IIT Bombay

Computer Programming

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Session: Representing Floating Point Numbers

Quick Recap of Relevant Topics

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- Architecture of a simple computer
- Representation of integers

Overview of This Lecture

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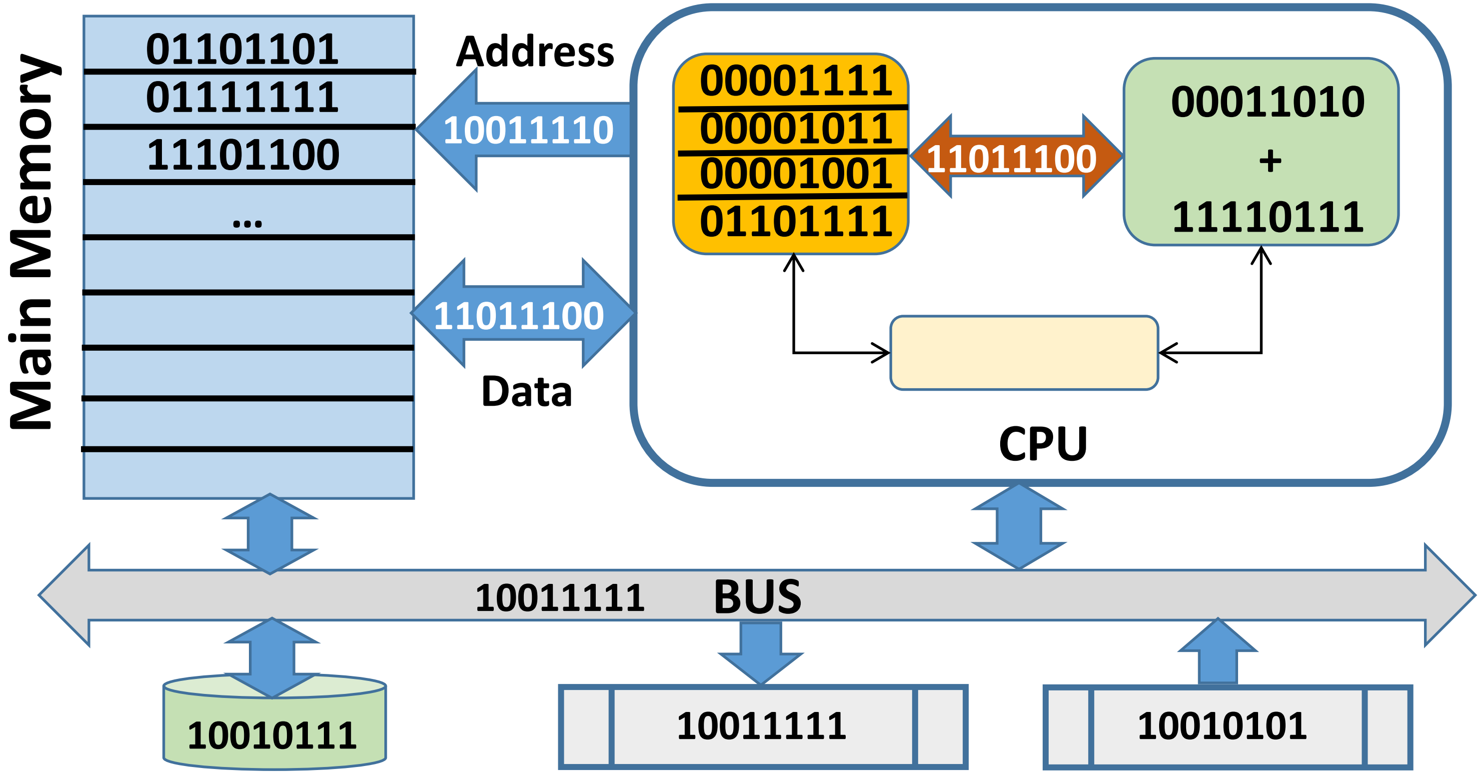
- A computer’s internal representation of numbers
  - Floating point numbers
- C++ declarations of floating point variables

Recap from Earlier Lecture

- Snapshot:



- How do we represent numbers like 3.14×10^{-23} in a computer?

Representing Floating Point Numbers

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- Numbers with fractional values, very small or very large numbers cannot be represented as integers
- Floating point number
  - Decimal: $\underset{\text{Sign}}{-}\ \underbrace{3.123}_{\text{Mantissa}} \times \underset{\text{Base/Radix}}{10}{}^{\underset{\text{Exponent}}{-11}}$
    - Mantissa = $-(3 \times 10^{0} + 1 \times 10^{-1} + 2 \times 10^{-2} + 3 \times 10^{-3})$
  - Binary: -1.1101×2^{110}
    - Mantissa = $-(1 \times 2^{0} + 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}) = -1.8125$
    - Exponent = $(1 \times 2^{2} + 1 \times 2^{1} + 0 \times 2^{0}) = 6$

Representing Floating Point Numbers

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- Normalized mantissa: single non-0 digit to left of radix point
  - $0.02345 \times 10^{12} = 2.345 \times 10^{10}$
  - $110.101 \times 2^{110} = 1.10101 \times 2^{1000}$
  - Binary: Implicit 1 always on left of radix point; need not be stored
- Floating point numbers represented by allocating fixed number of bits for mantissa and exponent
  - Cannot represent all real numbers
  - Finite precision artifacts
    - What is $0.101 \times 2^{111} + 1$ if we have only 3 bits to represent mantissa?

Floating Point Numbers in C++

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- float and double data types
- float
  - 32 bits (4 bytes): 1 sign, 8 exponent, 23 mantissa
  - Approximate range of magnitude: $10^{-44.85}$ to $10^{34.83}$
- double
  - 64 bits (8 bytes): 1 sign, 11 exponent, 52 mantissa
  - Approximate range of magnitude: $10^{-323.3}$ to $10^{308.3}$
- Special bit patterns reserved for 0, infinity, NaN (not-a-number: result of 0/0), …
- C++ declarations: float temperature; double verticalSpeed;

Floating Point Numbers in C++

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- Floating point constants can be specified in C++ programs as
  - 23.572 (can have non-normalized mantissa in programs)
  - 2357.2e-2 or 2357.2E-2 (scientific notation)
    - 2357.2×10^{-2} (base 10)
- C++ constant floating point declaration
  - const float pi = 3.1415
  - const double e = 2.7183
  - Values of pi and e cannot change during program execution

Summary

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- Binary representation of floating point numbers
  - Sign, mantissa and exponent
  - C++ declarations