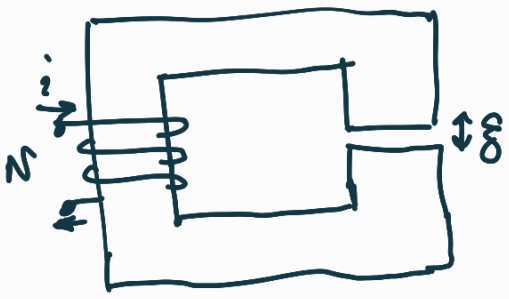
Tutorial - 4

1)

(a)



$$R_c = \frac{l_c}{\mu_0 \mu_r A_c}$$

$$= \frac{30 \times 10^{-2}}{4\pi \times 10^{-7} \times 70000 \times 9 \times 10^{-4}}$$

$$= 3789 \text{ AT/Wb}$$

$$R_g = \frac{l_g}{\mu_0 A_g} = \frac{0.05 \times 10^{-2}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}}$$

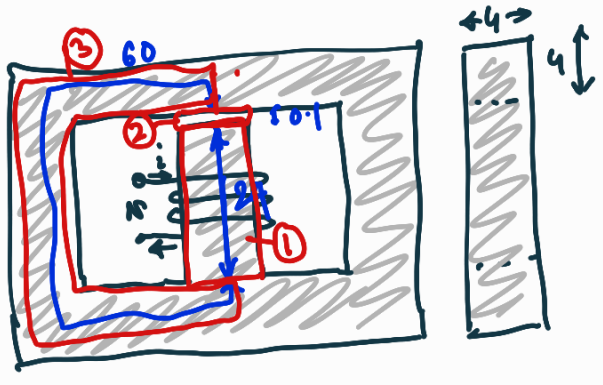
$$= 442097 \text{ AT/Wb}$$

(b) $\Phi = BA = 9 \times 10^{-4}$ Wb

(c) $\phi = \frac{Ni}{R_c + R_g} \Rightarrow i = \frac{9 \times 10^{-4} \times 445886}{800}$

$$= 0.8 \text{ A}$$

2)



cast steel				
H (AT/m)	500	600	800	900
B (Wb/m²)	0.5	0.6	0.9	1.0

$Ni = \sum Hl$, $N = 680$, $\Phi_{Tot} = 1.6$ mWb

$= H_1 l_1 + H_2 l_2 + H_3 l_3$

$$B_1 = \frac{1.6 \times 10^{-3}}{4 \times 4 \times 10^{-4}} = 1 \text{ Wb/m}^2 \Rightarrow H_1 = 900 \text{ AT/m}$$

$$H_2 = \frac{B_1}{\mu_0} = \frac{1}{4\pi \times 10^{-7}}$$

$B_3 = 0.5$ Wb/m² $\Rightarrow H_3 = 500$ AT/m

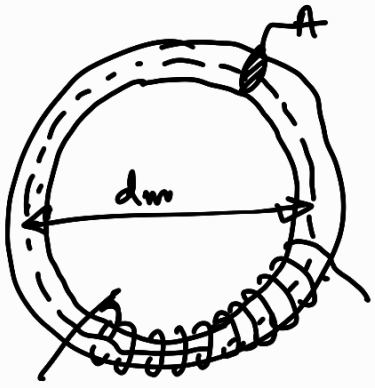
$$\therefore Ni = 900 \times 24 \times 10^{-2} + \frac{1}{4\pi \times 10^{-7}} \times 10^{-3} + 500 \times 60 \times 10^{-2}$$

$$= 1311.775$$

$$\Rightarrow i = 1.93 \text{ A}$$

3)

a)



$d_m = 50\,cm$,
$H = 40\ AT/cm$, $B = 1.0\ Wb/m^2$
$A = 20\,cm^2$
$N = 500$

$B = \mu H$
$\Rightarrow \mu = \frac{1}{4000}$

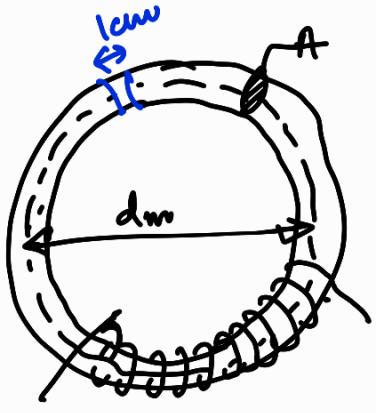
$$N i^{\circ} = H(\pi d_m)$$

$$L = \frac{N\phi}{i} = \frac{N^2(BA)}{\pi H d_m}$$

$$\therefore L = \frac{500^2 \times 1 \times 20 \times 10^{-4}}{\pi \times 40 \times 10^2 \times 50 \times 10^{-2}}$$

$$= 0.0796\ H = \boxed{79.6\ mH}$$

b)



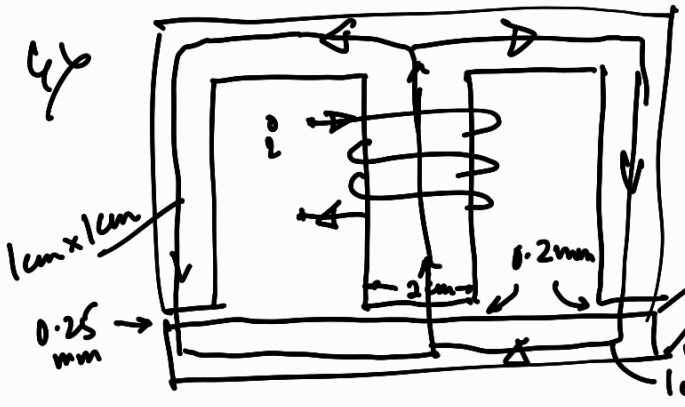
$$L = \frac{N\phi}{i^{\circ}}$$

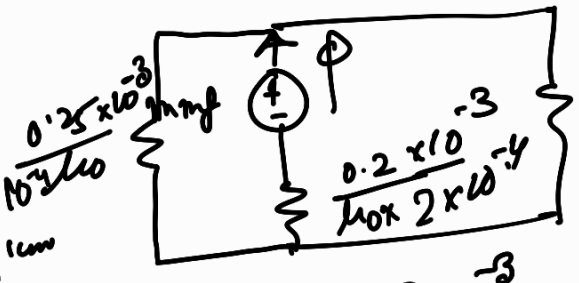
$$N i^{\circ} = \frac{B}{\mu}(\pi d_m - \xi) + \frac{B}{\mu_0}\xi$$

$$= \frac{N^2 BA}{\frac{B}{\mu}(\pi d_m - \xi) + \frac{B\xi}{\mu_0}} = \frac{N^2 A}{\frac{\pi d_m - \xi}{\mu} + \frac{\xi}{\mu_0}}$$

$$= \frac{500^2 \times 20 \times 10^{-4}}{\frac{50\pi - 1}{100\pi} \times 4000 + \frac{1}{100 \times 4\pi} \times 10^7} = \boxed{35.2\ mH}$$

4)



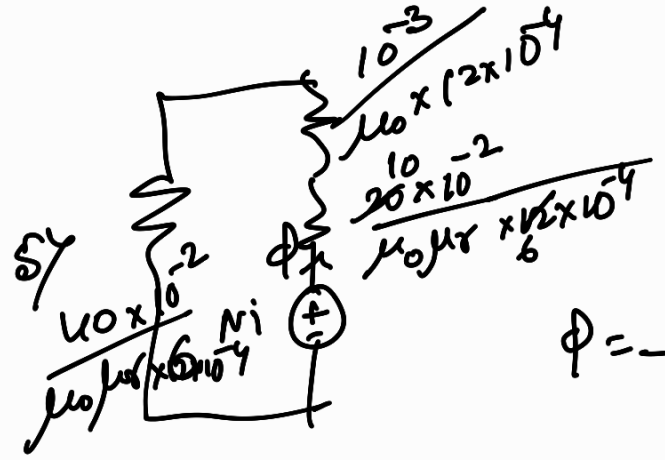


$$\phi = 0.8 \times 10^{-3} = \frac{mmf}{\frac{1}{\mu_0} + \left(\frac{2.5}{\mu_0} \| \frac{2}{\mu_0}\right)}$$

$$= \frac{\mu_0 (mmf)}{2.111}$$

$$\Rightarrow mmf = \boxed{1343.98\ AT}$$

5)

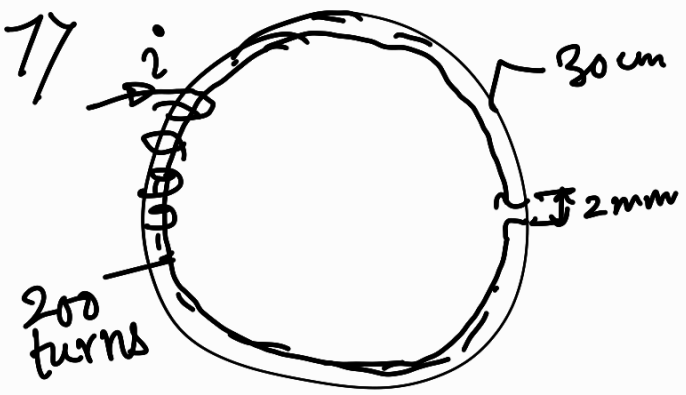


$$\phi = \frac{750 \times 0.5}{\frac{50}{6} \times 10^2 \mu_0\mu_r + \frac{10}{12\mu_0}} = 0.5 \times 10^{-3}$$

$$\Rightarrow \frac{10}{12} + \frac{5000}{6\mu_r} = \frac{750 \times 4\pi \times 10^{-7}}{0.5 \times 10^{-3}}$$

$\Rightarrow \mu_r = 7635.14$

77



$$B = \frac{\phi}{A} = \left(\frac{N i}{\frac{l}{\mu A} + \frac{g}{\mu_0 A}}\right) \frac{1}{A}$$

$$= \frac{N i}{\frac{l}{\mu} + \frac{g}{\mu_0}}$$

$$= \frac{200}{\frac{30\times10^{-2}}{1.25\times10^{-4}} + \frac{2\times10^{-3}}{4\pi\times10^{-7}}}$$

$$= 50.1 \text{ mWb/m}^2$$

8)

a) $$L = \frac{N\phi}{i} = \frac{N}{i}\left(\frac{N i}{\frac{g_1}{\mu_0 A_1} \,||\, \frac{g_2}{\mu_0 A_2}}\right)$$

$$= \frac{N^2 \mu_0 \left(\frac{g_1}{A_1} + \frac{g_2}{A_2}\right)}{\frac{g_1}{A_1} \cdot \frac{g_2}{A_2}}$$

$$= \mu_0 N^2 \left(\frac{g_1 A_2 + g_2 A_1}{g_1 g_2}\right) = \mu_0 N^2 \left(\frac{A_1}{g_1} + \frac{A_2}{g_2}\right)$$

b) $$B_1 = \frac{\phi_1}{A_1} = \frac{N \phi_2 \mu_0 A_1}{g_1 A_1}$$

$$= \frac{\mu_0 N i}{g_1}$$

5/ a) $R_c = 0$

$$R_g = \frac{g}{\mu_0 A_c} = \frac{2.3\times10^{-3}}{4\pi\times10^{-7}\times1.8\times10^{-3}}$$

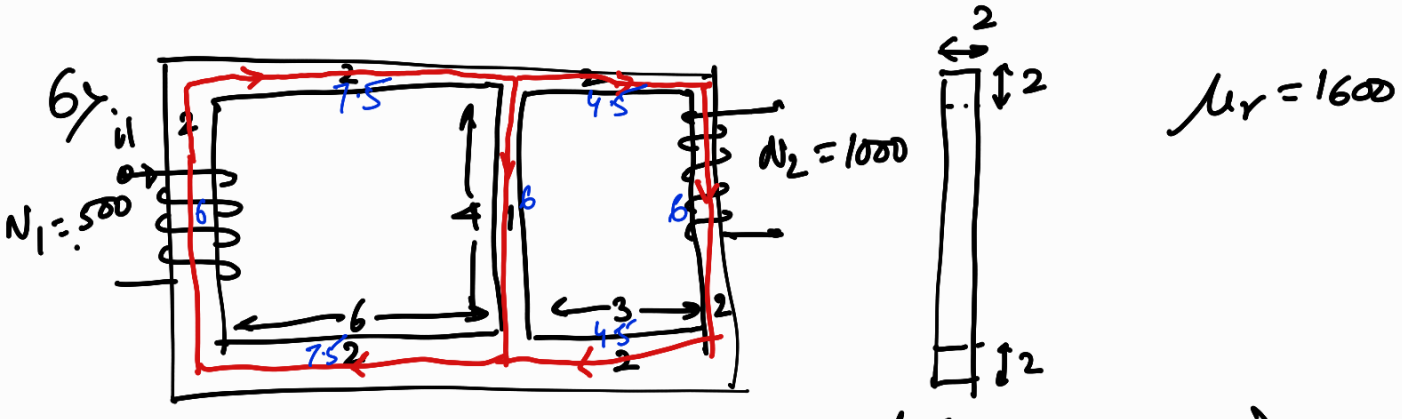
$$= 1.017\times10^{6}\ \text{AT/Wb}$$

b) $$\phi = \frac{83\times1.5}{1.017\times10^{6}} = 1.22\times10^{-4}\ \text{Wb}$$

c) $N\phi = 0.0101\ \text{Wb T}$

d) $$L = \frac{N\phi}{i} = 6.7\ \text{mH}$$

6/



$$L_1 = \frac{N_1\phi_1}{i_1} = \frac{N_1}{i_1}\left(\frac{N_1 i_1 \mu_0\mu_r}{\frac{600}{4} + \frac{1500}{4} + \left(\frac{600}{2}\,||\,\frac{1500}{4}\right)}\right)$$

$$= \frac{N_1^2\mu_0\mu_r}{150 + 375 + (300\,||\,375)}$$

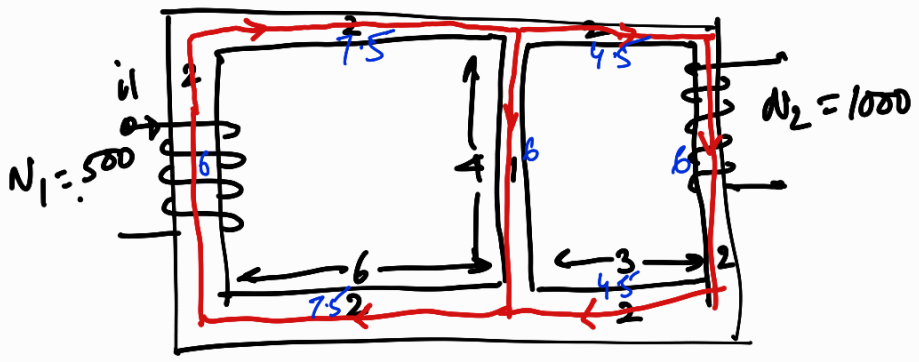
$$= \frac{500^2\times4\pi\times10^{-7}\times1600}{525 + \frac{300\times375}{675}}$$

$$= 0.727\ \text{H}$$

$$L_2 = \frac{N_2^2\mu_0\mu_r}{\frac{1500}{4} + \left(\frac{600}{2}\,||\,\frac{2100}{4}\right)}$$

$$= \frac{1000^2\times4\pi\times10^{-7}\times1600}{375 + 300\,||\,525}$$

$= 3.553\ H$



$$M = \frac{N_2 \phi_2}{i_1}, \quad \phi_2 = \frac{6/2}{6/2 + \frac{15}{4}} \times \frac{N_1 i_1 \text{ flux}}{\frac{2100}{4} + \left(\frac{600}{2} \,||\, \frac{15000}{4}\right)}$$

$$\Rightarrow M = \frac{3}{6.75} \times \frac{500 \times 1000 \times 4\pi \times 10^{-7} \times 1.6}{525 + 300 \,||\, 375}$$

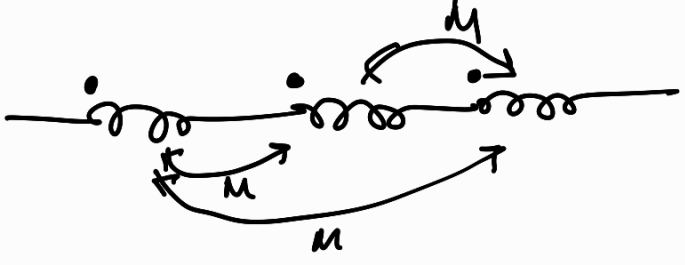
$$= 0.646\ H$$

10) 

$M = L$

$$-L\frac{di}{dt} - M\frac{di}{dt} - L\frac{di}{dt} - M\frac{di}{dt} = -L_{eq}\frac{di}{dt}$$

$\therefore L_{eq} = 4L$

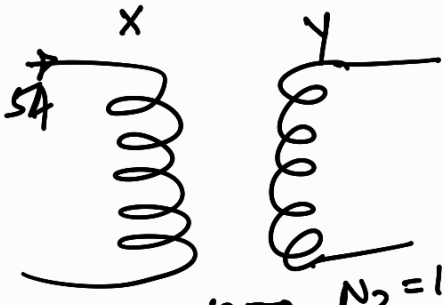


$M = L$

$L + 2M + L + 2M + L + 2M$

$= 3L + 6M$

$= 9L$

11) a) 

$N_1 = 12000 \quad N_2 = 15000$

$\phi_2 = 0.05\ mWb$

$\lambda_2 = (45\%\ \phi_2) \times N_2 \Rightarrow M = \frac{0.45 \times 0.05 \times 10^{-3} \times 15000}{5}$

$= 67.5\ mH$

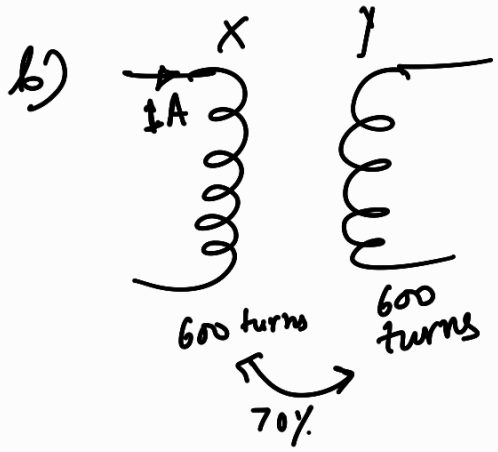
$L_1 = \frac{N_1 \phi_2}{i_1} = \frac{12000 \times 0.05 \times 10^{-3}}{5} = 120\ mH$

$$L_2 = \frac{N_2 \phi_y}{i_2} = \frac{1500 \times 0.075 \times 10^{-3}}{5}$$

$$= 225 \text{ mH}$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{67.5}{\sqrt{120 \times 225}} = 0.4108$$

b)



$$M = \frac{0.075 \times 10^{-3} \times 0.7 \times 600}{1}$$

$$= 31.5 \text{ mH}$$

$$L = \frac{0.075 \times 10^{-3} \times 600}{1}$$

$$= 45 \text{ mH}$$

$$|v_y| = \left| M \frac{di_x}{dt} \right| = 31.5 \times 10^{-3} \times 200$$

$$= 6.3 \text{ V}$$

$k = 0.95$

(2)

a)

$20\angle 0^\circ$ V

$\omega = 1000$ rad/s

$v = 20\sqrt{2} \sin \omega t$

time domain eqn: $v - i_a R_a - L_a \frac{di_a}{dt} = 0$ —(i)

$v_b = k\sqrt{L_a L_b} \frac{di_a}{dt}$ —(ii)

frequency domain eqn: $V - I_a R_a - j\omega L_a I_a = 0$

$V_b = k\sqrt{L_a L_b}\, j\omega I_a$

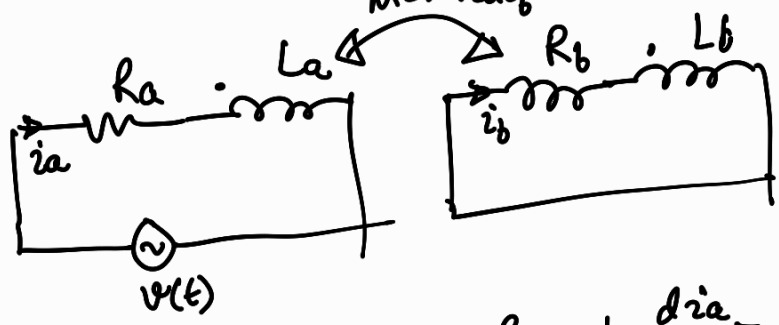
$= k\sqrt{L_a L_b}\, j\omega \frac{V}{R_a + j\omega L_a}$

$= \frac{j\omega k V \sqrt{L_a L_b}}{R_a + j\omega L_a}$

$= \frac{10^{-3} \times 0.95 \times 20 \sqrt{31 \times 120} \times 10^{-3}\, j}{20 + 120j}$

$\Rightarrow V_b = 9.4 + 1.566j$

b)



Time domain: $v - i_a R_a - L_a \frac{di_a}{dt} - k\sqrt{L_a L_b}\frac{di_b}{dt} = 0$

$-i_b R_b - L_b \frac{di_b}{dt} - k\sqrt{L_a L_b}\frac{di_a}{dt} = 0$

Freq. domain: $V - I_a(R_a + j\omega L_a) - jk\sqrt{L_a L_b}\,\omega I_b = 0$

$I_b(R_b + j\omega L_b) + jk\sqrt{L_a L_b}\,\omega I_a = 0$

$\therefore \quad (20 + 120j) I_a + 57.94j\, I_b = 20$

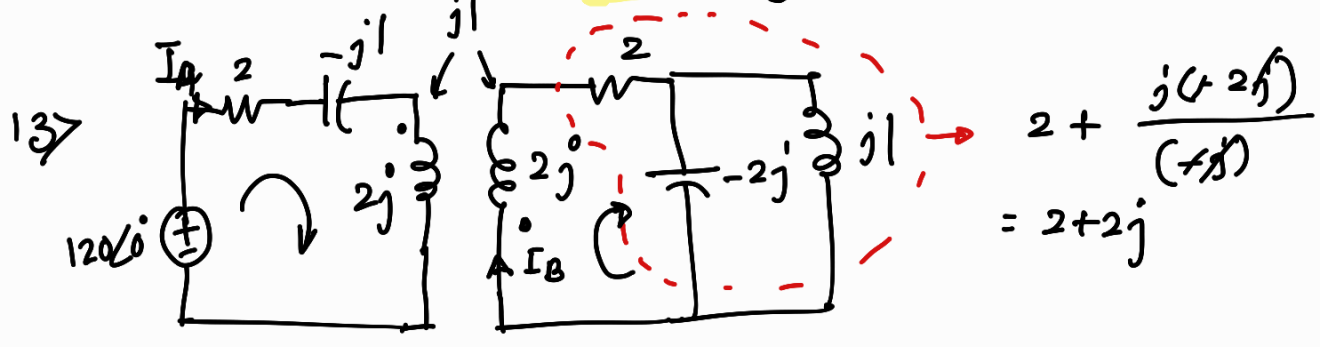
$57.94j\, I_a + (4 + 31j) I_b = 0$

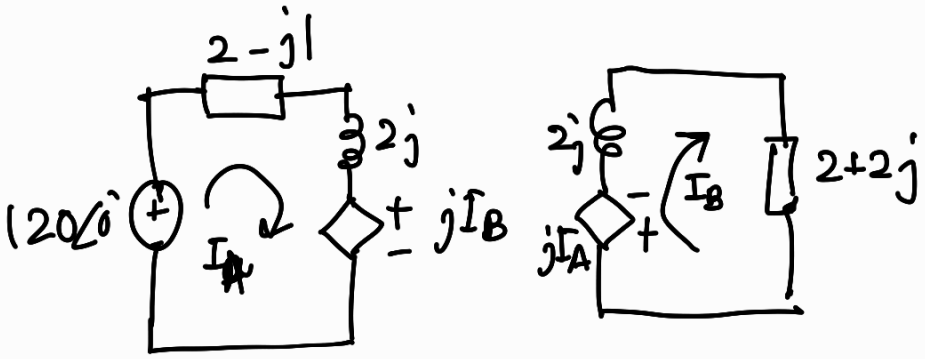
$$I_a = \frac{\begin{vmatrix} 20 & 57.94j \\ 0 & 4+31j \end{vmatrix}}{\begin{vmatrix} 20+120j & 57.94j \\ 57.94j & 4+31j \end{vmatrix}}$$

$$= \frac{20(4+31j)}{(20+120j)(4+31j) + 57.94^2}$$

$= 0.511 - 0.2j$ A

13)





$120 - (2+j)I_A - jI_B = 0$

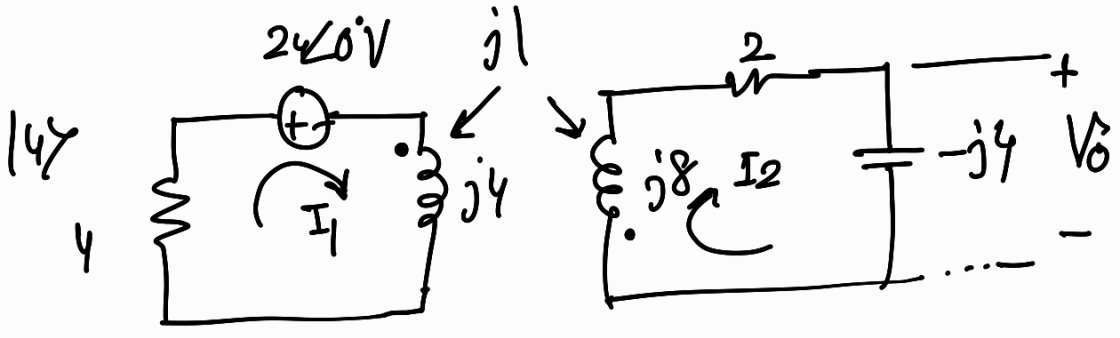
$-jI_A - (2+4j)I_B = 0$

$\therefore (2+j)I_A + jI_B = 120$

$jI_A + (2+4j)I_B = 0$

$$I_A = \frac{\begin{vmatrix} 120 & j \\ 0 & 2+4j \end{vmatrix}}{\begin{vmatrix} 2+j & j \\ j & 2+4j \end{vmatrix}} = \frac{120(2+4j)}{(2+j)(2+4j)+1}$$

$$Z_3 = \frac{120}{I_A} = \frac{120[1+(2+j)(2+4j)]}{120(2+4j)} = 2.25\angle 20.85^\circ\ \Omega$$



$$-4I_1 - 24 - 4jI_1 - jI_2 = 0 \Rightarrow (4+4j)I_1 + jI_2 = -24$$

$$-8jI_2 - jI_1 - (2-4j)I_2 = 0 \Rightarrow jI_1 + (2+4j)I_2 = 0$$

$$I_1 = \frac{\begin{vmatrix} -24 & j \\ 0 & 2+4j \end{vmatrix}}{\begin{vmatrix} 4+4j & j \\ j & 2+4j \end{vmatrix}} = \frac{-24(2+4j)}{(4+4j)(2+4j)+1}$$

$$= 4.29\angle 137.2^\circ A$$

$$I_2 = \frac{\begin{vmatrix} 4+4j & -24 \\ j & 0 \end{vmatrix}}{\begin{vmatrix} 4+4j & j \\ j & 2+4j \end{vmatrix}} = \frac{24j}{(4+4j)(2+4j)+1}$$

$$= 0.96\angle -16.26^\circ A$$

$$V_o = 3.84\angle -106.26^\circ V$$