FULL TITLE

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1. Rank Nullity Theorem

It is also called then fundamental theorem of Linear Maps because of its importance in linear transformation.

1.1. Stating and Proof.

Theorem 1. ***Rank-Nullity Theorem:*** *Suppose V is finite-dimensional and $T \in \mathcal{L} : (V, W)$. Then range T is finite-dimensional and*

$$Rank(T) \ + Nullity(T) \ = dim(V), \text{ or}$$
$$dim(Range(T)) \ + dim(Ker(T)) \ = dim(V)$$

Proof. Let $\phi_1, \phi_2, \ldots, \phi_l$ be the minimum vectors will that span $Ker(T)$, where l is the $dim(Ker(T))$.
And $v_1, v_2, \ldots, v_{n-l}$ be the vectors that will span the remaining vector space $V$where, n is the $dim(V)$. Their linear transformation must be independent and should span the entire range of T in W, let $dim(Range(T))$ be k. *We will prove this claim later.*
Then it becomes quite easy to see why the rank nullity theorem holds. □

Claim 1. $w_1, w_2, \ldots, w_{n-l}$ *are linearly independent and span the entire* $Range(T)$.

Proving Claim. □

1.2. Applications.

We can now prove that some of the mapping can not be surjective or injective.

Theorem 2 (A map to a smaller dimensional space is not injective)**.** *Suppose V and W are finite-dimensional vector spaces such that $dim(V)$ and $dim(W)$, then there is no injective linear mapping from V to W.*

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Proof. Let $T \in \mathcal{L}:(V, W)$. Then

$$\begin{aligned} dim(null\, T) &= dim(V) - dim(Range(T)) \\ &\geq dim(V) - dim(W) \\ &> 0. \end{aligned}$$

$\square$

Proof. $\square$

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