

DSE'23 - class 9: PCA & SVD

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Problems

Problem 1: SVD

Find SVD decomposition of matrix:

$$X = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

Problem 2: more SVD

The researcher Herodot received SVD of the matrix X:

$$X = U \begin{bmatrix} 7 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix} V'$$

If it is possible find SVD for the following matrices:

- (a) X' , $10X$
- (b) $X'X$, XX'
- (c) $(X'X)^4$, $(XX')^{-1}$, $5I + X'X$
- (d) $X'(XX')^{-1}X$
- (e) $X(X'X)^{-1}X'$

Problem 3: PCA by hands

Consider two variables: $x^1 = (1, 0, 0, 3)^T$, $x^2 = (3, 2, 0, 3)^T$. Find first and second principle components.

Problem 4: PCA

Consider observations from each vector as pairs: $(x_1, y_1), \dots, (x_n, y_n)$. The researcher Ivan wants to find the line, the sum of distances to which from each point is minimal. Is it true, that this line necessarily goes through the point of averages (\bar{x}, \bar{y}) ?