BITS Pilani
Pilani Campus

Mechatronics DECAZG511

Lecture

Modelling

A model is an abstract of the physical world

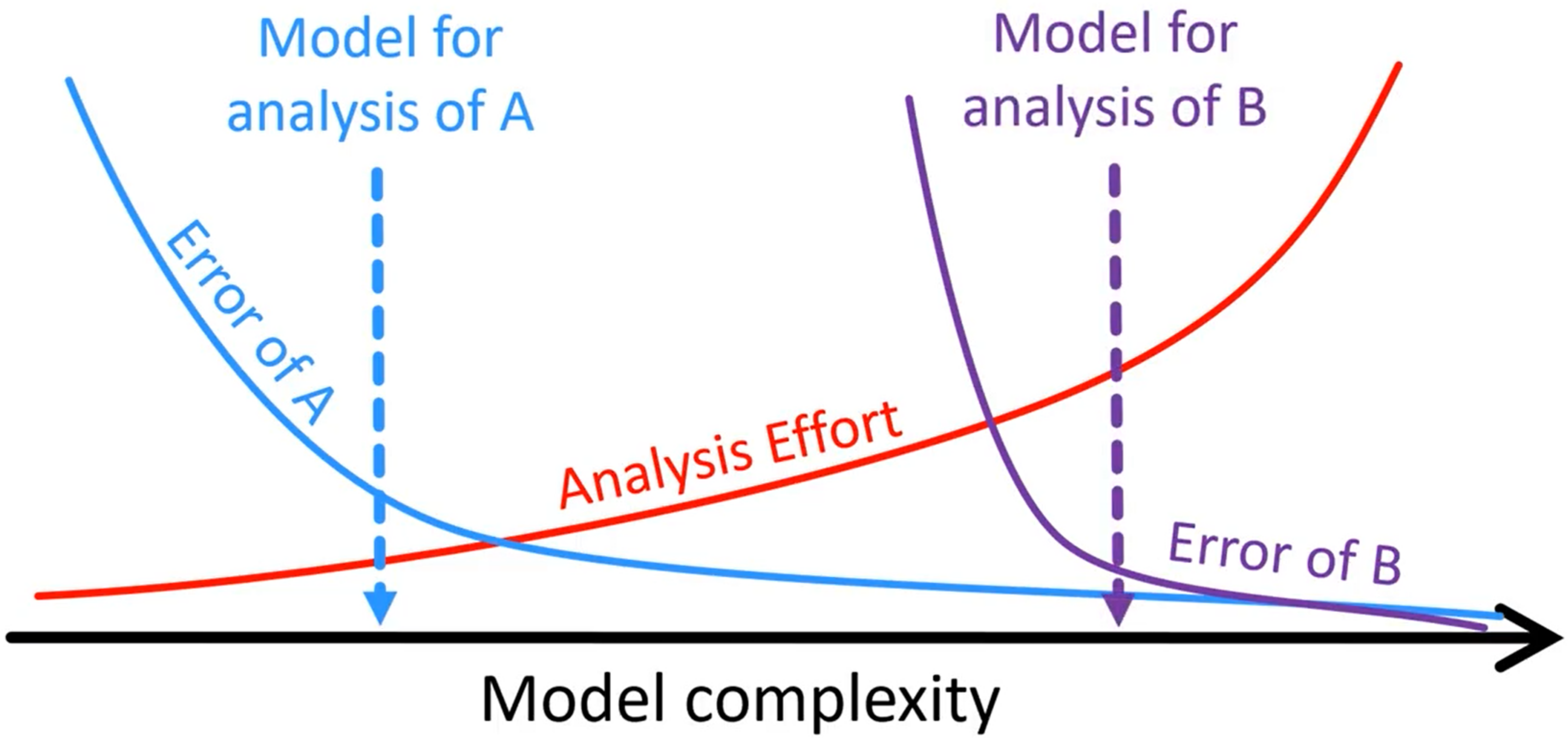
”All models are wrong but some are useful.”

George Box, *Robustness in the strategy of scientific model building*, in Launer, R. L.; Wilkinson, G. N., Robustness in Statistics, Academic Press, pp. 201–236, 1979

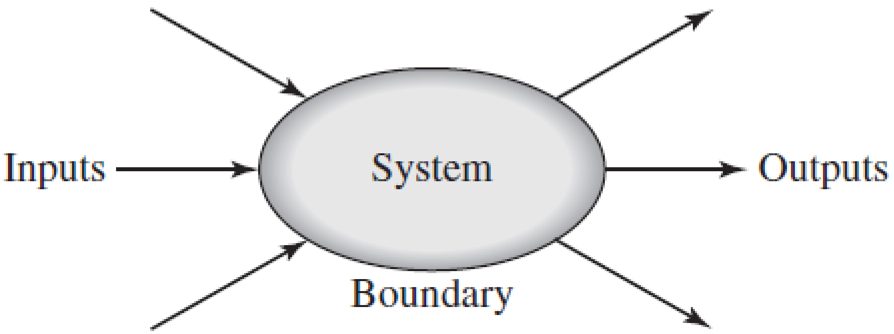
”Models and simulations can never replace observations and experiments – but they constitute an important and useful complement.”

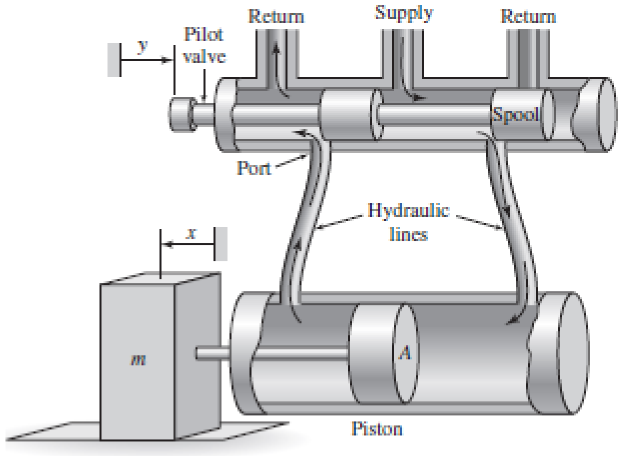
Lennart Ljung and Torkel Glad, *Modeling and Identification of Dynamic Systems, Studentliteratur, 2016*

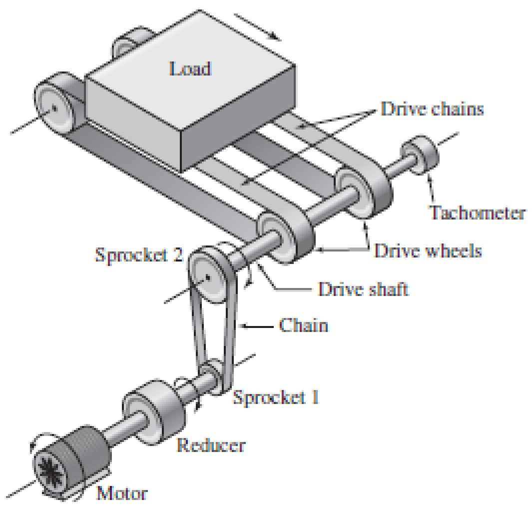
All models have errors!



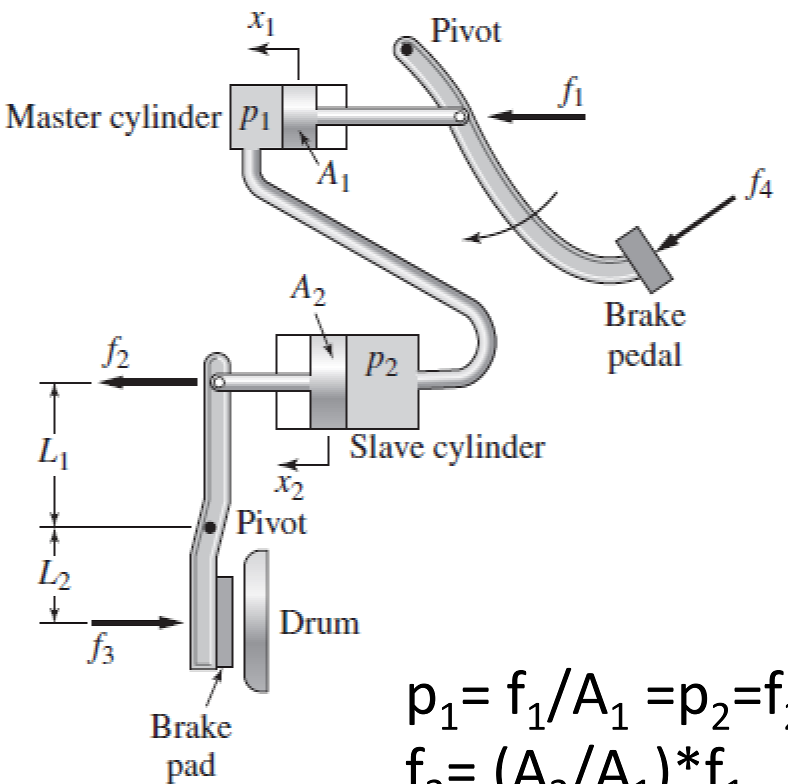
Input / Output concept

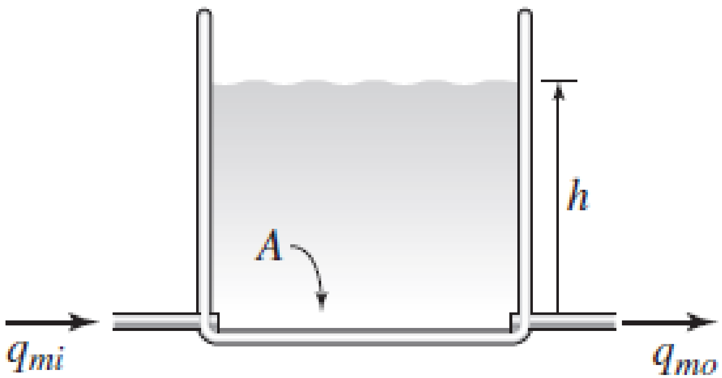






Static and dynamic systems





$p_1 = f_1/A_1 = p_2 = f_2/A_2$

$f_2 = (A_2/A_1)*f_1$

$f_2*L_1 = f_3*L_2$

$f_3 = f_1*(A_2/A_1)*(L_1/L_2)$

$dh/dt \;\alpha\; q_{mi} - q_{m0}$

Mechanical systems

Modeling Mechanical Systems

Approach

1. Choose coordinates and orientation
2. Draw free-body diagrams for each inertia
   - Note assumptions
3. Generate equations of motion using Newton's 2^{nd} Law and Euler's 2^{nd} Law

$$\sum F = ma \qquad \sum M = J\alpha$$

4. Double check

Mechanical systems

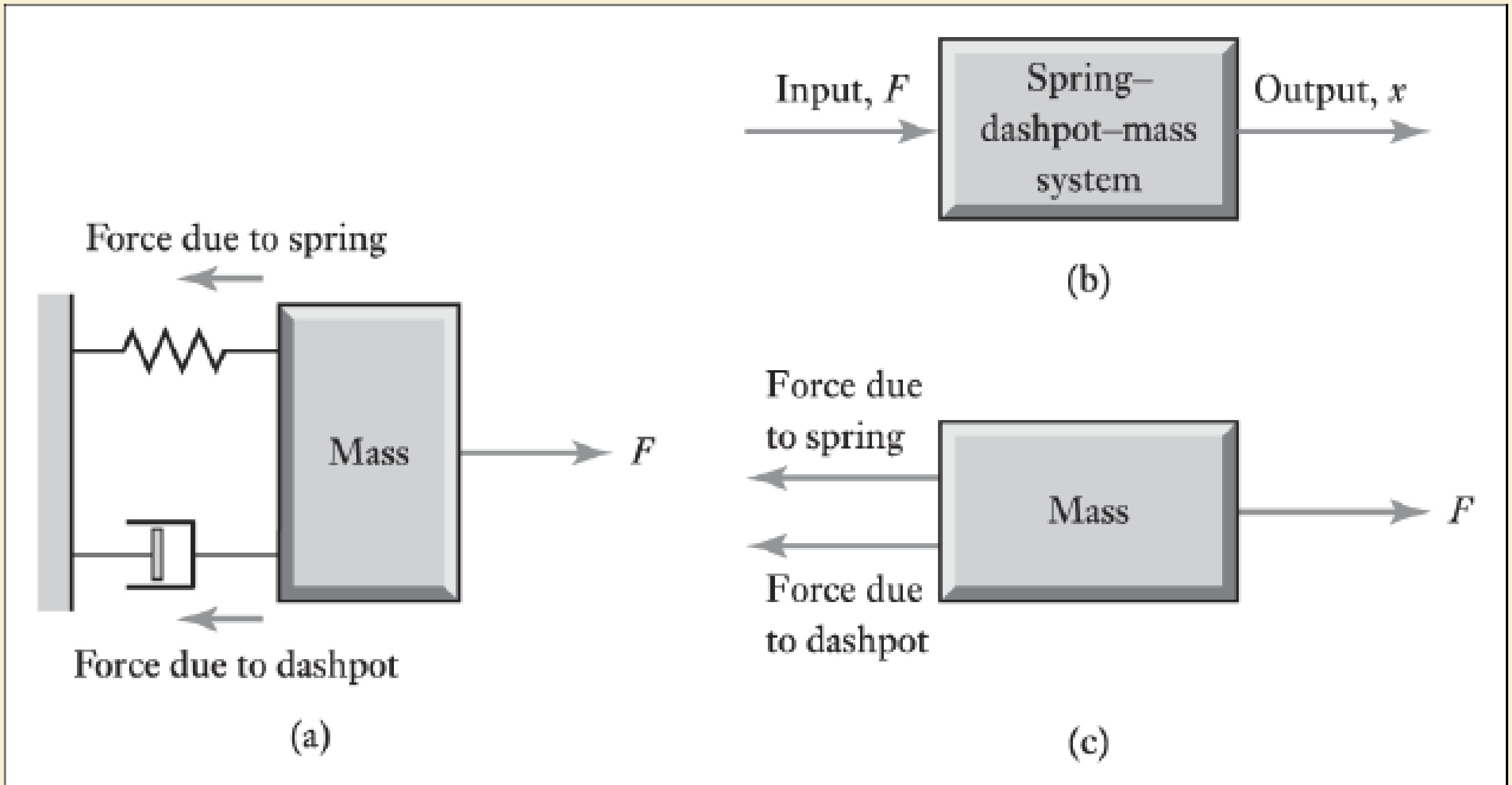
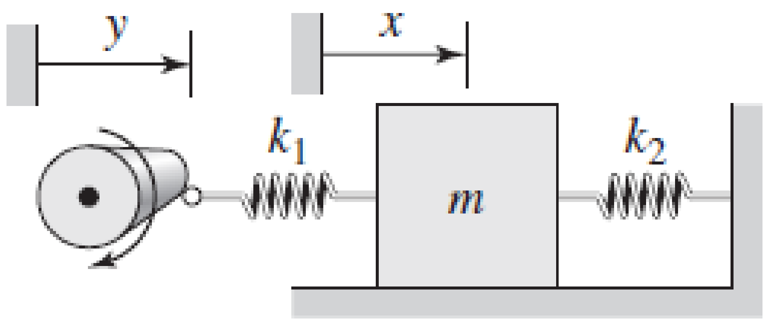
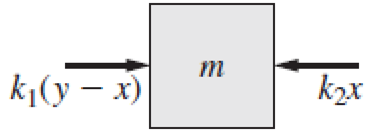


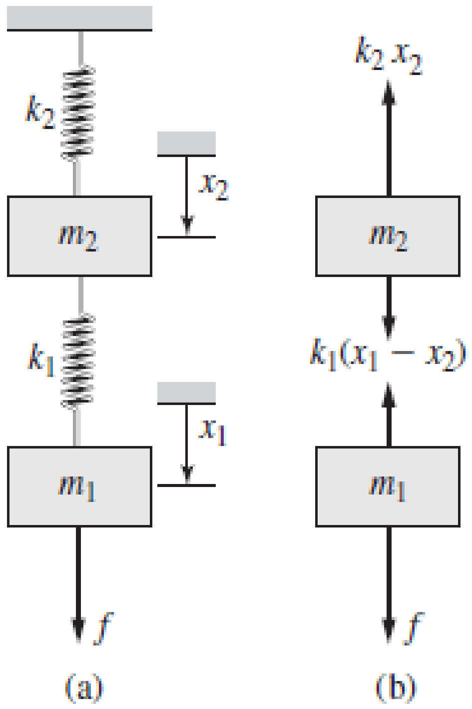
Figure 10.2 (a) Spring–dashpot–mass, (b) system, (c) free-body diagram

Spring Mass system





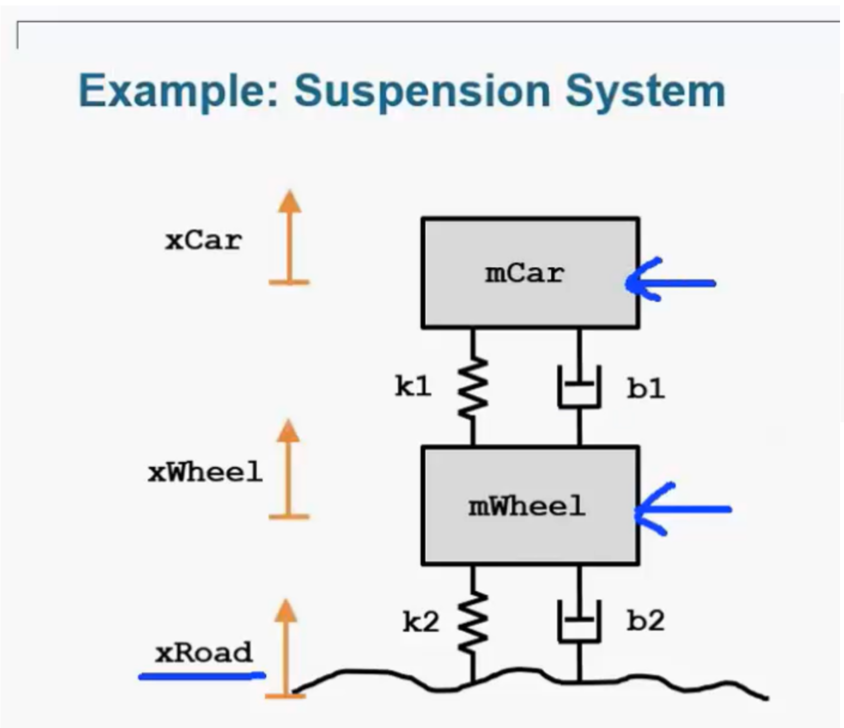
Spring Mass system



$$m_1\ddot{x}_1 = f - k_1(x_1 - x_2)$$

$$m_2\ddot{x}_2 = k_1(x_1 - x_2) - k_2x_2$$

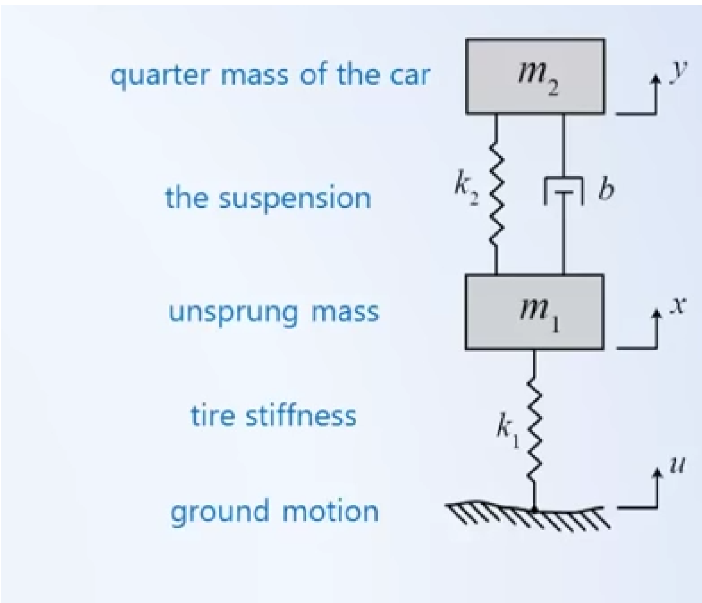
Mechanical systems

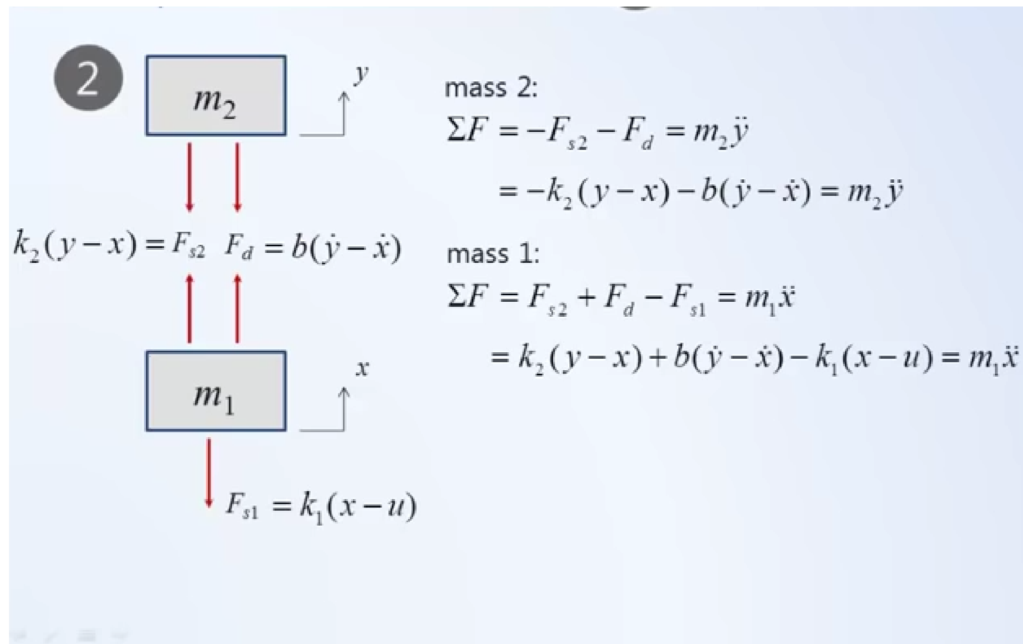


$$m_{car}\ddot{x}_{car} + b_1(\dot{x}_{car} - \dot{x}_{wheel}) + k_1(x_{car} - x_{wheel}) = 0$$

$$m_{wheel}\ddot{x}_{wheel} - b_1(\dot{x}_{car} - \dot{x}_{wheel}) - k_1(x_{car} - x_{wheel}) + b_2(\dot{x}_{wheel} - \dot{x}_{road}) + k_2(x_{wheel} - x_{road}) = 0$$

Quarter body

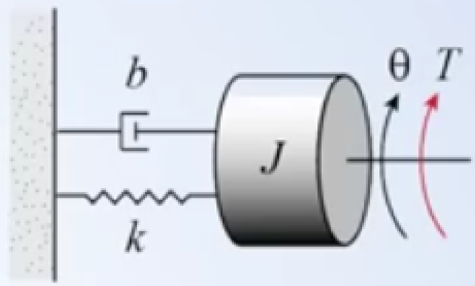




Rotational systems

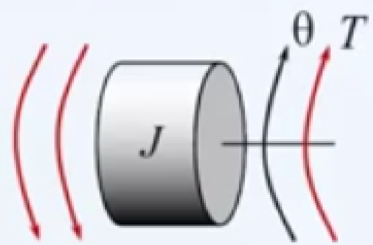
Example

1. → coordinate (θ) and orientation defined
→ assume linear spring and damper, parameters lumped together



- Driveline with wheel locked

2.

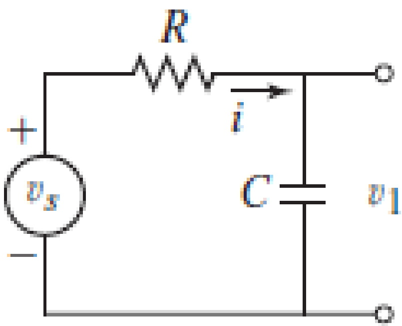


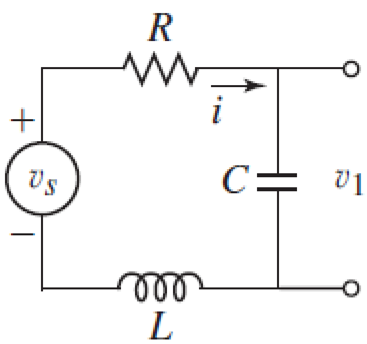
$b\dot{\theta} = T_d \quad T_s = k\theta$

3. $\Sigma M = J\alpha$

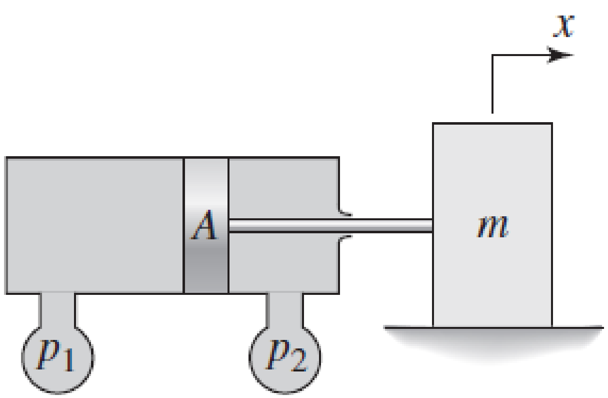
$$\Sigma M = T - T_d - T_s = J\ddot{\theta}$$
$$= T - b\dot{\theta} - k\theta = J\ddot{\theta}$$

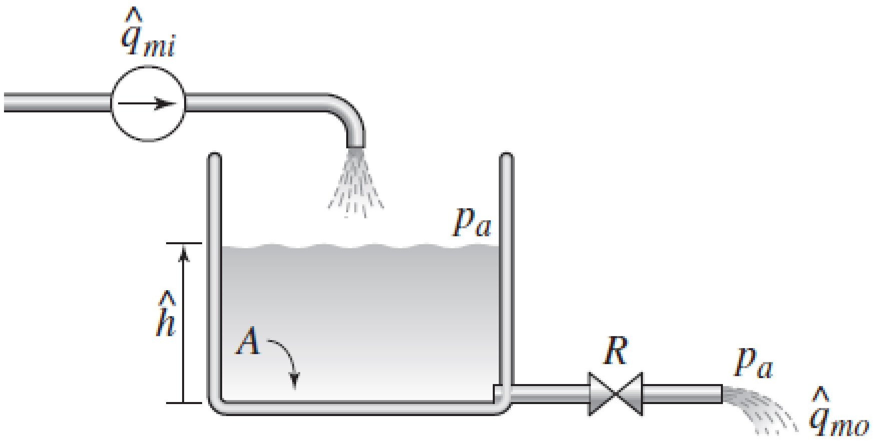
Electrical Systems





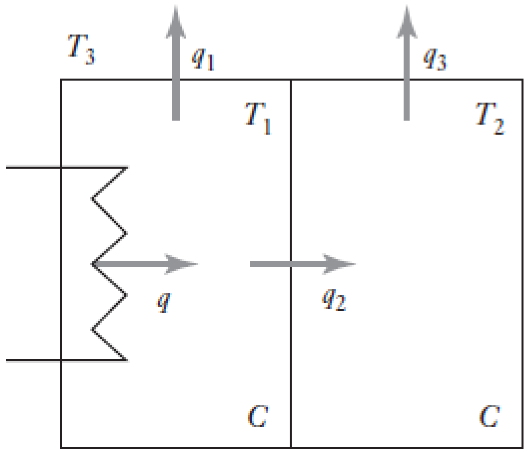
Hydraulic Circuits



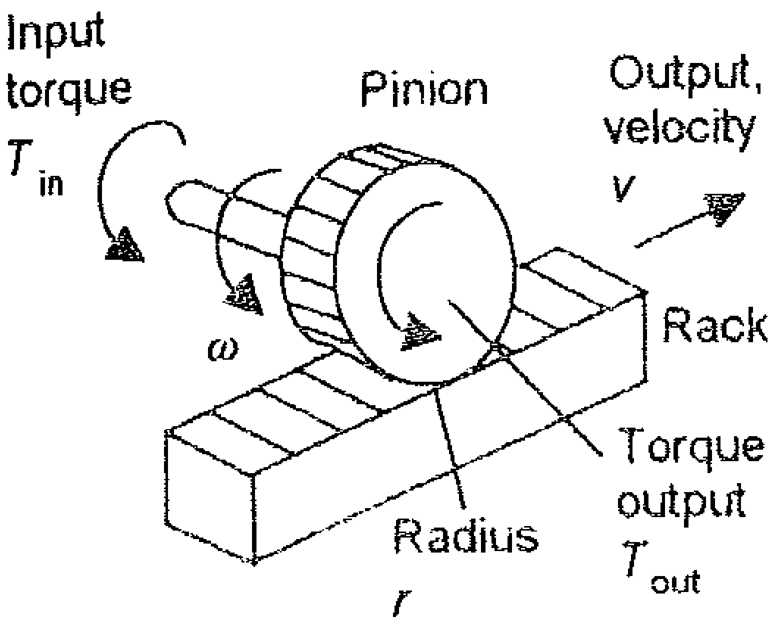


Exercise

17.10 Figure 17.26 shows a thermal system involving two compartments, with one containing a heater. If the temperature of the compartment containing the heater is T_1, the temperature of the other compartment T_2 and the temperature surrounding the compartments T_3, develop equations describing how the temperatures T_1 and T_2 will vary with time. All the walls of the containers have the same resistance and negligible capacitance. The two containers have the same capacitance C.



Rotational – Translational



Electro- Mechanical Systems

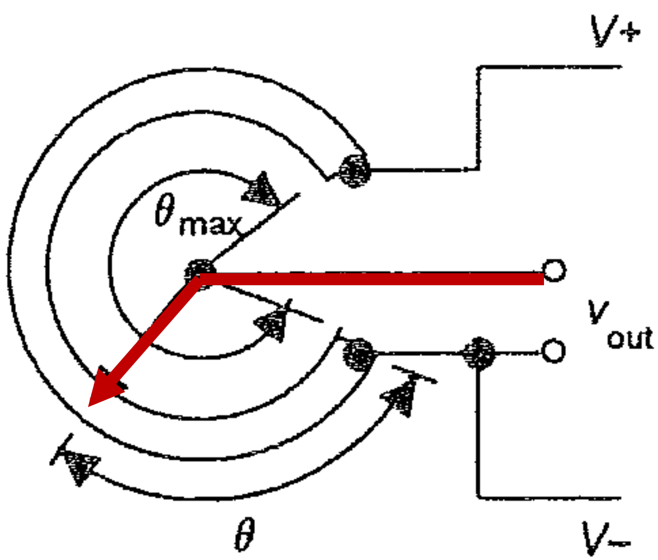
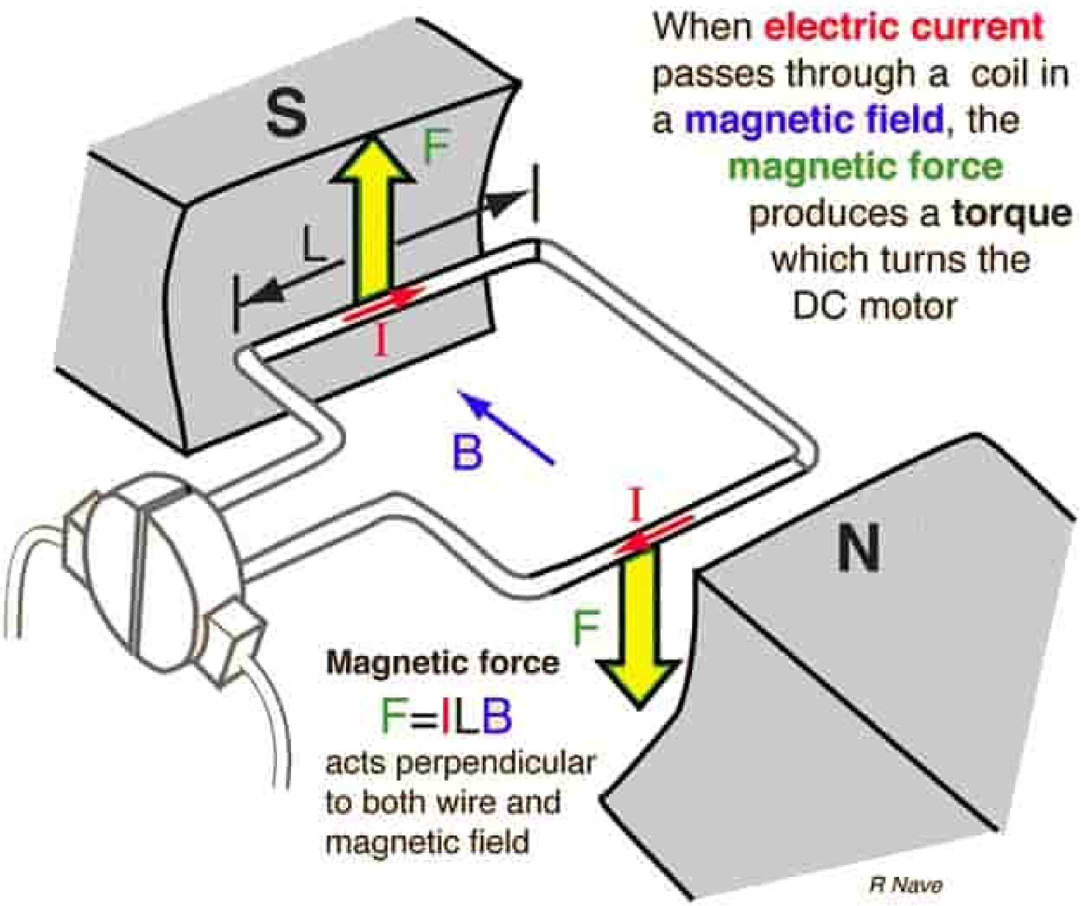


Fig. 9.5 Rotary potentiometer

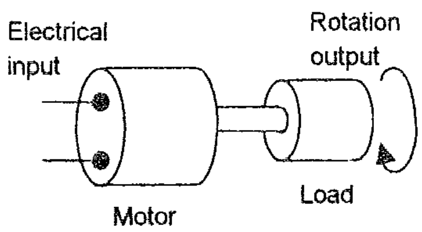
$$\frac{v_o}{V} = \frac{\theta}{\theta_{max}}$$

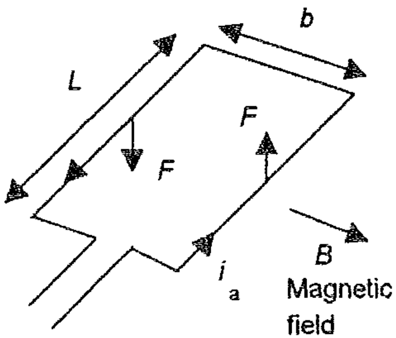
DC Motor

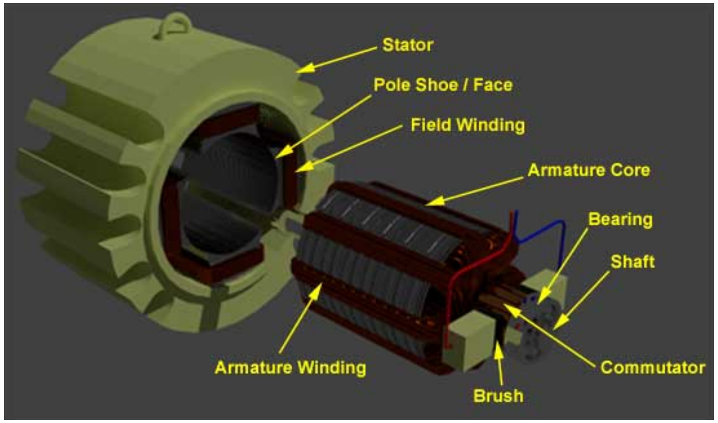


When electric current passes through a coil in a magnetic field, the magnetic force produces a torque which turns the DC motor
S
F
L
I
B
N
Magnetic force
F=ILB
acts perpendicular to both wire and magnetic field
R Nave

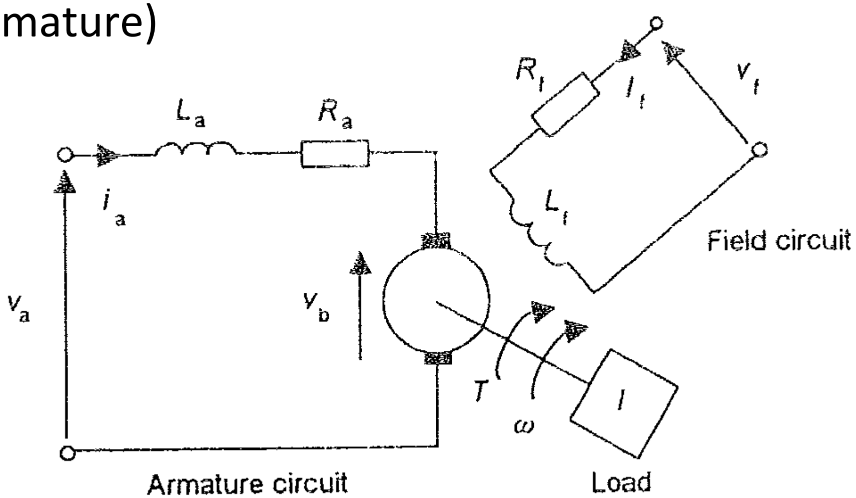
DC Motor





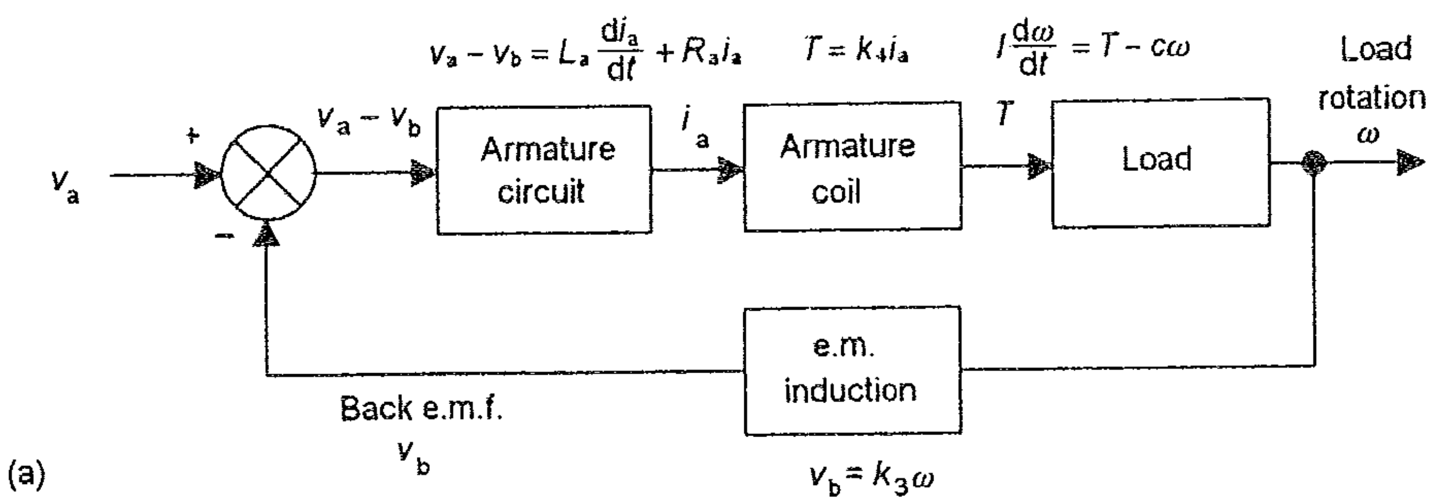


Rotating part (Armature)



Stationary part (Electro magnet)

DC Motor



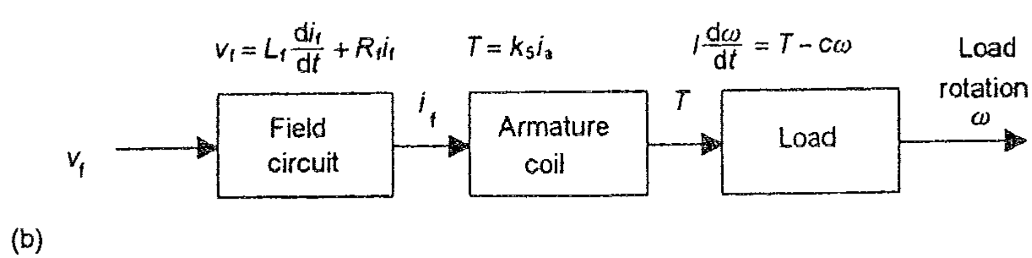


Fig. 9.9 D.C. motors:
(a) armature-controlled,
(b) field-controlled

Linearization

	Linearity	Non linear	Error
1	-2	-2	0
1.01	-2.04	-2.040199	0.000199
1.02	-2.08	-2.080792	0.000792
1.03	-2.12	-2.121773	0.001773
1.04	-2.16	-2.163136	0.003136
1.05	-2.2	-2.204875	0.004875
1.06	-2.24	-2.246984	0.006984
1.07	-2.28	-2.289457	0.009457
1.08	-2.32	-2.332288	0.012288
1.09	-2.36	-2.375471	0.015471
1.1	-2.4	-2.419	0.019
2	-6	-7	
3	-10	-10	
4	-14	-5	
5	-18	14	

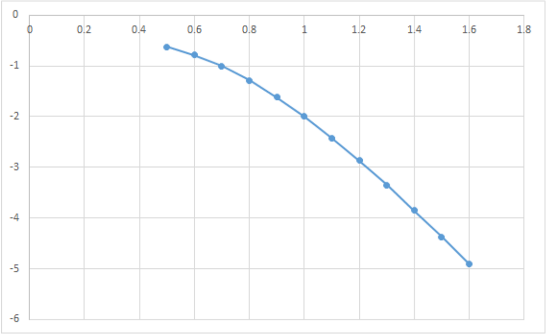
$$f(x) = f(x_s) + \left.\frac{\partial f}{\partial x}\right|_{x_s} (x - x_s) + \frac{1}{2}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_s} (x - x_s)^2 + \text{high order terms}$$

F(x)= x^3-5x^2+3x -1

L(x) at 1 is

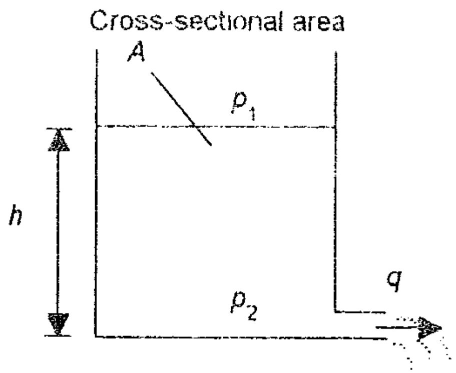
= -4x +2

Curve of F(x)= x^3-5x^2+3x +1



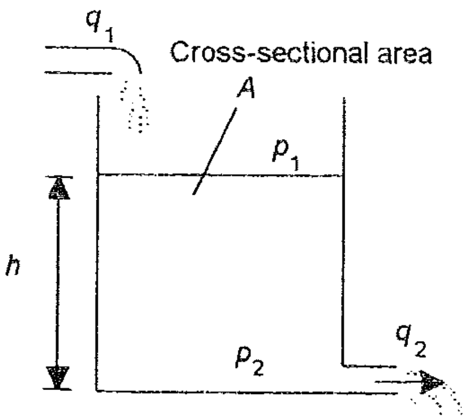
Dynamic response of systems

Natural Response



$$RA\frac{dh}{dt} + (\rho g)h = 0$$

Forced Response

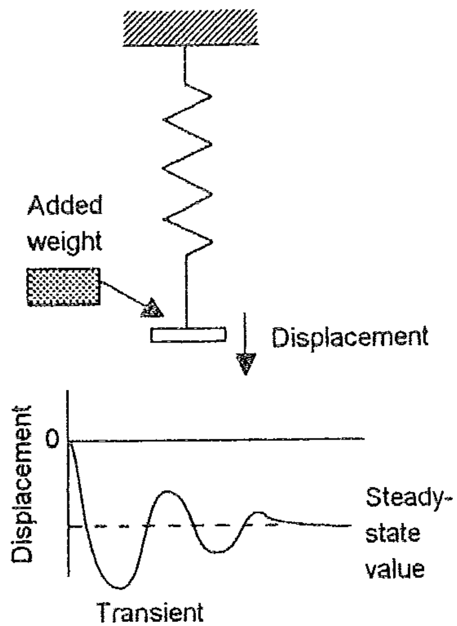


$$RA\frac{dh}{dt} + (\rho g)h = q_1$$

$$RC\frac{dT}{dt} + T = T_L$$

Dynamic response of systems

Transient and steady state response



Forms of Inputs

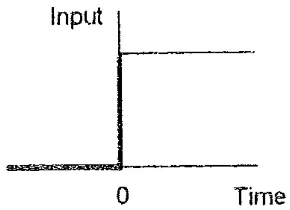


Fig. 10.4 Step input at time 0

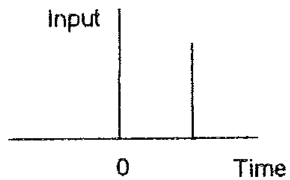


Fig. 10.5 Impulse

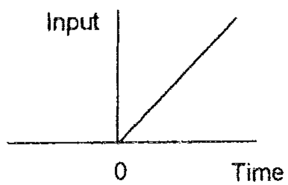


Fig. 10.6 Ramp input at time 0

Recap

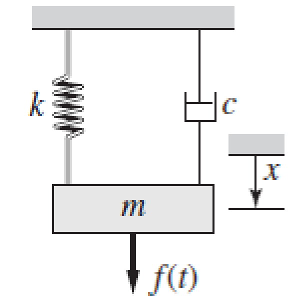
$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = g(x)$$

$$y = y_c + y_p.$$

TABLE 3.4.1 Trial Particular Solutions

$g(x)$	Form of y_p
1. 1 (any constant)	A
2. $5x + 7$	$Ax + B$
3. $3x^2 - 2$	$Ax^2 + Bx + C$
4. $x^3 - x + 1$	$Ax^3 + Bx^2 + Cx + E$
5. $\sin 4x$	$A \cos 4x + B \sin 4x$
6. $\cos 4x$	$A \cos 4x + B \sin 4x$
7. e^{5x}	Ae^{5x}
8. $(9x - 2)e^{5x}$	$(Ax + B)e^{5x}$
9. $x^2 e^{5x}$	$(Ax^2 + Bx + C)e^{5x}$
10. $e^{3x} \sin 4x$	$Ae^{3x} \cos 4x + Be^{3x} \sin 4x$
11. $5x^2 \sin 4x$	$(Ax^2 + Bx + C) \cos 4x + (Ex^2 + Fx + G) \sin 4x$
12. $xe^{3x} \cos 4x$	$(Ax + B)e^{3x} \cos 4x + (Cx + E)e^{3x} \sin 4x$

Behavior of second order system



The characteristic equation is $s^2 + cs + 16 = 0$. The roots are

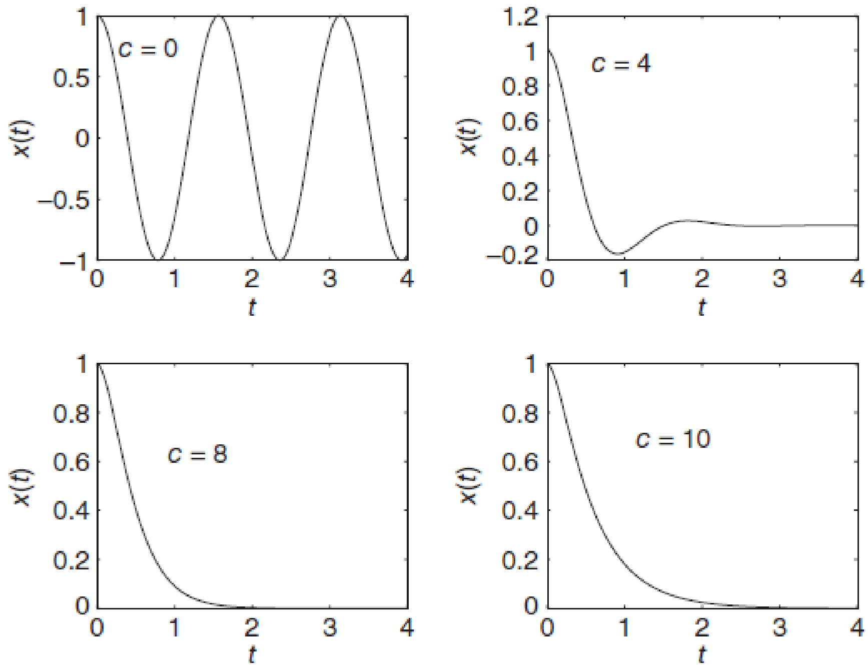
$$s = \frac{-c \pm \sqrt{c^2 - 64}}{2}$$

For $c = 0$ $\quad x(t) = \cos 4t$

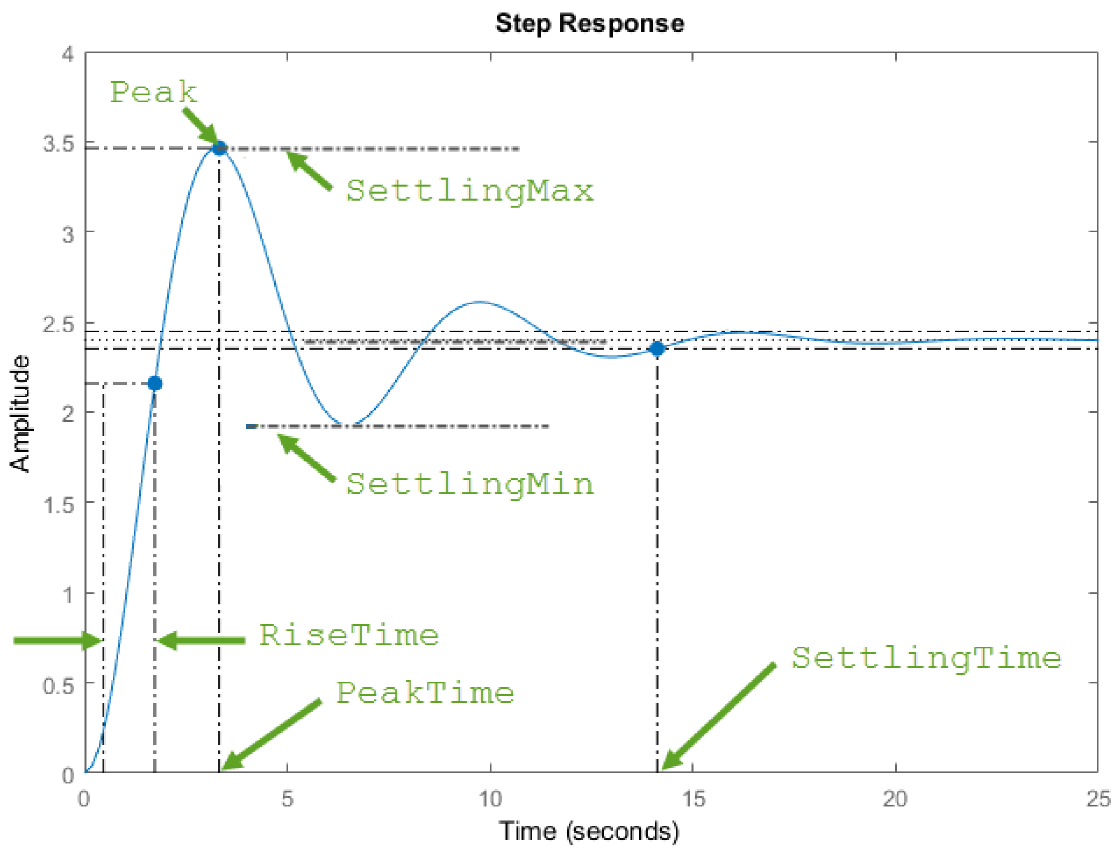
For $c = 4$ $\quad x(t) = 1.155e^{-2t} \sin\left(\sqrt{12}t + 1.047\right)$

For $c = 8$ $\quad x(t) = (1 + 4t)e^{-4t}$

For $c = 10$ $\quad x(t) = \frac{4}{3}e^{-2t} - \frac{1}{3}e^{-8t}$



Step response of systems



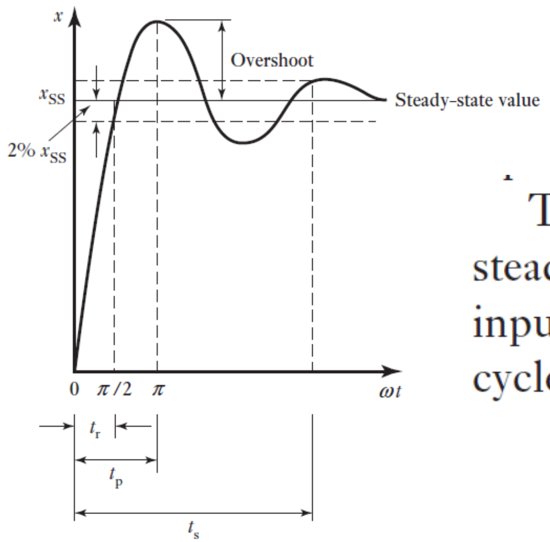
Rise time: From Zero to 90% of steady state value

Peak time : First peak to occur

Overshoot : Amplitude of first Peak

Settling times

Important parameters



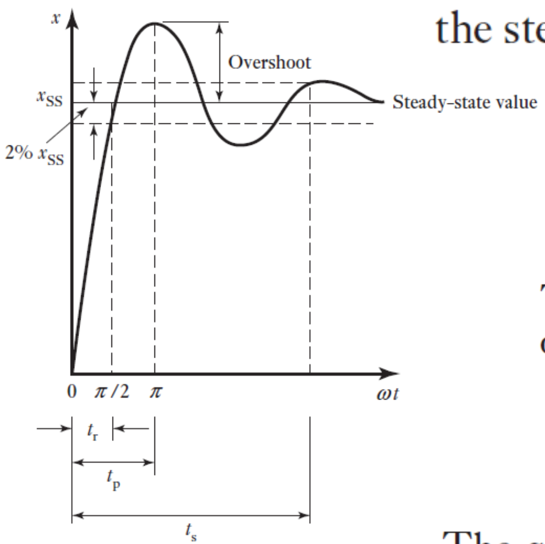
The **rise time** t_r is the time taken for the response x to rise from 0 to the steady-state value x_{SS} and is a measure of how fast a system responds to the input. This is the time for the oscillating response to complete a quarter of a cycle, i.e. $\frac{1}{2}\pi$. Thus

$$\omega t_r = \frac{1}{2}\pi$$

The **peak time** t_p is the time taken for the response to rise from 0 to the first peak value. This is the time for the oscillating response to complete one half cycle, i.e. π. Thus

$$\omega t_p = \pi$$

Important parameters



The **overshoot** is the maximum amount by which the response overshoots the steady-state value and is

$$\text{overshoot} = x_{SS} \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

The **subsidence ratio** or **decrement** is the amplitude of the second overshoot divided by that of the first overshoot and is

$$\text{subsidence ratio} = \exp\left(\frac{-2\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

The **settling time** t_s is the time taken for the response to fall within and remain within some specified percentage, e.g. 2%, of the steady-state value, this being given by

$$t_s = \frac{4}{\zeta\omega_n}$$

$$\text{number of oscillations} = \frac{\text{settling time}}{\text{periodic time}} \qquad \frac{2}{\pi}\sqrt{\frac{1}{\zeta^2} - 1}$$

Example problem

To illustrate the above, consider a second-order system which has a natural angular frequency of 2.0 Hz and a damped frequency of 1.8 Hz. Since $\omega = \omega_n\sqrt{(1 - \zeta^2)}$, then the damping factor is given by

$$1.8 = 2.0\sqrt{1 - \zeta^2}$$

and $\zeta = 0.44$. Since $\omega t_r = \frac{1}{2}\pi$, then the 100% rise time is given by

$$t_r = \frac{\pi}{2 \times 1.8} = 0.87 \text{ s}$$

The percentage overshoot is given by

$$\begin{aligned}\text{percentage overshoot} &= \exp\left(\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}\right) \times 100\% \\ &= \exp\left(\frac{-0.44\pi}{\sqrt{1 - 0.44^2}}\right) \times 100\%\end{aligned}$$

The percentage overshoot is thus 21%. The 2% settling time is given by

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.44 \times 2.0} = 4.5 \text{ s}$$

The number of oscillations occurring within the 2% settling time is given by

$$\text{number of oscillations} = \frac{2}{\pi}\sqrt{\frac{1}{\zeta^2} - 1} = \frac{2}{\pi}\sqrt{\frac{1}{0.44^2} - 1} = 1.3$$

Thank you