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Author(s): Jan K. Brueckner
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NETWORK STRUCTURE AND AIRLINE SCHEDULING*

JAN K. BRUECKNER†

This paper provides a simple analysis of the effects of network structure on the scheduling, traffic, and aircraft size choices of a monopoly airline. The analysis shows that switching to a hub-and-spoke network leads to increases in both flight frequency and aircraft size, while stimulating local traffic in and out of the hub. In addition, HS networks are shown to be preferred by the airline when travel demand is low, when flights are expensive to operate, and when passengers place a high value on flight frequency but are not excessively inconvenienced by the extra travel time required for a connecting trip. The welfare analysis shows that the flight frequency, traffic volumes, and aircraft size chosen by the monopolist are all inefficiently low under both network types. Moreover, in the most plausible case, the monopolist's network choice exhibits an inefficient bias toward the HS network, apparently reflecting an excessive desire to economize on the number of flights.

I. INTRODUCTION

UNDER THE REGULATED SYSTEM that existed prior to 1980, U.S. airlines faced constraints on both their fares and route structures. Fare changes required regulatory approval, as did entry or exit on particular airline routes. Service quality, however, was unconstrained, making this arena the principal locus of competition between the carriers. But paradoxically, many observers viewed service-quality competition as leading to some undesirable outcomes. In particular, competition on flight frequency, an important element of service quality, was thought to generate excessive frequencies and correspondingly low load factors (percentages of seats filled), raising the carriers' cost per passenger (see Douglas and Miller [1974] and Morrison and Winston [1986]).

With the airlines free to compete on price following deregulation, less vigorous competition on service quality might have been expected, leading to a reduction in the flight frequencies that were widely viewed as excessive. However, as documented by Morrison and Winston [1986, 1995], exactly the opposite occurred. Morrison and Winston show that a route-weighted

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measure of flight frequency *rose* by 9.2 percent between 1977 and 1983, generating passenger benefits in excess of $10 billion per year.

While this outcome appears to present a puzzle, an explanation comes from considering another important impact of deregulation. In particular, by giving the airlines freedom of entry and exit, deregulation allowed a rationalization of route structures, leading to the growth of hub-and-spoke (HS) networks. By concentrating traffic on the spoke routes in and out of hub airports, the airlines could exploit 'economies of traffic density,' or increasing returns at the route level. While HS networks thus reduce cost per passenger, helping to account for the decline in real airfares since deregulation, their growth has also affected service quality. Although time-consuming hub layovers for connecting passengers represent a negative effect, Morrison and Winston argue that HS networks are partly responsible for the documented increase in flight frequency, which has been an important source of passenger benefits. According to this view, upward pressure on frequencies from HS networks more than offset downward pressure from softer service-quality competition, leading to a resolution of the above puzzle.

Theoretical research has shown how exploitation of economies of density in HS networks puts downward pressure on airfares, and a number of empirical studies have confirmed this prediction.[1] However, only two theoretical papers, Berechman and Shy [1998] and Brueckner and Zhang [2001], analyze the connection between network structure and airline scheduling. As a result, the theoretical basis for Morrison and Winston's network-based explanation for flight frequency growth remains relatively undeveloped. To deepen our understanding of scheduling in networks, the present paper offers a model that improves upon previous work. The model generalizes the framework of Brueckner and Zhang [2001] (which in turn built upon Berechman and Shy [1998]) by eliminating several restrictive assumptions. The result is a framework that captures nearly all the key elements of the airline's optimization problem.

In the model, a passenger has a preferred departure time and dislikes 'schedule delay,' which equals the difference between the actual and preferred departure times. Since schedule delay falls on average as flight frequency increases, the passenger's willingness-to-pay for air travel rises with frequency, shifting the demand curve outward. Frequency is costly,

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[1] Contributions to the theoretical literature include Spiller [1989], Bittlingmayer [1990], Brueckner and Spiller [1991], Berechman, Poddar and Shy [1998], Zhang and Wei [1993], Hendricks, Piccone and Tan [1995, 1997], Oum, Zhang and Zhang [1995], Nero [1996], Zhang [1996], Park [1997], Berechman and Shy [1998], Nero and Black [1998], Pels, Nijkamp and Rietveld [1997, 2000] and Brueckner [2001]. Lederer [1993a,b] analyzes HS networks from an operations-research perspective. Empirical papers include Caves, Christensen and Tretheway [1984], Brueckner, Dyer and Spiller [1992], and Brueckner and Spiller [1994].

however, and the airline balances the demand-side gains from greater frequency against these higher costs in finding the optimum.[2]

Although the analysis handles demand somewhat differently than in Brueckner and Zhang [2001] (hereafter BZ), the main departure from their model lies in the assumptions on airline costs. The present model assumes that each flight entails a fixed cost as well as a variable cost per seat. As a result, cost per passenger falls with aircraft size, capturing the well-known economies from operation of larger aircraft. By contrast, BZ assumed a fixed cost but no variable cost, so that a flight could costlessly accommodate additional passengers (in effect, aircraft are so large they never become full). Because of these unrealistic assumptions, the relevance of BZ's analysis could be questioned, suggesting the need for the present generalization.

To focus on the effect of network structure on scheduling, the model portrays the simplest possible case, where a monopoly airline serves just three cities. In a point-to-point, or 'fully-connected' (FC) network, the airline operates three routes, connecting each pair of cities. In a hub-and-spoke (HS) network, one city serves as the hub, and the airline operates only two routes. Passengers in one of the city-pair markets must then make a connecting trip, passing through the hub. The goal of the analysis is to compare the solutions to the airline's optimization problem for the FC and HS cases. In addition to comparing flight frequencies, the analysis compares aircraft sizes, passenger volumes and fares between the two network structures.

If passenger volumes in each of the three city-pair markets were fixed along with aircraft sizes, then the effect of network structure on flight frequency would be transparent. Because of the need to send passengers in one of the three city-pair markets through the hub, the traffic volume on each of the two HS routes would exceed that on each of the three FC routes. With fixed aircraft sizes, more frequent flights would then be required to accommodate the greater traffic per route in the HS case. However, since the model portrays the airline as choosing aircraft sizes and passenger volumes along with flight frequency, this simple argument does not apply directly. However, the analysis shows that adjustments along these margins do not overturn the above conclusion: flight frequency is shown to be higher in the HS than in the FC network.

In addition, the analysis shows that the airline operates larger aircraft in the HS case while carrying more passengers in at least two of the three city-pair markets. In these two markets, where 'local' passengers travel nonstop between the hub and the two nonhub cities, the higher traffic volume is accompanied by higher, not lower, fares. The reason is that higher flight

frequencies raise a passenger's willingness-to-pay, shifting the demand curve outward. Like a similar finding in BZ, this result contradicts the standard view, which argues that lower cost per passenger in an HS network should lead to lower fares for local passengers. This view, however, overlooks the effect of higher frequency, which means that the HS network offers a better quality product to local passengers. By contrast, the effects of the HS network on fares and traffic in the third city-pair market, where passengers must connect at the hub rather than traveling nonstop, are ambiguous. The reason is that the longer HS travel time dilutes the gain from higher flight frequency, making the demand shift ambiguous for these passengers, while the cost of serving them rises (they require seats on two flights rather than one).

The paper also analyzes the airline's choice between the HS and FC networks, showing how parameter changes alter the identity of the preferred network. For example, the analysis shows that the HS network is preferred when the demand for air travel is low. Finally, the paper presents a welfare analysis, contrasting the profit-maximizing and socially optimal values of flight frequency and the other variables. While the monopolist's decisions regarding these variables are generally inefficient, the same conclusion applies to the choice of network type (HS vs. FC), with the social planner sometimes preferring a different type than the airline.

Overall, the results of the analysis appear to recapitulate many of the observed characteristics of HS networks, suggesting that the model may capture some essential features of the optimization problem confronting a network carrier. However, a drawback to the analysis is that, for reasons of tractability, it focuses on the monopoly case. It is likely, however, that many of the conclusions would carry over to an oligopoly version of the model. Such an extension could be a task for future research.

Section 2 of the paper presents the setup of the model, while section 3 analyzes the FC network. Section 4 analyzes the HS network, and Section 5 compares flight frequency and the other choice variables between the two network types. Section 6 analyzes the airline's network choice, and Section 7 presents the welfare analysis. Section 8 offers conclusions.

II. MODEL SETUP

The monopoly airline serves three symmetrically located cities, A, B and H, as shown in Figure 1. Demand for travel exists between each pair of cities, yielding three city-pair markets: AH, BH and AB. While round-trip travel occurs in both directions in each market (i.e., A to B and back, B to A and back), the analysis focuses on the demand for one-way travel in a single direction in each market, recognizing that symmetric one-way trips also occur in the other direction.

In an FC network, the airline operates flights between each pair of cities, so that nonstop travel occurs in each city-pair market. In an HS network, the
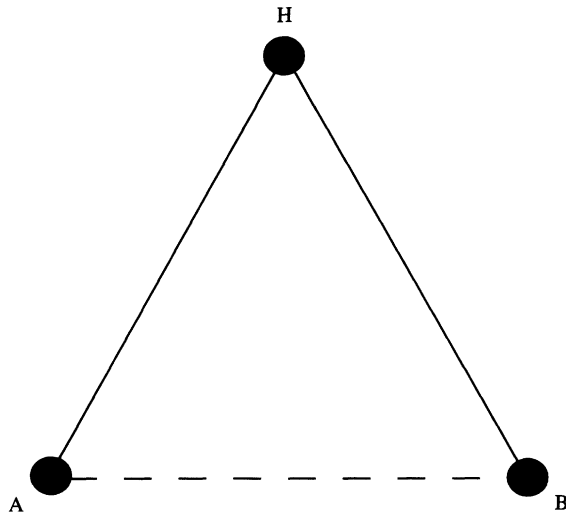
Figure 1
Network Structure

airline operates flights on only two routes, those connecting cities A and B to the hub H. Although passengers in markets AH and BH still enjoy nonstop service, AB passengers must make a connecting trip, changing planes at the hub. Thus, flights in the HS network carry both local traffic (passengers in markets AH or BH) and connecting traffic (passengers in market AB).

Travel demand is identical in the three city-pair markets, and it is derived as follows. Consumer utility is given by $u = C + B - time\ cost$, where C is consumption and B is travel benefit, which varies across consumers. Time cost consists of two components, the cost of schedule delay and the cost of actual travel time. The latter cost equals G for nonstop travel between any pair of cities. To derive the cost of schedule delay, suppose that airline flights are evenly spaced around the clock, with T denoting the number of available hours. Then, letting f denote the number of flights, the time interval between flights is T/f, and average time to the nearest flight is a quarter of this value, $T/4f$. Suppose that consumers' preferred departure times are uniformly distributed around the clock and that each hour of discrepancy between the preferred and actual flight time generates a schedule-delay cost of δ. Then, the average schedule-delay cost is $\delta T/4f$. Assuming this average value is relevant for each consumer, utility is then equal to $C + B - G - \delta T/4f$.

This formulation involves the crucial simplification that each consumer cares about *average* schedule delay even though desired departure times are heterogeneous. To justify this simplification, one approach is to imagine that consumers commit to travel before knowing their preferred departure times. If these times are randomly drawn from a uniform distribution, then average

schedule delay as computed above is relevant. Alternatively, consumers may know their preferred departure times when commiting to travel as well as the frequency of available flights, but they may not know the exact departure times. Viewing these (evenly spaced) times as randomly distributed around the clock, average schedule delay is again relevant.[3]

To derive the demand curve from this information, let Y denote the common level of consumer income and p denote the airfare, so that $C = Y - p$. Assuming that no benefits are enjoyed when a consumer does not travel, the utility from this option is Y. As a result, a consumer will undertake travel when

$$(1) \qquad Y - p + B - G - \delta T/4f \geqslant Y,$$

or when travel benefit satisfies $B \geqslant p + G + \delta T/4f$. Then, suppose that B (which is consumer-specific) has a uniform distribution with support $[\underline{B}, \bar{B}]$ and density $\eta = 1/(\bar{B} - \underline{B})$, implying that the total mass of consumers is unity. The number of consumers traveling then equals

$$(2) \qquad q = \int_{p+G+\delta T/4f}^{\bar{B}} \eta \, dB = (\bar{B} - p - G - \delta T/4f)\eta.$$

The inverse demand curve, which comes from solving (2) for p, is given by

$$(3) \qquad p = \alpha - \beta q - \gamma/f,$$

where $\alpha = \bar{B} - G$, $\beta = 1/\eta$, and $\gamma = \delta T/4$. The key feature of (3) is that, by reducing average schedule delay, an increase in flight frequency induces more consumers to travel, leading to an upward shift in the inverse demand function.

While (3) applies to nonstop travel, connecting travel through the hub for AB passengers involves an additional time cost of G as well as the cost of layover time. Letting the sum of these costs be denoted μ, the demand for connecting travel in market AB is given by (3) with α replaced by $\alpha - \mu$. Unrealistically, the layover portion of μ is taken to be independent of f.[4]

Linear demand curves are commonly used to simplify the analysis of a variety of economic models, and the preceding discussion shows that a linear function emerges in the present framework only after a number of strong

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[3] BZ took the opposite approach to handling heterogeneity in preferred departure times. In their model, consumers whose preferred departure times are closest to the actual flight time enjoy the highest utilities, while consumers for whom schedule delay is sufficiently large choose not to travel. While all passengers end up traveling when the fare is low, this latter effect is the only avenue by which price affects demand in BZ's model (the source of price sensitivity in the present framework is explained next).

[4] Note that a change in G affects both α and μ. Consequently, in the comparative-static calculations reported below, where α and μ are varied independently, this variation should be viewed as arising from changes in $\bar{B}$ and in the layover component of μ (G is held fixed).

assumptions are imposed. However, given that a general analysis of the effect of network structure on airline scheduling is not feasible, such assumptions are needed to make any headway on the problem.[5]

The assumptions on airline costs are easily stated. Let s denote the number of seats per flight, which is a choice variable for the airline. Then, the operating cost per flight is given by

$$(4) \qquad c(s) \equiv \theta + \tau s,$$

with θ equal to fixed cost and τ equal to marginal cost per seat. Given (4), cost per seat equals $\theta/s + \tau$, a decreasing function of s. Although this cost formulation is not entirely realistic, it captures the economies from operating larger aircraft in the simplest possible fashion.

A final assumption is that all aircraft seats are filled, with the load factor equal to 100 percent. Under this assumption, f, q, and s are related by the equation

$$(5) \qquad s = q/f.$$

In other words, on a given route, seats per flight must equal total passengers divided by the number of flights. The analysis would, of course, be unaffected if the load factor were fixed at a value realistically less than 100 percent.

In contrast to this approach, a more realistic model would allow the load factor to be endogenous. One way of achieving this endogeneity would be to assume a fixed aircraft size, with passengers per flight (q/f) constrained to be less than or equal to the fixed s. However, by suppressing the aircraft-size decision, this approach would remove an important focus of the analysis. Apparently, a formulation with stochastic demand would be required to construct a proper model where both s and the load factor are endogenous. Then, flights would have empty seats when demand realizations are low.[6]

III. ANALYSIS OF THE FC NETWORK

To analyze the FC network, the first step is to note that total cost per route is equal to $fc(s)$, which in turn equals $f[\theta + \tau(q/f)]$ using (4) and (5). Simplifying, cost can be written $\theta f + \tau q$, or fixed cost per flight times f plus variable cost times total passengers. Recalling that the airline operates three nonstop

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[5] A slight generalization of the above setup could be achieved by allowing income and the utility from not traveling (u_0) to be consumer specific, and by assuming that $Y + B - u_0$ is uniformly distributed in the population.

[6] An alternative approach would view s as passengers (rather than seats) per flight. Then, (4) would be viewed as a reduced-form equation that subsumes the airline's choices regarding aircraft size and load factor, yielding a cost per passenger that falls with the number of passengers per flight. Once again, however, this approach prevents a focus on the key aircraft-size decision.

routes in the FC case and using (3), profit then equals

(6) $\pi = 3[q(\alpha - \beta q - \gamma/f) - \theta f - \tau q]$.

In maximizing (6), f is treated as a continuous variable, an approach that only makes sense if the number of flights is large. The first-order conditions for choice of q and f are then

(7) $\alpha - 2\beta q - \gamma/f - \tau = 0$

(8) $q\gamma/f^2 - \theta = 0$.

The first condition says that the number of passengers is set optimally when marginal revenue as a function of q equals the marginal cost of a seat, τ. The second condition says that f is set optimally, holding total traffic fixed, when fixed cost per flight (θ) equals the revenue gain from an extra flight, which is given by passengers q times the fare increase per passenger (γ/f^2) made possible by greater frequency. Note that, with q and f determined by (7) and (8), the optimal s can be recovered from (5).

For future reference, (7) can be rearranged to show that

(9) $q = \dfrac{\alpha - \tau - \gamma/f}{2\beta}$.

Eliminating q in (8) using (9) and rearranging, the following condition determining f then emerges:

(10) $2\beta\theta f^3/\gamma = (\alpha - \tau)f - \gamma$.

This condition is shown in Figure 2, where the S-shaped curve is the graph of the cubic expression on the LHS and the line represents the RHS. Note that since $\alpha - \tau$ must be positive for (7) to hold, the line is necessarily upward sloping. The Figure shows the economically relevant case where (9) has three distinct solutions. Other possibilities include the case of no positive real solutions (where the line lies below the curve everywhere in the first quadrant), and the case of two repeated positive solutions (where the line is tangent to the curve in the first quadrant). Note that the latter case can be disrupted, yielding the second case, by a slight parameter change. The following result identifies the optimum:

Lemma. The second positive solution in Figure 2 represents the optimum. The second-order condition for the airline's optimization problem holds at this solution, being violated at the first positive solution.

The lemma, as well as all subsequent results, are proved in the appendix.

Although the lemma is crucial in the later comparison of the FC and HS solutions, it also immediately generates a number of intuitively appealing
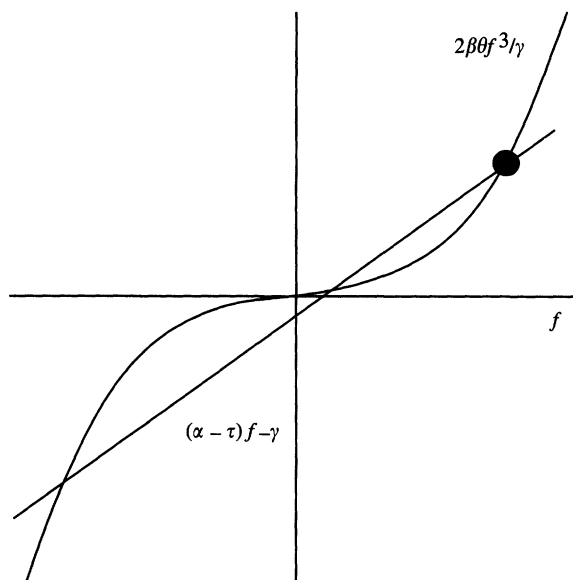
Figure 2
The f Solution

comparative-static results. Most of these results can be derived by inspection of Figure 2. For example, when the demand intercept α increases, the slope of the line in the Figure increases, and the second intersection moves to the right, raising f. With both α and f rising, the numerator of (9) then increases, raising q. Thus, a parallel outward shift in the demand curve raises both traffic and flight frequency, a natural conclusion. A full statement of the comparative-static results is as follows:

Proposition 1. Flight frequency f rises when demand increases (when α rises or β falls), when fixed or variable cost falls (when θ or τ falls), and when the disutility from schedule delay rises (when γ rises). Traffic q moves in step with flight frequency, except that it responds ambiguously to an increase in γ.

As a corollary to Proposition 1, the effects of parameter changes on aircraft size can also be derived. Recalling that $s = q/f$, it follows from (8) that $s = \theta f/\gamma$. Using this relationship along with the f effects from Proposition 1, it follows that s increases when demand rises (when α rises or β falls) and when variable cost falls (when τ falls), both natural results. However, since f moves in the same direction as γ and in the opposite direction to θ, the effects of both these parameters on aircraft size are ambiguous.

IV. ANALYSIS OF THE HS NETWORK

In the HS case, flight frequency on each of the two routes is denoted f_h. 'Local' traffic (passenger volume in each of markets AH and BH) is denoted q_h, while connecting traffic (passenger volume in market AB) is denoted Q. Note that since the HS network creates an asymmetry between the AB market and the other two markets, q_h and Q will not be equal.

As in past analyses of pricing in HS networks, the fare for connecting travel by AB passengers is set independently of fares for local passengers. In other words, the cost of a connecting trip is *not* simply the sum of the fares from A to H and from H to B. From (3), these local fares are given by $p_h = \alpha - \beta q_h - \gamma/f_h$. Analogously, the connecting fare is equal to $P = \alpha - \mu - \beta Q - \gamma/f_h$, where the demand intercept from (3) has been reduced by the extra travel cost term μ. It is important to note that while p_h and P are set independently using the two demand curves, the fares must satisfy an arbitrage condition. This condition says that an AB passenger must not be able to travel more cheaply by purchasing two local tickets, and it is written $P < 2p_h$. Because the condition is automatically satisfied under the maintained assumptions, as shown below, it need not be imposed explicitly as a constraint in the airline's optimization problem.

Since connecting passengers must travel on both HS routes, passenger volume on each is equal to $q_h + Q$, yielding an aircraft size of $s_h = (q_h + Q)/f_h$. Total airline costs are then $2f_h(\theta + \tau(q_h + Q)/f_h) = 2\theta f_h + 2\tau(q_h + Q)$. With the airline earning revenue from two local markets as well as the connecting market, HS profit is then[7]

$$
(11) \quad \begin{aligned} \pi^h = {} & 2q_h(\alpha - \beta q_h - \gamma/f_h) + Q(\alpha - \mu - \beta Q - \gamma/f_h) \\ & - 2\theta f_h - 2\tau(q_h + Q). \end{aligned}
$$

The first-order conditions for choice of q_h, Q, and f_h are

$$
(12) \quad \alpha - 2\beta q_h - \gamma/f_h - \tau = 0
$$

$$
(13) \quad \alpha - \mu - 2\beta Q - \gamma/f_h - 2\tau = 0
$$

$$
(14) \quad \gamma(2q_h + Q)/f_h^2 - 2\theta = 0.
$$

While the interpretation of (12) and (13) is the same as in the FC case, note that the marginal cost of a connecting passenger in (13) (which is equated to MR) is now 2τ since that passenger travels on two flights. Also, to interpret (14), note that an increase in f_h on both routes costs 2θ while allowing a fare increase of γ/f_h^2 for $2q_h + Q$ passengers, with the loss and gain equated at the optimum.

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[7] Note that the cost of operating the hub itself is assumed to be zero.

Using (12) and (13) to solve for q_h and Q yields

$$(15) \qquad q_h = \frac{\alpha - \tau - \gamma/f_h}{2\beta}$$

$$(16) \qquad Q = \frac{\alpha - \mu - 2\tau - \gamma/f_h}{2\beta}.$$

Note that (15) and (16) imply $q_h > Q$, so that local traffic exceeds connecting traffic. This result is a consequence of the higher costs of transporting a connecting passenger combined with the demand reduction due to longer travel time.

Substituting (15) and (16) into (14) and rearranging yields the condition that determines f_h:

$$(17) \qquad 2\beta\theta f_h^3/\gamma = (3\alpha/2 - \mu/2 - 2\tau)f_h - 3\gamma/2.$$

This condition has the same form as (10), except that the line on the RHS has a different slope and intercept. The condition generates a diagram like Figure 2, and as before, it can be shown that the second-order condition is violated at the first intersection in the positive quadrant, making the second intersection relevant. A key question, of course, concerns the location of this intersection relative to the one in Figure 2. The answer tells which network type has greater flight frequency. The next section considers this question, but before turning to that analysis, the comparative-static properties of the solution are worth noting:

Proposition 2. The qualitative effects of the parameters α, β, θ, τ, and γ on flight frequency and traffic levels in the HS network are the same as in the FC case. In addition, f_h, q_h and Q all decline when travel time for connecting passengers rises (when μ increases).

It can also be shown that the effects of parameter changes on aircraft size are qualitatively the same as in the FC case (the computations, however, are more complex). Finally, it is easily shown that the fare arbitrage condition mentioned above is satisfied.[8]

V. COMPARING THE FC AND HS SOLUTIONS

The main goal of the paper is to compare the FC and HS solutions, with a special focus on the difference in flight frequencies. As explained above, the

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[8] The condition requires that $2p_h$, which equals $2(\tau + \beta)$ using (15) and the demand relationship, must be less than the AB fare, which equals $\alpha - \mu - \beta Q - \gamma/f_h$. Noting from (13) that the latter expression must equal $2\tau + \beta Q$, and rearranging, the arbitrage condition reduces to $2q_h > Q$, which is satisfied.

f_h solution is generated by a diagram analogous to Figure 2, where the cubic curve is the same but the line has a different position. Comparing (9) and (18), it is clear that, while the intercept of the HS line is more negative than in the FC case, the relationship between the slopes is ambiguous. Although a comparison between f and f_h might look infeasible under these circumstances, the following result can be established:

Proposition 3. Flight frequency is higher in the HS network than in the FC network, with $f_h > f$.

By showing that $f_h > f$, the model thus validates Morrison and Winston's contention that the growth of HS networks following deregulation spurred an increase in flight frequencies. More generally, the model confirms the link between network structure and flight frequency, which is asserted throughout much of the airline literature.

The source of the result in Proposition 3 can be seen by comparing the first-order conditions for flight frequency in the FC and HS cases. Rewriting (14) to make it comparable to (8), the equation becomes $\gamma[q_h + Q/2]/f_h^2 - \theta = 0$. Comparing this equation to (8), it is clear that $f_h > f$ holds provided that $q_h + Q/2 > q$. To see the underlying intuition, observe that an increase in flight frequency on each route raises passengers' willingness-to-pay, allowing the fare to be increased. While the higher fare is paid by q passengers on each FC route, the increase is effectively paid by $q_h + Q/2$ passengers on each HS route (the higher fare is earned across two routes for connecting passengers, so that Q must be divided by 2 to put the gain on a route basis). Under the assumption that more passengers are affected ($q_h + Q/2 > q$), the gain from a dollar fare increase is thus greater in the HS case. As a result, the 'productivity' of a higher frequency in raising the fare (γ/f_h^2) must be driven lower in the HS case to equate the overall gain $\gamma(q_h + Q/2)/f_h^2$ to the marginal flight cost θ. This can only happen if $f_h > f$, so that the productivity expressions satisfy $\gamma/f_h^2 < \gamma/f^2$.

The assumption $q_h + Q/2 > q$ is in fact validated by the analysis. Indeed, Proposition 3 can be used to compare the magnitudes of variables other than flight frequency between the FC and HS cases. First, using (9) and (15), the fact that $f_h > f$ holds immediately yields $q_h > q$, implying that passenger volumes in city-pair markets AH and BH are greater under the HS network than under the FC network. Note that this result validates the previous inequality. Intuitively, since higher frequencies shift the demand curve outward, more traffic is generated in these markets. By contrast, the comparison between q and Q is ambiguous, implying that traffic in market AB could be higher or lower under the HS network than in the FC case. The reason is that the numerator term $\alpha - \mu - 2\tau$ in (16) is less than the analogous term $\alpha - \tau$ in (9), offsetting the difference in the flight-frequency terms. Intuitively, while higher frequencies again raise the demand for AB travel in

the HS case, the longer connecting travel time reduces it, making the net demand shift ambiguous. This ambiguity, combined with a doubling in the cost of transporting an AB passenger, accounts for the ambiguous relationship between q and Q.[9]

To compare aircraft sizes for the two network types, recall that $s = \theta f/\gamma$ in the FC case, and let (14) be rewritten to read

$$(18) \qquad s_h = \frac{q_h + Q}{f_h} = \frac{q_h + Q}{q_h + Q/2} \frac{\theta f_h}{\gamma}.$$

Since $f_h > f$ and the first ratio term on the RHS of (18) exceeds unity, it follows that $s_h > s$, so that the HS network has larger aircraft than the FC network. This implication of the model appears to be realistic.

A final comparison focuses on the level of fares. To compare fares in markets AH and BH across the network types, (3) can be used to rewrite (7) and (12), the first-order conditions for q and q_h, as $p - \tau - \beta q = 0$ and $p_h - \tau - \beta q_h = 0$. With $q_h > q$, the result $p_h > p$ follows immediately, so that the fare in markets AH and BH is higher under the HS network. This conclusion shows that the upward shift in the demand curve due to higher frequency dominates the downward movement along the curve from higher traffic, leaving the fare higher than in the FC case. As mentioned above, this result stands in stark contrast to received wisdom regarding HS networks, which argues that their lower cost per passenger should lead to lower fares. Because of larger aircraft size, cost per passenger is indeed lower under the HS network in the present model. But because higher flight frequency raises willingness to pay, fares for local passengers end up higher in the HS case. A final point is that, matching the ambiguous comparison between q and Q, the fare in market AB could be higher or lower in the HS network. This discussion has established the following results:

Proposition 4. Traffic in city-pair markets AH and BH is higher in the HS network than in the FC network ($q_h > q$), and the fare is higher as well ($p_h > p$). The comparison of AB traffic levels between the network types is ambiguous ($Q > (<)q$), as is the fare comparison. Aircraft sizes are larger under the HS network ($s_h > s$).

VI.   THE CHOICE OF NETWORK TYPE

After solving the FC and HS optimization problems, the airline must make a global choice of network type, and this section analyzes the factors affecting that choice. To carry out the analysis, the following strategy is adopted.

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[9] Manipulation of (9) and (16) shows that $q > (<)Q$ as $\mu + \tau > (<)\gamma(1/f_h - 1/f)$.

First, consider a parameter combination that makes the airline indifferent between the network types, with $\pi^h = \pi$ holding. Then increase the value of a particular parameter λ, and using the envelope theorem, compute the derivative of the HS-FC profit differential, $\Delta = \pi^h - \pi$, with respect to that parameter. If the derivative Δ_λ is positive, then high (low) values of the parameter favor the HS (FC) network, with the opposite conclusion holding if the derivative is negative.

Several key steps in the required analysis depend on the relationship between the flight frequencies chosen under the two network types. While it has been shown that $f_h > f$ holds, the analysis hinges on the relationship between $2f_h$ and $3f$, which give *total* flights operated under the HS and FC networks. Given $f_h > f$, the relationship between these flight totals would appear to be ambiguous, and this conclusion is confirmed by a more-detailed investigation. However, given that the HS network is meant to save airline costs through the operation of fewer routes, a natural expectation is that HS operations would reduce the total number of flights, implying $2f_h < 3f$.

Since this assumption is both natural and useful in generating sharp results below, it will be invoked at several points in the analysis.[10] It should be noted, however, that since the flight-total relationship depends on parameter values, assuming that $2f_h < 3f$ holds is tantamount to assuming that a certain portion of the parameter space is relevant. However, only a partial characterization of this region is possible.[11]

Letting Φ denote a parameter expression that is discussed further below, the following results can be established.

Proposition 5. Suppose that $2f_h < 3f$ holds, so that the HS network has fewer total flights than the FC network. Then, the following results can be established:

(i) $\Delta_\mu < 0$. The HS (FC) network is then favored for small (large) values of the travel-cost parameter μ.

(ii) $\Delta_\theta > 0$. The HS (FC) network is then favored for large (small) values of the fixed-cost parameter θ.

(iii) $\Delta_\beta > 0$. The HS (FC) network is then favored for large (small) values of the demand-slope parameter β.

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[10] Since it can shown that π can be either larger or smaller than π^h when $2f_h < 3f$ holds, the existence of a point of profit indifference, where $\pi = \pi^h$, is not inconsistent with the flight-total assumption.

[11] The relevant analysis proceeds as follows. Consider a set of parameter values leading to identical flight totals under the two network types, with $2f_h = 3f$. Then, using (10) and (17), compute the derivatives $\partial f_h/\partial\lambda$ and $\partial f/\partial\lambda$ for a given parameter λ. The results show that, when $2f_h = 3f$ holds, $\partial f_h/\partial\alpha < (3/2)\partial f/\partial\alpha$ and $\partial f_h/\partial\tau > (3/2)\partial f/\partial\tau$. These inequalities indicate that $2f_h < 3f$ holds for large values of α and for small values of τ. While this part of the parameter space is therefore relevant for the ensuing results that invoke the flight-total assumption, no statement about the relevant ranges for other parameters is possible.

(iv) *If $\Phi > 0$, then $\Delta_\alpha < 0$. The HS (FC) network is then favored for small (large) values of the demand-intercept parameter α.*

(v) *If $\Phi > 0$, then $\Delta_y > 0$. The HS (FC) network is then favored for large (small) values of the parameter γ representing the disutility of schedule delay.*

The parameter expression Φ, which is presented in the appendix, must be positive for technical reasons to establish parts (*iv*) and (*v*). Satisfaction of this condition is guaranteed if the travel-cost parameter μ is sufficiently small.

When $\Phi > 0$ holds along with the flight-total requirement, the Proposition implies that the HS network is favored when the additional travel time it requires is low (when μ is small), when demand is low (when α is small or β large), when the fixed cost of a flight is high (when θ is large), and when the disutility from schedule delay is high (when γ is large).[12] While the effects of μ and θ are intuitively transparent, an increase in γ favors the HS network because its higher flight frequency becomes more valuable to passengers as the disutility of schedule delay rises. To understand the demand effect, observe that in concentrating traffic on fewer routes, the HS network reduces cost per passenger by allowing the operation of larger aircraft. This traffic-collection role, however, becomes less crucial as demand rises, making point-to-point service more economical. Thus, for an airline that restricts its service to high demand routes, an FC network is likely to be profit maximizing (Southwest Airlines, which operates a point-to-point network serving relatively large markets, offers an example). Finally, note that the effect of the variable-cost parameter τ on network choice is ambiguous.[13]

VII. WELFARE ANALYSIS

The apparent accuracy of the model's predictions suggests that it may capture important elements of the real-world optimization problem solved by the airlines. As a result, it may be useful to explore the model's welfare implications. In contrast to the monopolist, the goal of the social planner is to maximize consumer benefits minus airline costs. As usual, it can be shown that benefits in the present problem are measured by the area under the demand curve. In the FC case, this area equals $q(\alpha - \beta q/2 - \gamma/f)$, which differs from revenue in (6) only in the replacement of β by $\beta/2$. With a similar observation applying to the HS case, it follows that the only difference

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[12] Recall that analysis is already restricted to a certain portion of the parameter space via the flight-total assumption (see footnote 10). The above statements apply within that restricted parameter area.

[13] It should be noted that part (*i*) of Proposition 5 does not require $2f_h < 3f$, and that the conclusion in part (*ii*) is reversed if $2f_h > 3f$ holds.

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between the profit- and welfare maximization problems is that β is replaced by $\beta/2$ in the latter problem. This difference means that the first-order conditions for choice of traffic ((7), (12) and (13)) now require equality between demand (rather than marginal revenue) and variable cost τ. Note, however, that the first-order conditions for choice of flight frequency ((8) and (15)) are the same as in the monopoly problem. Therefore, for a given traffic level, the monopolist and the planner would make the same frequency choices. However, traffic levels, and hence frequencies, differ between the two regimes.

To derive these differences, observe that with β replaced by $\beta/2$ in the welfare maximization problem, the cubic curve in Figure 2 rotates downward in the first quadrant. As a result, the second intersection moves to the right, so that the socially optimal flight frequency in the FC network is higher than the frequency chosen by the monopolist. Using the HS version of Figure 2, the same conclusion applies in the HS case. Moreover, since the denominators in the traffic expressions (9), (15) and (16) are replaced by β, while the increase in flight frequency raises the numerators, it follows that the socially optimal traffic levels are also higher than those chosen by the monopolist. Finally, the increase in frequency implies an increase in aircraft sizes, using (18) and the analogous expression for the FC case. This discussion has established the following results:

Proposition 6. Under each network type, the social optimum has greater flight frequency, higher traffic, and larger aircraft than the monopoly solution.

These results can best be understood by recognizing that the quantity (i.e., traffic) choices of a monopolist are, as usual, too small. Once traffic has been raised by the planner, however, the benefit of an increase in flight frequency grows because more passengers are affected, making higher frequencies optimal. With aircraft size and frequency linked by the same conditions as in the monopolist's problem ((18) and its FC analog), higher frequencies then imply larger aircraft at the social optimum.[14]

It should be noted that the under-provision of frequency is a result that is closely linked to the assumed monopoly market structure. As is well known (see Panzar [1979] and Schipper et al. [1998a, b]), schedule competition may

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[14] BZ's analysis generates the opposite frequency comparison, with the socially optimal frequency lower than that chosen by the monopolist. The differing conclusions are due to the fact that travel demand is effectively fixed in BZ's welfare analysis, with all consumers assumed to travel under both the profit and welfare maximizing solutions (see footnote 3). Under these circumstances, an increase in frequency allows fares to be increased with no reduction in traffic. Since this fare increase generates more monopoly revenue but constitutes a pure transfer from consumers to the monopolist (with no welfare significance), the incentives in BZ's problem model favor excessive frequency. Building on this result, Wojahn [2001] shows that hub congestion, by limiting flight frequency, may be welfare improving.
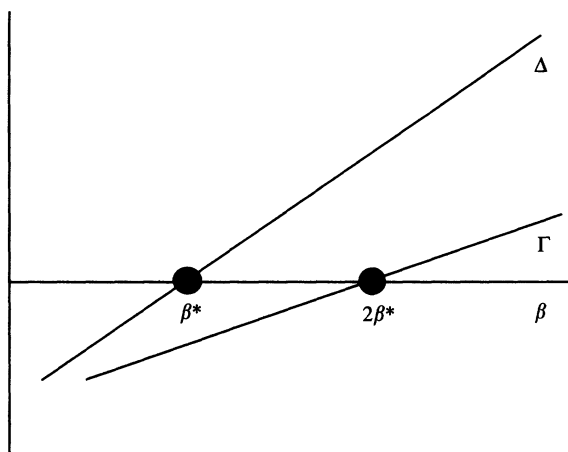
Figure 3
Bias in Network Choice

lead to the opposite outcome, with flight frequencies being excessive. Analysis of such an outcome in the present setting must await an oligopoly version of the model.

A final exercise is to compare the network choices of the monopolist and social planner. To carry out this exercise, let W^h and W denote the HS and FC welfare functions, and let $\Gamma = W^h - W$ denote the welfare differential between the network types, analogous to the profit differential $\Delta = \pi^h - \pi$. Then, the key observation, which follows from the above discussion, is that Γ is equal to Δ evaluated at $\beta/2$ rather than β. This fact in turn implies that, if Γ is evaluated at a β value twice as large as that used in Δ, then the magnitudes of the expressions are the same. Given this information, consider Figure 3, which shows graphs of Δ and Γ as functions of β. The graphs assume that $2f_h < 3f$ holds under both the monopoly and planning solutions, which restricts attention to the relevant portion of the parameter space. Then, given Proposition 5, Δ is upward-sloping where it crosses the horizontal axis, a property shared by Γ.[15] Letting β^* denote the β value where Δ intersects the horizontal axis, it follows that Γ's intersection occurs at $2\beta^*$.

Referring to Figure 3, it is clear that for β values below β^*, the monopolist and the planner both prefer the FC network, while for values above $2\beta^*$, both agents prefer the HS network. However, between β^* and $2\beta^*$, the monopolist prefers the HS network while the planner favors the FC network. Thus, over

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[15] Although Δ's slope as drawn is positive overall, the slope could turn negative at points away from the axis intersection. However, the curve cannot intersect the axis again since its slope would be negative at such a point, in violation of Proposition 5.

this range of β values, the monopolist's choice exhibits an *inefficient bias* toward the HS network. Note, however, that the other parameters of the problem aside from β are held fixed in Figure 3. But changing their values just leads to a new version of the Figure, with the positions of both the Δ and Γ curves shifted but the slope pattern unchanged. In this new Figure, the monopolist's choice again exhibits an inefficient bias toward the HS network over a particular range of β values. It is clear that, as the other parameters vary, this exercise traces out a region of the entire parameter space over which an inefficient HS bias exists. This discussion has thus established the following result:

Proposition 7. When $2f_h < 3f$ holds under both the monopoly and planning solutions, the monopolist's network choice exhibits an inefficient bias toward the HS network.

The intuitive explanation for this result is not completely transparent, but it appears that the monopolist excessively favors the network type in which fewer flights are operated.[16]

Although the absence of airline competition in the model makes a strong conclusion unwarranted, Proposition 7 suggests that the current structure of airline networks may exhibit too much 'hubbing.' In other words, a greater reliance on point-to-point service may be preferable from society's point of view.

VIII. CONCLUSION

This paper has provided a simple analysis of the effects of network structure on the scheduling, traffic, and aircraft-size choices of a monopoly airline. The results are realistic, suggesting that the model captures some essential features of the actual optimization problem solved by a network carrier. The analysis shows that switching from an FC to an HS network leads to increases in both flight frequency and aircraft size, while stimulating local traffic in and out of the hub. In addition, HS networks are shown to be preferred by the airline when travel demand is low, when flights are expensive to operate, and when passengers place a high value on flight frequency but are not excessively inconvenienced by the extra travel time required for a connecting trip. The welfare analysis shows that the flight frequency, traffic volumes, and aircraft size chosen by the monopolist are all

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[16] BZ also found that the monopolist's network choice shows an inefficient bias toward the HS network. Since the total number of flights is unambiguously lower in the HS network in their model, this result is consistent with Proposition 7. Note that, if $2f_h > 3f$ holds, then Proposition 7 is reversed, with network choice exhibiting an inefficient bias toward the FC network, which is then the network with fewer total flights.

inefficiently low under both network types. Moreover, in the most plausible case, the monopolist's network choice exhibits an inefficient bias toward the HS network, apparently reflecting an excessive desire to economize on the number of flights.

Although the model captures in a simple way nearly all the key elements of an airline optimization problem, one important factor is missing: competition from other carriers. To incorporate such an extension, a second airline could be added to the model, operating parallel routes in the FC case and operating a competing hub at city H in the HS case. Such a model would need to analyze schedule competition between the carriers, and this analysis could draw on previous treatments of this kind of interaction (see Panzar [1979] and Schipper et al. [1998a, b]).

A narrower but still useful generalization would increase the number of cities served by the airline. In this case, the HS network would serve n endpoints in addition to the hub, with $n > 2$. Even with these endpoints radially arrayed around the hub city, this setup would still involve an asymmetry in that the nonhub cities, which would be directly connected in the FC case, would not be equidistant from one another. However, since this alteration would complicate the algebra without changing the fundamental nature of the problem, results similar to those derived above would be likely to emerge.

APPENDIX

Proof of the Lemma: This result can be established by recasting the optimization problem as a two-stage problem, where q is chosen first conditional on f, with f then chosen in a second stage. Substituting in (9) into (6) and simplifying, profit as a function of f can be written

$$(a1) \qquad \pi = \frac{3}{4\beta}(\alpha - \tau - \gamma/f)^2 - 3\theta f.$$

The second derivative of $(a1)$ has the same sign as $-2[(\alpha - \tau)f - \gamma] + \gamma$. Using (10) to eliminate the terms in brackets, this expression has the sign of $-4\beta\theta f^3/\gamma + \gamma$ at the optimum. Noting that the cubic curve in (10) must be steeper than the line at the second solution, it follows that $6\beta\theta f^3/\gamma > (\alpha - \tau)f$ holds (both slope expressions have been multiplied by f). Substituting for the RHS expression using (10), the last inequality reduces to $-4\beta\theta f^3/\gamma + \gamma < 0$, indicating satisfaction of the second-order condition. At the first intersection, the above slope condition is reversed, reversing the last inequality and violating the second-order condition.

Proof of Proposition 1: The effects on f of changes in α, β, θ and τ on f follow from inspection of Figure 2, and the impacts on q then follow from inspection of (9). To find γ's effect on f, differentiation of (10) yields $\partial f/\partial \gamma = (2\beta\theta f^3/\gamma^2 - 1)/\Psi = (\alpha - \tau)f/\gamma\Psi > 0$, where $\Psi = 6\beta\theta f^2/\gamma - (\alpha - \tau) > 0$ from the proof of the Lemma and where the second equality uses (10). However, since the derivative of f/γ with respect to γ is ambiguous, so is the change in q from (9).

Proof of Proposition 2: The results in this proposition are established in analogous fashion to those in Proposition 1, relying on the analog to Figure 2 as well as (15) and (16).

Proof of Proposition 3: Insert the HS line from the RHS of (17) into Figure 2, and suppose that, at the f_h solution, the height up to that line is less than or equal to the height up to the FC line. Using (10) and (17), this inequality can be written $(3\alpha/2 - \mu/2 - 2\tau)f_h - 3\gamma/2 \leqslant (\alpha - \tau)f_h - \gamma$, or $\alpha - \mu - 2\tau - \gamma/f_h \leqslant 0$. Since the last inequality implies $Q \leqslant 0$ from (16), a contradiction, the FC line must instead lie below the HS line at f_h. Since the FC line thus lies below the cubic curve at f_h, and since the FC line is flatter than that curve, the f value where it intersects the curve must be smaller than f_h.

Proof of Proposition 5: The first step is to derive a second-stage profit expression analogous to $(a1)$ for the HS case. Recall that $(a1)$ was derived by choosing q optimally conditional on f, and then substituting the solution to yield profit purely as a function of flight frequency. When this same procedure is carried out for the HS case using the traffic solutions in (15) and (16), the HS profit expression reduces to

$$(a2) \qquad \pi^h = \frac{3}{4\beta}(\alpha - 4\tau/3 - \gamma/f_h)^2 + \Phi - 2\theta f_h,$$

where $\Phi \equiv \tau^2/6\beta + \mu(-2\alpha + \mu + 4\tau + 2\gamma/f_h)/4\beta$. After choosing f and f_h optimally and substituting the solutions into $(a1)$ and $(a2)$, the airline chooses the network type that yields the larger profit value.

Recall that the effect of a parameter λ on network choice is found by differentiating $\Delta = \pi^h - \pi$ with respect to that parameter. If the derivative Δ_λ is positive, then high (low) values of the parameter favor the HS (FC) network, with the opposite conclusion holding if the derivative is negative.

To prove part (i), differentiation of Δ using $(a1)$ and $(a2)$ along with the envelope theorem yields $\Delta_\mu = -(\alpha - \mu - 2\tau - \gamma/f_h)/2\beta$, which is negative from (16) given $Q > 0$. To establish part (ii), differentiation of Δ yields $\Delta_\theta = 3f - 2f_h > 0$, establishing the result. To prove part (iii), differentiation yields

$$(a3) \qquad \Delta_\beta = -(\pi^h + 2\theta f_h)/\beta + (\pi + 3\theta f)/\beta = (3f - 2f_h)\theta/\beta > 0,$$

where the indifference condition $\pi^h = \pi$ is used, establishing the result.

To prove part (iv), differentiation shows that Δ_α has the sign of

$$(a4) \qquad \frac{3}{4\beta}[(\alpha - 4\tau/3 - \gamma/f_h) - (\alpha - \tau - \gamma/f)] - \mu/2\beta.$$

To sign this expression, observe that if $2f_h < 3f$, then the first two terms in $(a2)$ must be smaller than the first term in $(a1)$ if $\pi^h = \pi$ is to hold. But if Φ is positive, then the first term in $(a2)$ by itself must be smaller than the first term in $(a1)$, making $(a4)$ negative and establishing part (iv). Note that $\Phi > 0$ is ensured when μ is sufficiently small (see the above expression).

To prove part (v), differentiation shows that Δ_γ has the sign of

$$(a5) \qquad \frac{3}{4\beta}\left[\frac{\alpha - \tau - \gamma/f}{f} - \frac{\alpha - 4\tau/3 - \gamma/f_h}{f_h}\right] + \mu/2\beta f_h.$$

With the numerator of the first term in brackets larger than the numerator of the second term by the above argument and $f_h > f$, the bracketed term is positive, making $(a5)$ positive and establishing part (v).

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