

Shubham Raj

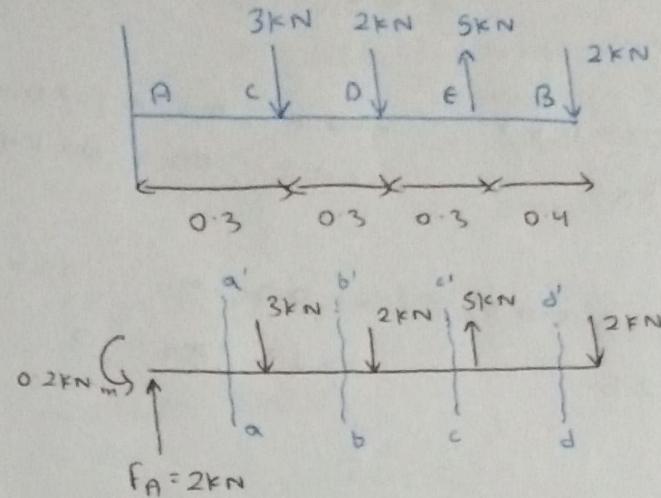
Signature: Shubham

CE19B030

Strength of Materials (CE2101) (CE2102)

1>

1>



$$\sum F_y \Rightarrow F_A = 0$$

$$\therefore F_A = 2\text{ kN}$$

$$\sum M_A = 0$$

$$\Rightarrow -3 \times 0.3 - 2 \times 0.6$$

$$+ 5 \times 0.3 - 2 \times 0.3$$

$$= -0.9 - 1.2 + 1.5 - 0.6$$

$$\therefore -0.2 \text{ kN m}$$

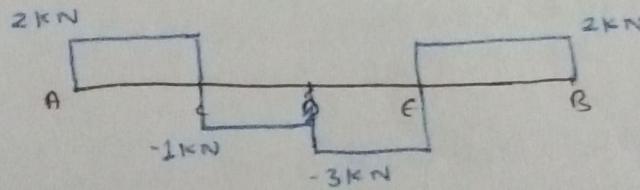
a) For shear force,

$$aa' \Rightarrow V_x = 2\text{ kN} \text{ (AC)}$$

$$bb' \Rightarrow V_x = -1\text{ kN} \text{ (AD)}$$

$$cc' \Rightarrow V_x = -3\text{ kN} \text{ (AE)}$$

$$dd' \Rightarrow V_x = 2\text{ kN} \text{ (AB)}$$



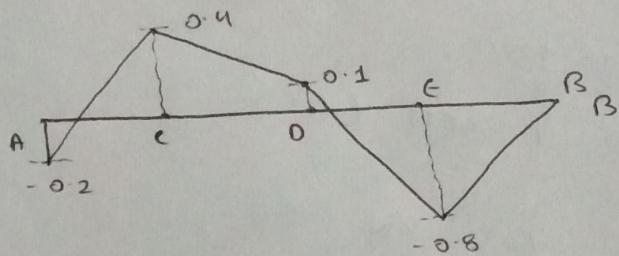
v) for Bending moment,

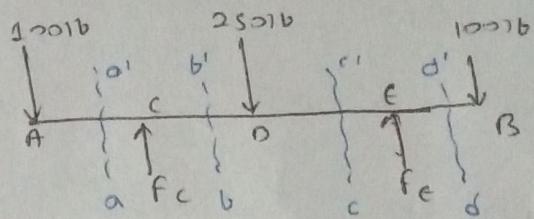
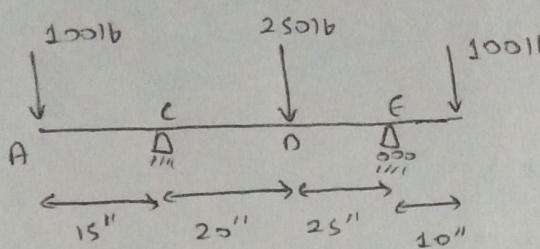
$$aa' \Rightarrow 2x - 0.2 \quad \text{at } x=0 \quad M_x = -0.2 \text{ kN-m}$$
$$x=0.3 \quad M_x = 0.4 \text{ kN-m}$$

$$bb' \Rightarrow 2x(-0.2) - 3(x-0.3)$$
$$= 0.7 - x \quad \text{at } x=0.3 \quad M_x = 0.4 \text{ kN-m}$$
$$x=0.6 \quad M_x = 0.1 \text{ kN-m}$$

$$cc' \Rightarrow 0.7 - x - 2(x-0.6)$$
$$\Rightarrow 1.9 - 3x \quad x=0.6 \quad M_{x1} = 0.1 \text{ kN-m}$$
$$x=0.9 \quad M_{x2} = -0.8 \text{ kN-m}$$

$$dd' \Rightarrow 1.9 - 3x + 5(x-0.9) \quad x=0.9 \quad M_{x1} = -0.8 \text{ kN-m}$$
$$\Rightarrow 2x - 2.6 \quad x=1.3 \quad M_x = 0$$





$$\text{B } \sum M_c = 0 \Rightarrow 100 \times 15 - 250 \times 25 + f_e \times 45 - 100 \times 55 = 0$$

$$\begin{aligned} \Rightarrow 45f_e &= 100(55-15) + 250 \times 25 \\ &= 100 \times 40 + 5000 \\ &\approx 9,000 \end{aligned}$$

$$f_e = 200 \text{ lb}$$

$$\sum F_y = 0 \Rightarrow f_e = 250 \text{ lb}$$

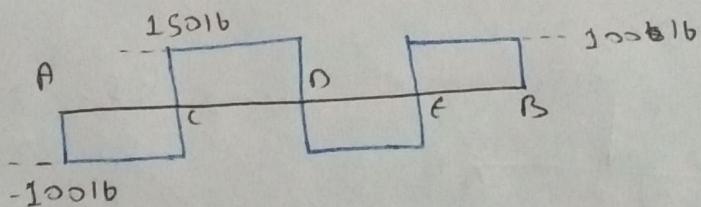
a) for shear force,

$$aa' \Rightarrow V_{S1} = -100 \text{ lb}$$

$$bb' \Rightarrow V_{S1} = \cancel{150 \text{ lb}} \quad 150 \text{ lb}$$

$$cc' \Rightarrow V_{S1} = \cancel{-150 \text{ lb}} \quad -100 \text{ lb}$$

$$dd' \Rightarrow V_{S1} = 100 \text{ lb}$$



b) for Bending moment,

$$aa' \Rightarrow M_x = -100x$$

$$\begin{aligned} bb' \Rightarrow M_x &= -100x + 250(x-15) \\ &= 150x - 3,750 \end{aligned}$$

$$\begin{aligned} cc' \Rightarrow M_{x1} &= 150x - 3,750 - 250(x-35) \\ &= 5000 - 100x \end{aligned}$$

$$\begin{aligned} dd' \Rightarrow M_{x1} &= 5000 - 100x + 200(x-60) \\ &= 100x - 7,000 \end{aligned}$$

for main $x=0$ $M_x = 0$

$x = 15$ $M_x = -1500 \text{ lb/in}$

for bb' $\Rightarrow x = 15$ $M_{x1} = -1500 \text{ lb/in}$

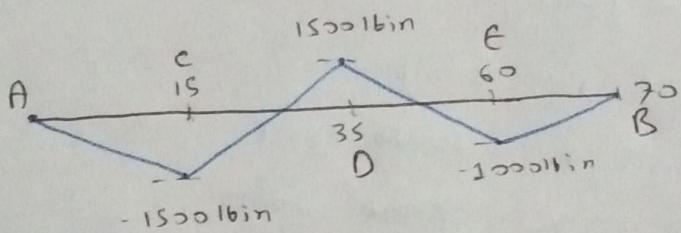
$x = 35$ $M_{x1} = 1500 \text{ lb/in}$

for cc' $\Rightarrow x = 35$ $M_x = 1500 \text{ lb/in}$

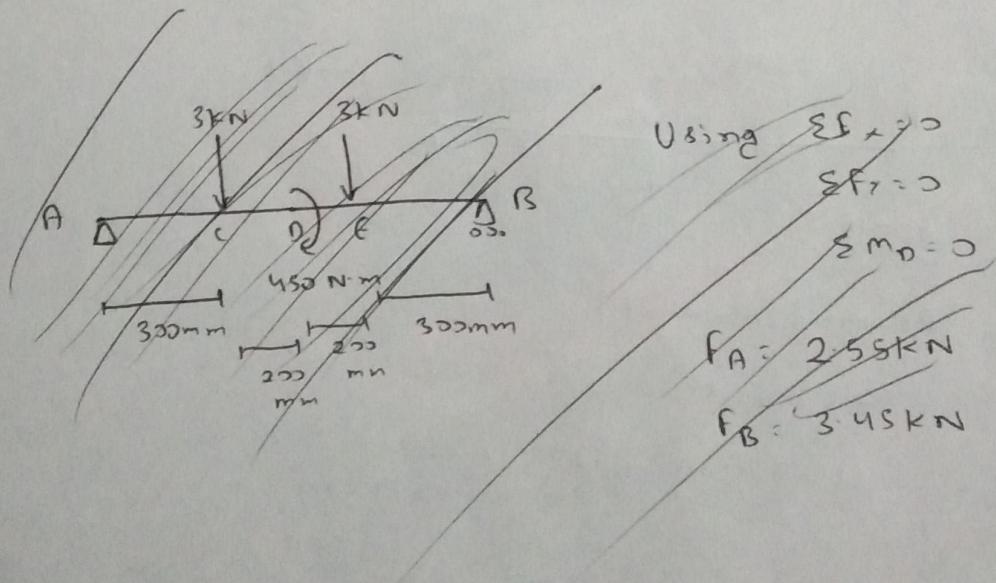
$x = 60$ $M_x = -1500 \text{ lb/in}$

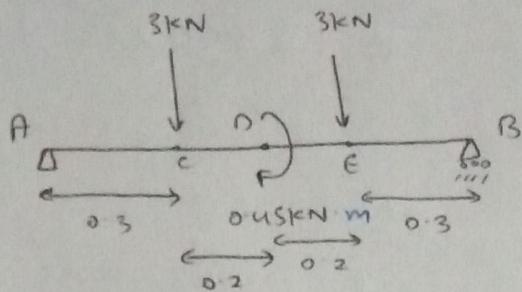
for dd' $\Rightarrow x = 60$ $M_x = -1500 \text{ lb/in}$

$x = 70$ $M_{x1} = 0$



III>

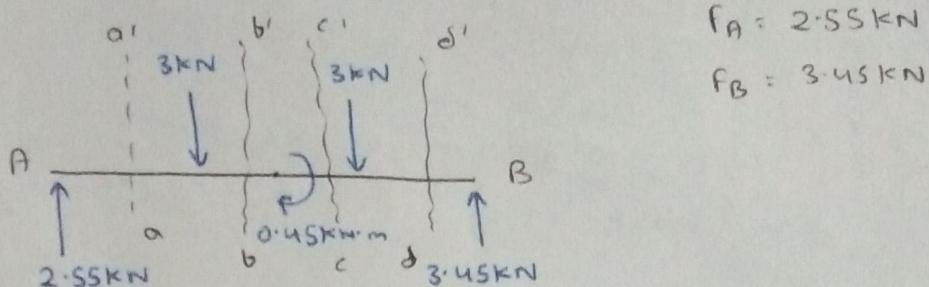




$$\text{Using } \sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_O = 0$$



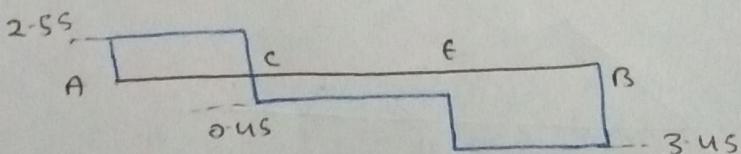
a) For shear,

$$aa' \Rightarrow V_x = 2.55\text{kN}$$

$$bb' \Rightarrow V_x = -0.45\text{kN}$$

$$cc' \Rightarrow V_x = -0.45\text{kN}$$

$$dd' \Rightarrow V_x = 3\text{kN} - 3.45\text{kN}$$



b) For moment diagram,

$$aa' \Rightarrow 2.55x \quad \text{at } x=0 \quad M_x = 0$$

$$x=0.3 \quad M_x = 0.765$$

$$bb' \Rightarrow 2.55x - 3(x-0.3) \quad \text{at } x=0.3 \quad M_x = 0.765$$

$$\Rightarrow 0.9 - 0.45x \quad x=0.5 \quad M_x = 0.675$$

$$CC' \Rightarrow 0.9 - 0.45x + 0.45 \quad \text{at } x = 0.5$$

$$\Rightarrow 1.35 - 0.45x \quad M_{x_0} = 1.125 \text{ kN m}$$

$$\text{at } x = 0.7$$

$$M_{x_0} = 1.035 \text{ kN m}$$

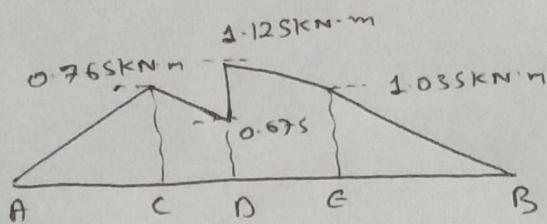
$$DD' \Rightarrow 1.35 - 0.45x - 3(x - 0.7) \quad \text{at } x = 0.7$$

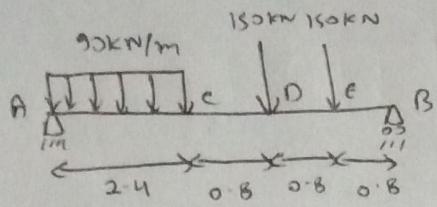
$$M_{x_0} = 1.035 \text{ kN m}$$

$$\Rightarrow 3.45 - 3.45x$$

$$\text{at } x = 1$$

$$M_{x_0} = 0$$





Using

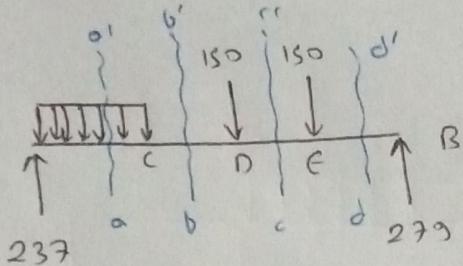
$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_A = 0$$

$$f_B = 279 \text{ kN}$$

$$f_A = 237 \text{ kN}$$



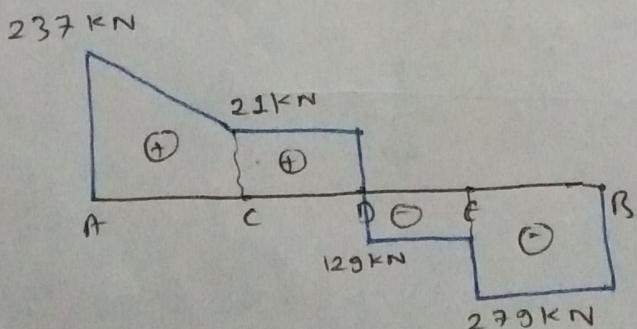
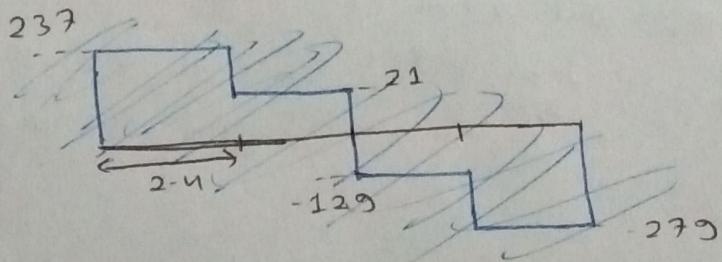
a) For shear force,

$$aa' \Rightarrow V_x = 237 - 90x \quad \begin{matrix} \text{at } x=0 & V_x = 237 \text{ kN} \\ x=2.4 & V_x = 21 \text{ kN} \end{matrix}$$

$$bb' \Rightarrow V_x = 237 - 90 \times 2.4 = 21 \text{ kN}$$

$$cc' \Rightarrow V_x = 237 - 90 \times 2.4 - 150 \quad (\text{or } 327) \\ \therefore -129 \text{ kN}$$

$$dd' \Rightarrow V_x = -279 \text{ kN}$$



b) For Bending moment,

$$aa' \Rightarrow M_{x_1} = 237x - 90x \cdot \frac{7}{2}$$

$$= 8237x - \frac{90x^2}{2}$$

$$= 237x - 45x^2 \quad \text{at } x_1 = 0 \quad M_x = 0$$

$$x = 2.4 \quad M_{x_1} = 568.8 - 259.2 \\ = 309.6 \text{ KN m}$$

$$bb' \Rightarrow M_{x_1} = 237x_1 - 90x_1 \cdot 2.4(x_1 - 1.2)$$

$$= 21x_1 + 259.2 \quad \text{at } x_1 = 2.4 \quad M_{x_1} = 309.6 \text{ KN m}$$

$$x_1 = 3.2 \quad M_x = 326.4 \text{ KN m}$$

$$cc' \Rightarrow M_{x_1} = 21x_1 + 259.2 - 150(x_1 - 3.2)$$

$$= 480 + 259.2 - 129x_1 \quad \text{at } x_1 = 3.2$$

$$= 739.2 - 129x_1 \quad M_x = 326.4 \text{ KN m}$$

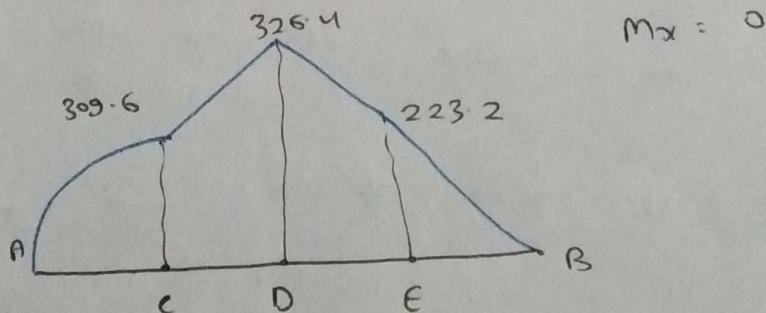
$$x_1 = 4$$

$$M_x = 223.2 \text{ KN m}$$

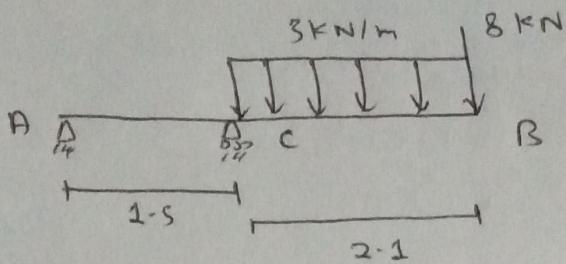
$$dd' \Rightarrow M_{x_1} = 739.2 - 129x_1 - 150(x_1 - 4) \\ = 1,339.2 - 279x_1 \quad \text{at } x_1 = 4$$

$$M_{x_1} = 223.2 \text{ KN m}$$

$$x_1 = 4.8$$



v)

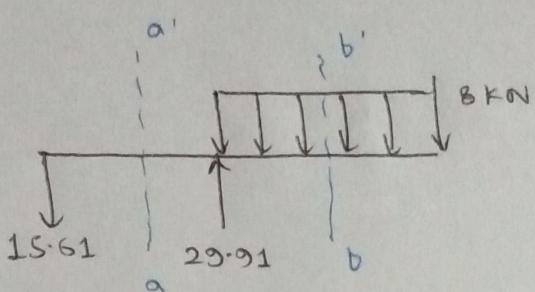


Using

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_c = 0$$



$$F_A = -15.61 \text{ kN}$$

$$F_B = 29.91 \text{ kN}$$

a) for shear force,

$$aa' \Rightarrow V_x = -15.61$$

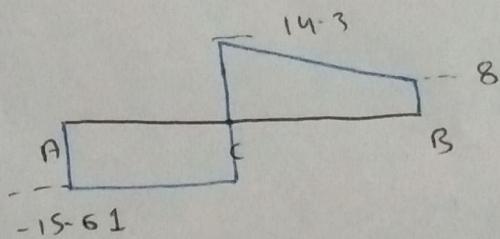
$$bb' \Rightarrow -15.61 + 29.91 - 3(x-1.5)$$

$$\Rightarrow 14.3 - 3x + 4.5$$

$$\Rightarrow 18.8 - 3x$$

$$\text{at } x = 1.5 \quad V_x = 14.3$$

$$x = 3.6 \quad V_x = 8$$



b) for moment,

$$aa' = -15.61x \quad \text{at} \quad x = 0 \quad M_x = 0$$

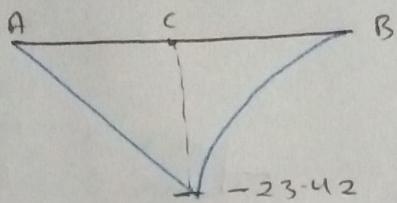
$$x = 1.5 \quad M_x = -23.42 \text{ kN-m}$$

$$bb' \Rightarrow M_{x_1} = -15.61x + 28.91(x-1.5) - \frac{3(x-1.5)^2}{2}$$

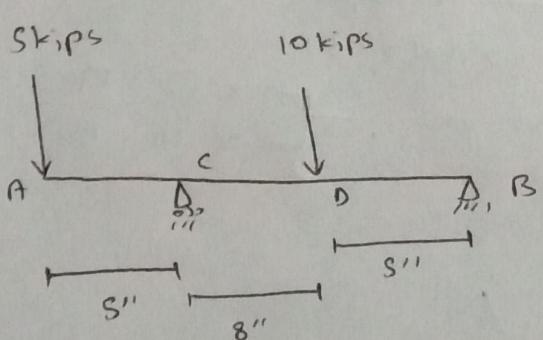
$$= 18.8x - 1.5x^2 - 48.24$$

$$\text{at } x=1.5 \quad M_{x_1} = -23.42$$

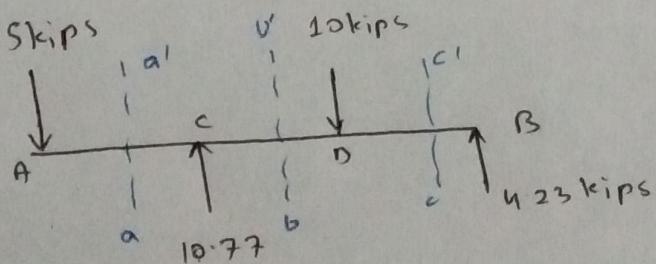
$$x = 3.6 \quad M_x = 0$$



v1)



$$\begin{aligned}\sum M_C &= 0 \\ \Rightarrow 5 \times 5 - 10 \times 8 + f_B \times 13 &= 0 \\ \therefore 13f_B &= 80 - 25 \\ f_B &= 4.23 \text{ kips} \\ f_A &= 10.77 \text{ kips}\end{aligned}$$

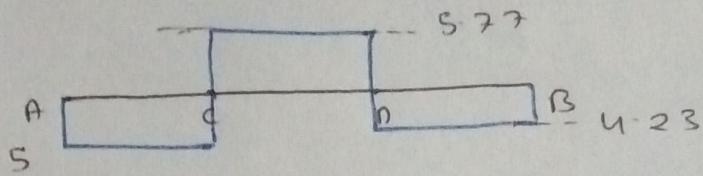


a) for Shear,

$$aa' \Rightarrow V_x = -5 \text{ kips}$$

$$bb' \Rightarrow V_{x1} = 5.77 \text{ kips}$$

$$cc' \Rightarrow V_{x1} = -4.23 \text{ kips}$$

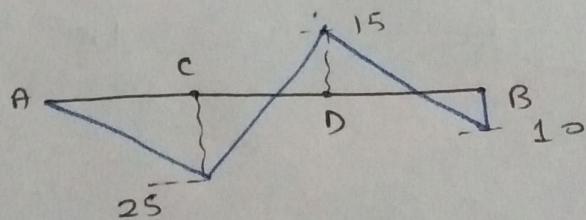


b) for Bending,

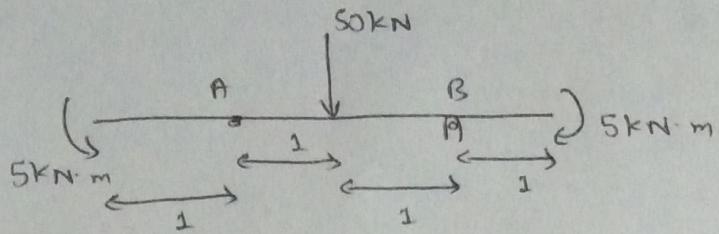
$$aa' \Rightarrow M_x = -5x \quad \begin{cases} x=0 & M_{x1}=0 \\ x=5 & M_{x1}=-25 \end{cases}$$

$$bb' \Rightarrow M_{x1} = -5x + 10(x-5) \\ = 5x - 50 \quad \begin{cases} x=5 & M_x=-25 \\ x=13 & M_x=15 \end{cases}$$

$$cc' \Rightarrow M_{x1} = 5x - 50 - 10(x-13) \\ = 80 - 5x \quad \begin{cases} x=13 & M_{x1}=15 \\ x=18 & M_x=-10 \end{cases}$$



VII)



Using $\varepsilon f_x = 0$

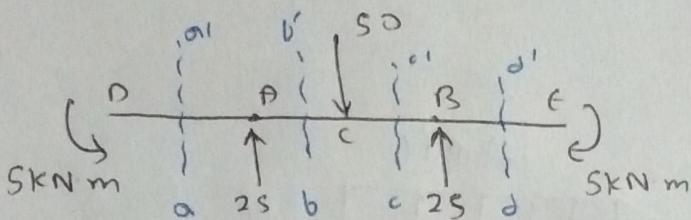
$$\sum F_y = 0$$

50 - 5

$$\sum m_B = 0$$

$$F_A = 2skN$$

$$F_B = 25 \text{ kN}$$



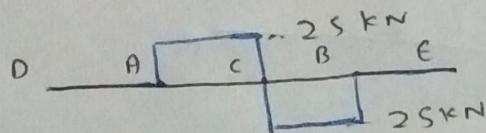
as for shear,

$$aa' \Rightarrow V_x = 0$$

$$60 \Rightarrow v_{2c} = 25 \text{ kN}$$

$$cc' \Rightarrow r_x = -25\text{ kN}$$

$$do' \Rightarrow v_x \in OKN$$



b) for moment,

$$aa' \Rightarrow M_x = -5kN \cdot m$$

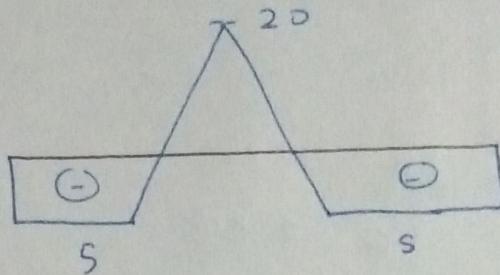
$$m_{21} = -5 + 25(x-1) \quad \begin{cases} x=1 & mx = -5 \\ x=2 & mx = 25 \end{cases}$$

$$= 25x - 30$$

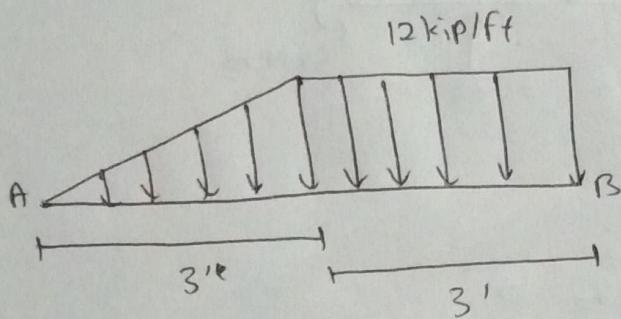
$$\Rightarrow M_{x1} = 25x - 30 - 50(x-2) \\ \qquad\qquad\qquad : 70 - 25x \quad \left[\begin{array}{l} x=2 \\ x=3 \end{array} \right] \quad m_{x1} = 20 \quad m_{x1} = -5$$

$$dd' \rightarrow m_2 22\bar{2}$$

$$\text{do} \Rightarrow M_x = 70 + 25x + 25(21 - 3) \\ = -5$$

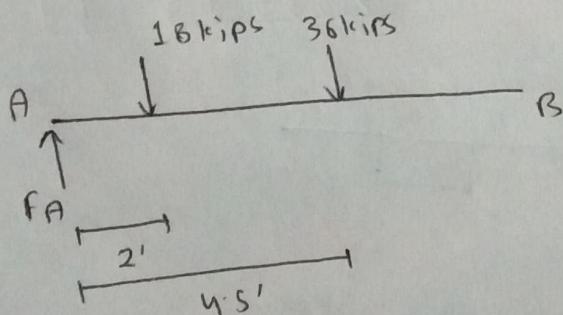


viii)



$$\text{Total load: } \frac{1}{2} \times 12 \times 3 = 18 \text{ kip (Best Triangle)}$$

$$\text{Total load: } 12 \times 3 = 36 \text{ (Rectangle)}$$

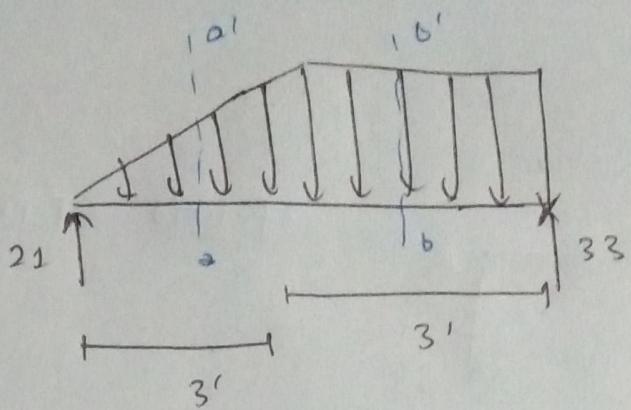


$$\sum M_A \Rightarrow -18 \times 2 - 36 \times \frac{9}{2} + F_B \times 6 = 0$$

$$\therefore 8F_B = 16 \times 11$$

$$F_B = 33 \text{ kips}$$

$$F_A = 21 \text{ kips}$$



a) For shear,

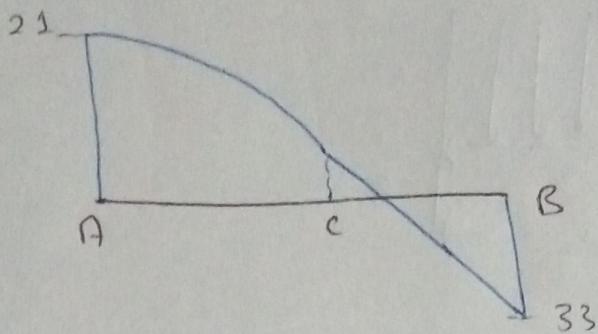
$$aa' \Rightarrow V_x = 21 - \frac{1}{2}x \cdot 4x^2 \\ \approx 21 - 2x^2 \quad \left. \begin{array}{l} x=0 \quad V_x=21 \\ x=3 \quad V_x=3 \end{array} \right.$$

$$bb' \Rightarrow V_x = 21 - \frac{1}{2}x^2 + 3 = 12(x-3) \\ \Rightarrow -12x + 39 \quad \left. \begin{array}{l} x=3 \quad V_x=3 \\ x=6 \quad V_x=-33 \end{array} \right.$$

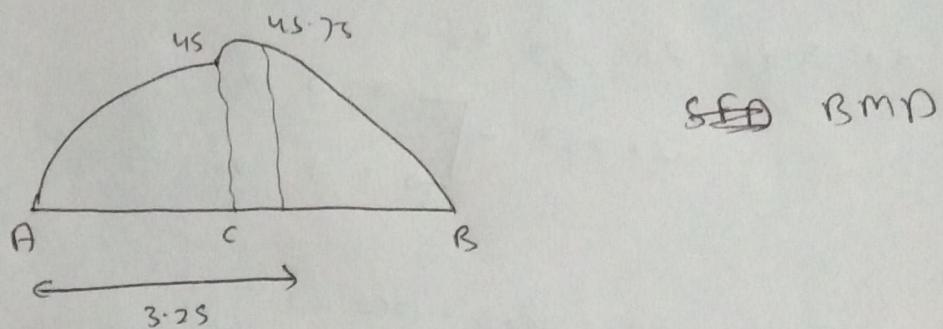
b) For moment,

$$aa' \Rightarrow M_{x1} = 21x - \frac{1}{2}x \cdot 2x^2 \cdot \frac{21}{3} \\ = 21x - \frac{2x^3}{3}$$

$$bb' \Rightarrow M_{x1} = 21x - 18(x-2) - \frac{12(x-3)^2}{2} \\ = 21x - 18(x-2) - 6(x-3)^2$$

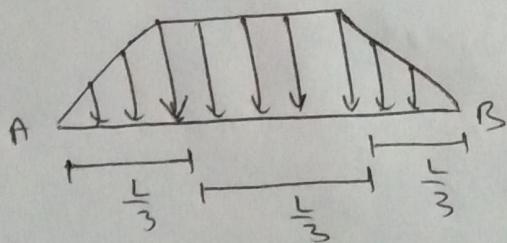


~~BMD~~ SFD

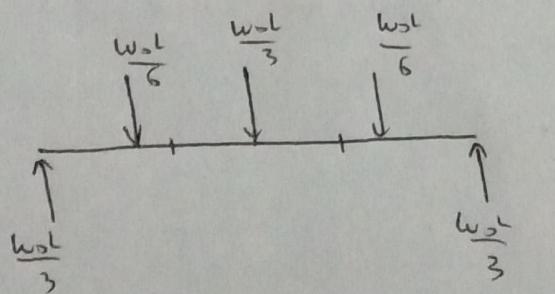


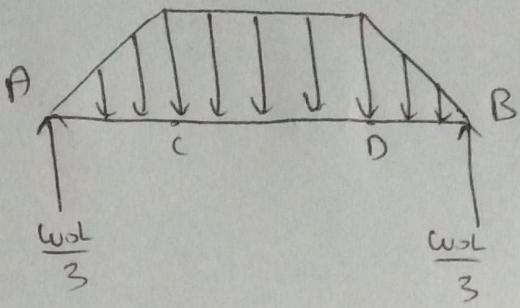
~~SFD~~ BMD

1x>



from symmetry,





a) for shear,

$$\begin{aligned} AC \rightarrow V_x &= \frac{w_0 L}{3} - \frac{1}{2} x \cdot \frac{w_0 \cdot 3x}{L} \\ &= \frac{w_0 L}{3} - \frac{3w_0 x^2}{2L} \end{aligned}$$

$$\begin{aligned} AD \rightarrow V_{xi} &= \frac{w_0 L}{3} - \frac{w_0 L}{6} - w_0 \left(xi - \frac{L}{3} \right) \\ &= \frac{w_0 L}{2} - w_0 \left(xi - \frac{L}{3} \right) \end{aligned}$$

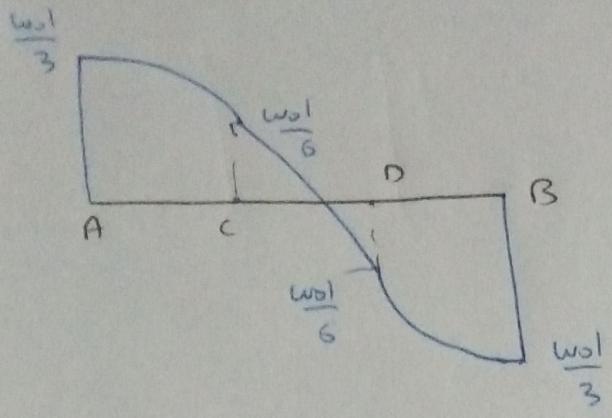
$$AB \rightarrow V_{xi} = \frac{w_0 L}{2} - w_0 \left(xi - \frac{L}{3} \right) - \frac{w_0 L}{3} + \frac{3w_0}{2} \left(xi - \frac{2L}{3} \right)^2$$

b) for Bending,

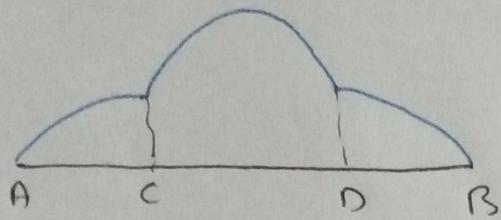
$$AC \rightarrow M_{xi} = \frac{w_0 b x}{3} - \frac{w_0 x^3}{24}$$

$$AD \rightarrow M_{xi} = \frac{w_0 b x}{3} - \frac{w_0 d}{6} \left(xi - \frac{2L}{3} \right) - \frac{w_0}{2} \left(xi - \frac{L}{3} \right)^2$$

$$AB \rightarrow M_{xi} = \frac{w_0 b x}{3} - \frac{w_0 d}{6} \left(xi - \frac{2L}{3} \right) - \frac{w_0}{3} \left(xi - \frac{L}{3} \right) - \frac{w_0}{24} \left(xi - \frac{2L}{3} \right)^3$$

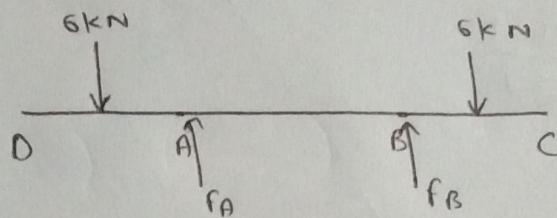
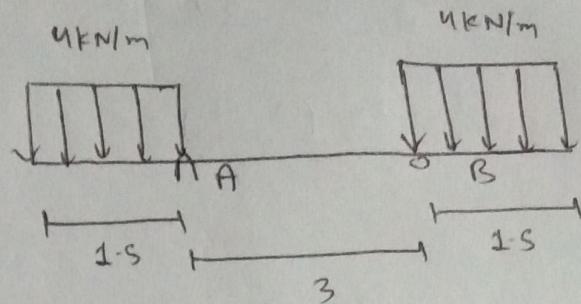


SFD



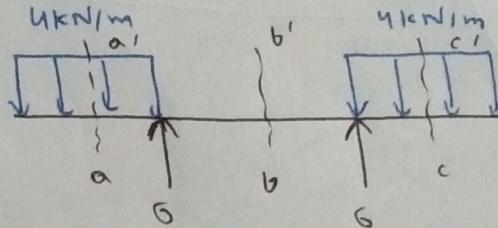
105

>>



By symmetry,

$$f_A = f_B = 6$$



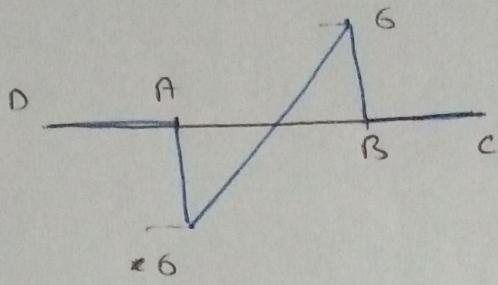
a) For Shear Force,

$$aa' \Rightarrow -4x \quad \begin{cases} x=0 & V_x = 0 \\ x=1.5 & V_x = -6 \end{cases}$$

$$bb' \Rightarrow 6 - 4x \times 1.5 = 0$$

$$\text{(Left)} \quad 6 - 4x \quad \begin{cases} x=0 & V_x = 6 \\ x=1.5 & V_x = 0 \end{cases}$$

$$cc' \Rightarrow 6 - 4(x - 1.5) \quad \begin{cases} x=1.5 & V_x = 0 \\ x=6 & V_x = 6 \end{cases}$$

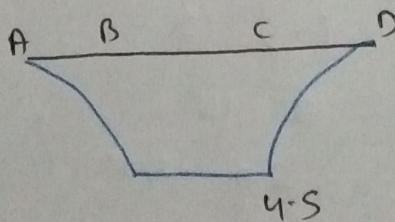


b) For Moment diagram,

$$aa' \Rightarrow -qx - \frac{x}{2} \Rightarrow -2qx^2 \quad \begin{cases} x=0 & M_x = 0 \\ x=1.5 & M_x = -4.5 \end{cases}$$

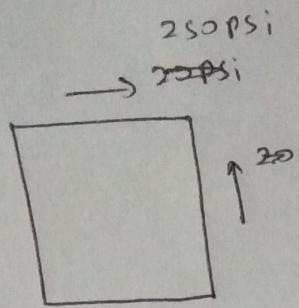
$$\begin{aligned} bb' &\Rightarrow -4x + 5(x - 0.75) + 6(x - 1.5) \\ &= -6(x - 0.75) + 6(x - 1.5) \\ &\Rightarrow 6[x - 1.5 - x + 0.75] \\ &\Rightarrow -4.5 \end{aligned}$$

$$\begin{aligned} cc' &\Rightarrow -4.5 - 4(x - 4.5) \frac{(x - 4.5)}{2} \\ &\Rightarrow -4.5 - \frac{4(x - 4.5)^2}{2} \quad \begin{cases} x=4.5 & M_x = -4.5 \\ x=6 & \end{cases} \\ &\quad M_x = -4.5 - \frac{4(2.25)^2}{2} \\ &\quad = -9 \end{aligned}$$



2>

a)



$$\sigma_x = \sigma_y = 0$$

$$\tau_{xy} = 250 \text{ psi}$$

$$\theta = +15^\circ$$

$$2\theta = -30^\circ$$

$$\sigma_a = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \tau_{xy} \sin 2\theta$$

$$= 250 \sin(-30^\circ)$$

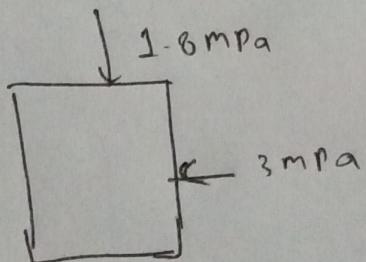
$$= -125 \text{ psi} \quad (\text{compression})$$

$$\tau_a = - \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= \tau_{xy} \cos 2\theta$$

$$= 216.5 \text{ psi}$$

b)



$$\sigma_x = -1.8 \text{ MPa}$$

$$\sigma_y = -3 \text{ MPa}$$

$$\tau_{xy} = 0$$

$$\theta = -15^\circ$$

$$2\theta = -30^\circ$$

$$\sigma_o = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\alpha + t_{xy} \sin 2\alpha$$

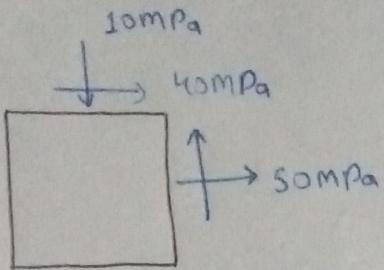
$$= \left(-\frac{1.6 + 3}{2} \right) + \left(\frac{-1.6 + 3}{2} \right) \cos(-30^\circ)$$

$$= -1.66 \text{ MPa}$$

$$t_o = - \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\alpha + t_{xy} \cos 2\alpha$$

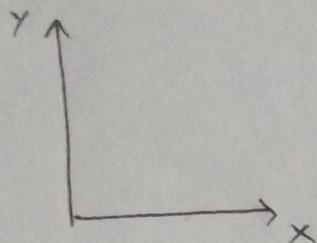
$$= 0.3 \text{ MPa}$$

3>



Sign conventions,

Tensile stress (+ve)

Shear stress producing
anti-clockwise rotation (+ve)

$$\sigma_x = 50 \quad \sigma_y = -10$$

$$\tau_{yx} = -40 \quad \tau_{xy} = 40$$

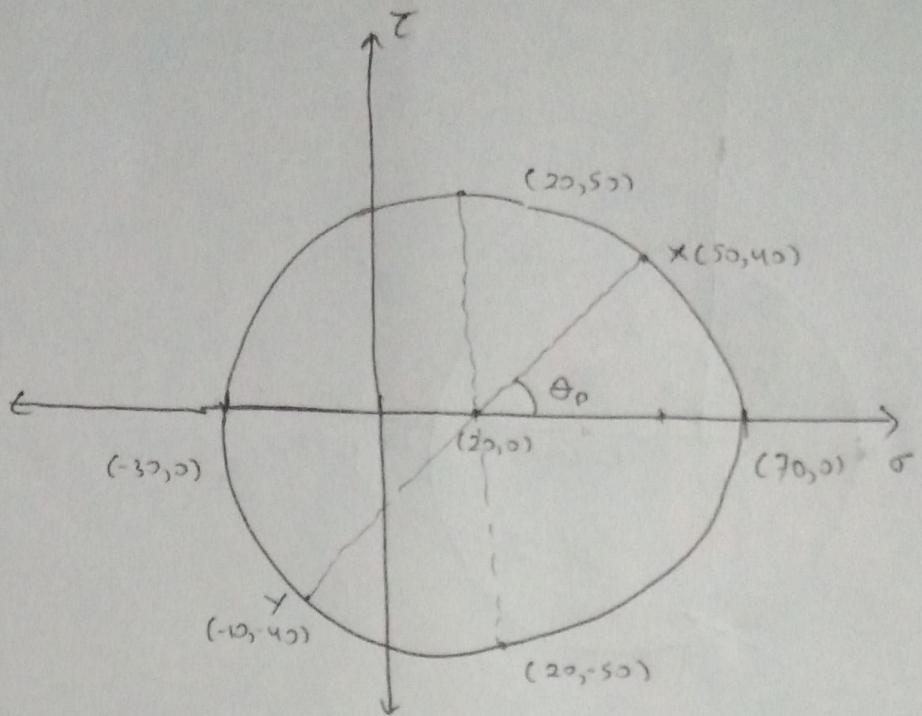
~~Because~~ we will use Mohr's circle,

$$\left[\sigma_x' - \left(\frac{\sigma_x + \sigma_y}{2} \right) \right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2$$

$$\text{Centre } C \text{ of circle} = \left(\frac{50 - 10}{2}, 0 \right) = (20, 0)$$

$$R = \sqrt{\left(\frac{50 + 10}{2} \right)^2 + 1600} = 50$$

$$X = (50, 40) \quad Y = (-10, -40)$$

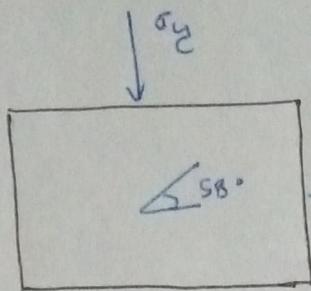


a) $\tan \theta_0 = \frac{45}{50} \Rightarrow \theta_0 = \tan^{-1}\left(\frac{45}{50}\right)$
 $= 38.66^\circ$

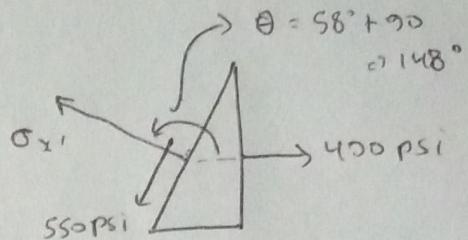
b) $\sigma_{\max} = 70 \text{ MPa}$ { $\sigma_{\min} = 30 \text{ MPa}$

c) $\tau = 50 \quad \sigma_{\text{average}} = 20$

4)



$$\sigma_x = 400 \text{ psi}$$



we know that,

$$\tau_{x'y'} = - \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\therefore 550 = - \left(\frac{400 - \sigma_y}{2} \right) \sin 296^\circ + 0$$

$$\therefore 550 = \left(\frac{400 - \sigma_y}{2} \right) \cdot 0.89$$

$$\therefore \frac{1100}{0.89} = (400 - \sigma_y)$$

$$\therefore 400 - \sigma_y = 1235.95$$

$$\therefore \sigma_y = -835.95 \text{ psi}$$