BITS Pilani
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Control Systems

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Lecture 03 - SI Control Strategies

Scope

Learning Outcomes

1. Overview of SI Engine Control Strategies
2. Calibration Requirements
3. Testing, Verification & Validation

Module Contents

1. Laplace Transform Examples
2. Building simple plant models

Introduction

Convolution & Laplace Transform

- Convolution – Mathematically tough to solve
- Convolution = Product of Laplace Transforms
- Laplace Transform is an integral transformation
- Transforms a real variable (often time “t”) into a complex variable “s” (often frequency)
- Laplace transform transforms a function from time domain to frequency domain

Laplace Transforms

Transform Function

- Laplace transform for a function f(t), for all t>= 0 is defined as f(s), given by
    - $F(s) = \int_0^{\infty} f(t)e^{-st}dt$
- We will never actually solve the integral, we will use tables to look up solutions
- Special Functions
    - Unit Impulse – Short Duration, Large Magnitude - $\delta(t)$
    - Unit Step – Changes State in a step - $\gamma(t) or\ u(t) or\ 1(t)$, defined in intervals t<0, t>=0
- https://lpsa.swarthmore.edu/LaplaceZTable/LaplaceZFuncTable.html

Laplace Transforms

Transform Function

- Linearity
  - $L[af(t) + bg(t)] = aL[f(t)] + bL[g(t)] = aF(s) + bG(s)$

Plant Model

By Definition

- The plant model is a mathematical equation or relation that can be used to predict a physical system behavior
- Usually derived by analyzing basic behavior – Physics, Fundamental Mechanics, Thermodynamics…
- For the scope of analyzing and predicting behavior, it is assumed that
  - The physical system exhibits predictable behavior – No randomness / Chaos is observed
  - The physical system is a Linear and Time Invariance System (LTI)
  - Even if the system is not LTI, can it be assumed to be so? What are the consequences / Limitations in such cases?

Plant Model

Linear and Time Invariance Systems

- An LTI is a system which exhibits the following properties
  - Homogeneity - Linearity
  - Super Position - Linearity
  - Time Invariance
- Plant Modelling is possible IF and ONLY IF, physical system conforms to the LTI rules

Plant Model

Linearity

- Homogeneity

- Super Position

$$x_1(t) + x_2(t) = y_1(t) + y_2(t)$$

Plant Model

Time Invariance

- Time Invariance
- Systems behaves the same way irrespective of the time

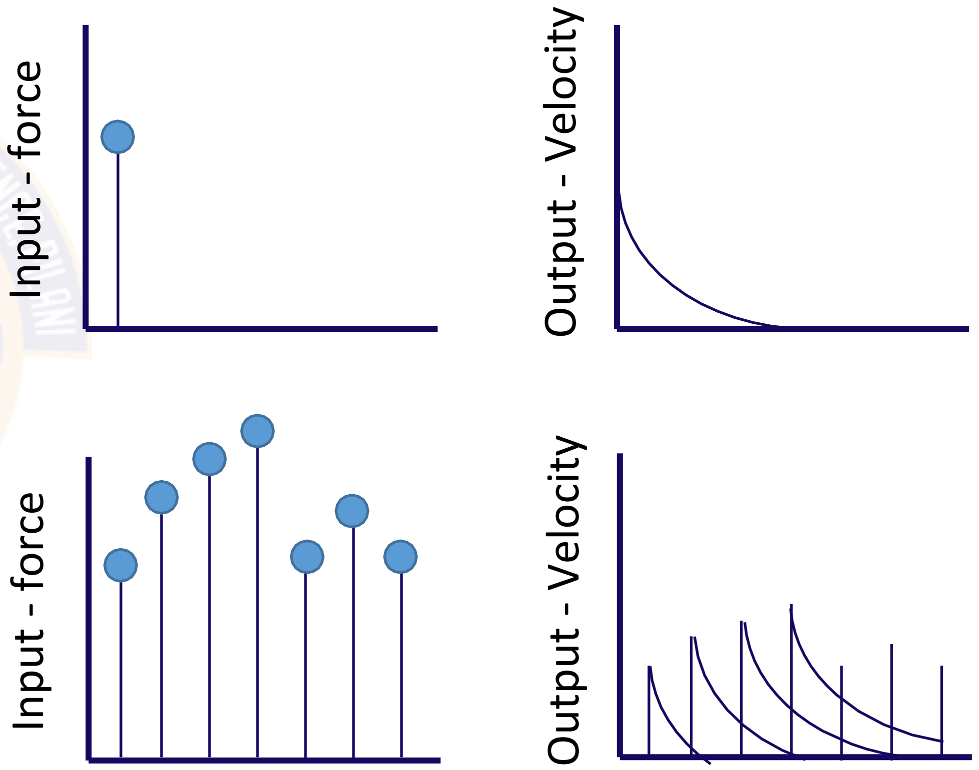
Plant Model

Physical Systems

- LTI System is a theoretical concept
- No real world system will meet all 3
- IC Engine
  - 5mg of fuel produces 10 HP
  - 10mg of fuel produces 20 HP?? Not Always
- Electric Motor
  - Linearity is observed
  - Time Invariance? Wear & Tear? Time does affect the system
- Why do we consider LTI systems then?
- “Because you can solve it!!!”
- Step 1 - Approximate physical systems into LTI systems

Plant Model

LTI Systems



- When ever an input force acts on the system
- There is a corresponding output – Velocity Profile
- Suppose Input was continuous – Hammering Effect
- Output response will look like
- This is obtained by
  - Resolving the continuous input into discrete impulses
  - And plotting corresponding outputs
  - LTI – Homogeneity

Plant Model

LTI Systems

- Output is resolved by calculating
  - Weighted average of the output function at each time “t”
  - Weighting is given by shifting for time “t”
- Defined as “Convolution”
- Suppose input defined by a function f(t), impulse response by g(t), then
- Convolution Function f(t) * g(t) produces the desired response function
- Mathematically, it is defined as the integral of the product of two functions after one is reversed and shifted
- Shifted – To compensate for varying inputs
- Reversed – To compute Delta alone at each interval

$$(f * g)(t) \triangleq \int_{-\infty}^{+\infty} f(\tau) * g(t-\tau)\,d\tau$$

Plant Model

LTI Systems

- Sounds Complicated?
- Well, it is sufficiently complicated.
- Alternate Method Exists
- f (g * t) = L[*f(t)*] x L[*g(t)*]
- The convolution of two functions is equal to the product of Laplace transforms of the two functions
- Makes things a lot more easier!!
- Laplace transforms can be looked up from tables
- Or solved using Mathematical Tools

Thank You!

In our next session:

Plant Model Development