DEEP LEARNING FOR PHYSICAL SYSTEMS ME504
ASSIGNMENT 4

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1. **Compare the principle directions obtained from Mode shape approach and PCA approach. Are they same or not ? Explain it.**

Mode Shape Approach:

- Mode shapes represent the direction of maximum vibration for each mode of the system.
- They are eigenvectors of the system's stiffness matrix.
- They are specific to the physical properties and dynamics of the system.

OUTPUT FROM THE PROGRAM:(for 5 block system)

```
Natural frequencies:
[1.34155582 3.42809166 5.55494417 7.15808839 9.00603664]

Mode shapes:
[[ 0.13544062 -0.29216762  0.47316387 -0.71786681 -0.39632955]
 [ 0.30908315 -0.52023248  0.38700491  0.15005882  0.67936391]
 [ 0.41725135 -0.46081267 -0.12399066  0.50700247 -0.58406147]
 [ 0.54013204 -0.05849401 -0.68849236 -0.43758425  0.19832832]
 [ 0.6482917   0.65439008  0.3700645   0.11669667 -0.03042436]]

Modal Participation Factors:
[[0.01834416 0.08536192 0.22388405 0.51533276 0.15707711]
 [0.09553239 0.27064184 0.1497728  0.02251765 0.46153532]
 [0.17409869 0.21234832 0.01537368 0.2570515  0.3411278 ]
 [0.29174262 0.00342155 0.47402173 0.19147998 0.03933412]
 [0.42028213 0.42822638 0.13694774 0.01361811 0.00092564]]
```

PCA Approach:

- PCA finds the directions of maximum variance in the data.
- Principal directions are eigenvectors of the covariance matrix of the data.
- PCA is a data-driven approach and does not consider the physical properties of the system.

OUTPUT FROM THE PROGRAM:(for 5 block system)

```
Covariance Matrix:
[[  4.04617093   8.13973208  19.15071876 -11.06002725  -7.49911859]
 [  8.13973208  16.37479965  38.52573767 -22.24959355 -15.08606957]
 [ 19.15071876  38.52573767  90.64125939 -52.34763309 -35.49368366]
 [-11.06002725 -22.24959355 -52.34763309  30.23208976  20.49850522]
 [ -7.49911859 -15.08606957 -35.49368366  20.49850522  13.89876518]]

Eigenvalues: [ 1.55193085e+02  2.84217094e-14  4.10346454e-15  5.42837145e-16
 -6.46718001e-15]

Eigenvectors: [[ 0.1614678  -0.98687798  0.0258388  -0.01029232 -0.01061479]
 [ 0.32482677  0.05314645 -0.70038663 -0.08189661  0.13163798]
 [ 0.76423475  0.12504008 -0.15851509  0.06392488 -0.59700106]
 [-0.44136501 -0.07221383 -0.39009487 -0.52575394 -0.72361327]
 [-0.29926224 -0.04896372 -0.57575158  0.84420579 -0.32020673]]
Reduced data using PCA:
[[ -1.06697026 -18.6847502 ]
 [  0.20408641   0.20408641]
 [  0.27152673   0.27152673]]
```

As we compare the principle directions obtained from Mode shape approach and PCA approach, they dont come out to be same. Mode shapes from modal analysis are specific to the physical properties and dynamics of the system. Principal directions from PCA are based solely on the distribution of data points and may not correspond to the actual physical behavior of the system

2. **Lets consider the case where displacement data is coming with noise of +/-8 percent. Does the noise affect the performance of the PCA, please explain in the report.**

Noise: 0

```
Input data with displacement and force values:
[[  0.26590593  -0.72058478]
 [  0.50263058 -12.89895255]
 [  1.08856062  -4.29230412]
 [  2.00396573  -2.85885432]
 [  2.7048049   25.94756958]]

Reduced data using PCA:
[[ -1.44790714 -29.25802092]
 [  2.83865394   2.83865394]
 [ -1.88219838  -1.88219838]]
```

Noise: +/- 8%

```
Input data with displacement and force values:
[[  0.28251299  -0.72058478]
 [  0.53955586 -12.89895255]
 [  1.09852128  -4.29230412]
 [  2.12900038  -2.85885432]
 [  2.88353535  25.94756958]]

Reduced data using PCA:
[[ -1.5315942  -29.2351875 ]
 [ -0.0912383   -0.0912383 ]
 [  1.76502881   1.76502881]]
```

Noise has an effect on the performance of the PCA, the principal component changes a little bit as the displacement incorporates some noise; as shown above, although it is minimal for 8% noise.
Noise in displacement data affects Principal Component Analysis (PCA) by increasing variance. As PCA aims to capture directions of maximum variance, increased variance due to noise can affect the accuracy of PCA thereby generating less meaningful principal directions.
PCA aims to reduce dimensionality while preserving important information. Noise can mislead PCA in selecting principal components, leading to suboptimal dimensionality reduction.
While PCA is robust to some noise as evident from the above example but excessive noise can significantly degrade its performance.

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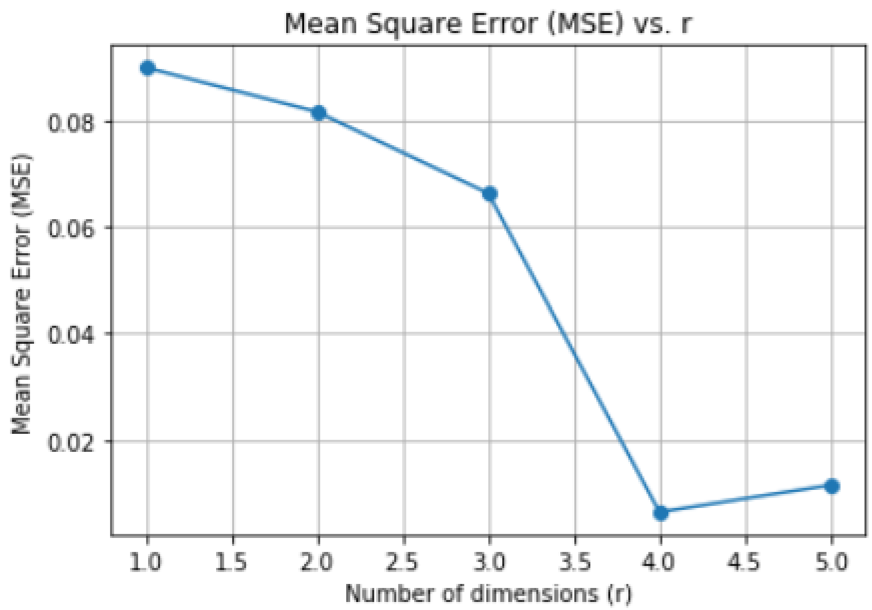
3. **For PCA approach, plot the mean square error with respect to ’r’ and explain its behaviour.**



In dimension reduction by PCA, the mean square error (MSE) generally decreases as the number of dimensions after reduction increases, the rate of decrease is not linear.
As we can see from the graph as r decreases (ie dimension reduced more) the MSE increases because as the data gets reduced not all the information gets captured; so the MSE from the original data increases.
But after a certain point adding more dimensions doesn't significantly reduce the MSE, as the additional dimensions might capture noise rather than actual data.

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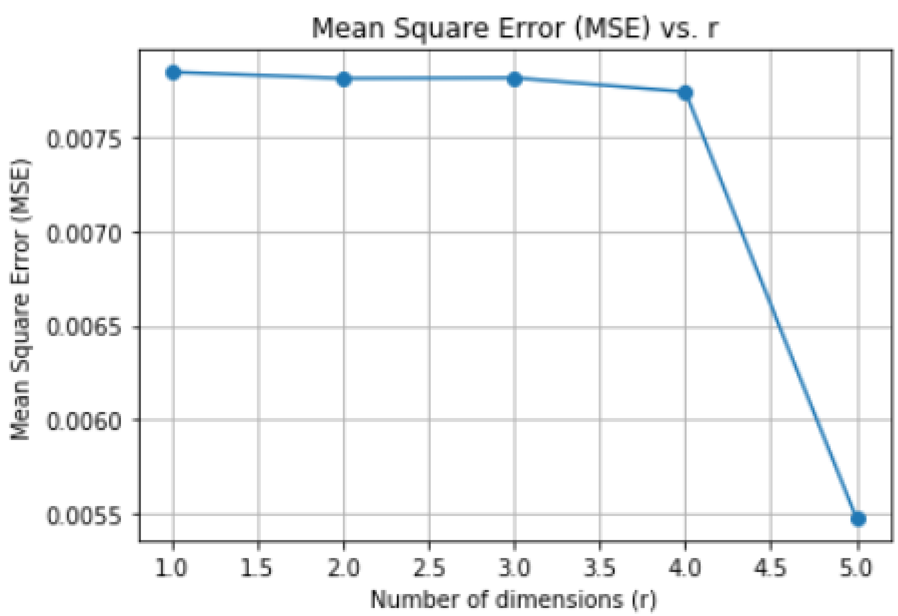
4. **Is it possible to find the a force distribution for which mode shape approach works fine, but the PCA approach fails? If yes, state such distribution and verify that indeed mode shape works better than PCA. If no, state the reason in the report.**

If the force applied is aligned perfectly with one of the mode shapes, modal analysis would easily capture this, providing the corresponding mode shape and natural frequency. However, PCA might not consider this direction if the force distribution doesn't capture significant variance in the data.

Here the force is perfectly aligned to the first node shape.

Force = np.array([-0.21201089, -0.52059953, 0.71596161, 0.37197045, 0.18183638])

PCA doesn't work efficiently in dimensional reduction as can be seen from the below graph, the MSE is less only for r=5 ie the initial number of spring-mass while for all reduced data ie r=1,2,3,4 the MSE is high and the same sort of.




```
Data- Displacement and Force:
[[ 0.02759069 -0.21201089]
 [ 0.05766241 -0.52059953]
 [ 0.09724419  0.71596161]
 [ 0.11920114  0.37197045]
 [ 0.1241125   0.18183638]]
```

```
Reduced data using PCA:
[[ 0.06267653  0.98019023]
 [-0.01292572 -0.01292572]
 [ 0.01166716  0.01166716]]
```

for r=3

5. **Is it possible to find the a force distribution for which PCA works fine, but the mode shape approach fails? If yes, state such distribution and verify that indeed PCA works better than mode shape and state reason in report why you thought of that distribution. If no, state the reason.**

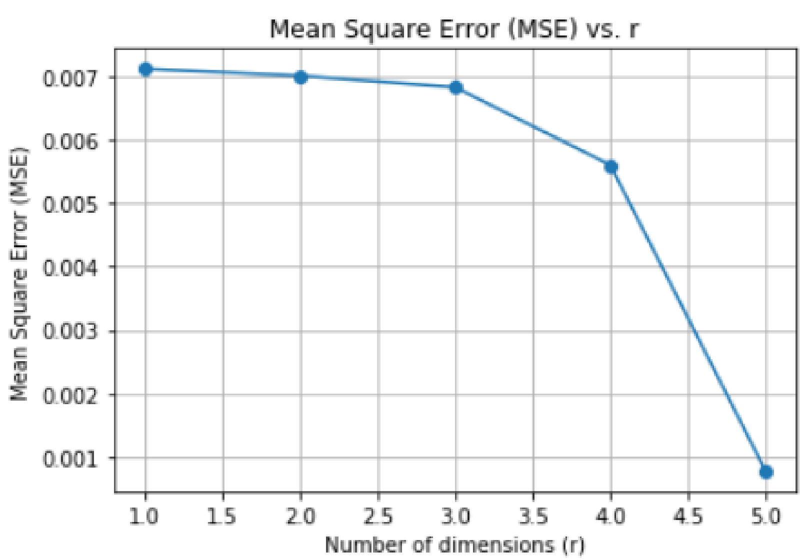
A force distribution where the data varies significantly along a direction that is not aligned with any of the mode shapes of the system PCA would work fine as it would identify this direction as one of the principal components since it captures the variance in the data. However, the Modal analysis relies on mode shape will fail as it doesn't align with any mode shape.

#force along the first mode
force1=np.array([-0.21201089, -0.52059953, 0.71596161, 0.37197045, 0.18183638])

#force along vector perpendicular to above vector
force2=np.array([-0.9772673,   0.11294021, -0.15532256, -0.08069623, -0.03944805])

```
Data- Displacement and Force:
[[ 0.02759069 -0.9772673 ]
 [ 0.05766241  0.11294021]
 [ 0.09724419 -0.15532256]
 [ 0.11920114 -0.08069623]
 [ 0.1241125  -0.03944805]]
```

```
Reduced data using PCA:
[[ 0.08718803 -0.98206226]
 [ 0.02338918  0.02338918]
 [-0.02086711 -0.02086711]]
```



6. **Summarize your learning about the PCA though this exercise and its comparison with the mode shape approach.**

Principal Component Analysis (PCA):

- PCA identifies the directions (principal components) in which the data varies the most. The data is linearly transformed onto a new coordinate system such that the directions capturing the largest variation in the data can be easily identified.
- PCA does not consider the physical interpretation of the data but focuses solely on variance.
- It is useful for capturing the overall variance in the data.

Modal Analysis:

- Modal analysis is the process of determining the dynamic characteristics of a spring-mass system in the form of natural frequencies, damping factors and mode shapes.
- Modal analysis is useful for understanding the physical behavior of the system

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