

# Computer Programming

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Session: Solving Simultaneous Equations

# Quick Recap

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- An Array handles a set of values of same type
  - is declared with a type and size
  - elements of an array are accessed by using index expressions
  - value of index must be between 0 and size-1

# Multidimensional Array (Matrix)



- We can declare and use arrays with more than one dimension

`int A[50][40];`

- Declares a two dimensional array with 50 rows and 40 columns
- Each element is accessed by a reference requiring two index expressions, e.g.,

`A[i][j] = 3782;`

- Row index 'i', can have a value from 0 to 49,
- Column index 'j' can have a value from 0 to 39
- All rules for index expression, apply to index for each dimension

# Simultaneous Equations

- Matrices are used to represent a system of simultaneous equations in multiple variables
- Consider the following equations in two variables

$$2x + 4y = 8 \quad \text{eq.1}$$

$$4x + 3y = 1 \quad \text{eq.2}$$

These equations can be represented as

$$\Rightarrow \begin{pmatrix} 2 & 4 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$$

# Gaussian Elimination Method



This method uses two properties of such systems to solve these equations :

***1. The system of equations is not affected if an equation is multiplied by a constant***

$$2x + 4y = 8 \quad \text{eq. 1}$$

Suppose we multiply eq.1 by 0.5, to get

$$1x + 2y = 4 \quad \text{eq. 1'}$$

we now have the same system of equations, but in the following form:

$$1x + 2y = 4 \quad \text{eq.1'}$$

$$4x + 3y = 1 \quad \text{eq.2}$$

# Simultaneous Equations ...

- Representing these equations in the form of matrices:

$$\begin{array}{rcl} 1x + 2y = 4 \\ 4x + 3y = 1 \end{array} \quad \longrightarrow \quad \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad \begin{array}{l} \text{eq.1'} \\ \text{eq.2} \end{array}$$

# Simultaneous Equations ...

***2. If an equation is replaced by a linear combination of itself and any other row, the system of equations remains same***

Multiply eq.1' by 4 to get  $4x + 8y = 16$

Subtract it from eq. 2, to get new eq.2'

$$\begin{array}{rcl} 4x + 3y & = & 1 \quad \text{eq.2} \\ - (4x + 8y & = & 16) \\ \hline 0x - 5y & = & -15 \quad \text{eq.2'} \end{array}$$

The system is now equivalent to

$$\begin{array}{l} 1x + 2y = 4 \\ 0x - 5y = -15 \end{array}$$



$$\begin{pmatrix} 1 & 2 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -15 \end{pmatrix} \begin{array}{l} \text{eq.1'} \\ \text{eq.2'} \end{array}$$

# Simultaneous Equations ...

If we multiply eq.2' by -0.2, (or equivalently, divide by -5), we will get

$$\begin{array}{l} 1x + 2y = 4 \\ 0x + 1y = 3 \end{array} \longrightarrow \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \begin{array}{l} \text{eq.1'} \\ \text{eq.2''} \end{array}$$

Starting from the last equation, and using back-substitution, we get the solution for all variables.

$$0 * x + 1 * y = 3 \quad \text{eq. 2'' directly gives us } y = 3$$

$$1 * x + 2 * 3 = 4 \quad \text{we back-substitute this value of } y \text{ in eq.1'}$$

$$\text{This gives us } x + 6 = 4, \text{ which in turn gives us } x = -2$$



# Simultaneous Equations ...



- **The essence of the method is to reduce the coefficient-matrix to an upper triangular matrix (all elements on the diagonal are 1), and then use back-substitution**
- The process is susceptible to round off errors
- There are other variations, such as:
  - Gauss Jordan elimination, Pivoting
  - L U decomposition
- A useful reference:
  - “Numerical recipes in C++”, [also in C, Fortran]
  - by William H Press, Saul A Teukolsky,  
William T Vetterling, and Brian P Flannery

# Simultaneous Equations ...

- In general, a system of linear equations in  $n$  variables can be represented by the following matrices

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} & \dots & a_{0n-1} \\ a_{10} & a_{11} & a_{12} & \dots & a_{1n-1} \\ a_{20} & a_{21} & a_{22} & \dots & a_{2n-1} \\ \vdots & & & & \\ \vdots & & & & \\ a_{n0} & a_{n1} & a_{n2} & \dots & a_{n-1n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_{n-1} \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ \vdots \\ b_{n-1} \end{pmatrix}$$

# Simultaneous Equations ...

- The Gaussian elimination technique essentially reduces the coefficient matrix to an upper triangular form:

$$\begin{pmatrix} 1 & a_{01} & a_{02} & \dots & a_{0n-1} \\ 0 & 1 & a_{12} & \dots & a_{1n-1} \\ 0 & 0 & 1 & & . \\ & & & & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ . \\ . \\ x_{n-1} \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ . \\ . \\ b_{n-1} \end{pmatrix}$$

# Summary

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- In this session,
  - We examined the properties of a system of simultaneous equations in many variables
  - Understood how matrices can be used to represent such systems
  - Studied Gauss elimination technique to solve the system
- In the next session, we will write a program to implement this algorithm