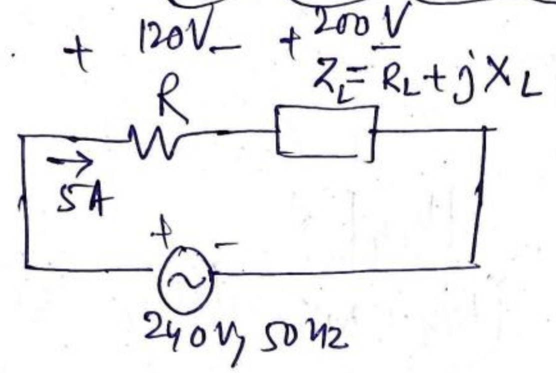
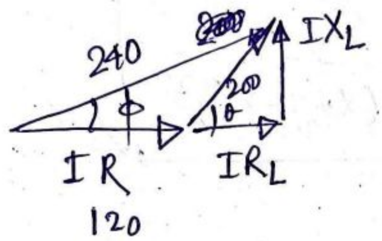
ET tutorial 2

1)



(a)



$120 = 5 \times R$

$\Rightarrow R = 24\,\Omega$

$\left(\frac{200}{5}\right)^2 = R_L^2 + X_L^2$

$\left(\frac{240}{5}\right)^2 = (24 + R_L)^2 + X_L^2$

$\Rightarrow (24 + R_L)^2 - R_L^2 = 704$

$\Rightarrow 24\,(24 + 2R_L) = 704$

$\Rightarrow R_L + 12 = 14.67\,\Omega$

$\Rightarrow \boxed{R_L = 2.67\,\Omega}$

$\boxed{X_L = \sqrt{40^2 - 2.67^2} = 39.91\,\Omega}$

$\boxed{Z_L = 40\,\Omega}$

(b) $P = 200 \times 5 \times \cos\theta$

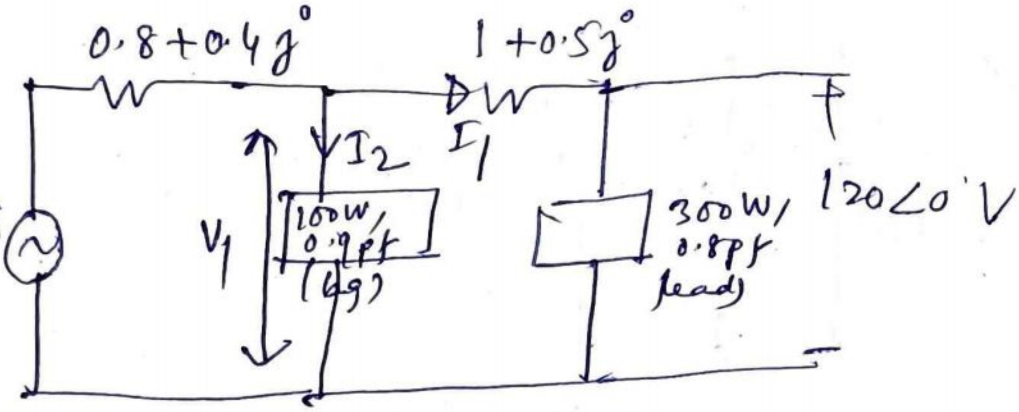
$= \cancel{200} \times 5 \times \frac{5 \times 2.67}{\cancel{200}}$

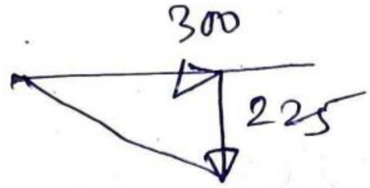
$\boxed{P = 66.67\,W}$

(c) $pf = \cos\phi = \frac{5(24 + 2.67)}{240}$

$\boxed{pf = 0.556}$

2)



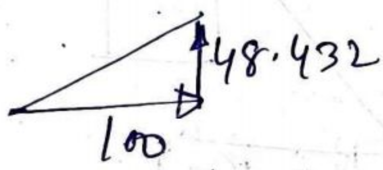


$$(300 - 225j) = 120\, I_1^*$$

$$\Rightarrow I_1 = 2.5 + 1.875j$$

$$V_1 = 120 + (2.5 + 1.875j)(1 + 0.5j)$$

$$= 121.5625 + 3.125j$$



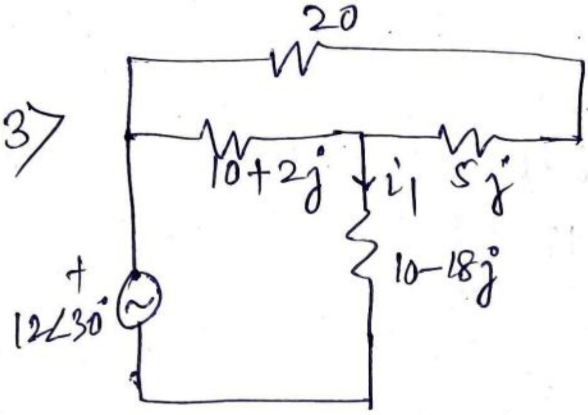
$$100 + 48.432j = (121.5625 + 3.125j)\, I_2^*$$

$$\Rightarrow I_2 = 0.832 - 0.377j$$

$$V_s = 121.5625 + 3.125j + (0.8 + 0.4j) \times (3.332 + 1.498j)$$

$$\boxed{V_s = 123.758 \angle 2.62^\circ}$$

3)



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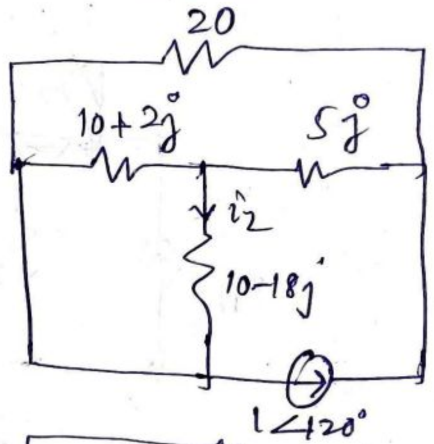
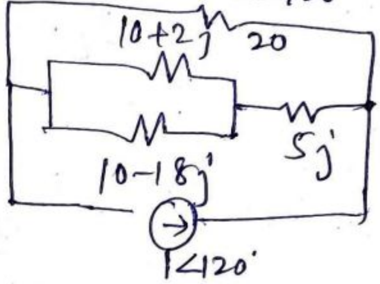




$i_c = i_1 + i_2$

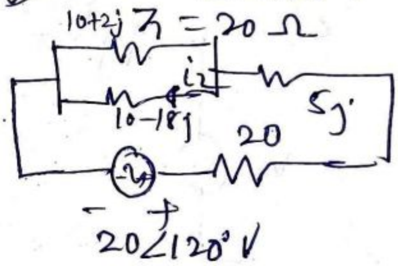
$$i_1 = \frac{12\angle 30^\circ}{(10-18j) + \frac{(10+2j)(20+5j)}{(30+7j)}}$$

$$i_2 = \frac{10+2j}{20-16j} \times \frac{20\angle 120^\circ}{20+5j + \frac{(10+2j)(10-18j)}{(20-16j)}}$$

⇓ Voltage-current source transformation

$V = 20\angle 120^\circ$ V

$Z_1 = 20\,\Omega$

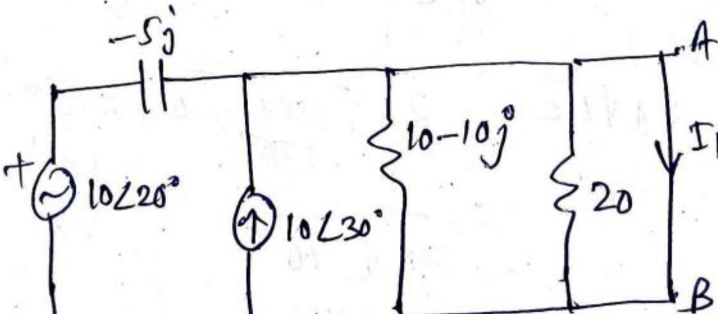


$\Rightarrow i_1 = 0.511\angle 74.804^\circ$

$i_2 = 0.2818\angle 162.98^\circ$

$\therefore \boxed{i_c = \cancel{12\angle 30^\circ}\ 0.59\angle 103.3^\circ}$

4)

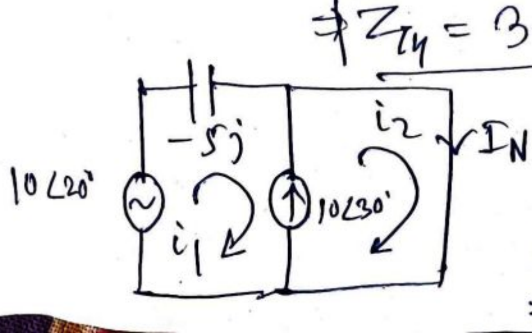


$Z_{Th} = Z_N$

$= 20 \,||\, (10-10j) \,||\, (-5j)$

$$\Rightarrow \frac{1}{Z_{Th}} = \frac{1}{20} + \frac{1}{10-10j} + \frac{1}{-5j} = 0.1 + 0.25j$$

$$\Rightarrow \boxed{Z_{Th} = 3.714\angle -68.2^\circ = Z_N}$$

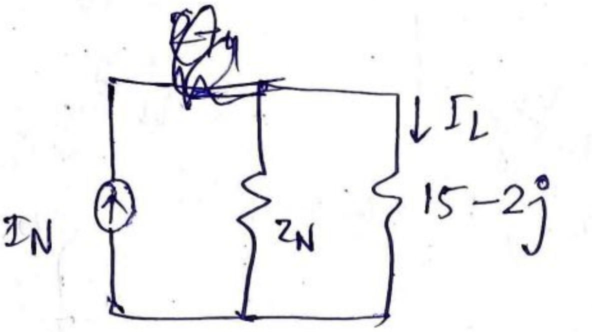


$i_2 = I_N, \quad i_2 - i_1 = 10\angle 30^\circ$

$10\angle 20^\circ - i_1(-5j) = 0$

$10\angle 20^\circ + (5j)(I_N - 10\angle 30^\circ) = 0$

$\Rightarrow I_N = 10\angle 30^\circ - 2\angle -70^\circ = \boxed{10.53\angle 40.78^\circ}$

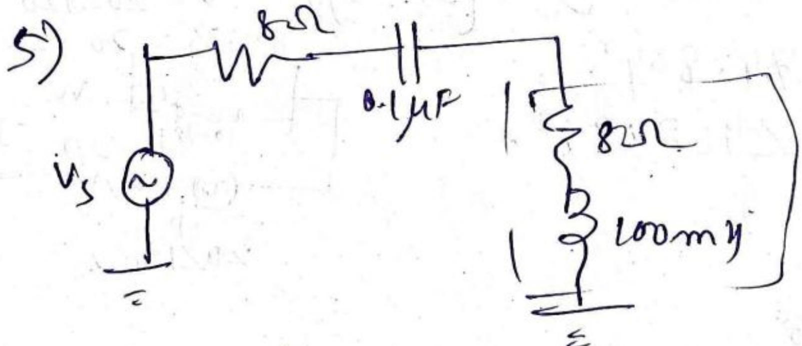


$$I_L = \frac{Z_N}{Z_N + Z_L} \times I_N$$

$$= \frac{3.714\angle -68.2^\circ}{3.714\angle -68.2^\circ + 15-2j} \times 10.53\angle 40.78^\circ$$

$$= \boxed{2.26\angle -9.02^\circ \text{ A}} \text{ (Ans)}$$

5)



$$X_L = -X_{Th},$$

$$\Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{\sqrt{LC}} = 2\pi f$$

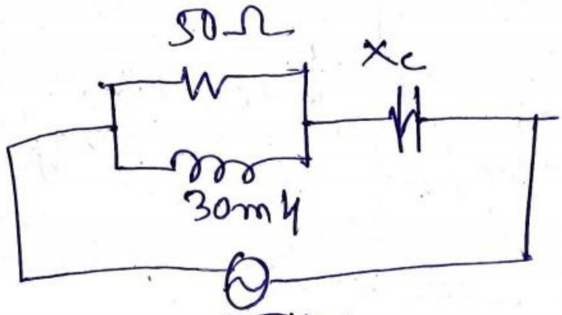
$$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{100}{1000} \times 0.1 \times 10^{-6}}}$$

$$= \frac{1}{2\pi\sqrt{10^{-8}}} = \frac{10^4}{2\pi}$$

$$= \boxed{1.59 \text{ kHz}}$$

$$P = \left(\frac{3.8}{16}\right)^2 \times 8 = \boxed{0.45 \text{ W}}$$

6)



~~200V~~
220V, 50Hz

(a)

$X_L = j \times 2 \times \pi \times 50 \times \frac{30}{1000}$

$= 9.425j$

$\text{Im}\left(\frac{50 \times 9.425j}{50 + 9.425j}\right) = 9.1016$

1.71565 + 9.1016j

$\Rightarrow X_C = 9.1016 = \frac{1}{wC}$

$\Rightarrow C = \frac{1}{2 \times \pi \times 50 \times 9.1016} \times 10^6 \ \mu F$

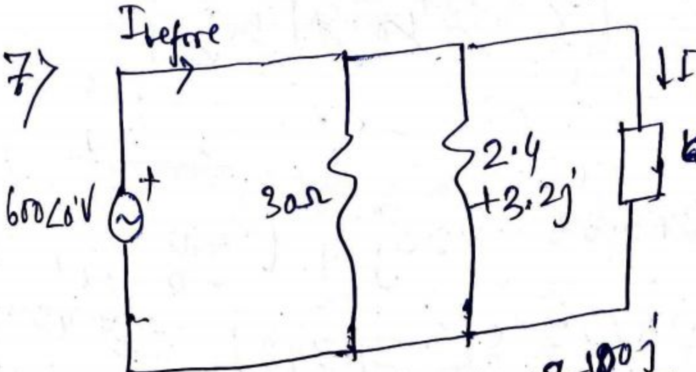
$= \boxed{349.73 \ \mu F}$.

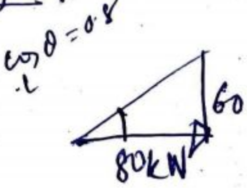
(b) $i = \frac{220}{\text{Re}\left(\frac{50 \times 9.425j}{50 + 9.425j}\right)} = \boxed{128.23 \text{ A}}$

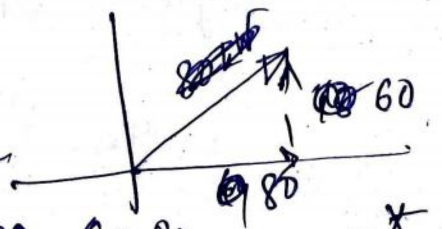
$i_L = \left| \frac{50}{50 + 9.425j} \times i \right| = \frac{50 \times 128.23}{\sqrt{50^2 + 9.425^2}}$

$= \boxed{126 \text{ A}}$.

7)







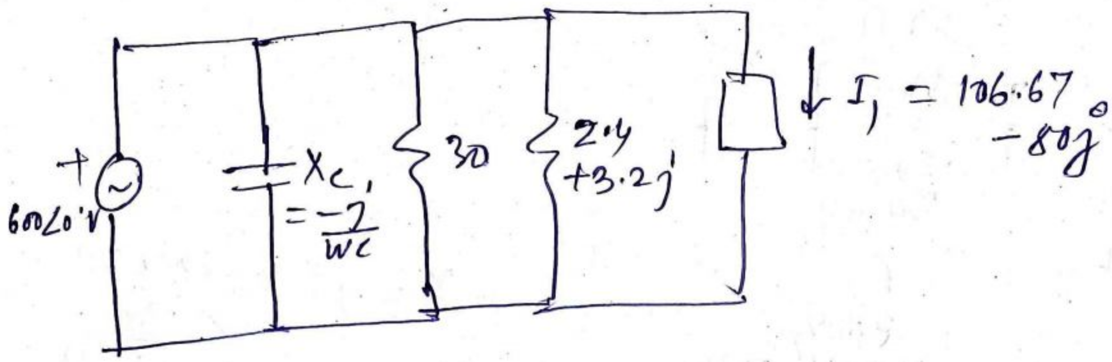
$I_{before} = \frac{600}{30} + \frac{600}{2.4 + 3.2j} + 133.33 - 100j$

$10^3 (80 + 60j) = 600 \ I_1^*$

~~106.67 − 80j~~

$= \boxed{328.04 \angle -42.1^\circ}$

$I_1 = \frac{10}{6}(80 - 60j) = 133.33$

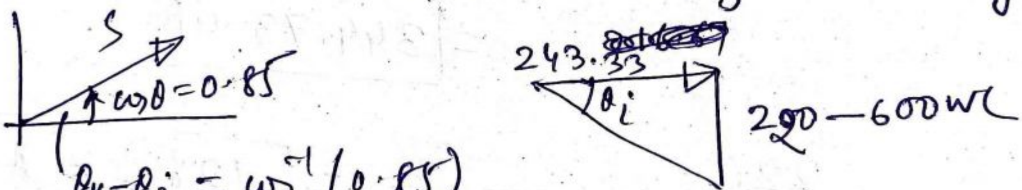


$$i = \frac{600\,wC}{-j} + \frac{600}{30} + \frac{600}{2.4+3.2j} + 133.33 - 100j$$

$$= 600\,wC\,j + 20 + \frac{600}{2.4+3.2j} + 133.33 - 100j$$

$$\cancel{+106.67 - 80j}$$

$$i = 243.33 - 220j + 600\,wC\,j$$



$$\theta_v - \theta_i = \cos^{-1}(0.85)$$

$$\Rightarrow \theta_i = -31.788^\circ$$

$$\Rightarrow \tan\theta_i = 0.6197 = \frac{220 - 600\,wC}{243.33}$$

$$C = \frac{220 - 0.6197 \times 243.33}{600 \times 2 \times \pi \times 50}$$

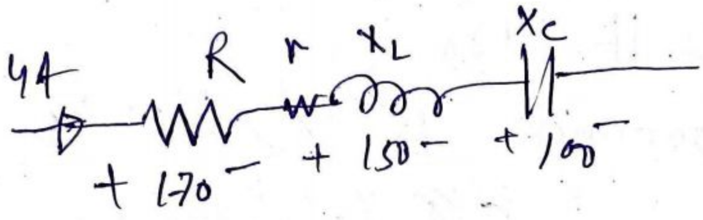
$$\boxed{\cancel{C = 348.71\,\mu F}}$$

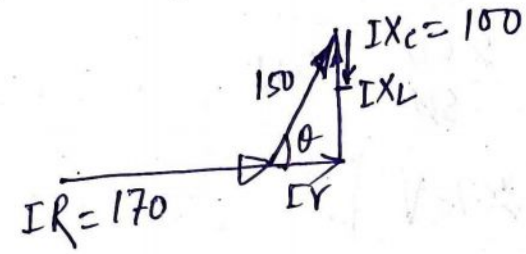
$$\boxed{C = 367.16\,\mu F}$$

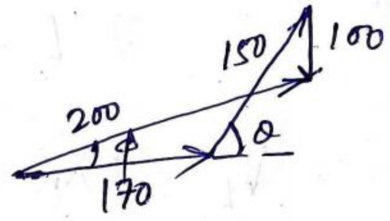
$$i_{after} = 243.33 - 220j + (220 - 0.6197 \times 243.33)j$$

$$\boxed{i_{after} = 286.265\angle -31.79^\circ}$$

8)







$$200^2 = (170 + 150\cos\theta)^2 + (150\sin\theta - 100)^2$$

$$\Rightarrow 20^2 = 17^2 + 15^2\cos^2\theta + 2\cdot 17\cdot 15\cos\theta + 15^2\sin^2\theta + 10^2 - 2\times 10\times 15\sin\theta$$

$$\Rightarrow \cancel{2\times 15\times 7}\; 30(17\cos\theta - 10\sin\theta) = 20^2 - 17^2 - 15^2 - 10^2$$

$$\Rightarrow 10\sin\theta - 17\cos\theta = \frac{107}{15}$$

$$(10\sin\theta)^2 = \left(17\cos\theta + \frac{107}{15}\right)^2$$

$$\Rightarrow 10(1 - x^2) = 17x^2 + \left(\frac{107}{15}\right)^2 + 2\times 17x \times \frac{107}{15}$$

$$\cancel{\cos\theta = \frac{\frac{107}{15} - 10}{(17 \pm 10)} \times (-0.17186)}$$

$\cancel{\theta = -141.67^\circ}$

$\theta = 80.74^\circ$

$\boxed{\cos\theta = 0.16}$

$\therefore$ $\boxed{\text{pf of the coil} = 0.16}$

pf of the circuit $= \cos\varphi = \dfrac{170 + 150\times 0.16}{200}$

$= \boxed{0.97}$

97)

@ resonance $\omega^2 = \frac{1}{LC}$ $\quad X_L = X_C$

$$\cancel{\Rightarrow LC = \frac{1}{(2\pi \times 50)^2 \times 2}}$$

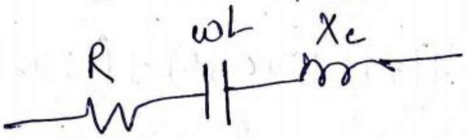
$IR = 1000$

$\Rightarrow I = 10A$

$$\Rightarrow \omega L = X_C$$

$$\Rightarrow L = \frac{200}{50} = 4H$$

$$V_L = IX_L = \boxed{2kV}$$



$$i = \frac{V}{\sqrt{R^2 + (\omega L - X_C)^2}}$$

$$V_L = \omega L \cdot \frac{V}{\sqrt{R^2 + (\omega L - X_C)^2}}$$

$$= \frac{V X_L}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\frac{\partial V_L}{\partial X_L} = 0 \Rightarrow \frac{1}{\sqrt{R^2 + (X_L - X_C)^2}} + \frac{X_L \left(-\frac{1}{2}\right) \times 2(X_L - X_C)}{\left[R^2 + (X_L - X_C)^2\right]^{3/2}} = 0$$

$$\Rightarrow R^2 + (X_L - X_C)^2 - X_L(X_L - X_C) = 0$$

$$\Rightarrow R^2 + \cancel{X_L^2} + X_C^2 - 2X_L X_C - \cancel{X_L^2} + X_L X_C = 0$$

$$\Rightarrow X_L X_C = (R^2 + X_C^2) \Rightarrow X_L = \frac{(R^2 + X_C^2)}{X_C}$$

$$X_L = \frac{R^2 + X_C^2}{X_C} = \frac{100^2 + 200^2}{200} = 250$$

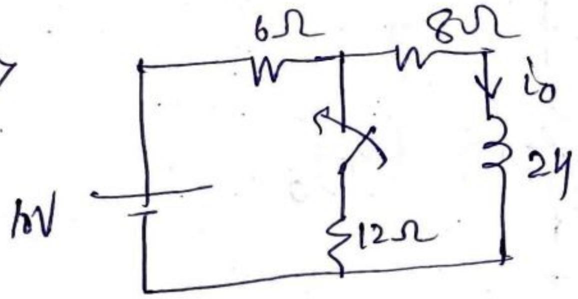
$$\cancel{\frac{100 \times 100}{200}}$$

$$\cancel{200 - 50}$$

$$\cancel{V_{max} = \frac{V\left(\frac{R^2}{X_C}\right)}{\sqrt{R^2 + \left(\frac{R^2}{X_C} - X_C\right)^2}} = \frac{1000 \times 50}{\sqrt{100^2 + 150^2}}}$$

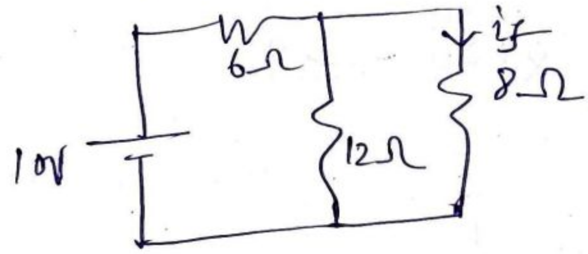
$$V_{max} = \frac{1000 \times 250}{\sqrt{100^2 + 50^2}} = \boxed{2.236 kV}$$

10)



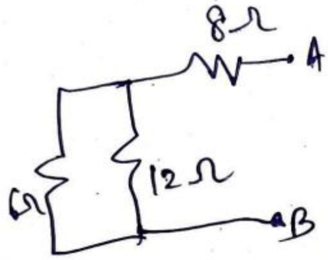
$i_0 = \frac{10}{14} = 5/7$

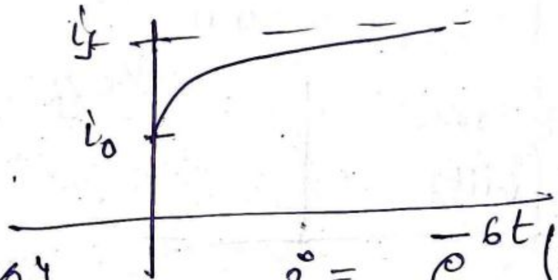
$i_f = \frac{12}{20} \times \frac{10}{6 + \frac{12 \times 8}{20}}$



$= \frac{120}{6 \times 20 + 12 \times 8}$

$= 5/9$





$R_{TH} = 8 + \frac{\cancel{6} \times \cancel{12}^{4}}{\cancel{18}_{3}}$

$= 12\,\Omega$

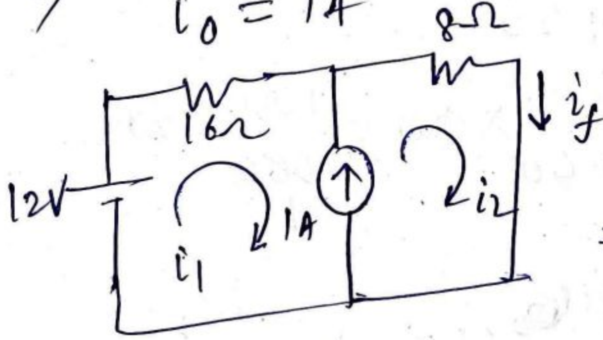
$\therefore \tau = L/R = 2/12 = 1/6$

$i = e^{-6t}(i_0 - i_f) + i_f$

$\Rightarrow i = \frac{5}{9} + \left(\frac{5}{7} - \frac{5}{9}\right) e^{-6t}$

$= \boxed{\frac{5}{9} + \frac{10}{63} e^{-6t}}$

11) $i_0 = 1A$



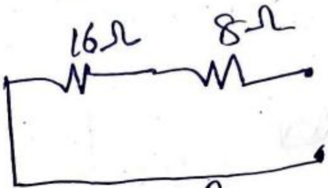
$i_2 - i_1 = 1\ ,\ i_f = i_2$

$12 - 16 i_1 - 8 i_2 = 0$

$\Rightarrow 12 - 16(i_f - 1) - 8 i_f = 0$

$\Rightarrow 28 = 24 i_f$

$\Rightarrow i_f = 7/6$



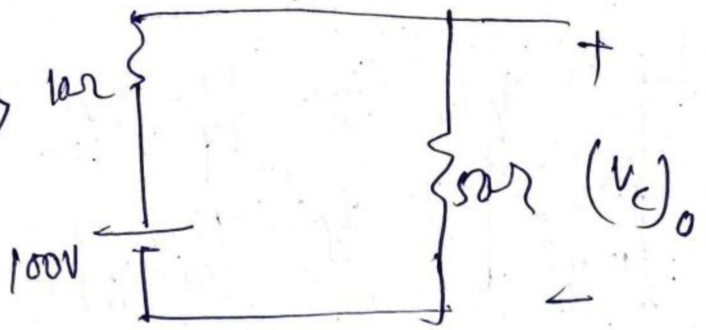
$R_{TH} = 24\,\Omega$

$\therefore \tau = L/R = 2/24 = 1/12$

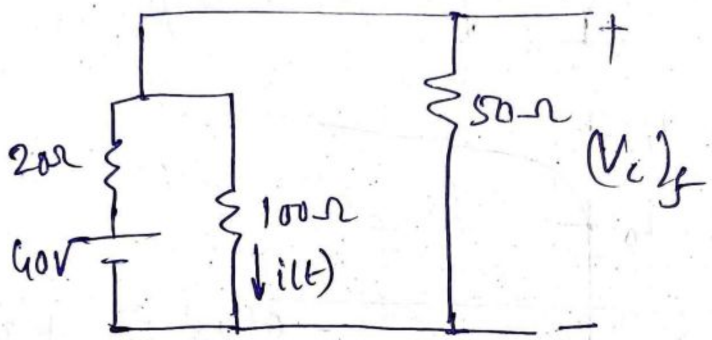
$\therefore i = e^{-12t}(1 - 7/6) + 7/6$

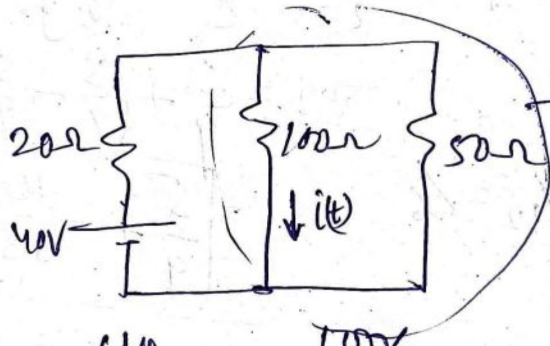
$= \boxed{\frac{7}{6} - \frac{1}{6} e^{-12t}}$

12)



$$(V_c)_0 = \frac{\cancel{50}}{3\cancel{00}} \times \overset{50}{\cancel{100}} = \frac{250}{3}$$

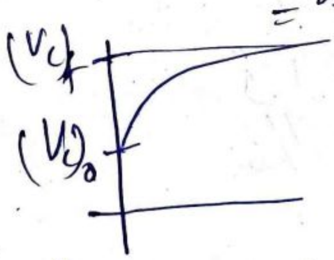




$$\frac{100 \times \cancel{50}}{\cancel{150}_{3}}$$

$$(V_c)_f = \frac{100/3}{\frac{100}{3} + 20} \times 40 = \frac{100}{100+60} \times 40 = \frac{100}{\cancel{160}_{4}} \times \cancel{40} = 25$$

$$R_{Th} = 20 \,||\, 100 \,||\, 50 = \frac{20 \times 100/3}{20 + \frac{100}{3}} = \frac{\cancel{20} \times \cancel{100}}{\cancel{160}_{8\,2}} = 25/2$$



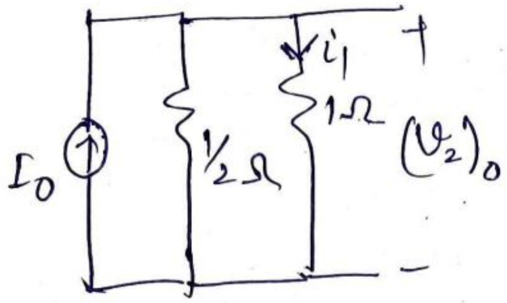
$$V_c = e^{-0.8t} \left(\frac{250}{3} - 25\right) + 25$$

$$= \boxed{\frac{175}{3} e^{-0.8t} + 25}$$

$$\tau = RC = \frac{25}{2} \times 0.1 \quad \frac{1}{\tau} = 0.8$$

$$\boxed{i(t) = \frac{V_c}{100} = \frac{1}{4} + \frac{7}{12} e^{-0.8t}}$$

137)



$$i_1 = \frac{1/2}{1/2+1} \times I_0$$

$$(v_2)_0 = I_0/3$$

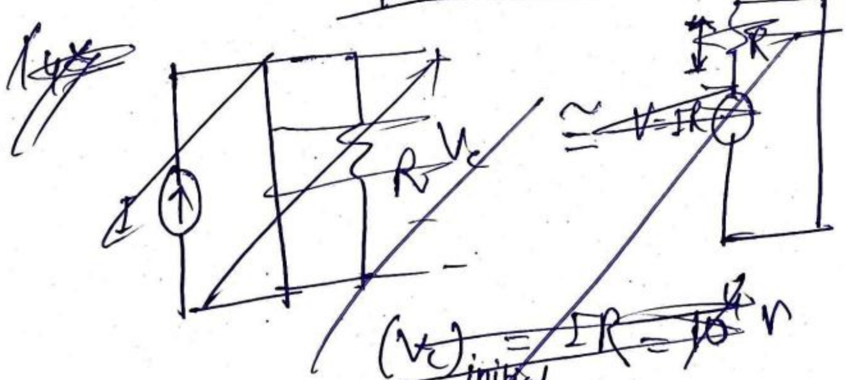
$$(v_2)_f = I_0$$

$$R_{Th} = 1\Omega \Rightarrow \tau = 0.5$$

$$\therefore v_2(t) = e^{-2t}\left(\frac{I_0}{3} - I_0\right) + I_0$$

$$= \boxed{I_0\left(1 - 0.67e^{-2t}\right)}$$

~~14)~~



$$\cancel{(V_c)_{initial} = IR = 10\ V}$$

16)

$$I_1 = I_2 e^{-t/\tau} = I_2(1/e)$$

$$I_2 = e^{-t/\tau}\left(I_1 - \frac{E}{R}\right) + E/R$$

$$= \frac{E}{R}\left(1 - \frac{1}{e}\right) + \frac{I_1}{e}$$

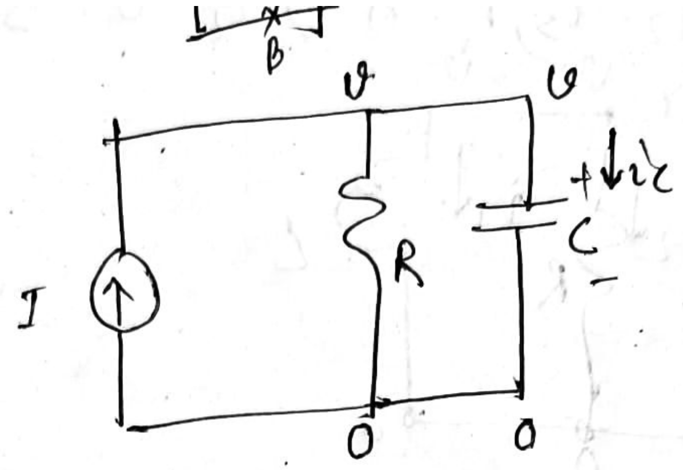
$$\Rightarrow I_2 = \frac{E}{R}\left(1 - \frac{1}{e}\right) + \frac{I_2}{e^2}$$

$$\Rightarrow I_2 = \frac{E/R\,(1 - 1/e)}{(1 - 1/e^2)}$$

$$= \frac{E/R}{1 + 1/e}$$

$$\therefore \boxed{\begin{aligned} I_2 &= \frac{Ee}{R(1+e)} \\ I_1 &= \frac{E}{R(1+e)} \end{aligned}} \checkmark$$

14)



$i_c = c\frac{dV}{dt}$

$I = \frac{v}{R} + i_c$

$\Rightarrow I = \frac{V}{R} + c\frac{dV}{dt}$

$(V)_{t=0^+} = (V)_{t=0} = 0$

$\left(dV/dt\right)_{t=0^+} = I/c = \frac{10}{10^{-6}} = 10^7\ V/s$

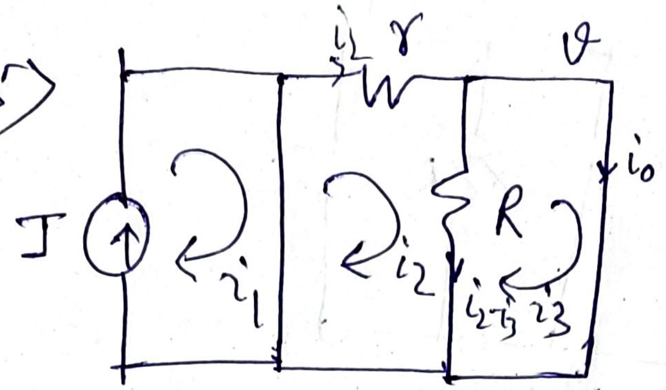
$\frac{dI}{dt} = \frac{1}{R}\frac{dV}{dt} + c\frac{d^2V}{dt^2}$

$\Rightarrow d^2V/dt^2 = -\frac{1}{Rc}\frac{dV}{dt}$

$= -\frac{1}{10^3 \times 10^{-6}} \times 10^7$

$= -10^{10}\ V/s^2$

15)

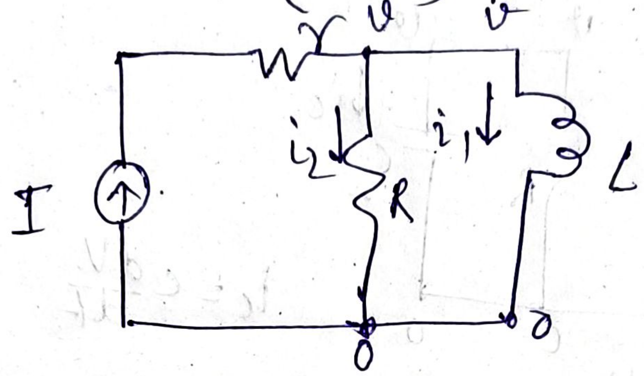


$i_3 = i_0$

$i_1 = I$

$-i_2 r - (i_2 - i_3) R = 0$

$\Rightarrow (i_2 - i_3) R = 0 \Rightarrow i_2 = 0 = i_3 = i_0$



$I = i_1 + i_2 = i_1 + \frac{v}{R}$

$v = L\, di_1/dt$

$I = i_1 + \frac{L}{R}\frac{di_1}{dt}$

$\left(\frac{di_1}{dt}\right)_{t=0^+} = \frac{IR}{L} = \frac{1\times 10^2}{1} = 100$

$\therefore (v)_{t=0^+} = 1\times 100 = 100V$

$0 = \frac{d^2 i_1}{dt^2} + \frac{L}{R}\frac{d^3 i_1}{dt^3}$

$dv/dt = L\frac{d^2 i_1}{dt^2}$, $0 = \frac{di_1}{dt} + \frac{L}{R}\frac{d^2 i_1}{dt^2}$

$\Rightarrow L\frac{d^2 i_1}{dt^2} = -R\left(\frac{di_1}{dt}\right)$

$\therefore \frac{dv}{dt} = -\frac{1\times 100\times 100}{1} = -10^4 = -\frac{IR^2}{L}$

$d^2v/dt^2 = L\frac{d^3 i_1}{dt^3} = -R\left(-\frac{IR^2}{L^2}\right) = \frac{IR^3}{L^2} = 10^6$