PS5

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1.

Denote $\Lambda_{ii} = \lambda_i, 1 \leqq i \leqq n(\lambda_i$ is the i^{th} eigenvalue), so $|\Lambda| = \Pi_i^n \lambda_i$. And because Γ is an orthogonal matrix of eigenvectors, $|\Gamma| = |\Gamma^T| = 1$. So $A = \Gamma\Lambda\Gamma^T \rightarrow |A| = |\Gamma||\Lambda||\Gamma^T| \rightarrow |A| = 1 * \Pi_i^n \lambda_i * 1 = \Pi_i^n \lambda_i$, i.e. $|A|$ is the product of the eigenvalues.

2.

Because when z is large the expression will cause overflow. The $exp(z)$ term will be *Inf* in R and the *expit* function will be *NaN*. To avoid this we can re-express it as $expit(z) = \frac{1}{1+exp(-z)}$. This function will return 1 when z takes very large values.

3.

Because we can only accurately present about 16 digits, x will lost accuracy after 4 decimal places. Thus their variance won't be the same and will be accurate to about 4 places. In this case they agree to 5 digits.

4.

(a)

Because in this case each task takes little time and the communication overhead of starting and stopping the tasks will reduce efficiency. So it's better to separate the computation into p tasks.

(b)

Assume each element in X, Y takes up a bytes.

A:

(1) The memory used=$4n^2a$ bytes.
(2) The communication cost= $n^3 + 2n^2$

B: (1) The memory used=$(4n^2 + p^3)a$ (2) The communication cost=$p^2(2n^2/p + m^2) = 2n^2p + n^2$

We can conclude that A is better for minimizing memory use and B is for minimizing communication.