On the synthesis of elements in the early Universe

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1 Introduction

The story is about the nucleons in the early phase of the universe. And we are interested in a time at which universe was older from seconds to minutes, and in a temperature $10^{12}K$ to 10^8K. The major components in the universe were photons, neutrinos, electrons, positrons and some small amount of neutrons and protons as nucleons. We will see what will happen to the nucleons as the universe cools and how the heavier elements gets synthesise.

2 Nucleosynthesis

In this Report, our Target is to find the evolution of the mass fraction of different nuclei that's been formed in the early universe from 10^{10} to 10^8 K.
At around 10^{12} K, the stable nucleons in the universe are neutrons and protons and the weaker -interactions allow the neutron-proton conversion through following processes:-

$$n + \nu \rightleftharpoons p + e^- \tag{2.0.1}$$

$$n \rightleftharpoons p + e^- + \nu^- \tag{2.0.2}$$

$$n + e^+ \rightleftharpoons p + \nu \tag{2.0.3}$$

The differential equation governing the neutron fraction $X_n = n_n/n_N$ is

$$\frac{dX_n}{dt} = -\lambda(n \to p) + \lambda(p \to n)(1 - X_n) \tag{2.0.4}$$

where n_N being the number density of nucleons and λ is the reaction rates.
At about 1 MeV, the neutrinos decouples from the plasma and at the same time the weak interactions starts to freeze out and possible lighter nuclei starts to produce from p, n reactions such as Deutron (H^2), Tritium (H^3), He^3 and α particles He^4. We shall take into account 11 nuclear reactions as given below along with their reaction rates. We shall consider following 11 reactions along with their reaction rates.

2.1 Reactions and their Reaction rates

1) $p + n \rightleftharpoons d + \gamma$;
$$4.742 \times 10^4 \times \left(1 - 0.850T_9^{1/2} + 0.490T_9 - 0.0962T_9^{3/2} + 8.47 \times 10^{-3}T_9^2 - 2.80 \times 10^{-4}T_9^{5/2}\right)$$

2) $p + d \rightleftharpoons He^3 + \gamma$;
$$2.65 \times 10^3 T_9^{-2/3} \exp\left(\frac{-3.720}{T_9^{1/3}}\right) \times \left(1 + 0.112T_9^{1/3} + 1.99T_9^{2/3} + 1.56T_9 + 0.162T_9^{4/3} + 0.324T_9^{5/3}\right)$$

3) $d + d \rightleftharpoons He^3 + n$

$$3.95 \times 10^8 T_9^{-\frac{2}{3}} \exp\left(-\frac{4.259}{T_9^{\frac{1}{3}}}\right) \times \left(1 + 0.098 T_9^{\frac{1}{3}} + 0.765 T_9^{\frac{2}{3}} + 0.525 T_9 + 9.61 \times 10^{-3} T_9^{\frac{4}{3}} + 0.0167 T_9^{\frac{5}{3}}\right)$$

4) $d + d \rightleftharpoons {}^3H + p$:

$$4.17 \times 10^8 T_9^{-\frac{2}{3}} \exp\left(-\frac{4.258}{T_9^{\frac{1}{3}}}\right) \left(1 + 0.098 T_9^{\frac{1}{3}} + 0.518 T_9^{\frac{2}{3}} + 0.355 T_9 - 0.010 T_9^{\frac{4}{3}} - 0.018 T_9^{\frac{5}{3}}\right)$$

5) $He^3 + n \rightleftharpoons H^3 + p$

$$7.21 \times 10^8 \left(1 - 0.508 T_9^{\frac{1}{2}} + 0.228 T_9\right)$$

6) $H^3 + d \rightleftharpoons He^4 + n$

$$1.063 \times 10^{11} T_9^{-\frac{2}{3}} \exp\left(-\frac{4.559}{T_9^{\frac{1}{3}}}\right) \left(1 + 0.092 T_9^{\frac{1}{3}} - 0.375 T_9^{\frac{2}{3}} - 0.242 T_9 + 33.82 T_9^{\frac{4}{3}} + 55.42 T_9^{\frac{5}{3}}\right)$$
$$+ 8.047 \times 10^8 T_9^{-\frac{2}{3}} \exp\left(-0.4857 T_9\right)$$

7) $5.021 \times 10^{10} T_9^{-\frac{2}{3}} \exp\left(-\frac{7.144}{T_9^{\frac{1}{3}}}\right) \times \left(T_9^{0.270}\right)^2 \times (1 + 0.058 T_9^{\frac{1}{3}} + 0.603 T_9^{\frac{2}{3}} + 0.245 T_9 + 6.97 T_9^{\frac{4}{3}}$

$$+ 7.19 T_9^{\frac{5}{3}}) + 5.212 \times 10^8 T_9^{-\frac{1}{2}} \exp\left(-\frac{1.762}{T_9}\right)$$

8) $He^3 + He^4 \rightleftharpoons Be^7 + \gamma$

$$4.817 \times 10^6 T_9^{-\frac{2}{3}} \exp\left(-\frac{14.964}{T_9^{\frac{1}{3}}}\right) (1 + 0.0325 T_9^{\frac{1}{3}} - 1.04 \times 10^{-3} T_9^{\frac{2}{3}} - 2.37 \times 10^{-4} T_9$$
$$- 8.11 \times 10^{-5} T_9^{\frac{4}{3}} - 4.69 \times 10^{-5} T_9^{\frac{5}{3}}) + 5.938 \times 10^6 T_9^{-\frac{3}{2}} \left(\frac{T_9}{1 + 0.1071 T_9}\right)^{\frac{5}{6}} \exp\left(-12.859 \frac{(1 + 0.1071 T_9)^{\frac{1}{3}}}{T_9^{\frac{1}{3}}}\right.$$

9) $H^3 + He^4 \rightleftharpoons Li^7 + \gamma$:

$$3.032 \times 10^5 T_9^{-\frac{2}{3}} \exp\left(-\frac{8.090}{T_9^{\frac{1}{3}}}\right) \left(1 + 0.0516 T_9^{\frac{1}{3}} + 0.0229 T_9^{\frac{2}{3}} + 8.28 \times 10^{-3} T_9 - 3.28 \times 10^{-4} T_9^{\frac{4}{3}}\right.$$
$$\left.- 3.01 \times 10^{-4} T_9^{\frac{5}{3}}\right) + 5.109 \times 10^5 T_9^{-\frac{3}{2}} \left(\frac{T_9}{1 + 0.1378 T_9}\right)^{\frac{5}{6}} \exp\left(-8.068 \frac{(1 + 0.1378 T_9)^{\frac{1}{3}}}{T_9^{\frac{1}{3}}}\right)$$

10) $Be^7 + n \rightarrow Li^7 + p$

$$2.675 \times 10^9 T_9^{-\frac{2}{3}} \left(1 - 0.560 T_9^{\frac{1}{2}} + 0.179 T_9 - 0.0283 T_9^{\frac{3}{2}} + 2.21 \times 10^{-3} T_9^2 - 6.85 \times 10^{-5} T_9^{\frac{5}{2}}\right)$$

$$+ 9.391 \times 10^8 \left(\frac{1}{1 + 13.076 T_9}\right)^{\frac{3}{2}} + 4.467 \times 10^7 T_9^{-\frac{3}{2}} \exp\left(-0.07486 T_9\right)$$

11) $Li^7 + p \rightleftharpoons He^4 + He^4$

$$1.096 \times 10^9 T_9^{-\frac{2}{3}} \exp\left(-\frac{8.472}{T_9^{\frac{1}{3}}}\right) - 4.830 \times 10^8 T_9^{-\frac{3}{2}} \left(\frac{T_9}{1 + 0.759 T_9}\right)^{\frac{5}{6}} \exp\left(-8.472 \left(1 + 0.759 T_9\right)^{\frac{1}{3}}\right)$$

$$+ 1.06 \times 10^{10} T_9^{-\frac{3}{2}} \exp\left(-30.442 T_9\right) + 1.56 \times 10^5 T_9^{-\frac{2}{3}} \exp\left(-8.472 T_9^{\frac{1}{3}} - \left(\frac{T_9}{1.696}\right)^2\right) \times (1 + 0.049 T_9^{\frac{1}{3}}$$

$$- 2.498 T_9^{\frac{2}{3}} + 0.860 T_9 + 3.518 T_9^{\frac{4}{3}} + 3.08 T_9^{\frac{5}{3}}) + 1.55 \times 10^6 T_9^{-\frac{3}{2}} \exp\left(-4.478 T_9\right)$$

2.2 Rate equation and nuclear abundances

Given the reaction rates, the abundance of nuclei of type i is evolved by ,

$$\frac{1}{A_i} \frac{dX_i}{dt} = \pm \sum_j \frac{X_j}{A_j} \lambda_k(j) \pm \sum_{j \geq k} \frac{X_j}{A_j} \frac{X_k}{A_k} [jk] \tag{2.2.1}$$

where X_i is the mass fraction, $\lambda_k(j)$ is inverse mean lifetime for the reaction between a lepton or photon(k) with a nucleus j. If j = i, the sign is negative , otherwise positive. Also $[jk] = \rho_B N_A < \sigma v >_{jk}$ is the reaction rate for the reaction between nuclei of type "j" and "k" and where σ being the cross-section of the reaction and v is the relative velocity.
The differential equations above have derivative with respect to time but all the quantities i.e the reaction rates are known as a function of temperature. Thus it is convenient to choose temperature as independent parameter of time. We define temperature as function through the differential equation,

$$\boxed{\frac{dt}{dT} = -\frac{1}{HT}} \rightarrow (1) \tag{2.2.2}$$

And the Hubble constant is given by

$$H = \sqrt{\frac{8\pi G\rho}{3c^2}} \tag{2.2.3}$$

Considering neutrinos, photons , electrons and positrons as the major components in the universe ,

$$\rho(T) = a_B T^4 \epsilon(m_e/k_B T) \tag{2.2.4}$$

$$\epsilon(x) = 1 + \frac{21}{8}\left(\frac{4\mathcal{S}}{11}\right)^{4/3} + \frac{30}{\pi^4}\int_0^\infty dy \frac{y^2\sqrt{y^2+x^2}}{exp(\sqrt{y^2+x^2}+1)} \tag{2.2.5}$$

$$\mathcal{S}(x) = 1 + \frac{45}{2\pi^4}\int_0^\infty y^2 dy \left(\sqrt{y^2+x^2} + \frac{y^2}{3\sqrt{y^2+x^2}}\right)\frac{1}{\exp\sqrt{y^2+x^2}+1} \tag{2.2.6}$$

where

$$a_B = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} \tag{2.2.7}$$

Method Used - This integration has been numerically solved using Gauss-Laguerre quadrature method where the integration of a particular kind can be approximated using roots and weights of the Laguerre polynomials.
Also, using the entropy conservation and evolving the Friedman equation, the neutrino temperature is given by

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T\mathcal{S}(m_e/k_B T) \tag{2.2.8}$$

$$\boxed{\frac{dT_\nu}{dT} = \left(\frac{4}{11}\right)^{1/3} \mathcal{S}(m_e/k_B T)} \tag{2.2.9}$$

Coupled Equation (6) and (12) is solved using solve $-$ ivp method to get photon temperature T, neutrino Temperature T_ν and time (t).

Numerical Evolution

The set of ODEs are solved by taking the initial conditions as thermal equilibrium distribution for p, n and other components as zero. The system is evolved in a temperature range of 10^11K to 10^8K using the 'radau' method in solve-ivp. .

3 Results

The set of ODE's are evolved in a temperature range of 10^11 and 10^8K using "radau" method in Solve-ivp. The formation of H^3 and He^3 didn't happen until the deutrons become abundant enough. This delay in the synthesis of heavier nuclei is called the deuterium bottleneck. The reaction producing He^4 nuclei will start a bit later once H^3 and He^3 become abundant for enough collision to happen. This is the He^3 bottleneck. The p-n collision decreases as the X_n decreases due to neutron decay and the deutrons also decreases and saturates. This is because

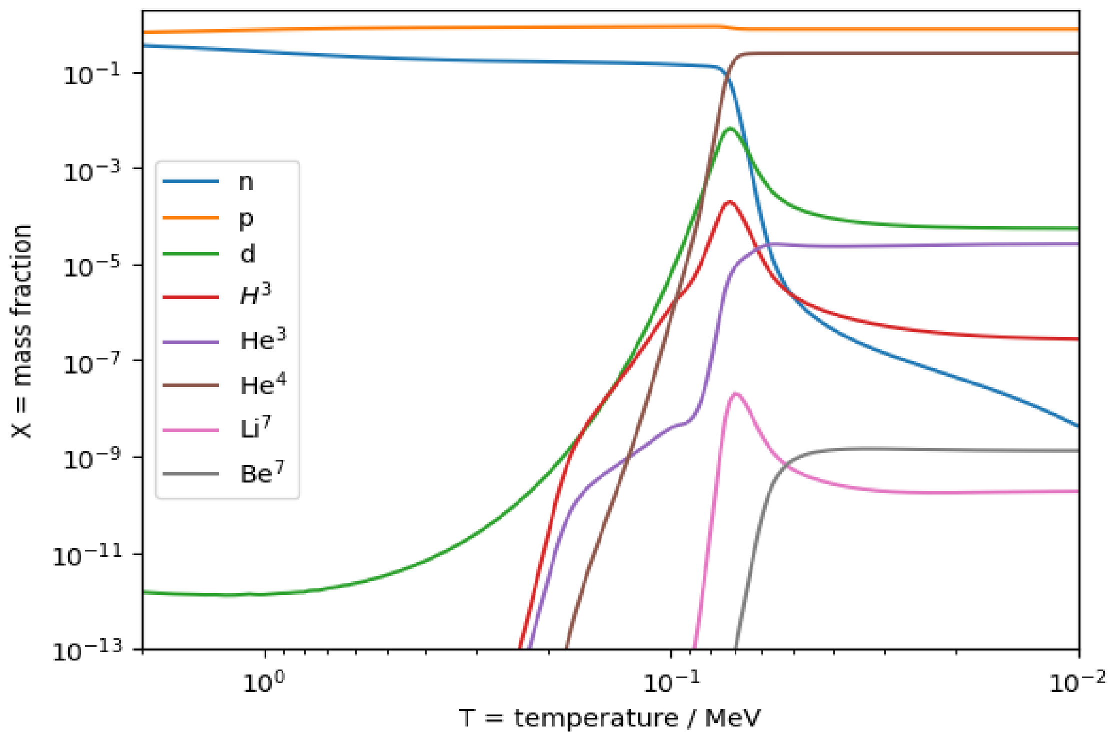

Figure 1: Evolution of mass fraction of different nuclei in the early universe.

whatever the amount of deutrons are produced from $p - n$ collisions, equal amount of deutrons get converted to He^4. As the universe cools more, there wont be enough collisions between the particles and the abundances saturates.

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5 References-

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