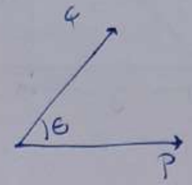
Scalar: quantity - determined by its magnitude; no. of units measured in a suitable scale

vector: determined by both magnitude & dirⁿ.

Scalar Product (Dot Product)

$\vec{Q}\cdot\vec{P} = |\vec{Q}||\vec{P}|\cos\theta \quad ; \quad \theta \in [0,\pi]$



- is a scalar quantity.

$\vec{P}\cdot\vec{Q} = \vec{Q}\cdot\vec{P}$

$\vec{P}\cdot(\vec{Q}+\vec{R}) = (\vec{P}\cdot\vec{Q} + \vec{P}\cdot\vec{R})$

$\vec{P}\cdot\vec{P} = |\vec{P}|^2$

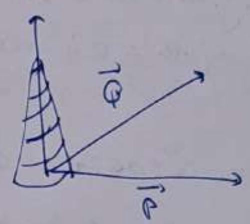
$(K\vec{P})\cdot\vec{Q}$
$= K(\vec{P}\cdot\vec{Q})$
$= \vec{P}(K\vec{Q})$

if $\vec{P}\cdot\vec{Q} = 0$ & $\vec{P} \neq \vec{Q} \neq 0$

then $P \perp Q$

Vector Product (cross Product)

- is another vector which is $\perp$ to the plane formed by the two planes.



$\vec{P}\times\vec{Q} = PQ\sin\theta$

$\vec{P}\times\vec{Q}$ is a vector

$\vec{P}\times\vec{Q} = -\vec{Q}\times P$

if $P \parallel Q$, then $\vec{P}\times\vec{Q} = 0$

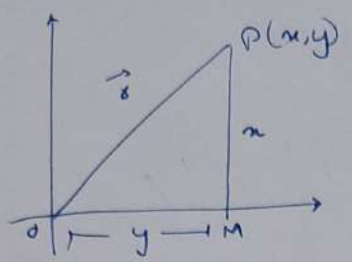
$\vec{P}\times\vec{P} = 0$

$i\times i = 0$	$i\times j = k$
$j\times j = 0$	$j\times k = i$
$k\times k = 0$	$k\times i = j$

$\vec{P}\times(\vec{Q}+\vec{R}) = (\vec{P}\times\vec{Q}) + (\vec{P}\times\vec{R})$

$(K\vec{P})\times\vec{Q} = K(\vec{P}\times\vec{Q})$

Components

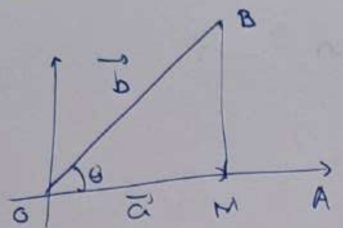


$\vec{OP} = \vec{OM} + \vec{MP}$ (Δ Law)

$\vec{r} = x\hat{i} + y\hat{j}$ | $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{r}| = \sqrt{x^2+y^2}$ | $|\vec{r}| = \sqrt{x^2+y^2+z^2}$

Projections of Vectors.



OM is projection of OB on OA

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \hat{a}\,\vec{b}$$

$$\left[\hat{a} = \frac{\vec{a}}{|\vec{a}|}\right]$$

- If i, j, k are unit vectors & mutually $\perp$ then
  $i \cdot j = j \cdot k = k \cdot i = 0$

- $\vec{a}\cdot\vec{a} = a^2\cos\theta = a^2\cos 0 = a^2$
  $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

- Angle b/w two non-zero vectors $\vec{a}$ & $\vec{b}$ $\quad \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$

- If $\vec{a}$ & $\vec{b}$ are $\perp$, then $a_1b_1 + a_2b_2 + a_3b_3 = 0$
  — " — $\parallel$, then $a_1 = tb_1$, $a_2 = tb_2$, $a_3 = b_3t$

$|\vec{a} \pm \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 \pm 2\vec{a}\vec{b}$

$|\vec{a} \cdot \vec{b}| \leq |\vec{a}|\,|\vec{b}|$

<u>Vector Product</u>: $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ $\Rightarrow$ if you interchange rows, sign of product changes.

$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = [\vec{a}\ \vec{b}\ \vec{c}] \Rightarrow [\vec{b}\ \vec{c}\ \vec{a}] \Rightarrow [\vec{c}\ \vec{a}\ \vec{b}]$

- changing cyclic order $\Rightarrow$ change of sign.
  $[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{b}\ \vec{a}\ \vec{c}]$
- $(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c})$
- $[\lambda\vec{a}\ \vec{b}\ \vec{c}] = \lambda[\vec{a}\ \vec{b}\ \vec{c}]$
- Necessary & suff. cond^n for three non-zero, non-collinear vectors $\vec{a}, \vec{b}, \vec{c}$ to be coplanar is that $[\vec{a}\ \vec{b}\ \vec{c}] = 0$
- $[\vec{a}+\vec{b}\ \ \vec{c}\ \ \vec{d}] = [\vec{a}\ \vec{b}\ \vec{d}] + [\vec{a}\ \vec{b}\ \vec{c}]$
- $[\vec{a}\ \vec{b}\ \vec{c}]$ = Volume of Parallelopiped.
- if any two values are zero, then Scalar Triple product $= 0$.

Vector Tripple product

$\vec{a}\times(\vec{b}\times\vec{c})=0$ if $\begin{cases} \text{any one of three is zero} \\ \vec{a} \text{ is } \parallel \text{ to } \vec{b} \\ \vec{c} \text{ is } \perp \text{ to both } \vec{a}\times\vec{b} \end{cases}$

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \begin{vmatrix} \vec{a}\cdot\vec{c} & \vec{a}\cdot\vec{d} \\ \vec{b}\cdot\vec{c} & \vec{b}\cdot\vec{d} \end{vmatrix}$$

$$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\cdot\vec{c}$$

$$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\cdot\vec{a}$$

- $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a}\ \vec{c}\ \vec{d}]\,\vec{b} - [\vec{b}\ \vec{c}\ \vec{d}]\,\vec{a}$

$$= [\vec{a}\ \vec{b}\ \vec{d}]\,\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\,\vec{d}$$

Reciprocal Vectors

if $\vec{a}', \vec{b}', \vec{c}'$ are reciprocals of $\vec{a}\ \vec{b}\ \vec{c}$.

iff $\vec{a}' = \dfrac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$; $\vec{b}' = \dfrac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$; $\vec{c}' = \dfrac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$

- $\vec{a}$ is a constant Vector

$\vec{a} = |\vec{a}|$; $\dfrac{da}{dt} = 0$

- Curves that can be represented as

1) Intersection of two surfaces : $F_1(x,y,z)=0$, $F_2(x,y,z)=0$

2) Parametric form $x=f_1(t)$, $y=f_2(t)$, $z=f_3(t)$

3) St. Line $\vec{r} = \vec{a} + t\vec{b}$

Vector Differential Calculus.

Vector f^n → let $S \subset R$. if for each scalar $t \in S$ $\exists$ a unique vector $\vec{f}(t)$, then $\vec{f}(t)$ is said to be vector function of scalar variable 't'.

$f(t)$ is a vector qty.

$\vec{f}(t) = f_1(t)\hat{i} + f_2(t)\hat{j} + f_3(t)\hat{k}$

Scalar field	Vector field
if to each pt (x,y,z) of a region R in space, there corresponds a unique no. or scalar $\phi(x,y,z)$, then ϕ is called a scalar function of position.	-If to each pt (x,y,z) of a region R in space, there corresponds a vector $\vec{V}(x,y,z)$, then $\vec{V}$ is called a vector f^n of position
$\phi(x,y,z) = x^2 - yz$ (example)	ex-velocity at any point (x,y,z) within a moving field.
ex-Temp. at any point within or on the earth's surface at a certain time defines a scalar field.	$\phi(x,y,z) = xy^2\hat{i} + 2yz^3\hat{j} + x^2zk$

$$\frac{d}{dt}(\vec{A} \pm \vec{B}) = \frac{d\vec{A}}{dt} \pm \frac{d\vec{B}}{dt}$$

$$\frac{d}{dt}(A \cdot B) = A \cdot \frac{dB}{dt} + \bar{B}\frac{dA}{dt}$$

$$\frac{d}{dt}(\vec{A} \times \vec{B}) = A \times \frac{dB}{dt} + B \times \frac{dA}{dt}$$

$$\frac{d}{dt}(\phi A) = \phi\frac{dA}{dt} + A\frac{d\phi}{dt}$$

$$\frac{d}{dt}[A\,B\,C] = \left[\frac{dA}{dt}\,B\,C\right] + \left[A\,\frac{dB}{dt}\,C\right] + \left[A\,B\,\frac{dC}{dt}\right]$$

$$\frac{d}{dt}\left\{\vec{A}\times(\vec{B}\times\vec{c})\right\} = \frac{d\vec{A}}{dt}\times\vec{B}\times\vec{c} + \vec{A}\times\left(\frac{d\vec{B}}{dt}\times\vec{c}\right) + \vec{A}\times\left(\vec{B}\times\frac{d\vec{c}}{dt}\right)$$

$\vec{r} \rightarrow$ vector fn of a scalar Variable 't'
$\hookrightarrow x\hat{i}+y\hat{j}+z\hat{k}$

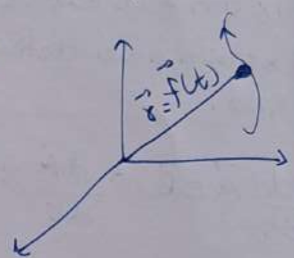
$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k \quad \Bigg| \quad \frac{d^2r}{dt^2} = \frac{d^2x}{dt^2}i + \frac{d^2y}{dt^2}j + \frac{d^2z}{dt^2}\hat{k}$$

- If f_1, f_2, f_3 are constant functions, then $\vec{f} = f_1\hat{i}+f_2j+f_3k$ is called constant vector function.

$$\frac{d\vec{f}}{dt} = 0 \qquad \vec{f}.\frac{d\vec{f}}{dt} = 0 \qquad |\vec{f}| = f$$

Differential Geometry : study of space curves & surfaces.

- A curve in space may be defined as the locus of a point whose co-ordinate may be expressed as a function of a single parameter



- tangent to a curve → a limiting position of a straight Line 'L' through P & a point Q of curve C. $\Rightarrow \frac{d\vec{r}}{dt}$

- unit tangent Vector $\frac{d\vec{r}}{ds} = T \quad \Big| \quad \frac{ds}{dt} = \left|\frac{d\vec{r}}{dt}\right|$

Gradient, Divergence & Curl.

- total differentiation $d\vec{r} = \frac{\partial r}{\partial x}dx + \frac{\partial r}{\partial y}dy + \frac{\partial r}{\partial z}dz$

- Vector Differential operator → del → (∇)

$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$ | $\nabla^2 = i\frac{\partial^2}{\partial x^2} + j\frac{\partial^2}{\partial y^2} + k\frac{\partial^2}{\partial z^2}$

- Can be treated to behave as ordinary vector.

- Gradient of a Scalar field.

$\nabla f = \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}\right)$ → is defined as a vector field.

- If f is a scalar fuⁿ, ∇F is a vector point function.
- If f is constant, $\nabla f = 0$
- $\nabla(f.g) = g\nabla f + f\nabla g$ (F & g are scalar fields)

$\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$

if ϕ is function of $r \Rightarrow \nabla\phi = \frac{d\phi}{dr}\nabla r$

Divergent of a vector point function.

$$\cancel{div V = \nabla \cdot \vec{v} \Rightarrow \left(i\frac{d\vec{v}}{dx} + j\frac{d\vec{v}}{dy} + k\frac{d}{} \right.}$$

$$div V = \nabla \cdot \vec{V} = \left(\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z} \right) \cdots \quad V = V_1 i + V_2 j + V_3 k$$

$[\nabla \cdot \vec{V} = 0 \rightarrow$ Solenoidal vector$]$

Curl of Vector

f = any vector point function

- curl f = $\nabla \times f \rightarrow$ vector quantity. $\Rightarrow$ also called Rotation of f.

$$\Rightarrow f = x\hat{i} + y\hat{j} + z\hat{k} \qquad \nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$\Rightarrow$ irrotational Vector $\nabla \times f = 0$

$\Rightarrow$ Laplacian operator : $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Line Integral:

Any Integral - to be evaluated along a ~~line~~ curve is called a line Integral.

suppose → $r(t)$ is pos^n vector - define a piecewise smooth curve

$F(x,y,z) = F_1 i + F_2 j + F_3 k$ is a vector point function defined and continuous along C

If s denotes arc length of C, then $\frac{dr}{ds} = \vec{t}$ is a unit vector along tangent to curve C at $\vec{r}$.

- Component of F vector along this tangent is $F \cdot \frac{d\vec{r}}{ds}$

$$\boxed{\int_C \left(F \cdot \frac{d\vec{r}}{ds}\right) ds = \int_C F \cdot dr \text{ is example of line Integral.}}$$

If C is simple closed curve, then the tangent line integral of F around C is called circulation of $\vec{F}$ about C *

$$\oint_C \vec{F} \cdot dr = \oint_C (F_1 dx + F_2 dy + F_3 dz)$$