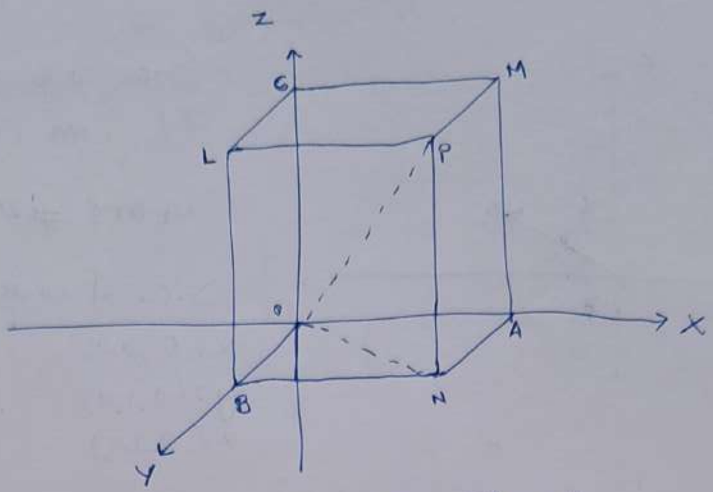
System of Co-ordinates.



$\vec{OP} = \vec{ON} + \vec{NP} \Rightarrow \vec{OA} + \vec{AN} + \vec{NP} \Rightarrow \vec{OA} + \vec{OB} + \vec{OC}$

$\therefore r = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{r} = (a, b, c) \rightarrow$ coordinates of P.
$\hookrightarrow$ Cartesian Co-ordinates.

- Distance b/w two points : $P(x_1, y_1, z_1)$; $Q(x_2, y_2, z_2)$

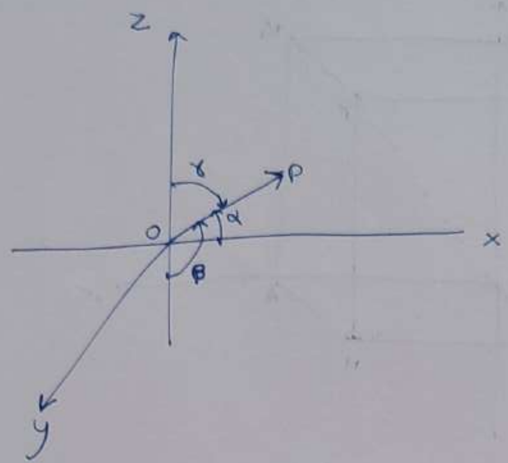
$$PQ = |\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Divison of line : P ——— R ——— Q (m_1, m_2) ; $R = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2} \ldots \right)$

(If ratio $\left(\frac{m_1}{m_2}\right)$ is positive , R divides PQ internally,
-ve ; —"— externally)
—"—

- Centroid of a triangle : $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$

Direction Cosines & Projections.



$(\cos\alpha, \cos\beta, \cos\gamma)$: d.c's of Line
$(l\ ,\ m\ ,\ n)$

$\alpha+\beta+\gamma \neq 360$

D.C's of co-ordinate axes:
$x : (1,0,0)$
$y : (0,1,0)$
$z\ (0,0,1)$

- If length of ~~the~~ Point P from O $= r$ then $P = (lr, mr, nr)$
  $x = lr,\ y = mr,\ z = nr$

- $l^2+m^2+n^2 = 1$

- If D.c's (l, m, n) of a given Line be proportional to any three nos $- a, b, c$, then a, b, c are D.R's of given Line.

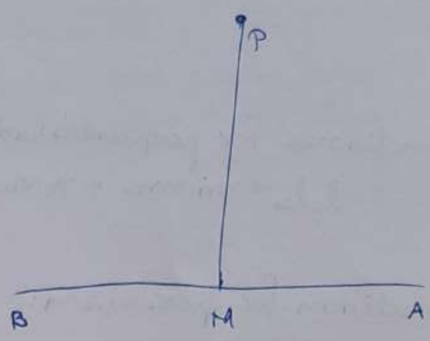
D.C's are unique ; D.R's are not unique.

- if a, b, c are DR's of a line, then $a\hat{i}+b\hat{j}+c\hat{k}$ is a vector parallel to that line.

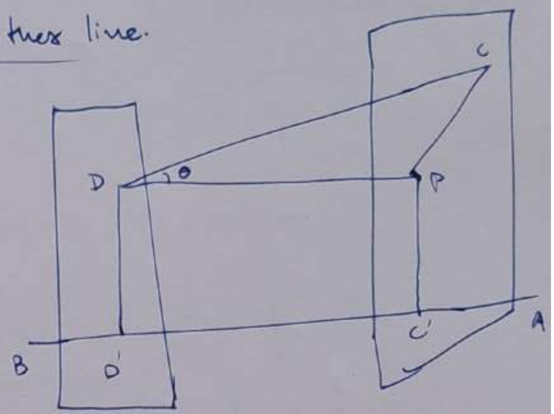
- If $a, b, c =$ given DR's $\Rightarrow \left(\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right) =$ D.C's.

Projection of a point on a given Line

- M is foot of ⊥ called the projection of the given point P on A.B.



- **Projection of a line segment on another line.**



- to find length of proj^n $C'D'$

$$C'D' = CD\cos\theta$$

- let $\vec{DC} = \vec{a}$ and let $\vec{b}$ be a unit vector along BA

then

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$

$$= DC\cos\theta$$

$$= \text{projection of } DC \text{ on } BA$$

- D.Cs of a line joining (x_1, y_1, z_1), (x_2, y_2, z_2) (P&Q)

$$\Rightarrow \frac{x_2-x_1}{PQ}, \frac{y_2-y_1}{PQ}, \frac{z_2-z_1}{PQ}$$

- Proj^n of Line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on another line whose dc's are l, m, n

$$\Rightarrow l(x_2-x_1) + m(y_2-y_1) + n(z_2-z_1)$$

Angle between 2 lines: $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

$$= \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

- condition for perpendicularity
  $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$ | $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

- condition for parallelism: $l_1 = l_2$, $m_1 = m_2$, $n_1 = n_2$ |

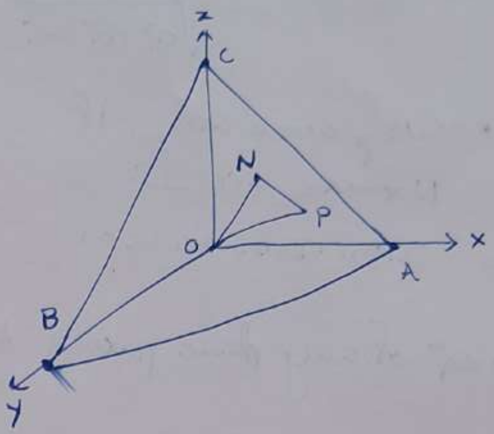
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Plane

equation (Normal form : eq^n in terms of p - length of $\perp$ from origin to the plane)

- $l, m, n \rightarrow$ d.Cs of normal to the plane - dir^n being from origin to the plane.

$\rightarrow eq^n : lx + my + nz = p.$



general eq^n: $ax + by + cz + d = 0$

a, b, c are D R's

- length of $\perp$ from origin to the plane is $\frac{-d}{\sqrt{a^2+b^2+c^2}}$, no. d being -ve.

- Intercept form : $Ax + By + Cz + D = 0$ (or) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
- plane through a given point (x_1, y_1, z_1) and $\perp$ to a line whose D.Rs are a, b, c. $\Rightarrow a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
- eq^n of plane throug 3 points: $\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}$

- Angle b/w two planes: $a_1x + b_1y + c_1z + d_1 = 0$ — (i)

$a_2x + b_2y + c_2z + d_2 = 0$ — (ii)

D.R's of normal to plane (i) $\Rightarrow a_1, b_1, c_1$

— " — (ii) $\Rightarrow a_2, b_2, c_2$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1^2+b_1^2+c_1^2)}\sqrt{(a_2^2+b_2^2+c_2^2)}} \quad \text{— (iii)}$$

- two planes are $\perp$ if Normals are $\perp$:

$\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

- two planes are $\parallel$ if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

- eq^n of any plane parallel to the plane $ax + by + cz + d = 0$ is $ax + by + cz + \lambda = 0$

- two Sides of Plane

$P(x_1, y_1, z_1)$ $Q(x_2, y_2, z_2)$, plane $\rightarrow ax + by + cz + d = 0$

suppose PQ meets R $\Rightarrow$ coordinates: $\left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, [y], [z]\right)$

& PR : QR = $m_1 : m_2$

& $\frac{m_1}{m_2} = \frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d}$

- Length of $\perp$ from (x_1, y_1, z_1) to a given plane

$$\pm \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$$

- To find the distance b/w two parallel planes.

→ take a point on one of the two given planes, then the required distance is the length of ⊥ drawn from this point to other plane

- A plane through the Intersection of two given planes.

$$P + \lambda Q = 0$$

- Condition that line (d.R : l, m, n) may be parallel / ⊥ to a given plane $ax+by+cz+d=0$

~~ax+by+cz=0~~

line ∥ to plane

$al+bm+cn=0$

line ⊥ to plane

$$\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$$

- Angle b/w Line & Plane is defined to be the component of angle b/w line and normal to the plane.

- Eq^n of planes bisecting two given planes.

– if (x_1, y_1, z_1) be co-ordinates of any point on plane bisecting the angle b/w the given planes, then the ⊥ distances of point from both the planes should be equal numerically.

eq^n: $$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

• eq^n of pair of planes

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

• angle b/w two planes.

$$\tan\theta = \left[\frac{2\sqrt{f^2+g^2+h^2-bc-ca-ab}}{a+b+c}\right]$$

cond^n of $\perp$: $a+b+c=0$

• Area of $\Delta \Rightarrow \Delta^2 = \Delta_x^2 + \Delta_y^2 + \Delta_z^2$

The Straight Line

- General Equation → $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$ } line of Intersection of 2 planes.

- Symmetrical form :

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = r$$

r is actual Distance of any point $P(x, y, z)$ from (x_1, y_1, z_1)

- In terms of D.R : $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = r$

- Line through two points.

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

- The projection/Image of a Line on or in a given plane.

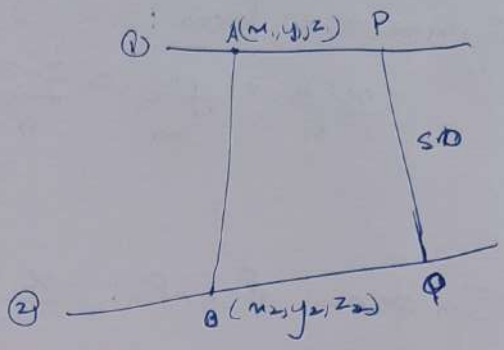
- The projⁿ of a line on a given plane is the line of intersection of the two planes - (i) the given plane and (ii) plane through given line & ⊥ to the given plane.

Shortest Distance

- Skew Lines : Neither Intersect , Nor parallel to each other.
- S.Dist: straight Line which is $\perp$ to each of the two skew Lines is S.D.

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \quad -(i)$$

$$\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2} \quad -(ii)$$

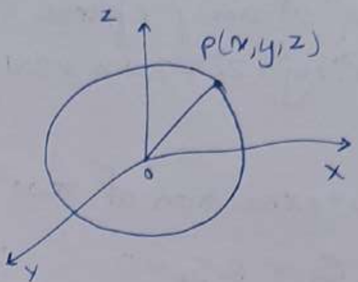


S.D eqn:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l & m & n \end{vmatrix} = 0$$

Sphere

- Locus of a point, which moves so that Distance from a fixed point remains constant.



- Standard form →

$$x^2+y^2+z^2=r^2$$

- $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

(or) → $x^2+y^2+z^2-2ax-2by-2cz+(a^2+b^2+c^2-r^2)=0$

→ comparing with general 2^{nd} degree eqn: $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

$u=-a,\ v=-b,\ w=-c$

$d=a^2+b^2+c^2-r^2$

→ coeff. of $x^2, y^2, z^2 \to 1$; $xy, yz, zx \to 0$

- Diametric form: $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0$

- Touching spheres: $C_1C_2 = r_1+r_2$

  $C_1C_2 = |r_1-r_2|$ ... internally touching

- 4 point form: $(x_1,y_1,z_1), (x_2,y_2,z_2), (x_3,y_3,z_3), (x_4,y_4,z_4)$

→ (4 non-coplanar points)

→ substitute it eqn: $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

→ find u, v, w, d → will give eqn of sphere.

The Conicoid

Locus / General equation:

$$ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$$

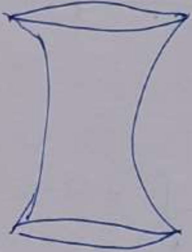
of 2^{nd} Degree in x,y,z is conicoid or a quadric.

(i) Every straight Line meets the surface in two points and every plane section of such a surface is a conic.

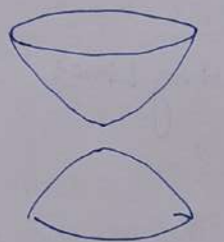
(ii) Gen. eq^n → 10 constants → atleast one is non-zero.
- Conicoid can be determined so as to pass through nine given points, no four of which are coplanar.

① ellipsoid : $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

② Hyperbolloid of one Sheet : $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$



③ Hyperbolloid of two sheets : $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

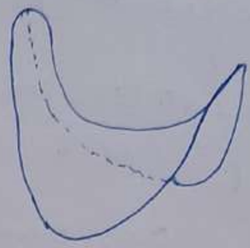


④ $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$

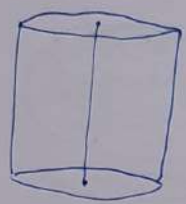
cone

Elliptical Paraboloid : $\frac{x^2}{a^2}+\frac{y^2}{b^2} = \frac{2z}{c}$

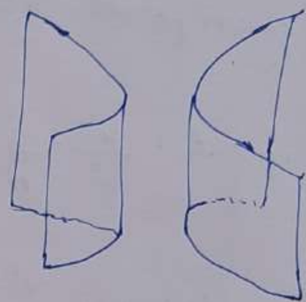
Hyperbollic Parabolloid $\frac{x^2}{a^2}-\frac{y^2}{b^2} = \frac{2z}{c}$



Elliptical cylinder $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$



Hyperbolic cylinder : $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$



$\frac{x^2}{a^2}-\frac{y^2}{b^2} = 0$

Pair of Intersecting Lines