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CONGESTION PRICING AND CAPACITY OF LARGE HUB AIRPORTS: A BOTTLENECK MODEL WITH STOCHASTIC QUEUES[1]

BY JOSEPH I. DANIEL[2]

This paper models and estimates congestion prices and capacity for large hub airports. The model includes stochastic queues, time-varying traffic rates, and endogenous, intertemporal adjustment of traffic in response to queuing delay and fees. Relative costs of queuing and schedule delays are estimated using data from Minneapolis–St. Paul. Simulations calculate equilibrium traffic patterns, queuing delays, schedule delays, congestion fees, airport revenues, airport capacity, and efficiency gains. The paper also investigates whether a dominant airline internalizes delays its aircraft impose. It tests game-theoretic specifications with atomistic, Nash-dominant, Stackelberg-dominant, and collusive-airline traffic.

KEYWORDS: Congestion pricing, bottleneck model, queuing theory, peak-load pricing, landing fees, airport capacity.

1. INTRODUCTION

AIRPORT CONGESTION COSTS U.S. AIRLINES and their passengers about five billion dollars annually in increased operating costs and travel time, according to FAA estimates. The twenty-one most congested hub airports in the U.S. each experiences over twenty thousand plane-hours of flight delay annually. Air traffic may double by the next century (Transportation Research Board (TRB) (1990)). Estimates of European airport and airspace congestion costs are about \$6.5 billion annually (Reed (1992)). Since 1988, about twenty per cent of all intra-European flights have experienced over fifteen minutes of flight delay each (Mecham (1992)). The FAA estimates that meeting capacity requirements by building new airports in metropolitan areas with the highest traffic volumes will cost thirty to sixty billion dollars over the next twenty years (TRB (1990)).

Current airport landing-fee structures are largely to blame for this state of affairs. Nearly all major airports in the U.S. assess landing fees proportional to aircraft weight, independent of airport congestion levels. Social costs of landings or takeoffs consist primarily of additional delay costs imposed on other aircraft and travelers using the airport at the same time. Social costs are essentially independent of aircraft weight. Weight-based fees do not provide appropriate

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[2] This paper is an extension of my doctoral thesis (Daniel (1992)). I am grateful to my thesis advisor, Herbert Mohring of the University of Minnesota, for his guidance and advice. My committee, Edward Green, Christina Kelton, David Kelton, and Michael Taaffe, provided many helpful suggestions. My colleagues at the University of Delaware, James Mulligan, Evangelos Falaris, Sridhar Iyer, provided many suggestions and encouragement. Steve Thompson provided outstanding assistance in procuring the traffic data and performing data entry. I also wish to thank a co-editor and two anonymous referees whose extensive comments and suggestions greatly improved this paper.

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incentives for airlines to choose social-cost-minimizing arrival and departure schedules. Such fees result in too many arrivals and departures close to the most popular travel times, whereas optimal scheduling would spread traffic out more. By inducing peak spreading, congestion pricing can improve efficiency of capacity utilization, thereby reducing future capacity requirements.

This paper models peak-load congestion pricing and provides an empirical application of the model to airport runway pricing. The paper combines stochastic queuing theory with a bottleneck model of traffic patterns adjusting intertemporally in response to queuing delay and congestion fees. This provides a technique for calculating equilibrium congestion prices. Previous models of congested transportation systems do not simultaneously include endogenously determined traffic rates, intertemporal traffic adjustments, and stochastic queuing systems. Equilibrium traffic patterns and congestion fees in this model significantly differ from those in models with exogenous traffic patterns and/or nonstochastic queues. The model is particularly applicable to hub airports that experience rapid fluctuation and severe peaking of traffic rates and queue lengths. Such airports can benefit from pricing mechanisms that spread traffic peaks out over time. While peak spreading reduces congestion delay, it also increases layover delay—an important aspect of hub airport pricing that has not received attention in previous studies. The bottleneck model captures the tradeoff between congestion and schedule delay.

This paper also investigates whether an airline with a large share of traffic at a hub airport internalizes congestion delays its own aircraft impose on one another. At many hub airports, a single airline accounts for over fifty per cent of the traffic. One might expect such airlines to internalize delays. Previous airport-congestion-pricing studies have ignored this issue. The paper presents specifications with purely atomistic airlines (which internalize none of the delays), a Nash-dominant airline (which internalizes delays imposed on itself) with an atomistic fringe, a Stackelberg-dominant airline (which may partially internalize delays imposed on itself) with an atomistic fringe, and joint-cost-minimizing (collusive) airlines (which internalize all delays). The paper provides formal statistical tests of these alternative specifications and compares simulated and actual traffic patterns.

Traffic data from the Minneapolis–St. Paul (MSP) airport give scheduled and actual arrival and departure times of every flight during the first week of May, 1990. Previous airport-pricing studies use traffic data aggregated by hour, which ignore the rapid fluctuations in traffic occurring at large hub airports. This study simulates changes in operating times of each aircraft in response to queuing delay and congestion pricing. While there are no reliable data on actual queue lengths, the queuing model infers expected queue lengths given airport capacity and actual traffic patterns. Using these queue lengths, it is possible to estimate relative costs of queuing and schedule delay. These estimates provide parameters for the simulation model that generates equilibrium traffic patterns, queuing delays, layover costs, congestion fees, airport revenues, capacity implications, and efficiency gains from reduced congestion.

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Simulated traffic patterns for the no-congestion-fee case are quite similar to actual traffic patterns at MSP. With congestion fees, the model predicts significant intertemporal traffic adjustments. Simulations indicate congestion pricing would reduce net social costs of landings and takeoffs at MSP by about 24%, holding capacity constant. Current capacity could accommodate 30% more traffic under congestion pricing with no increase in average social costs of landings and takeoffs. Moreover, while existing capacity could not accommodate 50% more traffic under the current pricing system, it could under congestion pricing with a modest (29%) increase in social cost of landings and takeoffs. If traffic were to double, building one additional runway and imposing congestion pricing would reduce social costs of landings and takeoffs below their current level and result in nearly equivalent costs to building two runways with the current pricing system.

2. BACKGROUND

Previous models of congested transportation systems fall into three categories: econometric models, bottleneck models, and queuing theoretic models. Econometric models estimate time-varying demand and delay functions and calculate equilibrium congestion fees. They use nonstructural specifications of delay functions and ignore intertemporal traffic adjustment in response to congestion fees (see e.g., Carlin and Park (1970), Morrison (1983), and Morrison and Winston (1989)). Bottleneck models generate equilibrium fees with intertemporal traffic adjustment, but generally employ simple deterministic queuing processes (see, e.g., Vickrey (1969), Arnott, de Palma, and Lindsey (1990b, 1993), and Henderson (1985)). Previous studies have not implemented bottleneck models empirically to estimate changes in traffic patterns resulting from congestion pricing.[3] Queuing-theoretic models capture the effects of stochastic arrivals on the evolution of queues, but assume exogenous arrival rates and do not calculate equilibrium congestion fees (see, e.g., Koopman (1972)). None of these models simultaneously include endogenously determined arrival rates, intertemporal traffic adjustment, and stochastic traffic and queuing processes.

Daniel (1991) applies a bottleneck model to airport congestion pricing. Airport traffic data is aggregated by ten minute intervals to develop a discrete-time bottleneck model with deterministic queues. Current queue length is a regression function of the previous period's queue length and arrival rate. The deterministic queuing system does not capture the stochastic nature of airport queues, and is inadequate for modeling optimal capacity because the regression coefficients in the queuing model are sensitive to changes in capacity.[4]

[3] Small (1982) estimated a piecewise linear schedule delay cost function for morning rush-hour commuters. He did not specify an explicit bottleneck model or cite Vickrey's (1969) paper.

[4] Several authors have extended the bottleneck model: Small (1983), Henderson (1985), Smith (1984), Daganzo (1985), Ben-Akiva, et al. (1984), and Arnott, et al. (1990a, 1993). While the bottleneck literature offers many interesting theoretical results and numerical examples, I am not aware of any previous empirical applications.

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Most queuing-theoretic models of transportation systems assume the queuing system has a constant arrival rate. Such systems have steady-state solutions and do not adequately model airport queues resulting from rapid fluctuation of traffic rates. Koopman (1972) applies a stochastic queuing model with time-dependent arrival rates and nonstationary queues to airport traffic. Koopman and the subsequent stochastic queuing literature do not develop pricing implications of time-dependent stochastic queuing systems, or model endogenous adjustment of traffic rates.

This paper develops a stochastic queuing model like that of Koopman embedded in a bottleneck model similar to that of Vickrey, Arnott, et al., and Henderson. The queuing model captures the stochastic nature of arrivals, departures, and queues, an essential element of airport capacity problems missing from bottleneck models. The bottleneck model provides endogenous adjustment of time-dependent traffic rates. This model is similar to Henderson's model in that congestion pricing lengthens peak periods and does not eliminate all congestion, whereas in Vickrey and Arnott, et al., the bottleneck capacity is the sole determinant of the duration of the peak period and the fee structure completely eliminates congestion. This model is more applicable to airports because it uses stochastic queuing rather than flow congestion. This paper significantly extends the work of Daniel (1991) by developing a bottleneck model with stochastic traffic and queues instead of a discrete-time deterministic model.

3. A BOTTLENECK MODEL OF EFFICIENT AIRPORT PRICING

A Stylized Model of Hub Airport Traffic

Hub-and-spoke air service networks exploit economies of scale in aircraft operation. Airline costs of providing seats on aircraft decrease as aircraft size increases. Larger aircraft, however, require longer headways between flights to maintain equivalent load factors (assuming constant demand density). Less frequent service increases passenger schedule-delay costs. Schedule delay refers to the time between the most-preferred travel time of a passenger and the closest available flight. When airlines choose aircraft size and service frequency, they trade off aircraft operating costs against passenger schedule-delay costs.

Hub-and-spoke networks enable airlines to simultaneously reduce their aircraft-operating cost and passenger schedule-delay costs by achieving higher load factors on larger aircraft with greater service frequency. Networks combine passengers with the same origination but different destinations on the same flight to the hub. They combine passengers with different originations but the same destination on flights from the hub. Each spoke of the network carries many more passengers to and from the hub than a direct route between individual city pairs would. Consequently, the network can provide more frequent service in larger aircraft at lower cost per passenger. Additional circuity of travel, inconvenience of making connections, and introduction of layover time at the hub are the costs of a hub system.

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To minimize costs, hub-and-spoke networks schedule arrivals and departures at hubs in "banks" of flights. Arrival banks consist of incoming flights from many spoke cities landing at approximately the same time. Departure banks consist of outgoing flights to spoke cities that depart at approximately the same time. At the hub, passengers change aircraft during interchange periods. Since passengers may combine virtually any origination and destination, it is necessary to schedule all arriving flights before any departing flights. The interchange period must be sufficiently long to permit exchanges of passengers between any aircraft in subsequent arrival and departure banks. Because all aircraft in an arrival or departure bank have similar mixes of connecting passengers, they all have the same preferred time of operation.

Layover costs result from the time aircraft spend at hubs after exiting arrival queues and before entering departure queues. Earlier arrivals and later departures experience greater layover costs. Interchange-encroachment costs result from the risk of passengers missing connecting flights due to inadequate layover times (or the costs of delaying flights to accommodate such passengers). Later arrivals and earlier departures experience greater interchange-encroachment costs. The sums of layover and interchange-encroachment costs attain a minimum in period τ^A for arrival banks and period τ^D for departure banks. By assumption, unit costs of layover and interchange-encroachment times are different, and the cost of time spent in each activity is proportional to the length of a flight's deviation from τ^A and τ^D. The period between τ^A and τ^D is more than adequate to accommodate transfers between any scheduled arrival and departure. If there were no capacity restriction at the hub airport, all aircraft in a given bank would seek to operate at precisely the most-preferred arrival and departure times, τ^A and τ^D respectively.

Limited airport capacity prevents all aircraft in a bank from landing or taking off at precisely the most-preferred times. Aircraft experience queuing delay while waiting for a turn to land or waiting for clearance to takeoff. In a bottleneck equilibrium, traffic rates and queue lengths adjust endogenously so that identical aircraft have the same *total* expected cost of queuing, layover, and interchange encroachment regardless of their scheduled arrival or departure times. Some aircraft arrive early (or depart late) and have long layovers, short waits in the queue, and little chance of interchange encroachment. Some aircraft arrive near τ^A (or depart near τ^D) and have short layovers, long waits in the queue, and some chance of interchange encroachment. Some aircraft arrive after τ^A (or depart before τ^D) and have very short layovers, short waits in the queue, and relatively large interchange-encroachment costs. If identical aircraft did not have the same total expected cost, high-cost aircraft would move to the lower-cost period, reducing queues in the high-cost period and increasing queues in the low-cost period. Equilibrium occurs when total expected costs are equal and there is no incentive to change scheduled times of operation.

Because networks need to coordinate arrivals and departures, hub airports experience peak demand during flight banks and slack demand between banks. Figure 1 shows actual traffic patterns at MSP. Each bar in the graph shows the

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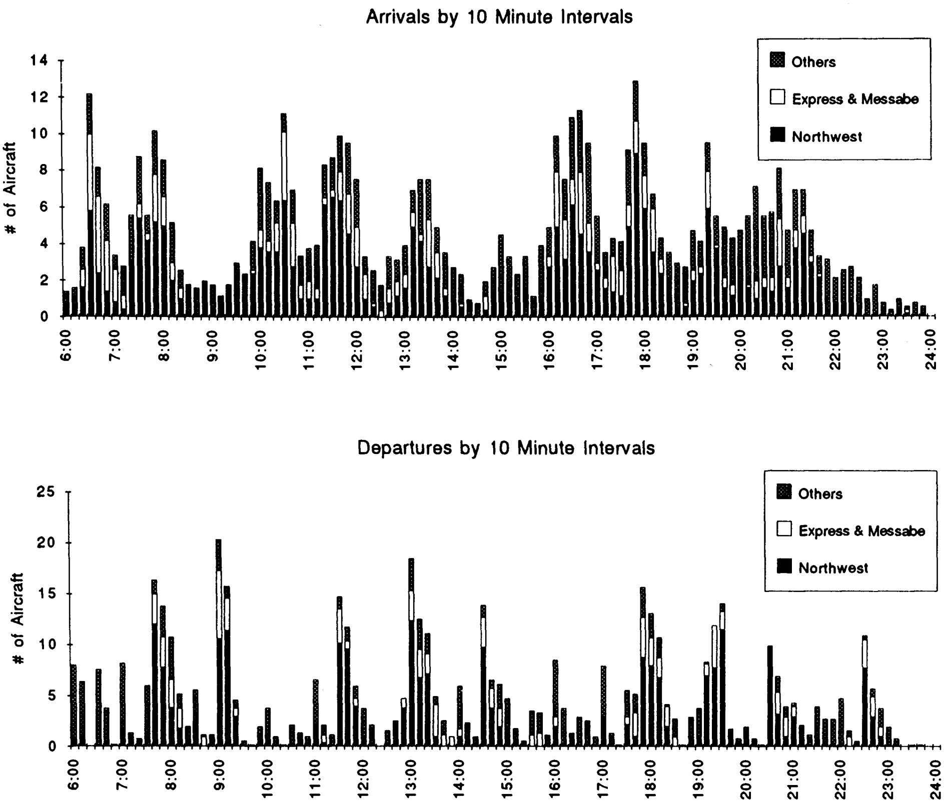

FIGURE 1.—Traffic by 10-minute intervals.

average number of arrivals or departures for ten minute intervals between 6:00 AM and 12:00 PM during the first week of May 1990. Shading of the bars indicates the number of aircraft operated by Northwest, Express and Messabe (Northwest's code-sharing affiliates), and other airlines or general aviation. The graph shows ten arrival and ten departure banks during the day spaced about 90–120 minutes apart. Inactive periods of about thirty minutes separate each arrival bank from the subsequent departure bank. Between banks, traffic rates typically drop to one or two aircraft per ten minutes. Arrival rates peak less severely than departure rates because arrival queues are more costly than departure queues. Table I summarizes bank schedules and numbers of aircraft operated by each type of airline. The number of aircraft in each bank varies because of fluctuations in demand during the day.

The stochastic bottleneck model developed below uses congestion pricing to attain optimal timing of traffic within banks. Without congestion fees, traffic rates are too high during bank peaks—resulting in unnecessarily long queues. Time-dependent congestion fees spread traffic more evenly over time and

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TABLE I

SCHEDULE OF ARRIVAL AND DEPARTURE BANKS

Bank Number	Arrivals			Number of Aircraft			
	Begin	Preferred	End	Northwest	Code Affiliates	Others	Total
1	6:20	6:56	7:10	14	15	10	39
2	7:20	8:13	8:20	28	8	23	59
3	9:50	10:48	11:00	29	12	20	61
4	11:20	12:07	12:40	30	11	21	62
5	12:55	13:41	14:10	20	8	17	45
6	14:30	15:11	15:40	5	1	18	24
7	16:00	17:14	17:30	33	16	24	73
8	17:40	18:22	18:45	31	11	16	58
9	19:00	19:34	19:55	17	4	14	35
10	20:15	21:27	22:00	30	10	47	87
Bank Number	**Departures**			**Number of Aircraft**			
	Begin	Preferred	End	Northwest	Code Affiliates	Others	Total
1	7:30	8:06	8:45	28	12	18	58
2	8:55	9:20	10:00	30	12	22	64
3	11:20	11:50	12:15	28	6	12	46
4	12:50	13:28	14:15	32	15	17	64
5	14:25	14:42	15:05	19	9	15	43
6	15:55	16:11	16:20	8	1	27	36
7	17:40	18:20	19:00	29	16	15	60
8	19:10	19:37	20:00	30	7	7	44
9	20:30	20:47	21:10	18	8	18	44
10	22:30	22:42	23:00	12	6	8	26

significantly reduce queue lengths. Some demand peaking still occurs, because increased layover times impose limits on peak spreading.

Figure 1 and Table I indicate Northwest operates about half of the traffic at MSP and its code-sharing affiliates operate another quarter. One might expect dominant airlines to fully internalize delays of their own aircraft even without congestion pricing. Nash-dominant airlines take other airline schedules as given and choose their aircraft schedules to minimize the sum of all their aircraft operating costs. As Nash-dominant airlines spread their traffic out to reduce delays, the atomistic fringe moves aircraft to the center of the bank. Both dominant and fringe airlines experience reduced queuing times. The fringe experiences greater reductions in costs because it has shorter average layover and interchange-encroachment times. By failing to account for the atomistic fringe response to changes in their schedules, Nash-dominant airlines may spread out their traffic too much and experience unnecessarily high costs.

Stackelberg-dominant airlines recognize that the atomistic fringe will respond to changes in their schedules. Atomistic airlines choose their schedules to minimize their own costs given dominant airline schedules. Stackelberg-dominant airlines choose their schedules to minimize their cost subject to the fringe response. Dominant airlines can schedule their aircraft among the fringe aircraft (mid-peak) or before or after the fringe aircraft (edge-peak). Stackel-

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berg-dominant airlines realize that they cannot reduce their cost by rearranging their traffic within the mid-peak. The fringe would offset any such rearrangement by reestablishing the initial joint arrival pattern, thereby equilibrating costs across all mid-peak aircraft. Therefore, Stackelberg-dominant airlines will not internalize any mid-peak delays. Stackelberg-dominant firms may reduce their costs, however, by shifting some of their aircraft to the edge-peak. Edge-peak aircraft experience greater layover or encroachment delay, but lower congestion costs. Dominant airlines will internalize costs that their edge-peak aircraft impose on one another, and a fraction of the cost they impose on *all* the mid-peak aircraft equal to their share of mid-peak traffic. The more dominant aircraft are in the edge-peak, the shorter the mid-peak length and the lower mid-peak costs.

If the number of fringe aircraft is fixed, the Stackelberg-dominant equilibrium is the same as the Nash-dominant equilibrium. Whenever dominant airlines have aircraft in the mid-peak, they can switch places with atomistic aircraft at the border between the mid- and edge-peak. Initially, the border aircraft has the same cost as all mid-peak aircraft, but the number of mid-peak aircraft decreases by one, the mid-peak period shrinks slightly, and remaining mid-peak aircraft attain slightly lower cost. This result is sensitive to new entry by atomistic aircraft. If lower mid-peak costs draw more atomistic aircraft into the bank, then the cost savings of dominant firms are reduced or eliminated. If fringe supply is sufficiently elastic, the Stackelberg equilibrium is the same as the atomistic equilibrium. Stackelberg-dominant airlines behave atomistically to keep the fringe aircraft costs high enough to discourage new entry.

It is also conceivable that all airlines achieve the joint-cost-minimizing traffic pattern even without congestion pricing. This outcome could result from a monopolist or social planner setting airline schedules or from a repeated game in which collusive airlines tacitly agree to abide by social-cost-minimizing schedules so long as all other airlines do. If one airline reneges, then all airlines revert to the atomistic equilibrium. While airlines would be better off in the current period by moving their aircraft to cheaper periods, they would not do so because of the penalty they would suffer in subsequent periods. Traffic patterns resulting from joint cost minimization are the same as those in atomistic equilibria with congestion fees. These equilibria are difficult to administer without fees, because aircraft have different costs. Airlines would need some mechanism to share high- and low-cost schedule times.

The Formal Model

Assume there are two independent $M(t)/D/S/K$ queuing systems at the airport, one for landings and one for takeoffs. In queuing-theoretic notation, $M(t)$ indicates that the arrival distribution is Poisson (Markovian) with time-dependent rates, D indicates the service-time distribution is deterministic, S indicates the number of servers (runways), and K is the maximum length of the queue. A time index, t, divides time into periods of length μ, the deterministic

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service time. The number of arrivals (or departures) in each service period is approximately Poisson-distributed with time-varying parameter, λ_t. A state vector, $\boldsymbol{q}_t$, represents the state of the queuing system at each period t. It consists of a column of elements, $q_{t0}, q_{t1}, q_{t2}, \ldots,$ and q_{tK}, which give the probability that the queue is of length $0, 1, 2, \ldots,$ and K at the beginning of period t. The state vector evolves according to the transition rule $\boldsymbol{q}_{t+1} = \boldsymbol{Q}_t \boldsymbol{q}_t$, where $\boldsymbol{Q}_t$ is a state-transition matrix determined by the $M(t)/D/S/K$ queuing model and arrival rates, λ_t. Appendix A derives $\boldsymbol{Q}_t$.

The model assumes Poisson-distributed arrival or departure rates, λ_t. These rates vary over time depending on random deviations of actual operation times from scheduled operation times of all aircraft. Assuming Poisson-distributed arrivals is standard for stochastic queuing models because it greatly simplifies the queuing system. Arrival or departure times refer to periods in which aircraft join landing or departure queues, not periods in which they actually land or take off. Let $\eta = \{1, 2, 3, \ldots, N\}$ denote the set of aircraft participating in a given bank. Let s_n denote the scheduled time at which aircraft n joins the landing or takeoff queue. Let $\boldsymbol{s} = (s_1, s_2,, \ldots, s_n)$ be the vector of scheduled arrival or departure times for all aircraft in a bank. Each aircraft is subject to an identical independently distributed random shock to its actual time of operation. Let $p_t^{s_n}$ be the probability that aircraft n with scheduled arrival time s_n actually arrives during period t. As shown in Section 5, airport traffic data determine the distribution of actual arrival or departure times about the scheduled time of operation. The expected number of arrivals during period t is $\lambda_t = \sum_{n \in \eta} p_t^{s_n}$. Section 5 also presents a test of the assumption that these random deviations from scheduled arrival times result in (approximately) Poisson-distributed arrivals with time-varying parameters λ_t. It compares the distribution of actual interarrival times with the expected distribution based on the λ_t's, and the distribution of simulated interarrival times with expected interarrival times based on the λ_t's. A negative exponential distribution closely approximates the actual and simulated interarrival times, indicating that the Poisson approximation is acceptable.

Service times are deterministic and occur at equally spaced intervals. This assumption simplifies calculation of layover and interchange-encroachment times, and it is in accord with actual airport practice during busy periods. There are constant streams of aircraft in the queuing system and aircraft separation rules dictate that operations take about the same time. Landings and takeoffs occur at regular intervals during such periods. When queues are empty, however, the assumption implies that aircraft must wait some fraction of the service interval before admission to service. Since equilibrium arrival rates are such that the queuing system stays busy throughout most of the banks, this simplification introduces small errors at the beginning and ending of banks. Koopman (1972) found that for busy queuing systems with time-varying arrivals, expected queue lengths are insensitive to whether service times are deterministic or stochastic.

The model uses a multiple-server queuing system in accord with the operation of multiple runway airports. It also permits discrete changes in capacity result-

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ing from adding runways. Nondiscrete changes in capacity occur when the length of the service interval changes, as might result from improved air-traffic-control procedures.

The assumption of finite queuing capacity, K, is flexible. Airports have limited airspace for stacking aircraft awaiting landing. Exceeding this limit results in diverting additional aircraft to another airport. The model can include such events by imposing a cost penalty for each diverted aircraft. In the present model, K is sufficiently large for both arrival and departure queues so that the probability of exceeding capacity is very small, so the model approximates an $M(t)/D/S/\infty$ queuing system. This is consistent with operations at MSP that rarely require diversion of aircraft due to capacity limits.

The model assumes that landing and takeoff queues operate independently. Some previous airport congestion models make the polar-opposite assumption of a single queuing system. In reality, MSP has separate queues for landings and takeoffs, but there is some interaction between queuing systems. Increasing the landing rate above nine aircraft per ten minutes requires substantial reduction in the number of departures. The model ignores this interaction to avoid complicating the queuing system. Since adjacent departure and arrival banks typically overlap, the airport usually operates under balanced traffic conditions with a maximum of about nine operations per ten minutes.

The model accounts for the effects of overlapping traffic and residual queues from one arrival or departure bank to the next. It calculates expected traffic rates (λ_t's) in each service interval using data from all banks simultaneously. The state of the queuing system at the beginning of each bank depends on the traffic rates and queues from the previous bank. This is important because congestion pricing and higher traffic levels cause traffic in a bank to spread out —resulting in more overlapping traffic and longer residual queues. The simulation model takes these effects into account in the pricing and capacity simulations.

Given the law of evolution for the queuing system, it is possible to specify expected queuing, layover, and interchange-encroachment costs for each period. As above, the most-preferred time for landing is τ^A (for arrival banks) and for takeoff is τ^D (for departure banks). Suppose an arriving aircraft joins the landing queue at time t and the length of the queue is k. Let $l(k)$ denote the waiting time required by a queue of length k. Then the aircraft will complete service at time $t + l(k)$. Completion of service before time τ^A costs an arriving aircraft and its passengers $\$c_b^A$ per period, and after time τ^A costs $\$c_a^A$ per period. Time spent in landing queues costs $\$c_q^A$ per period. If an arriving aircraft completes service before τ^A (i.e., $t + l(k) \leqslant \tau^A$), then its layover time is $[\tau^A - t - l(k)]$. If it completes service after τ^A, then its interchange-encroachment time is $[t + l(k) - \tau^A]$. To calculate expected layover and interchange-encroachment costs at time t simply weight $[\tau^A - t - l(k)]$ and $[t + l(k) - \tau^A]$ by the probability, $q_{tk}(s)$, that the queue is of length k at time t, sum over the range of k for which the aircraft is early or late, and multiply by the appropriate time cost. Expected queuing cost at period t is the waiting time in the queue,

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$l(k)$, weighted by its probability, $q_{tk}(s)$, summed over all values of k. The expected total arrival cost for aircraft that join the landing queue during period t is:

$$(1.1)\qquad C_t^A(s) = c_q^A \sum_{0 \leqslant l(k) \leqslant K} q_{tk}(s) l(k) + c_b^A \sum_{0 \leqslant l(k) \leqslant \tau - t} q_{tk}(s)\left[\tau^A - t - l(k)\right]$$
$$+ c_a^A \sum_{\tau - t < l(k) \leqslant K} q_{tk}(s)\left[t + l(k) - \tau^A\right].$$

The terms on the right hand side of (1.1) are expected queuing-delay cost, expected layover-delay cost, and expected interchange-encroachment cost.

For departing aircraft, expected costs differ because layover time ends (or interchange-encroachment time begins) when they join the departure queue, not when they complete service. Therefore, aircraft joining the departure queue before τ^D experience interchange-encroachment times of $[\tau^D - t]$ with unit cost of $\$c_b^D$ per period. If they join the departure queue after τ^D, then their layover time is $[t - \tau^D]$ with unit cost of $\$c_a^D$ per period. Time spent in takeoff queues costs $\$c_q^D$ per period. The expected total departure cost for aircraft that join the departure queue during period t is:

$$(1.2)\qquad C_t^D(s) = c_q^D \sum_{0 \leqslant k \leqslant K} q_{tk}(s) l(k) + c_b^D[\tau^D - t] \quad \text{if} \quad t \leqslant \tau^D, \quad \text{and}$$
$$C_t^D(s) = c_q^D \sum_{0 \leqslant k \leqslant K} q_{tk}(s) l(k) + c_a^D[t - \tau^D] \quad \text{if} \quad t > \tau^D.$$

The terms on the right-hand side of (1.2) are expected queuing-delay cost plus expected interchange-encroachment cost (top) and expected layover-delay cost (bottom).

These cost functions can be expressed as inner products of a vector of cost parameters, $\boldsymbol{c}_t$, and the vector $\boldsymbol{q}_t$. For arrivals, define the vectors $\boldsymbol{a}_t = (a_{t1}, a_{t2}, \ldots, a_{tK}), \boldsymbol{b}_t = (b_{t1}, b_{t2}, \ldots, b_{tK})$, and $\boldsymbol{c}_t^A = (c_{t1}^A, c_{t2}^A, \ldots, c_{tK}^A)$, such that:

$$(2.1)\qquad a_{tk} = \begin{cases} 1 & \text{if } \tau^A - t < l(k) \leqslant K, \\ 0 & \text{otherwise;} \end{cases}$$

$$b_{tk} = \begin{cases} 1 & \text{if } 0 \leqslant l(k) \leqslant \tau^A - t, \quad \text{and} \\ 0 & \text{otherwise;} \end{cases}$$

$$c_{tk}^A = c_q^A l(k) + c_b^A b_{tk}\left[\tau^A - t - l(k)\right] + c_a^A a_{tk}\left[t + l(k) - \tau^A\right].$$

For departures, define the vector $\boldsymbol{c}_t^D = (c_{t1}^D, c_{t2}^D, \ldots, c_{tK}^D)$, such that

$$(2.2)\qquad c_{tk}^D = \begin{cases} c_q^D l(k) + c_b^D[\tau^D - t] & \text{if} \quad t \leqslant \tau^D, \\ c_q^D l(k) + c_a^D[t - \tau^D] & \text{otherwise.} \end{cases}$$

In words, an element of $\boldsymbol{a}$ equals one if t and $l(k)$ are such that the aircraft completes service after the interchange and zero otherwise. Elements of $\boldsymbol{b}$ are the opposite of $\boldsymbol{a}$, indicating whether the aircraft completes service before the

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interchange. Elements of $\boldsymbol{c}$ are the sum of queuing and layover time costs or queuing and interchange-encroachment costs as appropriate for given values of t and k. The cost functions in vector notation are: $C_t^A(\boldsymbol{s}) = \boldsymbol{c}_t^A \cdot \boldsymbol{q}_t$ and $C_t^D(\boldsymbol{s}) = \boldsymbol{c}_t^D \cdot \boldsymbol{q}_t$. Let C denote either C^A or C^D and c denote either c^A or c^D in expressions that are identical apart from referring to either arrival or departure costs.

The expected queuing, layover, and interchange-encroachment costs of an aircraft scheduled to arrive (or depart) at s_n is the sum over all times, t, of $C_t(\boldsymbol{s})$ weighted by the probability of arriving at t given schedule time, s_n:

$$\sum_t p_t^{s_n} C_t(\boldsymbol{s}). \tag{3}$$

A social-cost-minimizing planner would choose $\boldsymbol{s}$ to minimize the sum of expected costs of all aircraft in the bank. The social planner's objective is

$$\min_{s_1 s_2 \cdots s_N} \sum_{n \in \eta} \sum_t p_t^{s_n} C_t(\boldsymbol{s}). \tag{4}$$

The social planner's first-order necessary conditions for minimization are

$$s_n: \quad \sum_t \frac{\partial p_t^{s_n}}{\partial s_n} C_t(\boldsymbol{s}) + \sum_{m \in \eta} \sum_t p_t^{s_m} \frac{\partial C_t(\boldsymbol{s})}{\partial s_n} = 0. \tag{5}$$

Atomistic airlines would choose their scheduled operation times, s_n, to minimize private costs. Their objective is:

$$\min_{s_n} \sum_t p_t^{s_n} C_t(\boldsymbol{s}). \tag{6}$$

A standard assumption in the congestion-pricing literature is that individual aircraft operators regard costs in each period as parametric, and ignore the effect of their own scheduling decision on $C_t(\boldsymbol{s})$. They observe average queue lengths in all periods during the bank and calculate costs accordingly, but they do not calculate how queue lengths and costs would change given some change in their own schedule. Their first-order necessary conditions for minimization are, therefore:

$$s_n: \quad \sum_t \frac{\partial p_t^{s_n}}{\partial s_n} C_t(\boldsymbol{s}) = 0. \tag{7}$$

The second term in the social planner's first-order condition represents the increase in costs imposed by the nth aircraft on all other aircraft. These costs are external to the nth aircraft operator's scheduling rule given by Equation (7). The social planner can achieve a decentralized implementation of the optimal arrival schedule by imposing a congestion fee equal to the external term. Such a congestion fee would be contingent upon the scheduled time of operation. Because the airport authority cannot directly observe schedule times, aircraft operators would have incentives to misrepresent their schedule times if these were the basis for toll assessments. This problem is particularly severe for (but

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not limited to) air freight, general aviation, and miscellaneous aircraft that do not operate on as strict schedules as commercial carriers. The solution is to impose a fee contingent on the actual arrival time. Let the fee for an aircraft arriving at time u be

$$(8) \qquad F_u = \sum_{m \in \eta} \sum_{v} p_v^{s_m} \frac{\partial C_v(\boldsymbol{s})}{\partial \lambda_u}.$$

This fee equals the expected increase in costs of all aircraft in the bank due to an increase in the arrival rate during the uth period.

The nth aircraft operator's new objective is to choose s_n to minimize the sum of its expected costs and fee:

$$(9) \qquad \min_{s_n} \sum_{t} p_t^{s_n}\left[C_t(\boldsymbol{s}) + F_t\right].$$

The new first-order condition for the nth aircraft is

$$(10) \qquad s_n\colon \ \sum_t \frac{\partial p_t^{s_n}}{\partial s_n} C_t(\boldsymbol{s}) + \sum_{m \in \eta} \sum_{v} p_v^{s_m} \sum_t \frac{\partial C_v(\boldsymbol{s})}{\partial \lambda_t} \frac{\partial p_v^{s_m}}{\partial s_n} = 0,$$

which is the same as the social planner's first-order condition.[5] It follows that a vector $\boldsymbol{s}$ which solves the system of N first-order conditions for individual aircraft operators as in Equation (10) also solves the social planner's problem, equation (5).

Calculation of congestion fees requires evaluation of derivatives $\partial C_t/\partial \lambda_u$, for all u and $t > u$. Letting x_{ij}^t denote the element in the ith row jth column of $\boldsymbol{Q}_t$, define the matrix $\boldsymbol{D}_t$ with $\partial x_{ij}^u/\partial \lambda_t$ as its (i, j) element. Since the cost function in vector notation is $C_t(\boldsymbol{s}) = \boldsymbol{c}_t \cdot \boldsymbol{q}_t$ and $\partial C_t(\boldsymbol{s})/\partial q_{tk} = c_{tk}$, the derivative $\partial C_t(\boldsymbol{s})/\partial \lambda_u$, is (see Appendix B):

$$(11) \qquad \frac{\partial C_t(\boldsymbol{s})}{\partial \lambda_u} = \boldsymbol{c}_t \boldsymbol{Q}_{t-1} \boldsymbol{Q}_{t-2} \cdots \boldsymbol{Q}_{u+2} \boldsymbol{Q}_{u+1} \boldsymbol{D}_u \boldsymbol{q}_u.$$

Finally, the fee is

$$(12) \qquad F_u = \sum_{m \in \eta} \sum_{v=u}^{\infty} p_v^{s_m} \boldsymbol{c}_v \boldsymbol{Q}_{v-1} \boldsymbol{Q}_{v-2} \cdots \boldsymbol{Q}_{u+2} \boldsymbol{Q}_{u+1} \boldsymbol{D}_u \boldsymbol{q}_u.$$

Expression (12) has a straightforward interpretation. The state of the queuing system before arrivals in period u is $\boldsymbol{q}_u$. Elements of the vector $\boldsymbol{D}_u \boldsymbol{q}_u$ give rates of change in the probability that the queue is of length $k = 0, 1, 2, \ldots, K$ in the next period with respect to changes in arrival rates during period u. Premultiplying $\boldsymbol{D}_u \boldsymbol{q}_u$ by each additional transition matrix, $\boldsymbol{Q}_.$, gives the analogous vector

[5] To see that the new first order condition of the nth aircraft is the same as the social planner's, note that:

$$\frac{\partial p_t^{s_n}}{\partial s_n} = \frac{\partial \lambda_t}{\partial s_n}, \quad \text{and} \quad \sum_t \frac{\partial C_v(\mathbf{s})}{\partial \lambda_t} \frac{\partial \lambda_t}{\partial s_n} = \frac{\partial C_v(\mathbf{s})}{\partial s_n}.$$

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for each subsequent period. Multiplying each of these vectors by the period cost vector $c_{.}$, gives the marginal increase in cost to aircraft arriving in that period with respect to the arrival rate in period u. Weighting these marginal costs by the expected number of arrivals and summing over all aircraft and all periods following u, results in the marginal cost that an arrival at u imposes on all aircraft in the bank.

A *no-fee atomistic bottleneck equilibrium* is a collection of scheduled arrival or departure times, expected arrival or departure rates, and probability distributions on queue lengths ($s_n \forall n \in \boldsymbol{\eta}$; $\lambda_t \forall t$; $\boldsymbol{q}_t \forall t$) which, given the distribution of actual arrival times about schedule arrival times (p_t^s), most preferred arrival time (τ), cost parameters (c_b, c_a, and c_q), and service time (μ), satisfies the following conditions:

$$(13.1) \quad \lambda_t = \sum_{n \in \eta} p_t^{s_n}, \quad \forall t,$$

$$(13.2) \quad \boldsymbol{Q}_t \text{ depends on } \lambda_t \text{ as specified in equation (A.1)}, \forall t;$$

$$(13.3) \quad \boldsymbol{q}_{t+1} = \boldsymbol{Q}_t \boldsymbol{q}_t, \quad \forall t;$$

$$(13.4) \quad C_t(\boldsymbol{s}) = \boldsymbol{c}_t \cdot \boldsymbol{q}_t;$$

$$(13.5) \quad \sum_t p_t^{s_n} C_t(\boldsymbol{s}) \leqslant \sum_t p_t^{s'_n} C_t(\boldsymbol{s}), \quad \forall s'_n, n \in \eta.$$

A *congestion-fee atomistic bottleneck equilibrium* is identical to the no-fee bottleneck equilibrium except for the congestion fee. The definition includes condition (12), and condition (14.5′) replaces condition (13.5). A congestion-fee bottleneck equilibrium is a collection ($s_n \forall n \in \boldsymbol{\eta}$; $\lambda_t \forall t$; $\boldsymbol{q}_t \forall t$; $F_t \forall t$) which, given $p_t^s, \tau, c_b, c_a, c_q$, and μ, satisfies: (13.1)–(13.4), (12), and

$$(14.5') \quad \sum_t p_t^{s_n} [C_t(s) + F_t] \leqslant \sum_t p_t^{s'_n} [C_t(s) + F_t], \quad \forall s'_n, n \in \eta.$$

Conditions (13) and (14) specify Nash equilibria in schedule times. Aircraft operators choose schedule times given other aircraft schedules to minimize their time costs. Equations (13.1) and (14.1) say that expected traffic rates are determined by random deviations about scheduled operation times. Equations (13.2), (13.3), (14.3), and (14.3) require that the queuing system evolve as described above and in Appendix A. Equations (13.4) and (14.4) provide that time costs are as outlined in Equations (1)–(3). Equations (13.5), (14.5), and (14.5′) are the cost-minimization conditions. Equation (12) says that congestion fees equal marginal costs that operations impose on all other aircraft in the bank.

If all aircraft have identical cost characteristics, then equilibrium conditions imply that sums of expected queuing, layover, and interchange-encroachment costs must be the same across all periods with scheduled aircraft operations.[6]

[6] More generally, groups of aircraft with identical cost characteristics will have identical expected costs in equilibrium, but expected costs will vary across groups with different cost characteristics.

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The algorithm for finding equilibrium schedules iteratively increases the intervals between scheduled operations in periods with increasing and greater than average expected costs, and decreases intervals between operations in periods with decreasing and lower than average expected cost until expected costs equilibrate.

A *no-fee bottleneck equilibrium with a Nash-dominant airline and an atomistic fringe* has the fringe satisfy inequality (13.5) and the Nash-dominant airline satisfy inequality (14.5′) with equation (12) modified to include only costs imposed on the dominant firm's own aircraft. Let Δ denote the set of dominant-airline aircraft, and let I_t denote the internal delay that a dominant aircraft operating at time t imposes on all other dominant aircraft. A no-fee Nash-dominant bottleneck equilibrium is a collection $(s_n \forall n \in \boldsymbol{\eta};\ \lambda_t \forall t;\ \boldsymbol{q}_t \forall t)$ which, given $p_t^s, \tau, c_b, c_a, c_q$, and μ, satisfies: (13.1)–(13.4) and,

$$(12')\qquad I_t = \sum_{m\in\Delta}\sum_{v=t}^{\infty} p_v^{s_m}\boldsymbol{c}_v\boldsymbol{Q}_{v-1}\boldsymbol{Q}_{v-2}\cdots\boldsymbol{Q}_{t+2}\boldsymbol{Q}_{t+1}\boldsymbol{d}_t\boldsymbol{q}_t,$$

$$(15.5)\qquad \sum_t p_t^{s_n}C_t(s) \leqslant \sum_t p_t^{s_n'}C_t(s),\quad \forall s_n', n\in\boldsymbol{\eta}\backslash\Delta,$$

$$(15.5')\qquad \sum_t p_t^{s_n}[C_t(s)+I_t] \leqslant \sum_t p_t^{s_n'}[C_t(s)+I_t],\quad \forall s_n', n\in\Delta.$$

A *no-fee bottleneck equilibrium with a Stackelberg-dominant airline and an atomistic fringe* has fringe aircraft and mid-peak Stackelberg-dominant aircraft behaving atomistically (as in (13.5)) and edge-peak Stackelberg-dominant aircraft fully internalizing delays they impose on one another while also internalizing a fraction of delays they impose on all other aircraft equal to the Stackelberg-dominant airline's share of mid-peak traffic. The Stackelberg-dominant airline chooses the fraction of its traffic to schedule mid-peak to minimize its cost given possible entry by additional atomistic aircraft. A no-fee Stackelberg-dominant bottleneck equilibrium is a collection $(s_n \forall n \in \boldsymbol{\eta};\ \lambda_t \forall t;\ \boldsymbol{q}_t \forall t)$ which, given $p_t^s, \tau, c_b, c_a, c_q$, and μ, satisfies (13.1)–(13.4), (15.5), (15.5′), and

$$(12'')\qquad I_t = \sum_{m\in\Sigma_\Delta}\sum_{v=t}^{\infty} p_v^{s_m}\boldsymbol{c}_v\boldsymbol{Q}_{v-1}\boldsymbol{Q}_{v-2}\cdots\boldsymbol{Q}_{t+2}\boldsymbol{Q}_{t+1}\boldsymbol{D}_t\boldsymbol{q}_t$$
$$+f*\sum_{m\in A}\sum_{v=t}^{\infty} p_v^{s_m}\boldsymbol{c}_v\boldsymbol{Q}_{v-1}\boldsymbol{Q}_{v-2}\cdots\boldsymbol{Q}_{t+2}\boldsymbol{Q}_{t+1}\boldsymbol{D}_t\boldsymbol{q}_t,$$

where Σ_Δ is the set of edge-peak Stackelberg-dominant aircraft, A is the set of mid-peak Stackelberg dominant aircraft, and f is the fraction of mid-peak aircraft operated by the Stackelberg-dominant airline.

A *no-fee bottleneck equilibrium with joint-cost-minimizing airlines* is identical to the congestion-fee atomistic bottleneck equilibrium with $\boldsymbol{F}_t$ interpreted as a shadow price of congestion instead of as a congestion fee. In this model, a pure monopolist, perfectly collusive airlines, or the social planner all choose the same joint-cost-minimizing schedules.

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4. THE TRAFFIC DATA FROM MINNEAPOLIS–ST. PAUL AIRPORT

Landing and takeoff data on all operations during the first week of May, 1990 come from the tower logs of MSP airport. The tower logs give flight numbers, aircraft types, destinations (if departing), and times (by minute) that aircraft contacted the tower to join landing or takeoff queues. While the airport does not have reliable data on actual lengths of landing and takeoff queues, the queuing model developed above enables estimation of expected queue lengths given airport capacity and actual traffic rates. Figure 2 shows the average number of arrivals and departures in each ten minute interval and the expected length of arrival and departure queues as inferred from the queuing model. The simulations use the precise times of operation for each aircraft, but the data are aggregated by ten minute intervals in the figures and tables to conveniently summarize the results.

The importance of arrival and departure banks in creating demand peaks at the airport is evident from Figure 2. There are ten arrival and ten departure banks during the day. Table I summarizes the number of aircraft and timing of the banks. Departures before 7:30 AM are not considered part of a departure bank because they are not preceded by an arrival bank and do not involve

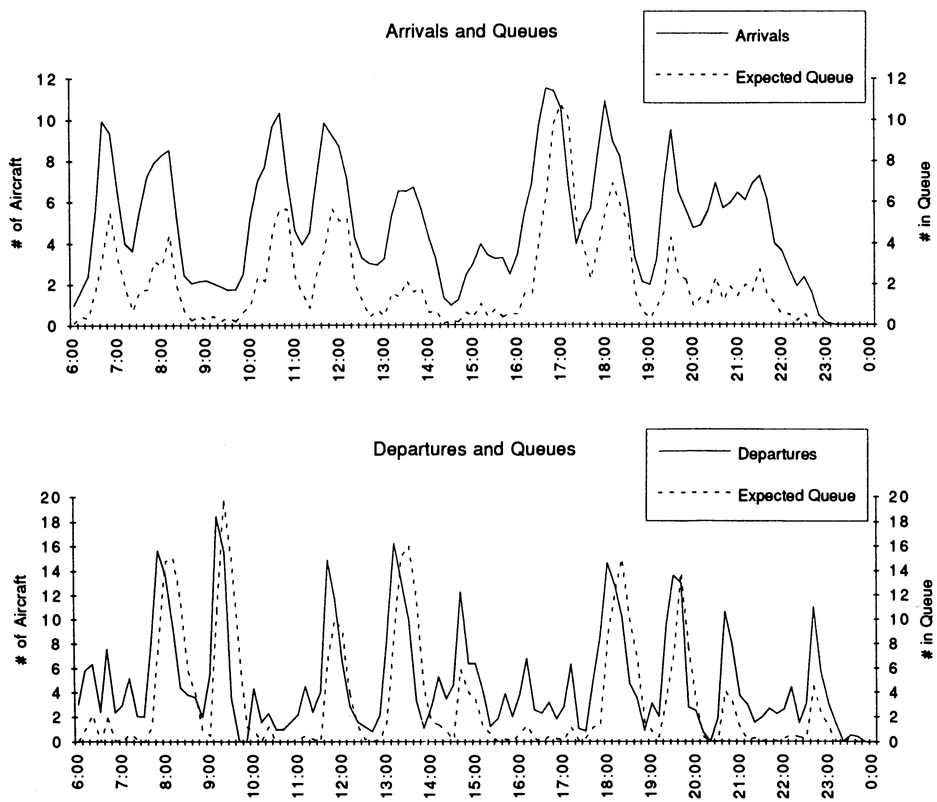

FIGURE 2.—Traffic rates and queues.

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exchange of passengers. Bank 6 is atypical because it is much smaller than the other banks, and consists primarily of non-hub flights. Bank 10 is atypical because most of the arriving aircraft and passengers do not participate in the subsequent departure bank. Between adjacent arrival and departure banks, there are generally thirty minute periods of inactivity during which the exchange of passengers occurs. Although flight banks differ in duration and number of aircraft, arrival banks all have similar bell-shaped traffic patterns. Arrival rates build at an increasing rate, level off at between ten to twelve aircraft per ten minutes, and then decline. The departure banks also have similarly shaped traffic patterns. Departure rates jump rapidly to between twelve to eighteen aircraft per ten minutes, then decline. Arrival queues develop only during arrival banks and return to nearly zero as the bank ends. Their peaks range from four to ten aircraft depending on the size of the bank. Departure queues are also significant only during departure banks. Their peaks range from ten to eighteen aircraft. This rapid fluctuation in traffic associated with flight banks suggests that hourly traffic data is inadequate for modeling congestion delays at MSP and similar hub airports. The fluctuation in traffic rates suggests that peak spreading induced by congestion pricing would significantly reduce congestion levels by spreading traffic out more evenly. Arrival peaks are lower than departure peaks—a fact attributable to lower cost of time spent in the takeoff queue versus the landing queue and greater randomness in the arrival process.

5. ESTIMATING PARAMETERS OF THE MODEL

Implementing the model requires estimation of its parameters, including: the current airport service time (μ), the distribution of actual arrivals and departures about their scheduled times (p_t^s), the most preferred arrival and departure times (τ^A and τ^D), the cost of time spent in the arrival and departure queues (c_q^A and c_q^D), the cost of layover time before the interchange period begins (c_b^A) and after the interchange period ends (c_a^D), and the cost of encroaching on the interchange period for arrivals and departures (c_a^A and c_b^D).

The Metropolitan Airports Commission (MAC) reports that MSP can serve 8.5 arriving aircraft and 8.5 departing aircraft during a ten minute interval, under optimal conditions and balanced numbers of arrivals and departures. Observations of the number of landings and takeoffs performed at the airport during the afternoon peak period on April 24, 1990 confirm a balanced traffic capacity of 8.5 operations of each type, but further indicate that the airport can accommodate more of one type at the cost of the other.[7] Regressions and simulations presented here assume capacity of nine operations per ten minutes. An integer number of service intervals per ten minutes facilitates comparisons between simulations and actual data which rounds or truncates some observations to the nearest five or ten minutes.

[7] A graph of the production possibilities set for landings and takeoffs is provided in Daniel (1991, p. 6).

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The distribution of actual aircraft arrival times from the mean arrival time of aircraft with the same flight number approximates the distribution of actual arrivals about scheduled times (p_t^s).[8] While the data give arrival times by minute, they round or truncate some departure times to even five or ten minutes. Only three per cent of observations deviate more than ten minutes from their scheduled departure time listed in the *Official Airline Guide* (Nelson (1990)). Consequently, the traffic data do not precisely identify the distribution of departures about scheduled departures. Given this lack of information, the distribution of actual departure times about scheduled times is negative exponential by assumption. A mean deviation parameter of 2.7 minutes results in ninety-seven per cent of actual departures occurring within ten minutes of their scheduled departure time.

To test the assumption that arrival times are Poisson-distributed with time varying arrival rates (λ_t), Figure 3 compares distributions of actual and simulated interarrival times with their expected distributions. Since the data truncate observations on arrival times to the minute, observed interarrival times are differences between subsequent arrival times measured in whole minutes. Observations of zero can result from actual interarrival times of 0 to 59 seconds. Observations of one can result from actual interarrival times of between 1 and 119 seconds. Observations of two can result from actual interarrival time of between 61 and 179 seconds, and so forth. Poisson-distributed arrivals imply that interarrival times have a negative exponential distribution, but it is necessary to adjust the distribution to account for truncation.[9] Substituting arrival rates for each ten minute interval into the adjusted negative exponential distribution gives expected numbers of observations in each cell during each ten minute period. Summing over all periods results in expected numbers of observations in each cell. The resulting distribution is approximate because it ignores variations in arrival rates within ten minute periods.

[8] The resulting probability histogram is available from the author upon request.

[9] Let X be a random variable denoting the exact interarrival time between two subsequent arrivals. By hypothesis, X is exponentially distributed. Let Y be a discrete random variable denoting the (integer) difference between arrival times which are truncated to the minute. Let y denote a realization of Y. Let Z be a random variable denoting the fraction of a minute which has already expired on the clock when the first arrival occurs. The distribution of Z can be assumed to be uniform on the interval $[0,1]$. Let z be a realization of Z. It follows that the clock's minute hand will jump one minute at $(1-z), (2-z), \ldots,$ and $(y-z)$ minutes from the first arrival. Given z, the probability of zero difference in truncated arrival times (i.e., $P(Y=0)$) is the exponential cumulative distribution function evaluated from 0 to $(1-z)$. The probability that the difference between the truncated arrival time $y = 1, 2, 3, \ldots$ is the exponential cumulative distribution function evaluated from $(y-z)$ to $(y+1-z)$. Summarizing, for given z:

$$P(Y=0|Z=z) = 1 - e^{(z-1)/\lambda} \quad \text{and}$$

$$P(Y=y>0|Z=z) = e^{(y-z)/\lambda} - e^{(y+1-z)/\lambda}.$$

Since by assumption, Z is uniformly distributed, it follows that:

$$P(Y=0) = 1 - \lambda + \lambda e^{-1/\lambda} \quad \text{and}$$

$$P(Y=y>0) = \lambda(e^{-(y-1)/\lambda} - 2e^{-y/\lambda} + e^{-(y-1)/\lambda}).$$

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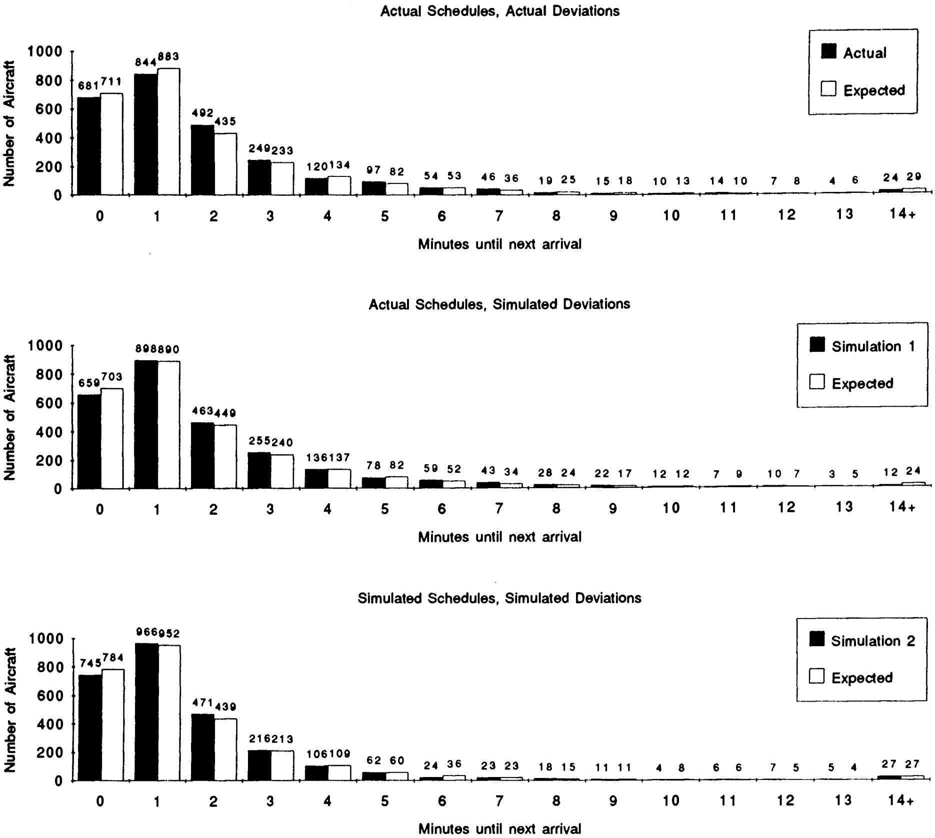

FIGURE 3.—Inter-arrival time distributions.

The top panel of Figure 3 uses interarrival times from actual observations. The middle panel uses interarrival times from simulated deviations about observed mean arrival times. The bottom panel uses interarrival times from simulated deviations about simulated arrival schedules. Chi-squared statistics from the comparisons are 24.5, 18.6, and 12.3 respectively, which are all below the ninety-five per cent critical value of 25.0. At this significance level, Chi-squared tests fail to reject the hypothesis that the distributions are the same.

Table I lists estimates of the most-preferred arrival and departure times (τ^A and t^D) for each bank. The estimate of τ^A is the time at which the expected queue reaches a maximum plus the expected queuing delay. The bottleneck model implies that equilibrium arrival queues attain maximum length during the period in which an aircraft joining the queue will complete service at precisely τ^A. The estimate of τ^D is the time at which the expected queue reaches a maximum. The bottleneck model implies that equilibrium departure queues attain maximum length at precisely τ^D. Setting τ^A and τ^D accordingly is consistent with defining them as the periods that minimize the sum of expected

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layover and interchange-encroachment costs. This approach to determining τ^A and τ^D assures that simulated traffic patterns and queue lengths vary in phase with the actual traffic data. It also improves cost-parameter estimates so that simulated traffic peaks attain similar levels as actual peaks.

Estimation of cost ratios, c_l/c_q and c_e/c_q, proceeds in two stages. Stage one uses the tower log data and the queuing model to generate traffic patterns for a typical day. For regularly scheduled flights, mean arrival times over five days approximate scheduled arrival times at the landing or takeoff queue. For unscheduled or general aviation flights, mean operating times of each group of five consecutive aircraft approximate their scheduled operating times. The queuing model estimates queuing, layover, and interchange-encroachment times of each flight based on its scheduled time of operation. The first stage is necessary because direct observations on expected queuing, layover, and interchange-encroachment times of each flight are unavailable. Stage two uses results of stage one in regression models that recover cost ratios by determining rates at which flights with identical cost parameters trade off queuing time against layover and interchange-encroachment time. In the bottleneck model, queue lengths adjust so that the sums of expected queuing cost (which is endogenous) and expected layover and interchange-encroachment costs (which are exogenous) are equal for each period with scheduled operations.

TABLE II

REGRESSION EQUATIONS

$$Q_{own} = \frac{C^*}{c_q} - \frac{c_l}{c_q}L_{own} - \frac{c_e}{c_q}E_{own} + \varepsilon \qquad \text{(Atomistic)}$$

$$Q_{own} = \frac{C^*}{c_q} - \frac{c_l}{c_q}(L_{own} + L_{firm}) - \frac{c_e}{c_q}(E_{own} + E_{firm}) - Q_{firm}L + \varepsilon \qquad \text{(Nash Dominant)}$$

$$Q_{own} = \frac{C^*}{c_q} - \frac{c_l}{c_q}(L_{own} + gL_{firm}) - \frac{c_e}{c_q}(E_{own} + gE_{firm}) - gQ_{firm} + \varepsilon \qquad \text{(Stackelberg Dominant)}$$

$$Q_{own} = \frac{C^*}{c_q} - \frac{c_l}{c_q}(L_{own} + L_{firm} + L_{oth}) - \frac{c_e}{c_q}(E_{own} + E_{firm} + E_{oth}) - (Q_{firm} + Q_{oth}) + \varepsilon \qquad \text{(Joint Cost Minimizing)}$$

Q_{own} is the aircraft's expected queueing time;
C^* is the bank's equilibrium cost level;
L_{own} is the aircraft's expected layover time;
E_{own} is the aircraft's expected encroachment time;
Q_{firm} is the additional expected queueing time imposed on the airline's other aircraft;
L_{firm} is the additional expected layover time imposed on the airline's other aircraft;
E_{firm} is the additional expected encroachment time imposed on the airline's other aircraft;
g is a dichotomous variable equal to 0 if the aircraft is mid-peak and 1 if the aircraft is edge-peak;
Q_{oth} is the additional expected queueing time imposed on other airline's aircraft;
L_{oth} is the additional expected layover time imposed on other airline's aircraft;
E_{oth} is the additional expected encroachment time imposed on other airline's aircraft; and,
ε is added as an error term to account for measurement errors and unobservable aircraft specific costs which affect scheduling.

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The equations in Table II show the relationship between equilibrium queuing, layover, and interchange-encroachment time. These equations are interpreted as regression equations with expected queue length as the dependent variable, expected layover and interchange-encroachment times as independent variables, cost ratios c_l/c_q and c_e/c_q as their coefficients, bank-specific equilibrium cost level as the constant term, and an error term to account for measurement error, stochastic variation in independent variables, and possible aircraft-specific costs that are unrelated to expected layover or interchange-encroachment times.

Figure 4 illustrates how the atomistic equilibrium endogenously determines the evolution of the arrival queue. The straight lines forming a "V" with vertex at time $0(\tau)$ give layover or interchange-encroachment costs of aircraft that complete service (for arrivals) or begin service (for departures) at time t along the horizontal axis. The curved line forming a "U" is the sum of expected layover and interchange-encroachment costs of aircraft scheduled to arrive at time s along the horizontal axis. It results from weighting layover or interchange-encroachment cost by the probability p_t^s that aircraft scheduled for s actually operate at t, then integrating over t. Expected queuing costs are vertical distances between this curve and the horizontal line at the equilibrium cost level. Queuing costs divided by the price of queuing time equals the expected queue length. In equilibrium, only s_n's along the horizontal portion of the cost curve will be selected by airlines.

Ordinary least squares estimates of regression equations in Table II reveal first-order autocorrelation of the error terms. Moreover, Dickey-Fuller unit-roots

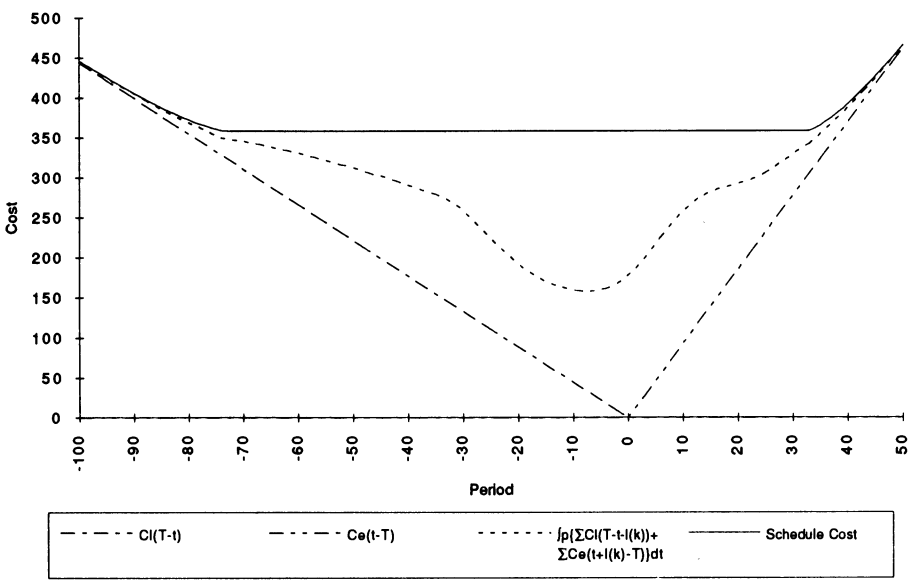

FIGURE 4.—Endogenous determination of queue lengths (for arrivals).

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tests generally fail to reject the hypothesis of nonstationarity of expected queue lengths, but do reject nonstationarity of the first-differenced model.[10] Autocorrelation and nonstationarity have natural interpretations in the context of queuing theory. The queuing model is a first-order Markov process. Transition matrices carry forward any unexplained variations in expected queue lengths into subsequent periods. When arrival rates are below capacity, the effects of unexplained variations diminish with time. When arrival rates equal or exceed capacity during peak periods, the queuing system becomes nonstationary and unexplained variations persist undiminished until arrival rates fall below capacity again. For these reasons, estimates presented here use an ARIM(1,1,0) specification.

Table III presents results from the atomistic model for individual banks and for pooled banks 1–4 and 7–9. These are also valid for the Stackelberg-dominant case in which the dominant firm schedules all its aircraft during the mid-peak period. Since the dominant firm does not schedule aircraft during the edge-peak, the atomistic and Stackelberg-dominant models are the same for these data. Estimated cost ratios are reasonably consistent across banks 1–4 and 7–9. These are major banks that interchange large numbers of passengers. While the pooled estimates differ significantly from bank-specific estimates in a number of cases, they nevertheless perform well in simulations of banks 1–4 and 7–9 presented in Section 7.

Cost-ratio estimates differ for banks 5, 6, and 10, due to different composition of traffic. Bank 6 has very few flights operated by Northwest or its code affiliates. General aviation contributes a high fraction of the flights. Bank 10 contains the largest number of arrivals, but the smallest number of departures. Most of these aircraft do not leave until the next morning and do not exchange passengers. Many of these are military or air-transport flights. Differences in cost-ratio estimates for bank 5 are harder to explain, except that it occurs in a relatively low-demand period.

Table IV presents pooled regression estimates of cost ratios for all four specifications of the model. Each set of estimates uses data from banks 1–4 and 7–9. The econometric model estimates relative values of the cost parameters, not their nominal values. Traffic patterns and queue lengths are homogeneous of degree zero in the cost parameters. Using Morrison's and Winston's (1989) estimates of hourly aircraft operating cost to assign nominal values, air-delay cost per hour is \$18.99* (Seats)$^{0.86}$ ($R^2 = 0.85$) in 1986 dollars. Assuming an aircraft size of one hundred, a load factor of sixty per cent, passenger costs of twelve dollars per hour, and five per cent annual inflation gives air-delay cost of about thirty-six dollars per minute or forty dollars per service interval in 1992 dollars. Departure-cost parameters based on pooled regressions assume the cost of layover time before the interchange is identical to the cost of layover time after the interchange. In either case aircraft are waiting at gates with their

[10] The OLS, AR(1), and unit-root test results are available from the author upon request.

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TABLE III

REGRESSION RESULTS

	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5	Bank 6	Bank 7	Bank 8	Bank 9	Bank 10	Banks 1–4, 7–9
Arrivals											
Layover	−0.14472	−0.06907	−0.12152	−0.07901	−0.03412	0.00673	−0.12061	−0.12881	−0.15445	−0.01283	−0.11095
	(0.00451)	(0.00222)	(0.00743)	(0.00587)	(0.00213)	(0.00475)	(0.00937)	(0.00437)	(0.00213)	(0.00323)	(0.00354)
Encroach-ment	−0.22834	−0.2101	−0.23972	−0.25854	−0.05961	−0.00045	−0.24983	−0.26535	−0.1365	−0.05569	−0.23114
	(0.01904)	(0.00698)	(0.01415)	(0.00683)	(0.00283)	(0.01187)	(0.08080)	(0.00948)	(0.00218)	(0.00372)	(0.00772)
AR(1)	0.33144	0.3997	0.41111	0.42758	0.2787	0.14402	0.32832	0.06724	0.04035	0.36446	0.31751
	(0.16521)	(0.08585)	(0.07029)	(0.12855)	(0.08093)	(0.25119)	(0.11576)	(0.07675)	(0.01083)	(0.12214)	(0.04737)
Durbin-Watson	2.29181	2.017	2.10395	2.117	1.68136	2.01986	2.17838	1.61817	1.90575	2.12956	2.09832
LM	1.83974*	3.15011*	10.0513*	5.55054*	4.94974*	2.96904*	11.4180^	10.5839*	1.22825*	11.3796^	71.2208
R-Squared	0.96953	0.97496	0.8992	0.9687	0.96084	0.14402	0.64956	0.96861	0.99617	0.82786	0.99867
Departures											
	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5	Bank 6	Bank 7	Bank 8	Bank 9	Bank 10	Banks 1–4, 7–9
Layover	−0.58629	−1.05169	−0.46073	−0.58953	−0.32823	−0.13929	−0.42033	−0.66114	−0.21759	−0.18486	−0.63963
	(0.03852)	(0.03473)	(0.04785)	(0.04719)	(0.04162)	(0.09306)	(0.03791)	(0.03771)	(0.03469)	(0.05338)	(0.02040)
Encroach-ment	−0.42691	−0.59484	−0.4366	−0.37241	−0.12439	−0.02333	−0.38207	−0.39356	−0.1887	−0.04617	−0.41455
	(0.02662)	(0.01671)	(0.03886)	(0.03051)	(0.01699)	(0.02483)	(0.03220)	(0.02549)	(0.01937)	(0.01555)	(0.01242)
AR(1)	0.07993	−0.07975	−0.08627	0.23614	−0.18006	0.31237	0.15315	0.26099	0.0000	0.22218	0.0347
	(0.04478)	(0.14628)	(0.03871)	(0.13238)	(0.17231)	(0.34294)	(0.04945)	(0.06289)	(0.22483)	(0.21831)	(0.02974)
Durbin-Watson	2.13069	1.89545	1.87356	1.91427	2.01377	1.65145	2.11619	1.90448	1.86012	2.08958	1.75116
LM	1.45772*	8.25526*	3.42027*	5.57422*	18.2047^	5.04084*	5.68804*	6.50230*	1.79239*	4.00049^	10.6269*
R-Squared	0.89814	0.96597	0.83916	0.7741	0.63163	0.19934	0.83958	0.89553	0.84383	0.43477	0.98067

Notes: Standard errors in parentheses. The LM statistic tests for autocorrelation in the error terms. * indicates absence of autocorrelation at the 95% confidence level. ^ indicates absence of autocorrelation at the 90% confidence level.

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TABLE IV

POOLED REGRESSION

	Arrivals				Departures		
	Atomistic / Stackelberg-Dominant	Nash-Dominant	Joint Cost Minimizing		Atomistic / Stackelberg-Dominant	Nash-Dominant	Joint Cost Minimizing
Layover	−0.11095	−0.22463	−0.83705	Layover	−0.63963	−0.24118	−0.40981
	(0.00354)	(0.01344)	(0.03787)		(0.02040)	(0.04248)	(0.10320)
Encroachment	−0.23114	−0.4588	−1.39411	Encroachment	−0.41455	−0.56184	−0.55674
	(0.00772)	(0.05242)	(0.12548)		(0.01243)	0.02102)	(0.05918)
Rho	0.31751	0.21989	0.37358	Rho	0.0347	0.00573	0.06656
	(0.04737)	(0.03567)	(0.05327)		(0.02974)	(0.05585)	(0.04865)
Durbin-Watson	2.09832	1.70396	2.19872	Durbin-Watson	1.75116	1.775	2.08209
R-Squared	0.99867	0.98758	0.9961	R-Squared	0.98067	0.96064	0.94909

continuing passengers on board and their noncontinuing passengers discharged. This assumption relates departure-cost parameters to arrival-cost parameters and enables assignment of cost levels to all parameters relative to the cost of time spent in the arrival queue. Table V presents the resulting cost parameters that the simulation model uses. Since the model is homogeneous of degree one in time costs, all cost estimates presented can be adjusted by multiplying all costs by the appropriate factor.

TABLE V

PARAMETER ESTIMATES

	Arrivals					Departures			
	Pooled Atomistic	Stackelberg Dominant	Nash Dominant	Joint Cost Minimizing		Pooled Atomistic	Stackelberg Dominant	Nash Dominant	Joint Cost Minimizing
Queue	40.00	40.00	40.00	40.00	Queue	6.94	6.94	37.26	81.70
Layover	4.44	4.44	8.99	33.48	Layover	4.44	4.44	8.99	33.48
Encroachment	9.25	9.25	18.35	55.76	Encroach	2.88	2.88	20.93	45.49

	Arrivals									
	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5	Bank 6*	Bank 7	Bank 8	Bank 9	Bank 10
Queue	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00
Layover	5.79	2.76	4.86	3.16	1.36	0.65	4.82	5.15	6.18	0.51
Encroachment	9.13	8.40	9.59	10.34	2.38	0.29	9.99	10.61	5.46	2.23

	Departures									
	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5	Bank 6*	Bank 7	Bank 8	Bank 9	Bank 10
Queue	6.94	6.94	6.94	6.94	6.94	6.94	6.94	6.94	6.94	6.94
Layover	4.07	7.30	3.20	4.09	2.28	0.97	2.92	4.59	1.51	1.28
Encroachment	2.96	4.13	3.03	2.58	0.86	0.16	2.65	2.73	1.31	0.32

Notes: All parameters given in dollars per minute. *Bank 6 parameters are based on an AR(1) regression because of the wrong sign in the ARIMA(1, 1, 0) regression.

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6. SPECIFICATION TESTS

Atomistic, Nash-dominant, and joint-cost-minimizing specifications of the model in Table II are testable against one another using J-tests. Standard J-tests add predicted values from one model to the list of independent variables in a second model. The J-test rejects the first model if the coefficient on the predicted value is not significantly different from zero. In cases here, hypotheses tests are more involved because the specifications are nonnested linear regression models subject to nonhomogeneous linear restrictions. Substituting the restrictions results in unrestricted models with different dependent variables.

Pesaran and Hall (1988) propose a solution involving adjustment of predicted values from the first model to account for differences in the dependent variables. Start with the common regression equation:

$$\begin{aligned} Q_{own} = \beta_i X + \varepsilon_i = \beta_{i1} &- \beta_{i2} L_{own} + \beta_{i3} E_{own} \\ &+ \beta_{i4} Q_{firm} + \beta_{i5} L_{firm} + \beta_{i6} E_{firm} \\ &+ \beta_{i7} Q_{oth} + \beta_{i8} L_{oth} + \beta_{i9} E_{oth} + E_i, \end{aligned}$$

where i indexes the alternative specifications. The atomistic restrictions are: $\beta_{14}, \beta_{15}, \beta_{16}, \beta_{17}, \beta_{18}, \beta_{19} = 0$. The Nash-dominant restrictions are: $\beta_{24} = 1$; $\beta_{25} = \beta_{22}$; $\beta_{26} = \beta_{23}$; and $\beta_{27}, \beta_{28}, \beta_{29} = 0$. The joint-cost-minimizing restrictions are: $\beta_{37} = \beta_{34} = 1$; $\beta_{38} = \beta_{35} = \beta_{32}$; $\beta_{39} = \beta_{36} = \beta_{33}$. To test the atomistic against the Nash-dominant model, regress Q_{own} on L_{own}, E_{own}, and $(\hat{\beta}_2 X - Q_{firm})$. To test the Nash-dominant against the atomistic model, regress $(Q_{own} + Q_{firm})$ on $(L_{own} + L_{firm})$, $(E_{own} + E_{firm})$, and $(\hat{\beta}_1 X + Q_{firm})$. To test the atomistic against the joint-cost-minimizing model, regress Q_{own} on L_{own}, E_{own}, and $(\hat{\beta}_3 X - Q_{firm} - Q_{oth})$. To test the joint-cost-minimizing model against the atomistic model, regress $(Q_{own} + Q_{firm} + Q_{oth})$ on $(L_{own} + L_{firm} + L_{oth})$, $(E_{own} + E_{firm} + E_{oth})$, and $(\hat{\beta}_1 X + Q_{firm} + Q_{oth})$.

Table VI presents the results of these J-tests. For arriving aircraft, the test fails to reject the atomistic specification in favor of the Nash-dominant model. Unfortunately, the test also fails to reject the Nash-dominant specification in

TABLE VI

J-TESTS OF ALTERNATIVE SPECIFICATIONS

Arrivals				Departures			
Atomistic v. *N*-Dominant	0.99672	(0.00554)	Do not reject Atomistic	Atomistic v. *N*-Dominant	0.90923	(0.02238)	Do not reject Atomistic
N-Dominant v. Atomistic	0.01098	(0.00364)	Do not reject Nash	*N*-Dominant v. Atomistic	0.01343	(0.01763)	Reject Nash
Atomistic v. Joint Cost Minimizing	0.99294	(0.00456)	Do not reject Atomistic	Atomistic v. Joint Cost Minimizing	0.97742	(0.01323)	Do not reject Atomistic
Joint Cost Minimizing v. Atomistic	0.0049	(0.03210)	Reject Joint Cost Minimizing	Joint Cost Minimizing v. Atomistic	0.0028	(0.00862)	Reject Joint Cost Minimizing

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favor of the atomistic model. However, the coefficient on the fitted Nash-dominant value (while significant) is very close to zero (0.011). All other J-tests strongly support the atomistic model. J-tests using the departure data fail to reject the atomistic model, but do reject the Nash-dominant specification. J-tests using arrival and departure data reject the joint-cost-minimizing specification in favor of the atomistic model. Since atomistic behavior is also consistent with the Stackelberg-dominant model, these tests also support the Stackelberg-dominant model.

7. SIMULATIONS OF ALTERNATIVE SPECIFICATIONS

Comparison of simulation results with actual traffic patterns also supports theoretical specifications in which dominant airlines behave atomistically. The timing of actual dominant-airline operations is not consistent with the Nash-dominant simulations. Figure 5 shows disaggregated traffic patterns that result from actual, atomistic, Stackelberg-dominant, Nash-dominant, and joint-cost-minimizing equilibria for bank 7. Each simulation uses cost parameters from Table V estimated under the hypothesis that it is the correct specification.

The dominant airline does not spread its traffic out more than the fringe traffic in the actual, atomistic, and joint-cost-minimizing equilibria. Stackelberg- and Nash-dominant airlines, however, shift traffic away from the mid-peak as they partially or fully internalize delays imposed on themselves. The Stackelberg-dominant equilibrium depicted in Figure 5 assumes that the dominant airline shifts fifty per cent of its traffic out of the mid-peak—so it is midway between the atomistic and Nash-dominant cases. Spreading traffic out over time is the only way to reduce internally imposed delays. Fringe airlines take advantage of peak spreading by shifting their operations to the middle of the bank. The Nash-dominant airline ignores the fringe reaction and shifts all of its traffic to the edges of the peak. The Stackelberg firm, however, takes fringe adjustment into account. Rearranging its traffic within the mid-peak cannot reduce its cost, because reaction by the fringe would reestablish the initial combined traffic pattern. By shifting some traffic out of the mid-peak, the Stackelberg firm can reduce costs of all aircraft in the mid-peak. Its edge-peak aircraft experience higher average cost than those remaining in the mid-peak, but congestion levels are lower so it may attain lower total cost than by acting atomistically.

Figure 6 illustrates traffic patterns resulting from a Stackelberg airline shifting progressively more traffic out of the mid-peak. These simulations use the pooled-atomistic parameters and assume there are no carryover effects from traffic in neighboring banks. As the Stackelberg airline treats more traffic nonatomistically, it schedules more traffic in the edge peak. Atomistic and Nash-dominant equilibria are limiting cases of the Stackelberg-dominant equilibrium. Table VII lists equilibrium average costs of the dominant and fringe aircraft assuming the Stackelberg firm shifts 0%, 33%, 66%, and 100% of its traffic out of the mid-peak, given a fixed number of fringe aircraft. Note that for arrivals and departures, the average cost of the dominant aircraft falls as it

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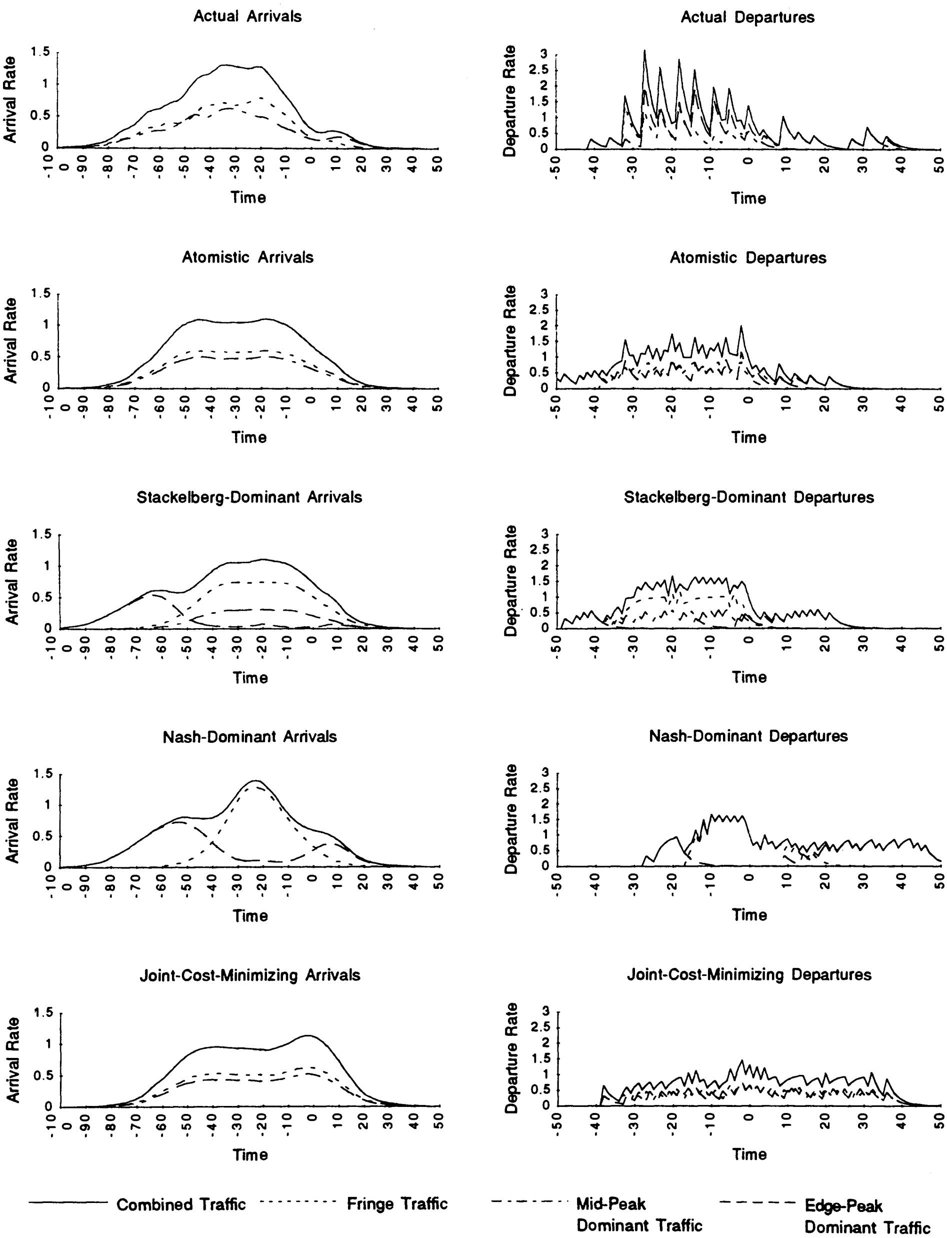

FIGURE 5.—Unaggregated traffic rates (bank 7).

shifts more traffic out of the mid-peak, but the cost for fringe aircraft falls much more. The fringe receives reduced congestion and lower average layover and interchange-encroachment costs. If lower costs draw additional fringe aircraft

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TABLE VII

COSTS AS FUNCTION OF STACKELBERG DOMINANT TRAFFIC SHIFTED TO THE EDGE PEAK

% of Dominant Aircraft Scheduled Non-Atomistically	Average Cost for Arrivals		Average Cost for Departures	
	Dominant	Fringe	Dominant	Fringe
0%	$290.0	$290.0	$119.0	$119.0
33%	$287.8	$270.0	$109.0	$105.0
66%	$284.3	$247.5	$104.0	$89.5
100%	$271.0	$232.0	$102.0	$76.0

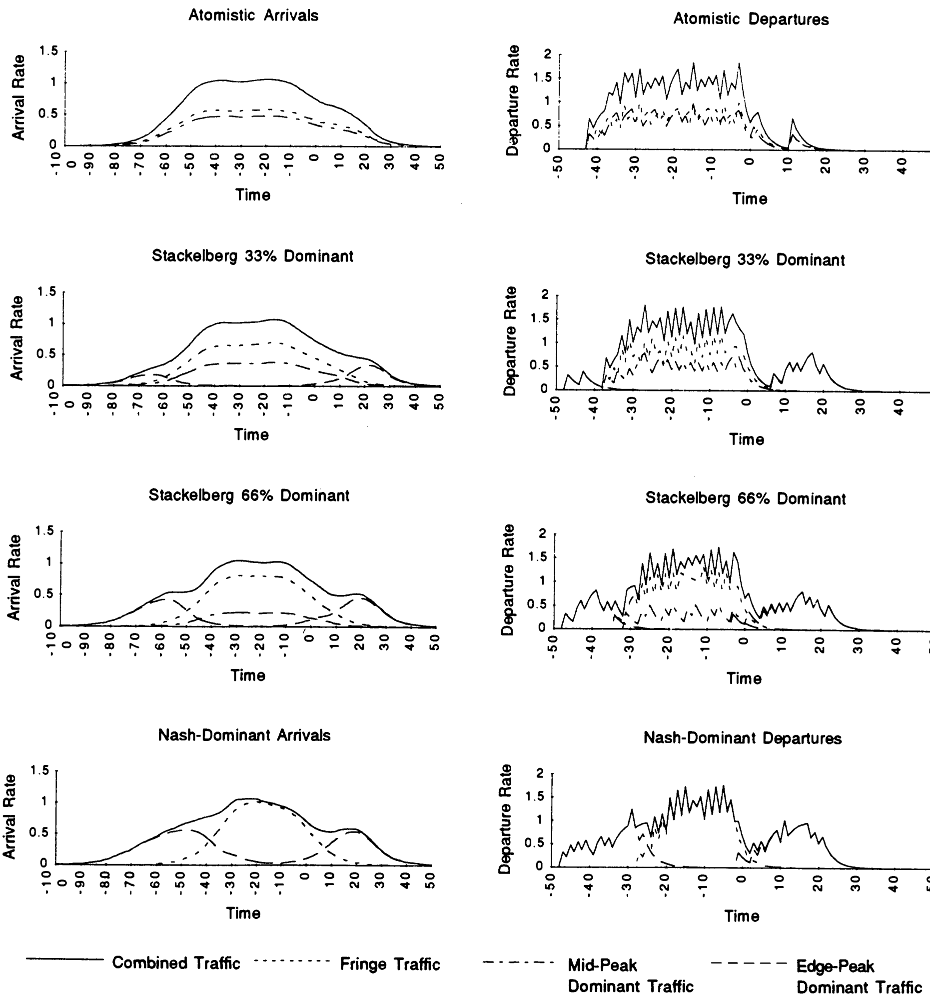

FIGURE 6.—Stackelberg adjustments.

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into the mid-peak, the dominant airline's cost savings evaporate. For example, six additional aircraft in the arrival peak would drive the average cost of the Stackelberg aircraft above $290 while leaving the cost of fringe aircraft at $246. Nine additional aircraft in the departure peak would drive the average cost for the Stackelberg aircraft above $119 while leaving fringe-aircraft cost at $91. This suggests that a dominant firm may behave atomistically to keep additional fringe aircraft from driving its cost up, placing it at a competitive disadvantage, and diminishing the attractiveness of its schedules.

Clearly, the data do not indicate that the dominant airline shifts any aircraft out of the mid-peak as implied by the dominant-airline simulation models. Furthermore, the joint-cost-minimizing equilibrium cannot generate traffic patterns with as sharp peaks as the actual patterns because full internalization of delays flattens the top of the peaks. This suggests that the atomistic equilibrium (or Stackelberg equilibrium in which the dominant firm treats virtually all of its aircraft atomistically) is the correct model.

Table VIII gives chi-square statistics for comparing simulated and actual traffic patterns. These statistics result from comparing actual and simulated numbers of arrivals and departures aggregated by ten minute intervals during major interchange banks (banks 1–4 and 7–9). "Actual" traffic levels are expected levels based on the distribution of actual arrivals about mean arrival times. The table includes atomistic simulation results for bank-specific and pooled regression parameters. Both perform well for arrival banks, with the only significant difference occurring in the ninth bank. For the sake of consistency, this paper uses pooled parameters for banks 1–4 and 7–9 in subsequent simulations. Stackelberg- and Nash-dominant arrival simulations also perform well in simulating combined traffic rates. As shown above, however, they imply that dominant-airline operations occur disproportionately during edge-peak periods and fringe-airline operations during mid-peak periods. Even the joint-cost-minimizing airlines simulations are reasonably close to actual traffic patterns, but their chi-squared statistics are much worse.

For departure simulations, chi-squared statistics are disappointing. The model overpredicts the size of departure peaks and queues because it assumes all aircraft participate in one of the banks. The atomistic simulation with bank-specific parameters compares acceptably for the seventh bank and barely so for the first and third banks. The pooled-parameter atomistic simulation survives a chi-squared test at standard significance levels only for bank 7. The Stackelberg simulation performs best in predicting combined traffic rates, but not traffic rates of dominant and fringe airlines individually. The Nash-dominant departure simulation predicts combined traffic rates almost as well as the atomistic simulation with bank-specific parameters, but suffers from the same problem as the Stackelberg-dominant simulation. The joint-cost-minimizing airlines model performs the worst of all.

Figure 7 plots actual and simulated traffic rates for the pooled-parameter atomistic case. It puts the chi-squared test statistics in perspective. Clearly the arrival simulation fits actual arrival rates well. The general timing and magni-

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TABLE VIII

CHI-SQUARED STATISTICS FOR COMPARISON OF ACTUAL AND SIMULATED TRAFFIC

	Arrivals							
	Bank 1	Bank 2	Bank 3	Bank 4	Bank 7	Bank 8	Bank 9	Banks 1–4 & 7–9
Separate	0.7663	1.9389	1.9256	5.0073	3.1060	0.9509	0.4661	19.9930
Atomistic	0.9978	0.9631	0.9926	0.8907	0.9892	0.9985	0.9996	1.0000
Pooled	1.3648	1.2769	1.6219	1.8329	3.1950	1.5627	7.3123	24.1154
Atomistic	0.9866	0.9890	0.9961	0.9975	0.9879	0.9916	0.3971	1.0000
Stackelberg	0.7722	0.5767	1.7951	1.3887	4.5917	8.7109	4.6876	27.8300
Dominant	0.9977	0.9991	0.9943	0.9992	0.9493	0.3673	0.6980	0.9998
Nash	0.8663	1.5292	1.0521	3.1642	2.8079	7.9722	5.1238	27.8995
Dominant	0.9967	0.9813	0.9993	0.9773	0.9930	0.4362	0.6449	0.9998
Joint Cost	3.2384	3.4980	7.0730	7.6020	17.3850	11.4344	5.0817	58.5825
Minimizing	0.8621	0.8354	0.6295	0.6677	0.0970	0.1783	0.6500	0.4909
	Departures							
	Bank 1	Bank 2	Bank 3	Bank 4	Bank 7	Bank 8	Bank 9	Banks 1–4 & 7–9
Separate	15.7300	15.8495	14.3551	27.4393	3.6605	19.0815	29.3918	126.4245
Atomistic	0.0464	0.0032	0.0452	0.0001	0.8179	0.0040	0.0001	0.0000
Pool	37.7357	12.0331	27.8834	26.3958	3.1835	15.5199	61.6504	185.3035
Atomistic	0.0000	0.0171	0.0002	0.0002	0.8675	0.0166	0.0000	0.0000
Stackelberg	6.0557	11.7928	7.5821	3.1182	1.8594	3.6028	50.6166	88.1499
Dominant	0.6410	0.0190	0.3709	0.7939	0.9672	0.7302	0.0000	0.0001
Nash	16.6410	6.3515	24.9551	17.6094	22.2053	26.6935	13.6711	128.3459
Dominant	0.0341	0.1744	0.0008	0.0073	0.0023	0.0002	0.0574	0.0000
Joint Cost	19.5808	16.5215	26.3529	17.0174	19.2206	22.9481	27.6294	149.3032
Minimizing	0.0120	0.0024	0.0004	0.0092	0.0075	0.0008	0.0003	0.0000

Notes: The top number in each cell is the chi-squared statistic. The lower number is the *P*-value.

tude of simulated departure peaks are similar. The simulated departure peaks are fatter and peak at a lower maximum departure rate. Maximum queue sizes, however, are virtually identical. The main difference between simulated and actual traffic is that simulations predict too much traffic during departure peaks. This results from the assumption that all airport traffic participates in one of the banks. Plots of Nash- and Stackelberg-dominant simulations show that they result in steeper and narrower traffic peaks and have more traffic in the edges of peaks. This matches combined traffic patterns better than the pooled-parameter atomistic simulation, but results from disproportionate fringe traffic in the mid-peak and dominant traffic in the edge-peak. Plots of simulations with the parameters from the joint-cost-minimizing specification appear to be the worst overall because they result in excessive peak spreading. This occurs even though these simulations use parameters estimated under the assumption that the joint-cost-minimizing specification is correct and therefore are best case parameters for that specification.[11]

[11] These graphs are available from the author upon request.

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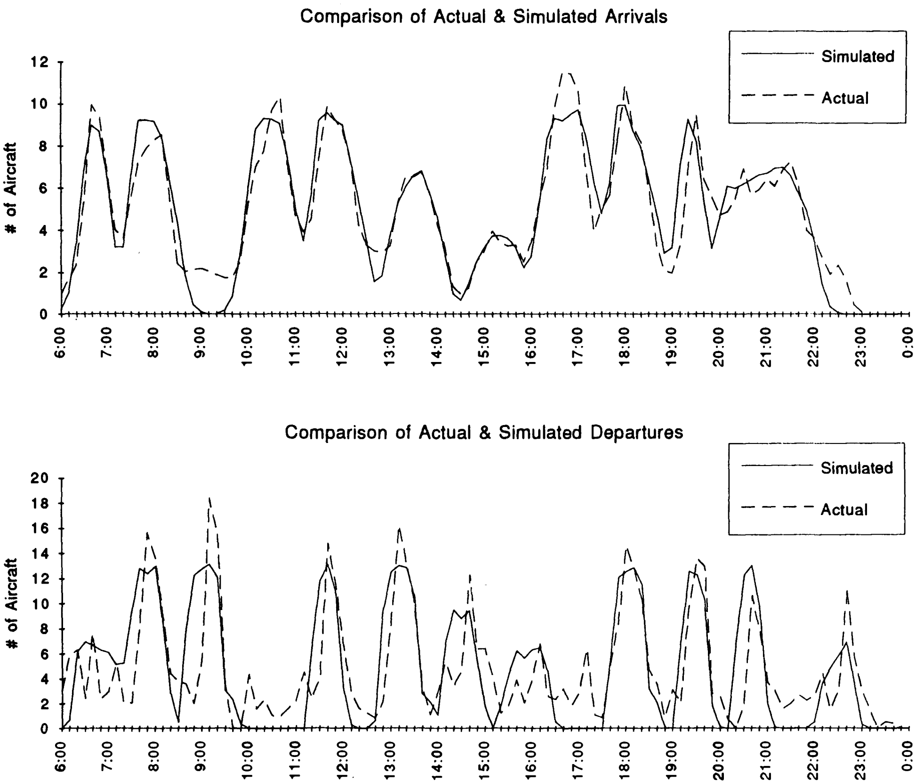

FIGURE 7.—Comparison of actual and simulated atomistic traffic (pooled parameters).

8. EQUILIBRIUM PRICING RESULTS

This section presents simulations of the effects of congestion pricing on equilibrium traffic patterns under the current airport capacity level. Figures 8–11 illustrate the effect of congestion pricing on arrival and departure patterns, queue lengths, and congestion costs and fees. These graphs are based on disaggregated traffic rates from the simulation model which are aggregated by ten minute intervals. In each figure, the top panel shows aggregate numbers of arrivals or departures by ten minute intervals between 6:00 AM and 12:00 PM. The second panel shows queue length sampled at the beginning of each interval. The bottom panel illustrates dollars of additional delay that arrivals or departures impose on all other aircraft. For cases with congestion pricing, external congestion levels in the bottom panel are the equilibrium fees. The first four rows of Table IX compare equilibrium average queue lengths, layover times, encroachment times, external congestion levels, airline costs, and social costs before and after imposition of congestion pricing.

The top panels of Figures 8 and 9 depict daily arrival patterns for no-fee and congestion-fee equilibria. The simulated no-fee equilibrium matches actual

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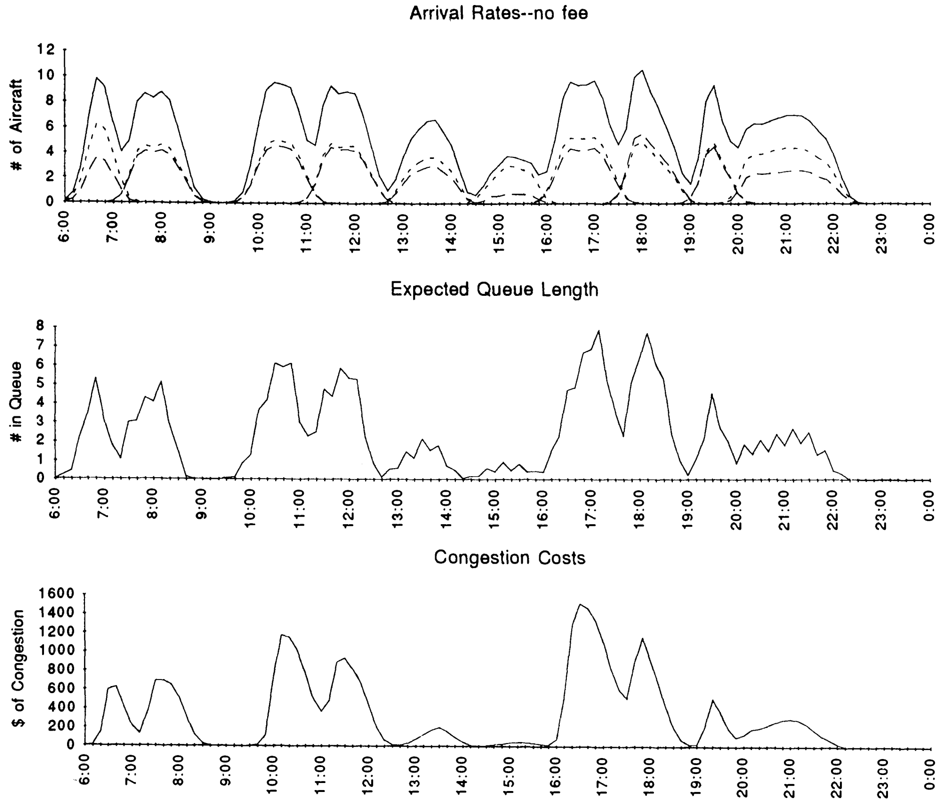

FIGURE 8.—Atomistic no-fee equilibrium (current capacity, current traffic).

arrival patterns well. The graph of congestion-fee equilibrium arrival rates illustrates the extent of peak spreading resulting from congestion pricing. Traffic shifts toward both sides of the peaks, thereby leveling traffic out and spreading it over somewhat longer periods. These arrival rates generally remain between four to seven aircraft per ten minutes, well below capacity. Although some aircraft arrive further in advance of the interchange than without congestion fees, shorter queue lengths enable more aircraft to arrive close to the beginning of the interchange without experiencing significant interchange-encroachment costs. These two effects virtually cancel each other out—average layover time for arrivals increases only from 21.2 to 22.2 minutes. Average interchange-encroachment time per aircraft increases slightly from 6.1 to 7.3 minutes. This slight lengthening of arrival periods in response to congestion pricing contrasts with deterministic models in which congestion pricing causes the traffic rates to equal service rates throughout the peak but does not increase peak-period duration. With stochastic traffic, queueing delay persists even if expected traffic rates equal service rates, so there is an additional tradeoff between queuing delays and schedule delays resulting in longer peak periods.

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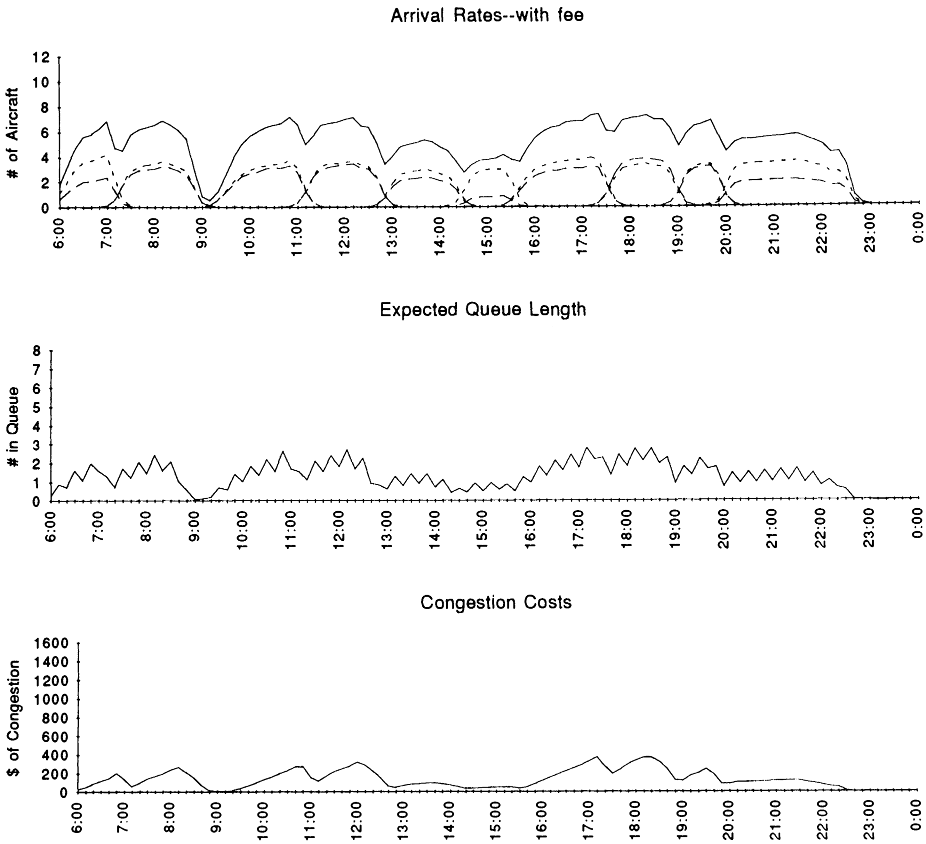

FIGURE 9.—Atomistic congestion-fee equilibrium (current capacity, current traffic).

The middle panels in Figures 8 and 9 illustrate the effect of congestion pricing on expected arrival queues. For the no-fee equilibrium, maximum expected queuing time for arrival during major interchange banks is between five and nine minutes. As with no-fee arrival rates, evolution of expected queues is similar to estimated queues based on actual arrival data depicted in Figure 2. For the congestion-fee equilibrium, expected queues decrease significantly because of peak spreading. The queue increases slowly over the bank period and attains maximum length of only about two or three minutes. As a result of congestion pricing, average arrival queues drop from 3.6 to 1.7 minutes. Reduction in queuing is more dramatic for large arrival peaks such as bank 7 in which average queue lengths fall from 5.1 to 2 minutes. Plots of expected queues are jagged during periods with low arrival rates and short queues because they alternately sample expected queues just before and after service epochs.

The bottom panels in Figures 8 and 9 show additional congestion costs that arriving aircraft impose. The congestion costs include the additional queuing delay and changes in schedule delays experienced by other aircraft. For the

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no-fee equilibrium, aircraft impose maximum external congestion costs during major interchange banks from \$600 to \$1600, depending on the size of the bank. During peak periods, aircraft impose external costs that are several times the costs they experience. Average external-congestion levels peak before arrival rates and queues because earlier arrivals contribute to queuing delay of relatively more aircraft than later arrivals. Graphs of external-congestion costs in congestion-fee equilibria show the dramatic effect that congestion pricing has on congestion levels. External-congestion costs remain quite low throughout most of the arrival banks and achieve peak levels of \$200 and \$400. Plots of external-congestion costs for congestion-fee equilibria are the social-cost-minimizing congestion-fee schedules that the airport would impose to attain these equilibria. The average congestion fee is \$168 for arrivals.

Plots of external-congestion costs illustrate the importance of endogenous treatment of traffic rates. Models that ignore intertemporal adjustments of traffic can overestimate external-congestion levels and result in excessive congestion fees. Allowing for adjustment of traffic rates in response to congestion fees results in much shorter equilibrium queues and lower external congestion.

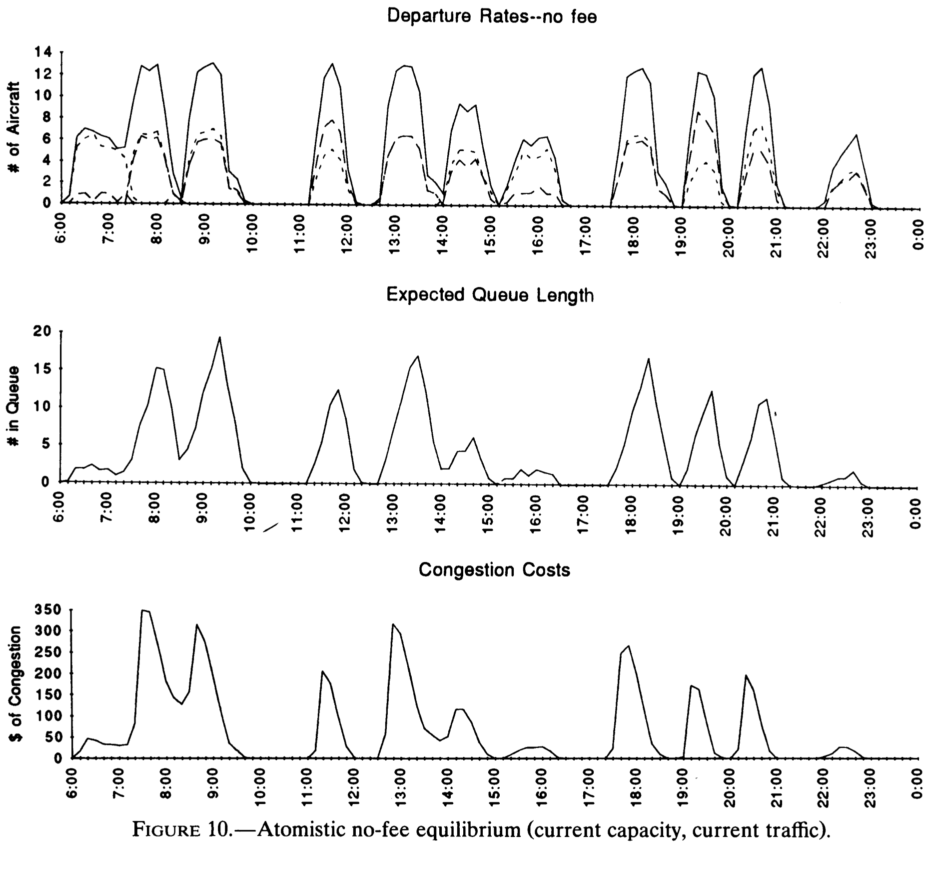

FIGURE 10.—Atomistic no-fee equilibrium (current capacity, current traffic).

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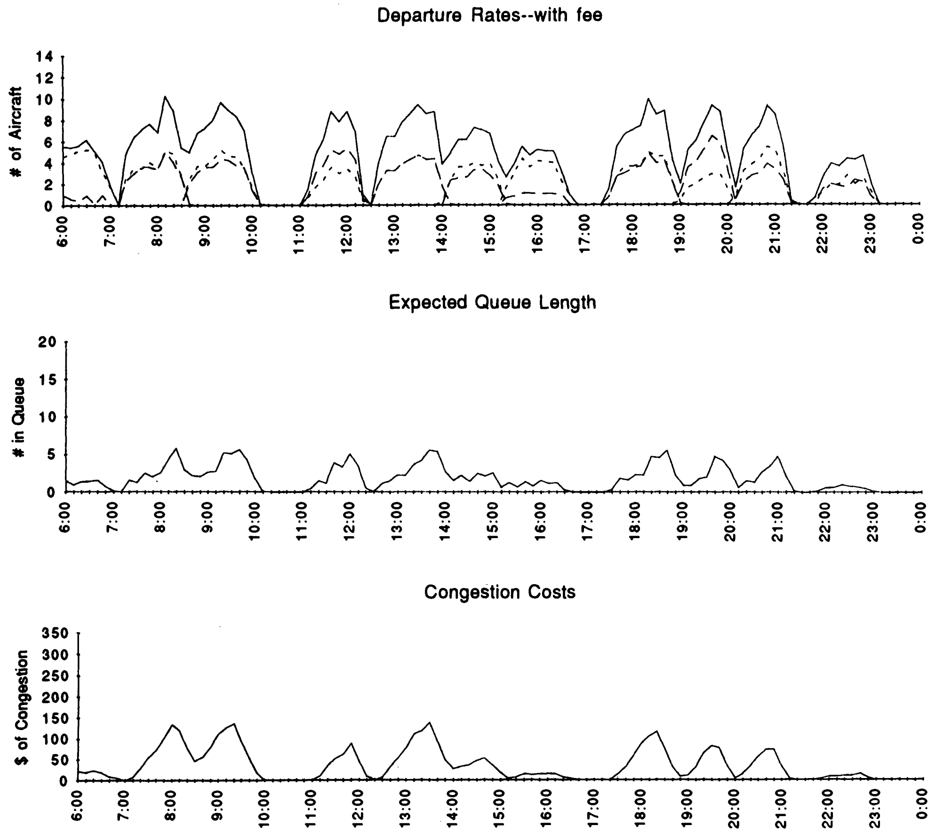

FIGURE 11.—Atomistic congestion-fee equilibrium (current capacity, current traffic).

The top panels in Figures 10 and 11 illustrate simulated daily departure patterns before and after imposition of congestion pricing. In the no fee equilibrium, departure rates fluctuate even more rapidly than arrival rates. Following the interchange, departure rates increase rapidly to about fourteen aircraft per ten minutes, remain at that level for several ten minute intervals, then drop rapidly to near zero between banks. Simulated departure peaks are somewhat lower and broader than actual peaks and have fewer departures between banks.

Imposition of congestion pricing causes departure rates to level out considerably. Departure rates peak at about ten aircraft per ten minutes, but they remain below capacity throughout most of the peak periods. Departure peaks are steeper than arrival peaks for both equilibria because the cost of waiting in departure queues on the taxi way is less than that of circling in the air in the landing queue. Average aircraft layover time following the interchange increases from about 0.7 to 4.3 minutes. Average interchange-encroachment time de-

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creases from 19.0 to 14.5 minutes because congestion pricing shifts departures away from the interchange.

The middle panels of Figures 10 and 11 illustrate the effect of congestion pricing on departure queues. In the no-fee equilibrium, departure queues increase rapidly to between sixteen and twenty-two minutes during major interchange banks. Maximum departure queues in these simulations are similar to those inferred from actual departure data as shown in Figure 2. Imposition of congestion pricing dramatically reduces departure queues. For the congestion-fee equilibrium, queues attain maximum lengths of about five to seven minutes during major interchange banks. Average queue lengths decline from 7.8 to 2.9 minutes.

In contrast to bottleneck models with deterministic queues, congestion pricing does not eliminate expected queues. With stochastic arrivals, eliminating queuing requires spreading out traffic so much that there is no possibility of two arrivals during the same service interval. Further reduction of expected queue lengths would require further spreading of arrival rates, thereby increasing layover costs more than queuing costs fall.

The bottom panels in Figures 10 and 11 show additional congestion costs departing aircraft impose on all other aircraft, including queuing delays and schedule delay. For the no-fee equilibrium, external-congestion costs achieve peak levels of $200 to $350 during major interchange banks. Average external-congestion costs imposed by departing aircraft in the no-fee equilibria are $136. In the congestion-fee equilibrium, the external-congestion costs achieve peak levels of only $75 to $150. The external-congestion costs for congestion-fee equilibria are the social-cost-minimizing departure fee schedules that the airport would impose. The average departure fee is $57. As with arrival rates, endogenous adjustment of departure rates in response to congestion pricing results in much shorter equilibrium queues and lower external-congestion levels than under current traffic patterns.

Table IX shows equilibrium costs of aircraft landings and takeoffs averaged over all banks. Column 8 lists private costs that are the sum of expected queuing, layover, and interchange-encroachment-time costs and (in the congestion-fee case) the fees imposed. The equilibrium private cost must be the same for identical aircraft in a given bank. Under the congestion-fee equilibrium, average private costs increase $122 ($336–$214) for arrivals plus $28 ($116–$88) for departures over the no-fee equilibrium. Eliminating current weight-based fees that average about $100 per aircraft would largely offset this increase. Nevertheless, imposition of congestion pricing would increase private costs on average. In such cases, however, increases in airport revenues more than offset increases in private costs, resulting in net resource savings from reduced congestion.

Columns 11 and 12 of Table IX show the effect of congestion pricing on social costs of aircraft landing and taking off averaged over all banks. Imposition of congestion fees would increase equilibrium private costs by $150 ($122 + $28) per aircraft and would generate $225 ($168 + $57) in airport revenues per

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TABLE IX

AVERAGE DELAYS AND COSTS UNDER VARIOUS CAPACITIES AND TRAFFIC LEVELS

Capacity and Traffic Level	No Fee or Fee	Arrivals or Departures	Average Queue (in minutes)	Average Layover (in minutes)	Average Encroachment (in minutes)	Average Congestion (in dollars)	Equilibrium Private Cost (in dollars)	Average Social Cost (in dollars)	Number of Aircraft	Avg Social Cost Landing & Takeoff (in dollars)	Resource Savings (in dollars)
Current Capacity	No Fee	Arrivals	3.57	21.22	6.12	506	214	214	537	302	73
		Departures	7.81	0.71	18.98	136	88	88	530		
Current Traffic	Fee	Arrivals	1.72	22.17	7.34	168	336	169	537	228	
		Departures	2.94	4.32	14.53	57	116	60	530		
Current Capacity	No Fee	Arrivals	7.65	25.17	12.82	1590	404	404	809	559	173
		Departures	14.60	2.16	24.72	380	155	155	796		
150% Current Traffic	Fee	Arrivals	3.49	34.33	13.42	645	938	293	809	386	
		Departures	4.54	7.69	19.79	148	241	93	796		
150% Current Capacity	No Fee	Arrivals	3.31	20.69	5.86	487	202	202	809	284	67
		Departures	7.11	0.82	18.97	108	81	81	796		
150% Current Traffic	Fee	Arrivals	1.56	22.09	6.66	150	309	159	809	216	
		Departures	2.68	3.81	16.27	51	108	57	796		
150% Current Capacity	No Fee	Arrivals	5.27	24.01	8.74	950	297	297	1074	411	121
		Departures	10.56	1.36	22.78	209	114	114	1060		
200% Traffic	Fee	Arrivals	2.20	26.03	10.11	296	511	214	1074	290	
		Departures	3.26	5.50	19.42	89	164	76	1060		
200% Current Capacity	No Fee	Arrivals	3.13	20.20	5.70	451	194	194	1074	273	65
		Departures	7.00	0.88	18.22	99	79	79	1060		
200% Current Traffic	Fee	Arrivals	1.48	21.46	6.38	140	292	152	1074	208	
		Departures	2.47	3.76	16.14	49	105	56	1060		

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aircraft, resulting in resource savings of $73 per aircraft. The sum of equilibrium landing and takeoff costs in the no-fee case is $302, so net resource savings would be twenty-four per cent of current costs.

9. EFFECTS OF HIGHER TRAFFIC VOLUME AND RUNWAY CAPACITY

This section compares the effects of building additional capacity with imposing congestion pricing as ways of coping with increases in traffic volumes. Airports can increase capacity in one of two ways—build additional runways or increase throughput of existing runways by improving air-traffic-control technology. Technological and safety factors are often not a matter of choice to the airport authority and obviously limit this second method. In the simulation model, building runways corresponds to increasing the number of servers in the queuing system and increasing throughput corresponds to reducing the length of service intervals. This section focuses on the effects of building additional runways.

Table IX summarizes equilibrium levels of delays and costs for simulations involving current capacity and current traffic, current capacity and 150% of current traffic, 150% of current capacity (one more runway) and 150% of current traffic, 150% of current capacity and 200% of current traffic, and 200% of current capacity (two more runways) and 200% of current traffic. The simulations assume that traffic in each bank increases by the same proportion.[12]

As expected, resource savings from imposing congestion pricing increase as congestion levels increase. At current capacity and traffic levels, resource savings are $73 per aircraft whereas at current capacity and 150% of current traffic volume, resource savings are $173 per aircraft. At 150% of current capacity and traffic levels, resource savings are $67 per aircraft; at 150% of current capacity and 200% of current traffic, resource savings are $121 per aircraft; and at 200% of current capacity and traffic, resource savings are $65.

Proportionate increases in both capacity and number of flights in a bank cause slight decreases in average social costs of landings and takeoffs. In no-fee equilibria, average social cost per aircraft is $302 at current levels, $284 at 150% of current levels, and $273 at 200% of current levels. This indicates that the simulation model exhibits slight economies of scale. The principle of massed reserves as described by Mulligan (1983, 1986) states that equi-proportional increases in arrival rates and capacity cause a steady-state (constant, exogenous arrival rate), multiserver queuing system to experience less than proportional increases in queue lengths. These scale economies arise from diminishing expected idle time per server as traffic volume and capacity of the queuing system increase proportionately. Scale economies diminish rapidly, however, with additional servers. This paper extends Mulligan's result to a stochastic queuing system with time-varying, endogenous arrival rates. As capacity and traffic levels increase proportionately, traffic takes advantage of the more efficient queuing system by shifting to shorter and proportionately more peaked

[12] Graphs of these simulations are available from the author upon request.

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banks. Consequently, average queuing time and schedule delay diminish simultaneously, resulting in lower average cost.

Current capacity at MSP could serve about 30% more traffic under congestion pricing, at the same average social cost per aircraft achieved currently without congestion pricing. Thus, congestion pricing presents a costless way of increasing the amount of traffic accommodated by about 30%.

Without congestion pricing, MSP could not practically serve 50% more traffic without increasing capacity. Average social cost of landings and takeoffs would nearly double. Maximum arrival queues would be around sixteen minutes. Maximum departure queues would range between twenty-eight to thirty-nine aircraft. External congestion levels would average $1590 for arriving aircraft and $380 for departing aircraft. Using congestion pricing, however, MSP could accommodate 50% more traffic without increasing capacity. Average social costs would increase a more modest 28%. In this case, arrival rates would be level, at around eight aircraft per ten minutes throughout the whole day. Departure rates would also be fairly smooth, remaining close to eight aircraft per ten minutes. Arrival and departure queues would remain about three and four minutes long respectively. Even with congestion pricing, MSP could not accommodate much more than 150% of current traffic without increasing capacity. At this traffic level, congestion pricing virtually exhausts the benefits of peak spreading and additional traffic would cause explosive growth in queue lengths.

Imposing congestion pricing and increasing capacity 50% by building an additional runway would enable MSP to accommodate more than twice as much traffic at current average social costs per aircraft. With 200% of current traffic and 150% of current capacity, congestion pricing would result in approximately the same landing and takeoff cost per aircraft ($290) as building two additional runways without imposing congestion pricing ($273). Clearly, significant savings are possible from using congestion pricing to delay the need for airport expansion and to reduce the size of eventual expansion projects.

Several simplifying assumptions create potential bias in estimating benefits from congestion pricing. Assuming all flights participate in some bank and all flights within a given bank have the same preferred arrival or departure times (τ^A or τ^D) biases estimates upward. Assuming that nonnetwork flights have as strong preferences for scheduling near the interchange as the hub-and-spoke network flights biases estimates downward, because the model overstates the cost of shifting these flights away from the peak. The model's ability to match existing traffic patterns provides a test of the seriousness of these biases. Since simulated traffic patterns match actual traffic patterns reasonably well, the biases must not be large. Assuming bank and route service frequencies are exogenous also biases estimates downward because some routes would shift to less frequent service with larger aircraft, thereby reducing congestion costs on the route. This bias should also be small because elimination of current landing fees would largely offset increases in equilibrium costs from congestion pricing. Assuming homogeneous aircraft and inelastic traffic demand also biases estimates downward, because congestion pricing would disproportionately increase

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costs of commuter flights, general aviation, and other smaller aircraft. These aircraft can adjust their schedules at lower cost than larger aircraft. Large aircraft would obtain most of the benefits of operating in peaks with lower congestion.

10. CONCLUSION

Flight banks associated with MSP's hub-and-spoke air service network create frequent and rapid fluctuation in airport traffic rates. Demand peaking means congestion pricing can generate large savings by smoothing out demand. The model predicts significant effects from intertemporal traffic adjustments. Changes in traffic patterns, queuing times, and congestion costs are missing from studies that do not model intertemporal traffic adjustment.

This paper focuses on airport pricing to alleviate congestion externalities. A related externality arises from the business stealing incentive which results in scheduling of flights together at popular times and too many flights serving each route (see, e.g., Panzar (1979), Dorman (1983), Borenstein (1988), and Borenstein and Netz (1991)). In monopolistically competitive markets, equilibrium locations and number of firms are generally not optimal. Even without airport congestion, equilibrium flight schedules will typically not be socially efficient. The gains from airport pricing could be higher if excessive crowding occurs from competitive reasons as well as because of congestion externalities. This is an important issue for further research that requires extending the model to include elastic demand for participating in a bank.

This paper raises a number of other issues for future research. It assumes all aircraft in a given bank have the same most-preferred arrival time. Relaxing this assumption would make the model more applicable to airports with more nonhub traffic. The model also assumes all aircraft have identical cost characteristics. Allowing for nonhomogeneous aircraft would enable investigation of the distributional consequences of congestion pricing for general aviation, commuter traffic, air freight, and commercial aviation. The model uses traffic rates for a typical day. A mix of different traffic conditions due to adverse weather conditions and weekly and seasonal demand fluctuations would probably increase gains from congestion pricing. The model ignores queuing of aircraft waiting for access to a terminal gate. At some airports, this is an important source of congestion delay. A queuing network could jointly model congestion delays on runways and gates. While the model captures the first-order effects of congestion pricing on a single airport, it ignores the second-order, system-wide effects on a network of airports. Additional research on the effect of congestion pricing on flight schedules between a network of airports is desirable. Finally, additional research on sources of economies and diseconomies of scale in airport operation is necessary to address airport capacity issues.

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APPENDIX A

(A.1)

$$Q_t = \underbrace{\left[\begin{array}{ccc}
\frac{(\lambda_t)^0(e)^{-\lambda_t}}{0!} & \cdots & \frac{(\lambda_t)^0(e)^{-\lambda_t}}{(0)!} \\
\frac{(\lambda_t)^1(e)^{-\lambda_t}}{(1)!} & \cdots & \frac{(\lambda_t)^1(e)^{-\lambda_t}}{(1)!} \\
\frac{(\lambda_t)^2(e)^{-\lambda_t}}{(2)!} & \cdots & \frac{(\lambda_t)^2(t)^{-\lambda_t}}{(2)!} \\
\vdots & \vdots & \vdots \\
\frac{(\lambda_t)^{K-1}(e)^{-\lambda_t}}{(K-1)!} & \cdots & \frac{(\lambda_t)^{K-1}(e)^{-\lambda_t}}{(K-1)} \\
1-\sum_{r=0}^{K-1}\frac{(\lambda_t)^r(e)^{-\lambda_t}}{r!} & \cdots & 1-\sum_{r=0}^{1-1}\frac{(\lambda_t)^r(e)^{-\lambda_t}}{r!}
\end{array}\right.}_{S+1 \text{ terms}}$$

$$\underbrace{\left.\begin{array}{ccccc}
0 & 0 & 0 & \cdots & 0 \\
\frac{(\lambda_t)^0(e)^{-\lambda_t}}{0!} & 0 & 0 & \cdots & 0 \\
\frac{(\lambda_t)^1(e)^{-\lambda_t}}{(1)!} & \frac{(\lambda_t)^0(e)^{-\lambda_t}}{(0)!} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{(\lambda_t)^{K-2}(e)^{-\lambda_t}}{(K-2)} & \frac{(\lambda_t)^{K-3}(e)^{-\lambda_t}}{(K-3)} & \cdots & \cdots & \frac{(\lambda_t)^{s-1}(e)^{-\lambda_t}}{(s-1)!} \\
1-\sum_{r=0}^{K-2}\frac{(\lambda_t)^r(e)^{-\lambda_t}}{r!} & 1-\sum_{r=0}^{K-3}\frac{(\lambda_t)^r(e)^{-\lambda_t}}{r!} & \cdots & \cdots & 1-\frac{(\lambda_t)^{s-1}(e)^{-\lambda_t}}{(s-1)!}
\end{array}\right]}_{K-S \text{ terms}}$$

At most S departures from the queue are possible during a period, but any number of arrivals may occur. The element in the ith row and jth column of Q_t gives the probability that there will be $i-1$ aircraft in the queue at the beginning of the next period, conditional upon there being $j-1$ aircraft in the queue at the beginning of the current period. Thus, the first row of W_t says that the probability of there being no queue in the next period is equal to the sum of: the probability of no arrivals during the current period times the probability that there are currently no aircraft in the queue; plus the probability of no arrivals times the probability that there is one aircraft currently in the queue; ...; plus the probability of no arrivals times the probability that there are S aircraft currently in the queue. The second row says that the probability of one aircraft being in the queue next period is equal to the sum of: the probability of one arrival and none currently in queue; plus the probability of one arrival and one currently in queue; ...; plus the probability of one arrival and S aircraft currently in the queue; plus the probability of no arrivals and $S+1$ currently in queue. The final row says that the probability of K aircraft in the queue in the next period is the sum of the probabilities of: K or more arrivals and no aircraft currently in queue; K or more arrivals and one

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aircraft currently in queue; ...; K or more arrivals and S aircraft currently in queue; $K-1$ or more arrivals and $S+1$ aircraft currently in queue; ...; and S or more arrivals and K aircraft currently in queue.

APPENDIX B

Let x_{ij}^{t} denote the element in the ith row jth column of Q_t. It follows that:

(A.2) $$q_{u+1k} = x_{k1}^{u} q_{u1} + x_{k2}^{u} q_{u2} + \cdots + x_{kK}^{u} q_{uK}, \quad \text{and}$$

$$\frac{\partial q_{u+1}k}{\partial \lambda_u} = \frac{\partial x_{k\,l}^{u}}{\partial \lambda_u} q_{ul} + \frac{\partial x_{k2}^{u}}{\partial \lambda_u} q_{u2} + \cdots + \frac{\partial x_{kK}^{u}}{\partial \lambda_u} q_{uK}.$$

Now define the matrix D_t with $\partial x_{ij}^{u} / \partial \lambda_t$ as its (i, j) element:

(A.3)
$$e^{-\lambda_t} \left[\underbrace{\begin{matrix}
\frac{0(\lambda_t)^{-1} - (\lambda_t)^0}{0!} & \cdots & \frac{0(\lambda_t)^{-1} - (\lambda_t)^0}{0!} \\
\frac{1(\lambda_t)^{0} - (\lambda_t)^1}{(1)!} & \cdots & \frac{1(\lambda_t)^{0} - (\lambda_t)^1}{(1)!} \\
\frac{2(\lambda_t)^{1} - (\lambda_t)^2}{(2)!} & \cdots & \frac{2(\lambda_t)^{1} - (\lambda_t)^2}{(2)!} \\
\vdots & \vdots & \vdots \\
\frac{(K-1)(\lambda_t)^{K-2} - (\lambda_t)^{K-1}}{(K-1)} & \cdots & \frac{(K-1)(\lambda_t)^{K-2} - (\lambda_t)^{K-1}}{(K-1)} \\
\sum_{r=0}^{K-1} \frac{(\lambda_t)^r - r(\lambda_t)^{r-1}}{r!} & \cdots & \sum_{r=0}^{K-1} \frac{(\lambda_t)^r - r(\lambda_t)^{r-1}}{r!}
\end{matrix}}_{S+1 \text{ terms}} \right.$$

$$\left. \underbrace{\begin{matrix}
0 & 0 & 0 & \cdots & 0 \\
\frac{0(\lambda_t)^{-1} - (\lambda_t)^0}{0!} & 0 & 0 & \cdots & 0 \\
\frac{1(\lambda_t)^{0} - (\lambda_t)^1}{(1)!} & \frac{0(\lambda_t)^{-1} - (\lambda_t)^0}{01} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{(K-2)(\lambda_t)^{K-3} - (\lambda_t)^{K-2}}{(K-2)} & \frac{(K-3)(\lambda_t)^{K-4} - (\lambda_t)^{K-3}}{(K-3)} & \cdots & \cdots & \frac{(S-1)(\lambda_t)^{S-2} - (\lambda_t)^{S-1}}{(S-1)} \\
\sum_{r=0}^{K-2} \frac{(\lambda_t)^r - r(\lambda_t)^{r-1}}{r!} & \sum_{r=0}^{K-3} \frac{(\lambda_t)^r - r(\lambda_t)^{r-1}}{r!} & \cdots & \cdots & \sum_{r=0}^{S-1} \frac{(\lambda_t)^r - r(\lambda_t)^{r-1}}{r!}
\end{matrix}}_{k-S \text{ terms}} \right]$$

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The derivatives $\partial q_{u+1}k/\partial\lambda_u$ can be written in vector-matrix notation:

$$\text{(A.4)}\qquad \begin{bmatrix} \dfrac{\partial q_{u+1\,1}}{\partial\lambda_u} \\ \dfrac{\partial q_{u+1\,2}}{\partial\lambda_u} \\ \vdots \\ \dfrac{\partial q_{u+1\,K}}{\partial\lambda_u} \end{bmatrix} = \mathbf{D}_u \begin{bmatrix} q_{u1} \\ q_{u2} \\ \vdots \\ q_{uK} \end{bmatrix}.$$

Using the chain rule and the Markov property of the queuing evolution, the derivatives $\partial q_{v\,k}/\partial\lambda_u$ for $v \geq u+2$ can be written as:

$$\text{(A.5)}\qquad \frac{\partial q_{v\,k}}{\partial\lambda_u} = \frac{\partial q_{v\,k}}{\partial q_{v-1\,1}}\frac{\partial q_{v-1\,1}}{\partial\lambda_u} + \frac{\partial q_{v\,k}}{\partial q_{v-1\,2}}\frac{\partial q_{v-1\,2}}{\partial\lambda_u} + \cdots + \frac{\partial q_{v\,k}}{\partial q_{v-1\,K}}\frac{\partial q_{v-1\,K}}{\partial\lambda_u}.$$

Noting that $\partial q_{v\;i}/\partial q_{v-1\;j} = x_{ij}^{v-1}$, the derivatives $\partial q_{v\;k}/\partial\lambda_u$ can be written in vector-matrix notation as:

$$\text{(A.6)}\qquad \begin{bmatrix} \dfrac{\partial q_{v\,1}}{\partial\lambda_u} \\ \dfrac{\partial q_{v\,2}}{\partial\lambda_u} \\ \vdots \\ \dfrac{\partial q_{v\,K}}{\partial\lambda_u} \end{bmatrix} = \mathbf{Q}_{v-1} \begin{bmatrix} \dfrac{\partial q_{v-1\,1}}{\partial\lambda_u} \\ \dfrac{\partial q_{v-1\,2}}{\partial\lambda_u} \\ \vdots \\ \dfrac{\partial q_{v-1\,K}}{\partial\lambda_u} \end{bmatrix}.$$

It follows that:

$$\text{(A.7)}\qquad \begin{bmatrix} \dfrac{\partial q_{t\,1}}{\partial\lambda_u} \\ \dfrac{\partial q_{t\,2}}{\partial\lambda_u} \\ \vdots \\ \dfrac{\partial q_{t\,K}}{\partial\lambda_u} \end{bmatrix} = \mathbf{Q}_{t-1}\mathbf{Q}_{t-2}\cdots\mathbf{Q}_{u+2}\mathbf{Q}_{u+1}\mathbf{D}_u\mathbf{q}_u.$$

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