Note: you may use calculator or computer to find extremum points if you can't solve FOC by hand, but you should clearly state the functions that you optimize.

1. You are guessing the value of binomial random variable X with $n = 3$ and real probability of success $p = 0.3$.

   (a) Find the entropy $H(X)$.

   (b) Now imagine that you wrongly assume that X has binomial distribution with $n = 3$ and $q = 0.5$. Find the cross-entropy $CE(\text{Bin}(n = 3, p = 0.3)||\text{Bin}(n = 3, q = 0.5))$.

2. You may play a favorable game infinitely many times. Your bet in round n is multiplied by a random varibales K_n with distribution given by

x	0	5	10
$\mathbb{P}(K_n = x)$	$1/2$	$1/4$	$1/4$

   The random variables K_n are independent. You optimize the long-term interest rate. Which proportion of you current wellfare should you bet in every round of this game?

3. There is a race with three horses, A, B or C and you may receive one of the two signals: $S = 1$ or $S = 2$. The joint distribution of the signal and winning horse is given by:

	$W = A$	$W = B$	$W = C$
$S = 1$	0.1	0.2	0.3
$S = 2$	0.2	0.1	0.1

   If the horse wins your bet on this horse is multiplied by inverse unconditional probability that this horse wins, for example a bet on a horse A is multiplied by $1/0.3$ if A wins.

   You optimize the long-term interest rate and you can bet infinitely many times.

   (a) For each signal find the optimal proportions to divide your bet.

   (b) Calculate the conditional entropy $H(W|S)$.

   (c) What is the highest long-term interest rate that you can achieve?