Tutorial 4 (DAA)

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DAA

Ans-1 $T(n) = 3T(n/2) + n^2$

$a = 3, b = 2, f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (case 3)

$\therefore$ A/c to master's theorem

$T(n) = \Theta(n^2)$

Ans-2 $T(n) = 4T(n/2) + n^2$

$a = 4, b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (case-2)

$\therefore$ according to master's theorem

$T(n) = \Theta(n^2 \log n)$

Ans-3 $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$n^{\log_b a} = n^0 = 1$

$1 < 2^n$ (case 3)

$\therefore T(n) = \Theta(2^n)$

Ans-4 $T(n) = 2^n T(n/2) + n^3$

$\therefore$ master theorem is not applicable as a is a function

Ans-5 $T(n) = 16T(n/4) + n$

$a = 16, b = 4$

$n^{\log_4 16} = n^2$

$\because n^2 > f(n)$ (case 1)

$\therefore T(n) = \Theta(n^2)$

Ans-6 $T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2, f(n) = n\log n$

$n^{\log_2 2} = n < f(n)$

$\therefore T(n) = \Theta(n\log n)$

Ans-7 $T(n) = 2T(n/2) + \frac{n}{\log n}$

$a = 2, b = 2, f(n) = \frac{n}{\log n}$

$n^{\log_2 2} = n > f(n)$

$\therefore T(n) = \Theta(n)$

Ans-8 $T(n) = 2T(\frac{n}{4}) + n^{0.51}$

$a = 2, b = 4, f(n) = n^{0.51}$

$n^{\log_4 2} = n^{0.5} < f(n)$

$\therefore T(n) = \Theta(n^{0.5})$

Ans-9 $T(n) = 0.5T(n/2) + \frac{1}{n}$

$\therefore$ Not applicable as $a < 1$

Ans-11 $T(n) = 4T(\frac{n}{2}) + \log n$

$a=4, b=2 \quad f(n) = \log n$

$n^{\log_2 4} = n^2 > f(n)$

$\therefore T(n) = \Theta(n^2)$

Ans-12 $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$

$\therefore$ MT not applicable as a is not constant

Ans-13 $T(n) = 3T(n/2) + n$

$a=3, b=2, f(n) = n$

$n^{\log_2 3} = n^{1.58} > f(n)$

$\therefore T(n) = \Theta(n^{1.58})$ or $\Theta(n^{\log_2 3})$

Ans-14 $T(n) = 3T(n/3) + \sqrt{n}$

$a=3, b=3, f(n) = \sqrt{3n}$

$n^{\log_3 3} = n > \sqrt{n}$

$\therefore T(n) = \Theta(n)$

Ans-15 $T(n) = 4T(n/2) + cn$

$a=4, b=2, \quad f(n) = c*n$

$n^{\log_2 4} = n^2$

$n^2 > cn$

$\therefore T(n) = \Theta(n^2)$

Ans-16 $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$n^{\log_4 3} = n^{0.79} < f(n)$

$\therefore\ T(n) = \Theta(n\log n)$

Ans-17 $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3,\ f(n) = n/2$

$n^{\log_3 3} = n$

$\Theta(n) = \Theta\left(\frac{n}{2}\right)$

$\therefore\ T(n) = \Theta(n\log n)$

Ans-18 $T(n) = 6T(n/3) + n^2\log n$

$a = 6,\ b = 3,\ f(n) = n^2\log n$

$n^{\log_3 6} = n^{1.63} < n^2\log n$

$\therefore\ T(n) = \Theta(n^2\log n)$

Ans-19 $T(n) = 4T(n/2) + n/\log n$

$a = 4,\ b = 2,\ f(n) = n/\log n$

$n^{\log_2 4} = n^2$

$n^2 > n/\log n$

$\therefore\ T(n) = \Theta(n^2)$

Ans-20 $T(n) = 64\,T(n/8) - n^2 \log n$

Master theorem is not applicable as $f(n)$ is not increasing function

Ans-21 $T(n) = 7\,T(n/3) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

$\therefore T(n) = \Theta(n^2)$

Ans-22 $T(n) = T(n/2) + n(2 - \cos n)$

Master's theorem is not applicable since regularity condition is isolated in case 3.