but it ignores income effects. Ignoring income effects results in a poor approximation to deadweight loss; see Hausman (1981). For this reason, we focus on exact surplus in our analysis, though we do find that average Marshallian surplus provides a useful upper bound for average equivalent variation for a normal good, as shown for individual demands by Willig (1976).

4. IDENTIFICATION

We consider identification of objects of interest when we know the CDF $F(r|x, q, G)$ of demand over a set $\bar{\chi}$ of prices and income. This corresponds to knowing the distribution of demand in cross-section data, where we only observe one price and income for each individual. If more than one value of x were observed for each individual, as in panel data, then one could identify some joint distributions of demand at different values of x. We leave consideration of this topic to future research.

We adapt a standard framework to our setting, as in Hsiao (1983), by specifying that a structure is a demand function and heterogeneity distribution pair (q, G), where for notational convenience we suppress the arguments of q and G.

DEFINITION 1: (q, G) and $(\tilde{q}, \tilde{G})$ are observationally equivalent if and only if, for all r and $x \in \bar{\chi}$,

$$F(r|x, q, G) = F(r|x, \tilde{q}, \tilde{G}).$$

The set $\bar{\chi}$ will correspond to the set of x that is observed. We allow $\bar{\chi}$ to differ from χ of Assumption 1 in order to allow the Slutzky conditions to be imposed outside the range of the data. We consider identification of an object $\delta(q, G)$ that is a function of the structure (q, G). Here $\delta(q, G)$ is a map from the demand function and the distribution of heterogeneity into some set. The identified set for δ we consider will be the set of values of this function for all structures that are observationally equivalent.

DEFINITION 2: The identified set for δ corresponding to (q_0, G_0) is $\Lambda(q_0, G_0) = \{\delta(\tilde{q}, \tilde{G}) : (q_0, G_0)$ and $(\tilde{q}, \tilde{G})$ are observationally equivalent$\}$.

The (q_0, G_0) in this definition can be thought of as the true values of the demand function and heterogeneity distribution. The identified set $\Lambda(q_0, G_0)$ is the set of δ that is consistent with the distribution of demand $F(r|x, q_0, G_0)$ implied by the true values. The set $\Lambda(q_0, G_0)$ is nonempty since it always includes the true value $\delta(q_0, G_0)$. The set $\Lambda(q_0, G_0)$ is sharp, given only knowledge of $F(r|x, q_0, G_0)$, because it consists exactly of those δ that correspond to some $(\tilde{q}, \tilde{G})$ that generates the same distribution of demand as the true values. In other words, sharpness of $\Lambda(q_0, G_0)$ holds automatically here because we are explicitly formulating the identified set in terms of all the restrictions on the distribution of demand that are implied by the model, and we are assuming that the distribution of demand is all we know.

Viewing demand as a stochastic process indexed by x helps explain identification. Here $q(x, \eta)$ is a function of x for each η, that varies stochastically with η, that is, $q(x, \eta)$ is a stochastic process. In this way, the structure (q, G) can be thought of as a demand process. In the language of stochastic processes, the distribution of $q(x, \eta)$ for fixed x is a marginal distribution, while the distribution of $(q(x^1, \eta), \dots, q(x^K, \eta))^T$ for some fixed set $\{x^1, \dots, x^K\}$ of prices and income is a joint distribution. In our notation, the marginal CDF of this stochastic process is $F(r|x, q, G)$. Thus, two demand processes will be observationally equivalent if and only if they have the same marginal distribution.

Objects $\delta(q, G)$ that depend only on the marginal distribution of the demand process are point identified, because they are the same for all observationally equivalent structures. For example, average demand $\bar{q}(x) = \int q(x, \eta)G(d\eta) = \int rF(dr|x, q, G)$ is identified, as are functionals of it, such as average surplus with constant income effect in equation (3.3).

Joint distributions of the demand process, such as the joint distribution of $(q(\tilde{x}, \eta), q(\bar{x}, \eta))^T$ for two different values of x, will not be identified. We will show this result for certain demand processes below and the intuition is straightforward. Intuitively, joint distributions are not identified from marginal distributions. Because joint distributions are not identified, distributions and averages of objects that depend on varying x for given η will not be identified. As we show rigorously below, such nonidentified objects will include average surplus, which depends on varying both price and income for a given η.

It will generally be impossible to identify demand functions for individuals from the marginal distribution of demand. Again the intuition is straightforward, with individual demands not identified because we only observe one price and income for each individual. More formally, we can think of the ability to identify individual demands as $q_0(\tilde{x}, \eta)$ being perfectly predictable for each $\tilde{x}$ if we know $q_0(\bar{x}, \eta)$ for some $\bar{x}$, that is, as $\operatorname{Var}(\tilde{q}_0(\tilde{x}, \tilde{\eta})|\tilde{q}_0(\bar{x}, \tilde{\eta})) = 0$ for any $(\tilde{q}, \tilde{G})$ that is observationally equivalent to the truth. This is a property of the joint distribution of the demand process, and so is not identified from the marginal distribution of the demand process.

The specific nonidentification results we show are for two goods where Theorem 1 provides the key to the proof. Combining its first and second conclusion implies that $Q(\tau|x)$ is a demand function and that $Q(\tilde{\eta}|x)$ for $\tilde{\eta} \sim U(0,1)$ gives the same conditional distribution of quantity as true demand. Thus, under the conditions of Theorem 1, the quantile demand is observationally equivalent to true demand. The joint distribution of the quantile process can differ from the true one. For example, the true demand may have $\mathrm{Var}(q(\tilde{x},\eta)|q(\bar{x},\eta)) > 0$ but $Q(\tilde{\eta}|x)$ will be one-to-one in $\tilde{\eta}$ for each x so $\mathrm{Var}(Q(\tilde{\eta}|\tilde{x})|Q(\tilde{\eta}|\bar{x})) = 0$.

For example, consider a true demand process that is linear in x with varying coefficients, where

$$q_0(x,\eta) = \eta_1 + \eta_2 p + \eta_3 y,$$

and η_3 varies across individuals. This demand process is a familiar specification. By Theorem 1, quantile demand will be observationally equivalent to this true demand. Thus, there is no way to distinguish nonparametrically this true, linear, varying coefficients process from quantile demand. Also, true average surplus will generally be different than average surplus for quantile demand. Intuitively, the true demand is linear in income y, but quantile demand will generally be nonlinear in y because of varying η_3. The nonlinearities in income of quantile demand lead to average surplus for quantile demand being different than average surplus for the true demand.

We prove nonidentification by showing numerically that the true average surplus is different than the average surplus for quantile demand in an example of a linear varying coefficients model.

THEOREM 2: *If* $q_0(x,\eta) = \eta_1 - p + \eta_2 y$, $\eta_1 \sim U(0,1)$, η_2 *is distributed independently of* η_1 *with two point support* $\{1/3, 2/3\}$, *and* $\Pr(\eta_2 = 1/3) = 1/2$, *the average equivalent variation* $\bar{S}$ *is not identified for* $p^0 = 0.1$, $p^1 = 0.2$, *and* $\bar{y} = 3/4$.

It can be seen from equation (3.1) that average surplus should be continuous in the demand function q and distribution G in an appropriate sense. Thus, the nonidentification result of Theorem 2 should hold for demand processes that are close to the example of Theorem 2. We also expect nonidentification of average surplus to be generic in the class of demand processes with varying income effects, though it is beyond the scope of this paper to prove this result.

5. INCOME EFFECT BOUNDS

Known bounds on income effects can be used to bound average surplus and deadweight loss using average demand. The idea is to extend Section 2, where constant income effects allow identification of average surplus from average demand, to identify bounds on surplus from average demand. To describe the result, for any constant B, let

$$\bar{S}_B = \int_0^1 \left[\bar{q}\big(p(t), \bar{y}\big)^T \frac{dp(t)}{dt}\right] e^{-Bt}\, dt \tag{5.1}$$

be the solution $\bar{s}_B(t)$ at $t = 0$ to the linear differential equation

$$\frac{d\bar{s}_B(t)}{dt} = -\bar{q}\big(p(t), \bar{y}\big)^T \frac{dp(t)}{dt} + B\bar{s}_B(t), \quad \bar{s}_B(1) = 0. \tag{5.2}$$

From Section 2, we see that $\bar{S}_B$ would be the average surplus if just the price of the first good were changing and the demand for the first good had a constant income effect $\partial q_1(p(t), y, \eta)/\partial y = B/\Delta p_1$.

THEOREM 3: *If* (i) $q(p(t), \bar{y} - s, \eta)^T dp(t)/dt \geq 0$ *for* $s \in [0, S(\eta)]$, (ii) *there are constants* $\underline{B}$ *and* $\bar{B}$ *such that* $\underline{B}s \leq [q(p(t), \bar{y}, \eta) - q(p(t), \bar{y} - s, \eta)]^T dp(t)/dt \leq \bar{B}s$; (iii) *all prices in* $p(t)$ *are bounded away from zero then*

$$\bar{S}_{\bar{B}} \leq \bar{S} \leq \bar{S}_{\underline{B}}, \quad \bar{S}_{\bar{B}} - \bar{q}(p^1, \bar{y})^T \Delta p \leq \bar{D} \leq \bar{S}_{\underline{B}} - \bar{q}(p^1, \bar{y})^T \Delta p.$$

Condition (i) is a restriction on the price path that is automatically satisfied when only the price of the first good is changing and $p_1^1 > p_1^0$. Also, the bounds in the conclusion are satisfied under weaker conditions than bounded income effects. To conserve space, here we give the more general result in the Supplemental Material.

The key ingredient for these average surplus bounds are bounds on the income effect $[\partial q(x, \eta)/\partial y]^T dp(t)/dt$. Economics can deliver such bounds. Consider again, and for the rest of this section, a price change in the first good, where $\underline{B}$ and $\bar{B}$ are bounds on $\Delta p_1 \partial q_1(x, \eta)/\partial y$ and $\Delta p_1 > 0$. If q_1 is a normal good, then the income effect is nonnegative, so we can take $\underline{B} = 0$. Then an upper bound for average equivalent variation and deadweight loss can be obtained from Marshallian surplus for average demand as

$$\bar{S} \leq \bar{S}_M = \int_0^1 \big[\bar{q}\big(p(t), \bar{y}\big)^T dp(t)/dt\big]\, dt, \quad \bar{D} \leq \bar{S}_M - \bar{q}\big(p^1, \bar{y}\big)^T \Delta p.$$

The upper bound on average deadweight loss could be useful for policy purposes, for example, to proceed with a tax if average public benefits (e.g., environmental benefits) exceed average deadweight loss and the appropriate separability conditions are satisfied.

Economics can also deliver upper bounds on income effects. If no more than a fraction π of additional income is spent on q_1, then

$$\partial q_1(x, \eta)/\partial y \leq \pi/p_1 \leq \pi/p_1^0,$$

so that

$$\bar{B} = \Delta p_1 \pi / p_1^0 = \pi\big(p_1^1/p_1^0 - 1\big)$$

is an upper bound on $[\partial q(x,\eta)/\partial y]^T\, dp(t)/dt$. For example, in the gasoline demand application below, we are quite certain that only a small fraction of any increase in income is spent on gasoline, making our choice of $\bar{B}$ very credible. The Slutzky condition also can limit the size of income effects relative to price effects. In the next section, we consider bounds based on the Slutzky condition.

The quantiles of the demand distribution are informative about income effects. Let $Q_1(\tau|x)$ denote the conditional quantile of the first good, where we continue to suppress dependence on q and G. By Hoderlein and Mammen (2007),

$$\frac{\partial Q_1(\tau|x)}{\partial y} = E\left[\frac{\partial q_1(x,\eta_i)}{\partial y}\middle| q_1(x,\eta_i) = Q_1(\tau|x)\right],$$

where η_i is a random variable with distribution G. Note that constancy of the income effect will also imply constancy of $\partial Q_1(\tau|x)/\partial y$ as τ varies. Thus, if $\partial Q_1(\tau|x)/\partial y$ varies with τ, the income effect for the first good is heterogeneous. Also, a necessary condition for $\underline{B}$ and $\bar{B}$ to bound $\Delta p_1 \partial q_1(x,\eta)/\partial y$ is $\underline{B} \leq \Delta p_1 \partial Q_1(\tau|x)/\partial y \leq \bar{B}$. This result can be used to guide the choice of bounds on income effects. For example, one might choose an upper bound that is much larger than derivatives of many quantiles, as we do in the gasoline application to follow. Note, though, that this approach does not serve to identify the bounds, because we cannot tell from the quantile derivative how the income effect varies over η with $q_1(x,\eta) = Q_1(\tau|x)$.

The conditional quantile is also informative about the surplus bounds. Let S^τ be the exact surplus obtained by treating $Q_1(\tau|x)$ as if it were a demand function, obtained as the solution $s^\tau(0)$ at $t=0$ to the differential equation

$$\frac{ds^\tau(t)}{dt} = -Q_1\big(\tau | p_1^0 + t\Delta p_1, \bar{p}_2, \bar{y} - s^\tau(t)\big)\Delta p_1, \quad s^\tau(1) = 0.$$

With two goods and scalar heterogeneity, the average surplus would be $\int_0^1 S^\tau\, d\tau$. It turns out that $\int_0^1 S^\tau\, d\tau$ is between the surplus bounds in general.

COROLLARY 4: *If the conditions of Theorem 1 are satisfied and only the first price is varying, then*

$$\bar{S}_{\bar{B}} \leq \int_0^1 S^\tau\, d\tau \leq \bar{S}_{\underline{B}}.$$

Surplus bounds are relatively insensitive to income effect bounds when a small proportion of income is spent on the good. This result is related to the Hotelling (1938) result that when expenditure is small, approximate consumer surplus is typically close to actual consumer surplus. Differentiate equation (5.1) with respect to B to obtain

$$\begin{aligned}\bar{y}^{-1}\frac{\partial \bar{S}_B}{\partial B} &= -\bar{y}^{-1}\int_0^1 \left[\bar{q}\big(p(t),\bar{y}\big)^T\,dp(t)/dt\right]te^{-Bt}\,dt \\ &= -\int_{p_1^0}^{p_1^1}\left[\bar{q}_1(p_1,\bar{p}_2,\bar{y})p_1/\bar{y}\right]\left(\frac{1-p_1^0/p_1}{\Delta p_1}\right) \\ &\quad\times \exp\left(-B\frac{p_1-p_1^0}{\Delta p_1}\right)dp_1.\end{aligned}$$

We see that the bounds are less sensitive to B when the income share is smaller.

6. GENERAL BOUNDS WITH TWO GOODS

It is possible to drop knowledge of income effects and obtain average surplus bounds based only on utility maximization. We do this by finding the supremum and infimum of average surplus over an approximation to the set of demand processes that are consistent with the distribution of demand. We focus here on the two good case. The approximation is based on a series expansion around the quantile demand where the coefficients of the series terms have a discrete distribution. We consider a demand process of the form

$$\tilde{q}(x,\eta) = Q(\tilde{\eta}|x)\exp\big(m(x)^T\eta\big),$$

where $m(x) = (m_1(x),\dots,m_J(x))^T$ are approximating functions, $\tilde{\eta} \sim U(0,1)$, and $\breve{\eta}$ has a discrete distribution with L points of support $\{\breve{\eta}^1,\dots,\breve{\eta}^L\}$. In this approximation, we draw the support points $\breve{\eta}^\ell$ at random, keeping only those where $\tilde{q}^\ell(x,\tilde{\eta}) = Q(\tilde{\eta}|x)\exp(m(x)^T\breve{\eta}^\ell)$ satisfies the Slutzky condition over a grid of values for $\tilde{\eta}$ and $x \in \chi$. Let $F(r|x) = Q^{-1}(r|x)$ be the CDF corresponding to the quantile $Q(\tau|x)$, $\varphi_\varkappa(\tilde{\eta}) \geq 0$ satisfy $\sum_{\varkappa=1}^{Y}\varphi_\varkappa(\tilde{\eta}) = 1$, $\rho_\ell^\varkappa \geq 0$ satisfy $\sum_{\ell=1}^{L}\rho_\ell^\varkappa = 1$, and suppose $\Pr(\breve{\eta} = \breve{\eta}^\ell|\tilde{\eta}) = \sum_{\varkappa=1}^{Y}\varphi_\varkappa(\tilde{\eta})\rho_\ell^\varkappa$. Integrating over $\tilde{\eta}$ gives

$$F(r|x,\tilde{q},\tilde{G}) = \sum_{\ell=1}^{L}\sum_{\varkappa=1}^{Y}\rho_\ell^\varkappa\Psi_\ell^\varkappa(r,x),$$

$$\Psi_\ell^\varkappa(r,x) = \int_0^{F(r\exp(-m(x)^T\breve{\eta}^\ell)|x)}\varphi_\varkappa(\tilde{\eta})\,d\tilde{\eta}.$$

The integration over $\tilde{\eta}$ here helps to smooth out the CDF to improve fit.

We allow the mixture probabilities $\rho_\ell^\varkappa$ to vary and look for the maximum and minimum average surplus subject to restrictions imposed by the data. Let

$\tilde{S}^{\ell}(\tilde{\eta})$ be the surplus for $\tilde{q}^{\ell}(x, \tilde{\eta})$ and $\bar{S}_{\ell}^{\varkappa} = \int_0^1 \varphi_{\varkappa}(\tilde{\eta})\tilde{S}^{\ell}(\tilde{\eta})\,d\tilde{\eta}$. We can get an approximate upper bound for surplus by solving the linear program

$$\max_{\rho_{\ell}^{\varkappa}} \sum_{\varkappa=1}^{Y} \sum_{\ell=1}^{L} \rho_{\ell}^{\varkappa} \bar{S}_{\ell}^{\varkappa} \quad \text{s.t.}$$

$$F(r_m|x_m) = \sum_{\ell=1}^{L} \sum_{\varkappa=1}^{Y} \rho_{\ell}^{\varkappa} \Psi_{\ell}^{\varkappa}(r_m, x_m), \quad (r_m, x_m) \in \Gamma,$$

$$\rho_{\ell}^{\varkappa} \geq 0, \quad \sum_{\ell=1}^{L} \rho_{\ell}^{\varkappa} = 1,$$

where Γ is a grid where the constraints are imposed. This is a linear program so computation is straightforward. However, as for other estimators of partially identified objects (e.g., Manski and Tamer (2002)), it may be important for consistent estimation to include some slackness in the constraints, by solving instead

$$\max_{\rho_{\ell}^{\varkappa}} \sum_{\varkappa=1}^{Y} \sum_{\ell=1}^{L} \rho_{\ell}^{\varkappa} \bar{S}_{\ell}^{\varkappa} \quad \text{s.t.}$$

$$\sum_{(r_m, x_m) \in \Gamma} \left[F(r_m|x_m) - \sum_{\ell=1}^{L} \sum_{\varkappa=1}^{Y} \rho_{\ell}^{\varkappa} \Psi_{\ell}^{\varkappa}(r_m, x_m) \right]^2 \leq \varepsilon,$$

$$\rho_{\ell}^{\varkappa} \geq 0, \quad \sum_{\ell=1}^{L} \rho_{\ell}^{\varkappa} = 1,$$

for some $\varepsilon > 0$. This quadratic program can be computed using standard software.

This approach provides approximate bounds for surplus for a series approximation to the set of all demand processes that are consistent with a quantile demand $Q(\tau|x)$. Approximation to the true bounds depends on large J, L, and Y. The choice of objects and the corresponding approximation and inference theory are beyond the scope of this paper. Note, though, that these bounds are of interest even for some fixed J. As long as $\breve{\eta}^{\ell} = 0$ for some ℓ, the average quantile surplus will be between the bounds, so that the bounds give a measure of how much surplus can vary away from the quantile surplus for other random utility specifications consistent with the data. Further, if they turn out to be wider than bounds based on knowing income effects, then we can be assured that using income effects produces narrower bounds, because increasing J will increase the width of the general bounds.

This series approximation approach provides a way of empirically implementing the RUM, that is, of finding identified sets for objects of interest under revealed stochastic preference conditions. This approach differs from Kitamura and Stoye (2012) where revealed stochastic preference inequalities are imposed. Here we impose the Slutzky conditions on a grid and then interpolate between points using a series approximation. This approach relies on and exploits smoothness in underlying demand functions.

7. EMPIRICAL APPLICATION

The previous results are based on the average and distribution of demand for fixed price and income. These objects are identified when prices and income in the data are independent of preferences, that is, when the data are (q_i, x_i) $(i = 1, \dots, n)$ with $q_i = q_0(x_i, \eta_i)$ and x_i and η_i are statistically independent. In that case,

$$E[q_i|x_i = x] = \bar{q}_0(x), \quad \Pr(q_i \le r|x_i = x) = F(r|x, q_0, G_0).$$

Here average demand is the conditional expectation of quantity given prices and income in the data, and similarly for the distribution of demand. The conditional expectation of quantity, and not some other function of quantity, such as the log, is special because it equals the average demand, which is used in bounds based on income effects. Average demand could also be recovered from the conditional expectation of the share of income spent on q.

The conditional expectation $E[q_i|x_i = x]$ could be estimated by nonparametric regression, as we do in the gasoline demand application below. Alternatively, if there are many goods, so that nonparametric estimation is affected by the curse of dimensionality, a semiparametric or parametric estimate of the conditional expectation of quantity could be used. Those estimators could have functional form misspecification, but are useful with high dimensional regressors.

Independence of η_i and x_i encompasses a statistical version of a fundamental hypothesis of consumer demand, that preferences do not vary with prices. It is also based on the individual being small relative to the market of observation, as would hold when different observations come from different markets. The independence of income from preferences has been a concern in some demand specifications where allowance is made for dynamic consumption, but is an important starting point and is commonly imposed in the gasoline demand application we consider.

Independence of η_i and x_i could be relaxed to allow for covariates. Consider an index specification where there are covariates w_i with possible value w and it is assumed that there is a vector of functions $v(w, \delta)$ that affect utility such that η_i and $(x_i^T, w_i^T)^T$ are independent. These covariates might be demographic variables that represent observed components of the utility. For example, one could use a single, linear index $v(w, \delta) = w_1 + w_2^T \delta$, with the usual scale and location normalization imposed. The demand function $q_0(x, v(w, \delta_0), \eta)$ would then depend on the index $v(w, \delta_0)$, as would the average demand

$$\bar{q}_0\big(x, v(w, \delta_0)\big) = \int q_0\big(x, v(w, \delta_0), \eta\big)G(d\eta)$$
$$= E\big[q_i|x_i = x, v(w_i, \delta_0) = v\big].$$

Here average demand is equal to a partial index regression of quantity q_i on x_i and $v(w_i, \delta)$. Similar approaches to conditioning on covariates are common in demand analysis.

Endogeneity can be accounted for if there is an estimable control variable ξ such that x_i and η_i are independent conditional on ξ_i and the conditional support of ξ_i given x_i equals the marginal support of ξ_i. In that case, it follows, as in Blundell and Powell (2003) and Imbens and Newey (2009), that

$$\int E[q_i|x_i = x, \xi_i = \xi]F_\xi(d\xi) = \bar{q}_0(x),$$
$$\int \Pr(q_i \le r|x_i = x, \xi_i = \xi)F_\xi(d\xi) = F(r|x, q_0, G_0),$$

where $F_\xi(\xi)$ is the CDF of ξ_i. Although conditions for existence of a control variable are quite strong (see Blundell and Matzkin (2014)), this approach does provide a way to allow for some forms of endogeneity.

8. ESTIMATION AND WELFARE ANALYSIS OF GASOLINE DEMAND

In this section, we estimate average consumer surplus and deadweight loss from changes in the gasoline tax in the United States while allowing for unrestricted multi-dimensional individual heterogeneity. We use data from the 2001 U.S. National Household Transportation Survey (NHTS). This survey is conducted every 5–8 years by the Federal Highway Administration. The survey is designed to be a nationally representative cross section which captures 24-hour travel behavior of randomly selected households. Data collected include detailed trip data and household characteristics such as income, age, and number of drivers. We restrict our estimation sample to households with either one or two gasoline-powered cars, vans, SUVs, and pickup trucks. We exclude Alaska and Hawaii. We use daily gasoline consumption, monthly state gasoline prices, and annual household income. The data we use consist of 8,908 observations. Summary statistics are given in Table I. Note that the mean price of gasoline was \$1.33 per gallon with the mean number of drivers in a household equal to 2.04.

TABLE I

SUMMARY STATISTICS

Variable	Mean	Median	Std. Dev.	Min	Max
Price ($)	1.33	1.32	0.08	1.14	1.46
Quantity (gallons)	4.90	2.65	7.53	0.01	195.52
Income (1,000$)	62.19	47.5	47.47	2.08	170.72
Number of drivers	2.04	2	0.78	1	7
Public transit availability	0.24	0	0.42	0	1
Observations	8,908				

To estimate average gasoline demand, we estimate up to a fourth degree polynomial with interaction and predetermined variables along with price and income for household i:

$$\widehat{\bar{q}(x, w)} = \sum_{j,k,\ell=1}^{4} \hat{\beta}_{j,k,\ell}(\ln p)^j(\ln y)^k\big(v(w, \hat{\delta})\big)^{\ell}. \tag{8.1}$$

We construct equation (8.1) taking the price of gasoline as predetermined assuming a world market for gasoline. We also allow for the gasoline price to be jointly endogenous using state tax rates as instruments and also distance of the state from the Gulf of Mexico, as in Blundell, Horowitz, and Parey (2012). Here we take a control function approach, where, in the first stage, we use the instruments z_i, along with household income, and the predetermined variables w_i. We then take the estimated residuals from this first stage $\hat{\xi}_i$ and use them as a control function in equation (8.1), constructing

$$E[\widehat{q_i|x, w}, \xi] = \sum_{j,k,\ell,m=1}^{4} \tilde{\beta}_{j,k,\ell,m}(\ln p)^j(\ln y)^k\big(w'\tilde{\delta}\big)^{\ell}(\hat{\xi})^m, \tag{8.2}$$

where $\tilde{\beta}_{j,k,\ell,m}$ are the coefficients from the regression of q_i on log price, income, the covariates index, and the first stage residual. The average demand is then estimated by averaging over the estimated residuals $\hat{\xi}_i$ holding p, y, and w fixed.

In Figure 1, we plot the OLS average demand estimate for monthly gasoline consumption. Note that it is generally downward sloping except at low prices. In Figure 2, we estimate the demand function using the control function approach and find it to be better behaved. In Table II, we consider the estimated price elasticities for OLS and IV. We see that the estimated price elasticity has the incorrect sign for the 75th quantile for three out of the four specifications, while the IV estimates all have the correct sign. However, the IV estimates are somewhat large except perhaps for the third and fourth order specification. In

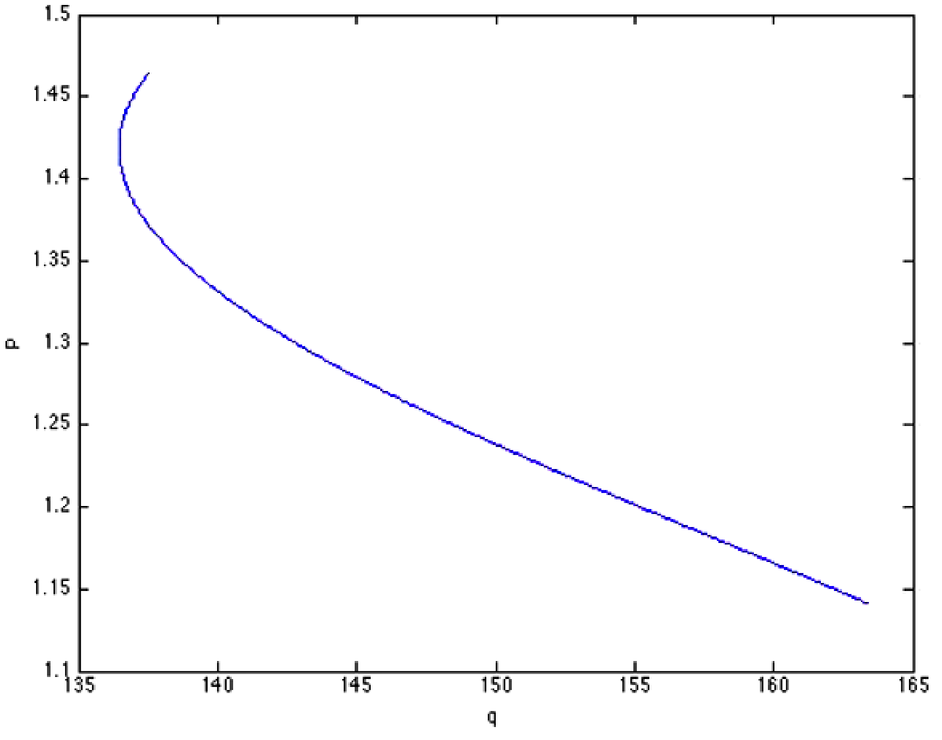

FIGURE 1.—Estimated demand: OLS. *Notes*: Demand estimated from 3rd order series regression evaluated at median income.

Table III, the estimated income elasticities for both OLS and IV are quite similar and also similar to previous estimates, for example, Hausman and Newey (1995).

To set bounds on income effects, we assume that gasoline is a normal good and so choose the lower bound $\underline{B}$ to be 0.0. To set the upper bound, we estimate a local linear quantile regression of log of gasoline demand on log price and log income and evaluate the income derivative of the gasoline quantile at median price and income. We find that this income effect is increasing in the quantile τ. We take the upper bound on the income effect to be 0.0197, which is 20 times the quantile derivative at $\tau = 0.9$. This income effect is very large, corresponding to more than two cents of every additional dollar of income being spent on gasoline. We are confident that very few would have such a large income effect for gasoline. Estimating linear, varying coefficients demand, we find a precisely estimated mean income effect of 0.000726. From the squared residual regression, the upper 95% confidence bound on the square root of the coefficient of y^2, which estimates the standard deviation of the income effect, is

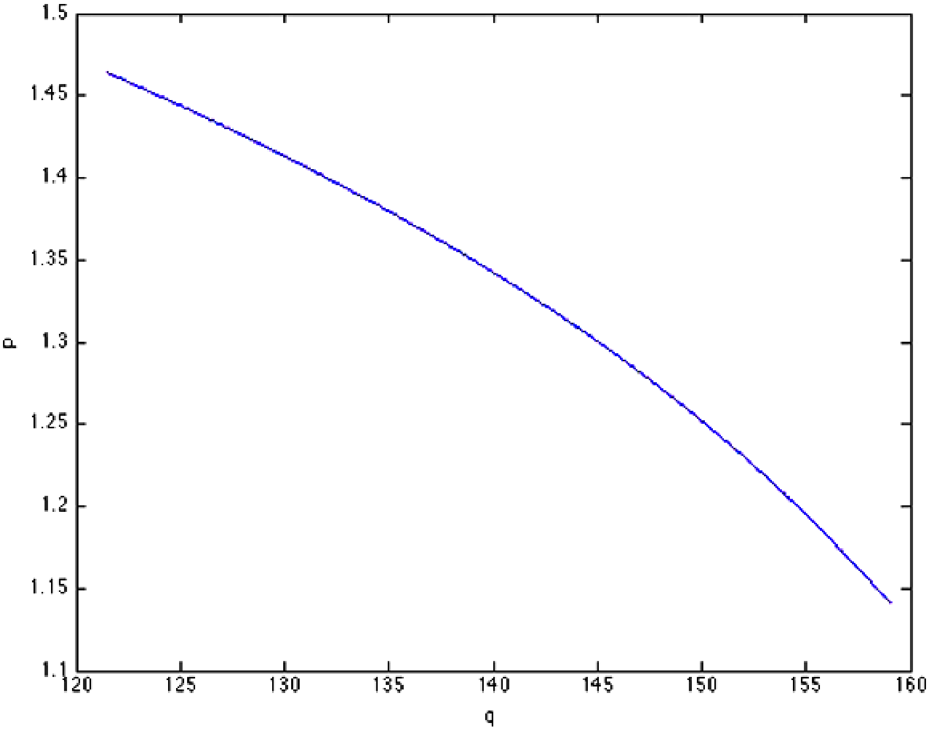

FIGURE 2.—Estimated demand: control function. *Notes*: Demand estimated from 3rd order power series control function regression evaluated at median income.

TABLE II
ESTIMATED PRICE ELASTICITIES

	OLS Estimates			Control Function Estimates		
Quantiles:	0.25	0.5	0.75	0.25	0.5	0.75
Order 1	−0.698	−0.656	−0.631	−1.111	−1.060	−1.043
	(0.254)	(0.244)	(0.243)	(0.282)	(0.280)	(0.291)
Order 2	−1.597	−0.798	0.069	−1.675	−1.350	−1.111
	(0.469)	(0.283)	(0.509)	(0.476)	(0.320)	(0.657)
Order 3	−1.214	−0.798	0.271	−1.037	−1.102	−0.872
	(0.753)	(0.570)	(0.721)	(0.860)	(0.670)	(0.952)
Order 4	−0.713	−0.583	0.140	−0.389	−0.588	−0.853
	(0.877)	(0.623)	(0.801)	(1.158)	(0.707)	(1.032)

TABLE III

ESTIMATED INCOME ELASTICITIES

	OLS Estimates			Control Function Estimates		
Quantiles:	0.25	0.5	0.75	0.25	0.5	0.75
Order 1	0.168 (0.022)	0.157 (0.019)	0.151 (0.017)	0.167 (0.022)	0.159 (0.019)	0.157 (0.018)
Order 2	0.221 (0.032)	0.244 (0.025)	0.261 (0.031)	0.210 (0.032)	0.233 (0.025)	0.262 (0.034)
Order 3	0.217 (0.057)	0.221 (0.04)	0.236 (0.039)	0.220 (0.059)	0.221 (0.041)	0.256 (0.043)
Order 4	0.266 (0.067)	0.305 (0.054)	0.275 (0.047)	0.267 (0.074)	0.294 (0.058)	0.277 (0.052)

0.00241. Here 0.0197 is well out in the distribution of income effects, implying the bound is approximately correct; see the Supplemental Material.

In Figure 3, we graph the bounds on the monthly average equivalent variation for a price increase from the stated price on the lower axis to \$1.40 per gallon. We use the estimates from the third order power series, with a control function, evaluated at median income. Note that the lower bound and upper bound estimates are almost the same and it is difficult to distinguish between them. This result follows from the small share of gasoline expenditure in overall household expenditure. The results demonstrate that although the welfare function is not point identified, in this type of situation the upper and lower bound estimates are very similar.

In Figure 4, we graph the bounds on deadweight loss for a price increase from the price on the lower axis to \$1.40. Again we use the third order power series control function estimates evaluated at the median income. Again, the lower and upper bound estimates are quite similar and difficult to distinguish except for very low gasoline prices. Since deadweight loss is a second order calculation compared to the first order calculation of equivalent variation (e.g., Hausman (1981)), the closeness of the bounding estimates allows for policy evaluation, even in the absence of point identification.

We now estimate confidence sets for our estimated bounds. We use the Chernozhukov, Hong, and Tamer (2007) confidence interval for the identified set with 0.95 coverage probability. Let the estimated set identification regions be given by $\hat{\Theta} = [\hat{\theta}_\ell, \hat{\theta}_u]$ and the joint estimated asymptotic variance matrix $\hat{\theta}_\ell$ and $\hat{\theta}_u$ be $\hat{\Sigma}$. It will follow from Hausman and Newey (1995) that the bounds are joint asymptotically normal. We form $\hat{\Sigma}$ by bootstrapping assuming the data vector is i.i.d. across individuals. The results are given in Table IV for the equivalent variation estimates with the estimated standard errors in parentheses and the 95% confidence intervals given in brackets. Concentrating on

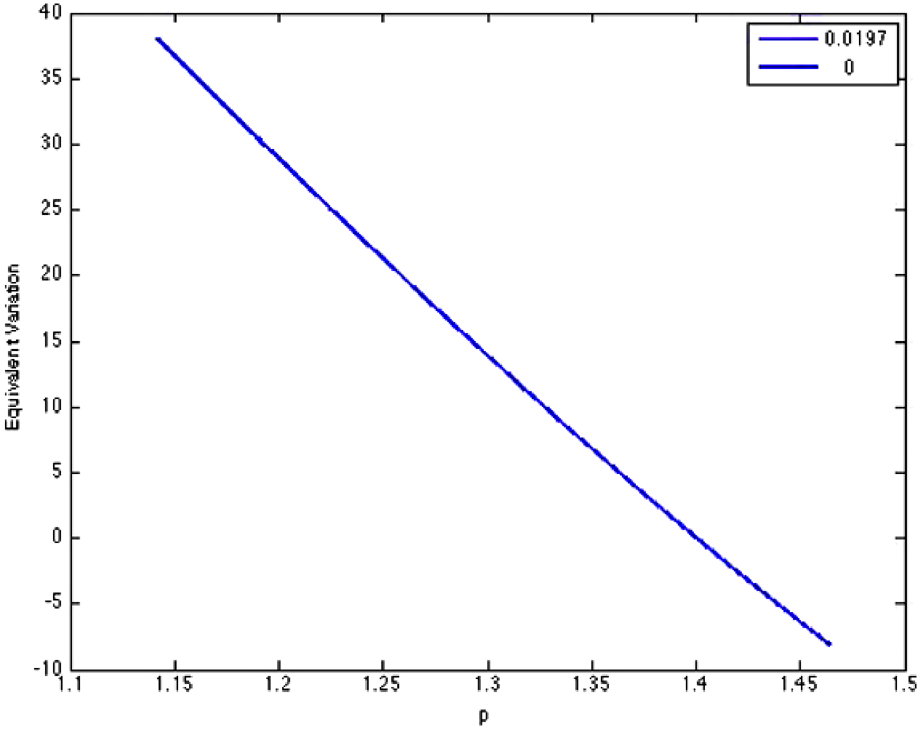

FIGURE 3.—Equivalent variation bounds. *Notes*: Graph shows change in equivalent variation for a price increase from p to \$1.40, evaluated at upper and lower bounds of income derivative and at median income and estimated from 3rd order power series control function estimated demand.

the third order estimates which we plotted before, we see that the estimated standard errors are quite small at both the lower bound and the upper bound and the 95th percentile confidence interval goes from \$13.72 to \$16.24, which is small enough for reliable policy analysis. In Table V, we give the standard errors and bounds and the DWL estimates. Here we find that the standard errors are reasonably small but the estimate confidence intervals are sufficiently large to impact the policy analysis.

We also estimated the general bounds described in Section 6. We used a third order power series in $\ln p$ and $\ln y$ to estimate the quantile of $\ln q$ and for $m_j(x)$, where J corresponded to a cubic in logs specification, analogous to the main empirical specification with income effect bounds given above. We estimated the conditional quantile at 99 values of τ and imposed the Slutzky condition on a grid. We set $\Upsilon = 4$, chose $\varphi_{\varkappa}(\tilde{\eta})$ to be linear b-splines on $[0, 1]$ with knots at 0, 1/3, 2/3, and 1, and drew $L = 1{,}000$ coefficients, with more details given in the Supplemental Material. We calculated the general

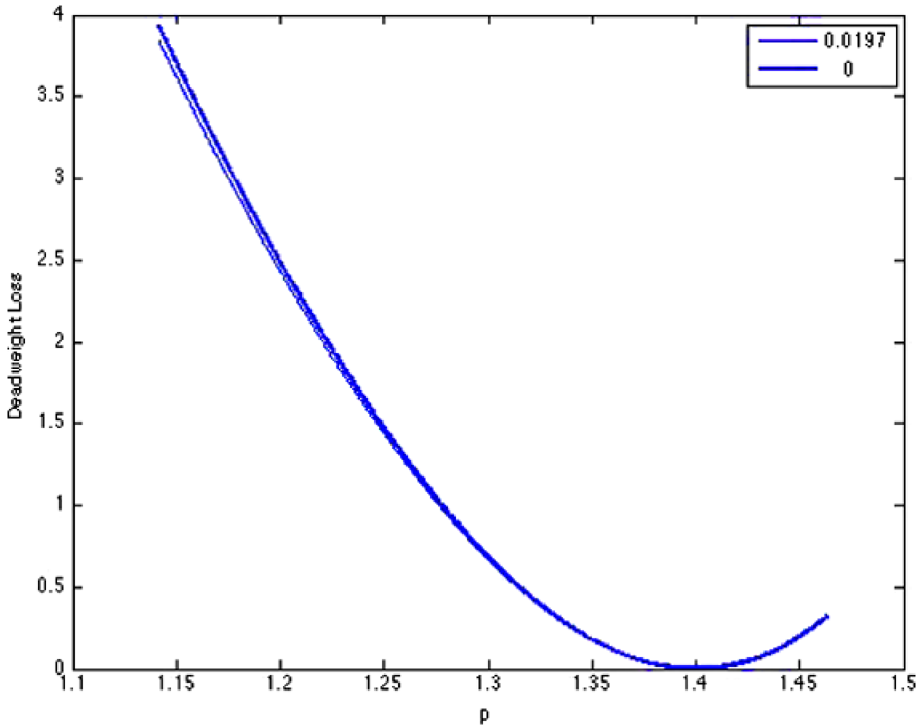

FIGURE 4.—Deadweight loss bounds. *Notes*: Graph shows change in deadweight loss for a price increase from p to \$1.40, evaluated at upper and lower bounds of income derivative and at median income and estimated from 3rd order power series control function estimated demand.

TABLE IV
BOUNDS ON EQUIVALENT VARIATION ESTIMATES

	From \$1.20 to 1.30		From \$1.20 to 1.40	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Order 1	16.777	16.794	32.281	32.343
	0.349	0.349	0.502	0.501
	[16.104, 17.468]		[31.355, 33.270]	
Order 2	15.147		28.829	28.884
	0.443	0.443	0.753	0.751
	[14.275, 16.005]		[27.405, 30.309]	
Order 3	14.972	14.987	28.845	28.900
	0.647	0.646	0.884	0.882
	[13.715, 16.244]		[27.163, 30.583]	
Order 4	14.625	14.639	28.546	28.601
	0.660	0.659	0.924	0.922
	[13.340, 15.925]		[26.788, 30.360]	

TABLE V

BOUNDS ON DEADWEIGHT LOSS ESTIMATES

	From $1.20 to 1.30		From $1.20 to 1.40	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Order 1	0.646	0.663	2.467	2.529
	0.175	0.175	0.669	0.668
	[0.319, 0.990]		[1.219, 3.776]	
Order 2	0.735		2.821	2.876
	0.233	0.233	0.728	0.727
	[0.277, 1.178]		[1.452, 4.245]	
Order 3	0.485	0.500	2.434	2.489
	0.393	0.393	1.04	1.039
	[−0.272, 1.257]		[0.447, 4.475]	
Order 4	0.321	0.335	1.827	1.882
	0.498	0.497	1.137	1.135
	[−0.641, 1.298]		[−0.343, 4.052]	

surplus bounds for a price change from 1.20 to 1.40 accounting for the distribution constraints at five quantile values for r, including the median, and replacing $F(r|x)$ in the constraints by a smoothed version of $\hat{Q}^{-1}(r|x)$. The results for three values of ε are reported in Table VI. We find that, for the smallest value of ε, the bounds are informative but substantially wider in percentage terms than those we obtained with bounds on income effects. Here the bounds based on income effects turn out to be more informative than general bounds.

We have used our bounds approach to estimate household gasoline demand functions allowing for unrestricted heterogeneity. While the welfare measures are not point identified, we find that the lower and upper bound estimates are close to each other and provide precise information about exact surplus with general heterogeneity.

TABLE VI

GENERAL AVERAGE SURPLUS BOUNDS

	General Bounds		
ε	Max CS	Min CS	Dif. CS
0.001	49.06	24.95	24.11
0.0001	32.97	27.08	5.89
0.00001	30.26	27.44	2.82

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Dept. of Economics, MIT, E52-518, 50 Memorial Drive, Cambridge, MA 02139, U.S.A.; jhausman@mit.edu

and

Dept. of Economics, MIT, E52-318A, 50 Memorial Drive, Cambridge, MA 02139, U.S.A.; wnewey@mit.edu.

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