1. Consider a small dataset of five observations. The predictor is continuous, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, $x_5 = 5$, and the response variable is discrete, $y_1 = A$, $y_2 = A$, $y_3 = B$, $y_4 = C$, $y_5 = A$.

   (a) Construct the classification tree. Use Gini impurity index as the splitting criterion. Split the node if there are more than two observations inside.

   (b) What are the forecasted probabilities for $x = 4.7$?

2. Consider a small dataset of five observations. The predictor is continuous, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, $x_5 = 5$, the response variable is also continuous, $y_1 = 100$, $y_2 = 120$, $y_3 = 130$, $y_4 = 60$, $y_5 = 70$.

   (a) Construct the regression tree. Use residual sum of squares as the splitting criterion. Split the node if there are more than two observations inside.

   (b) What is the forecasted value of y for $x = 4.7$?

3. Random variables y_1, y_2, ..., y_n is a random sample from uniform distribution on $[0; 1]$. Consider one of the bootstrap samples, y_1^*, y_2^*, ..., y_n^*.

   (a) Find $\mathbb{E}(y_1^*)$. Explain in one or two lines why it is equal to or different from $\mathbb{E}(y_1)$.

   (b) Find $\mathbb{V}\text{ar}(y_1^*)$. Explain in one or two lines why it is equal to or different from $\mathbb{V}\text{ar}(y_1)$.

   (c) Find $\mathbb{C}\text{ov}(y_1, y_1^*)$.

   (d) Find $\mathbb{C}\text{ov}(y_1^*, y_2^*)$. Explain in one or two lines why it is equal to or different from $\mathbb{C}\text{ov}(y_1, y_2)$.

   (e) Find $\mathbb{V}\text{ar}(\bar{y}^*)$.