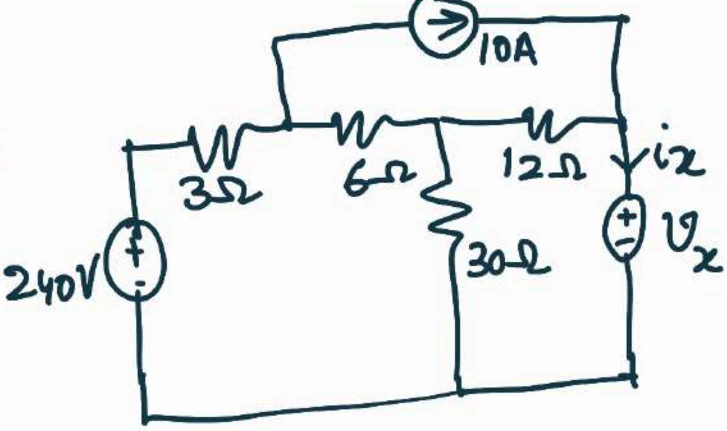
EE11003 – Mid Spring 2023-24

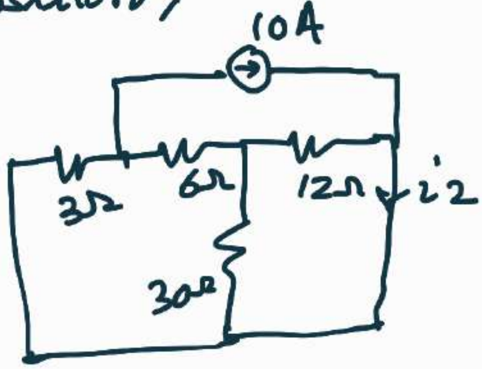
1) (a)



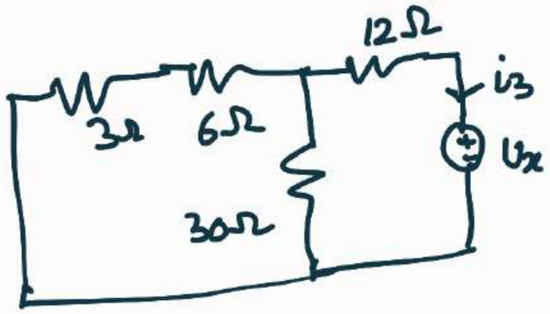
$i_x = 0$. To find v_x.

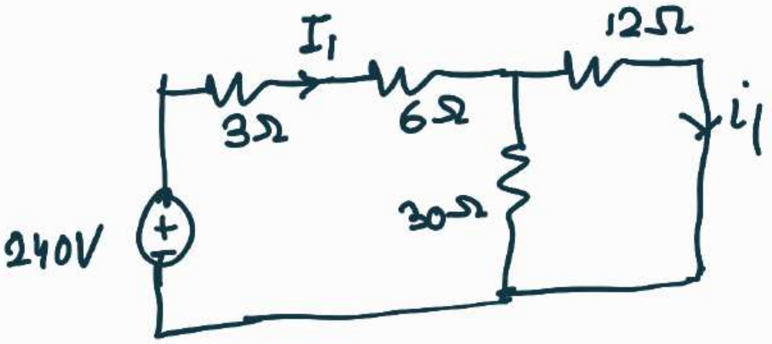
Applying method of superposition, $i_x = i_1 + i_2 + i_3$

(+)



(+)



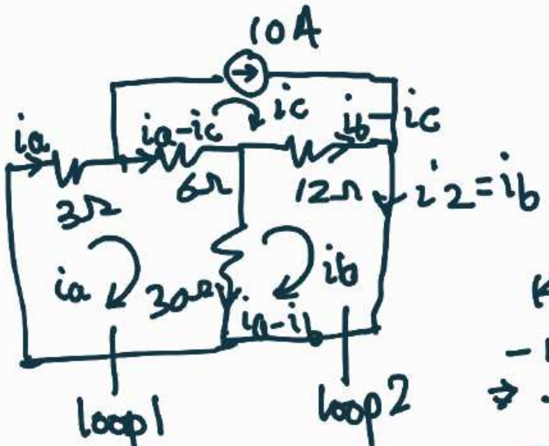


$$i_1 = \frac{30}{30+12} I_1$$

$$= \frac{30}{42} \times \frac{240}{9 + \frac{30\times 12}{42}}$$

$$= \frac{30 \times 240}{9\times 42 + 30\times 12}$$

$$= \frac{400}{41} A$$



$i_c = 10A$

KVL in loop 1,

$$-3i_a - 6(i_a - i_c) - 30(i_a - i_b) = 0$$

$$\Rightarrow -39 i_a + 60 + 30 i_b = 0 \quad \text{—(i)}$$

KVL in loop 2,

$$-12(i_b - i_c) + 30(i_a - i_b) = 0$$

$$\Rightarrow -42 i_b + 120 + 30 i_a = 0 \quad \text{—(ii)}$$

$$\Rightarrow i_b = \frac{360}{41} A = i_2$$

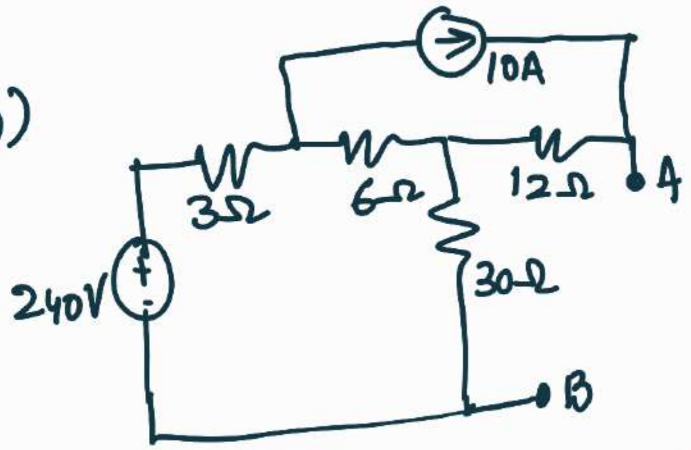
$$-i_3 = \frac{v_x}{12 + \frac{\overset{10}{\cancel{30}} \times 9}{\underset{13}{\cancel{39}}}} = \frac{13 v_x}{246}$$

$$i_x = i_1 + i_2 + i_3 = 0$$

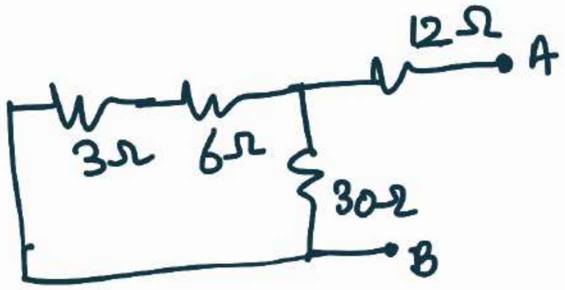
$$\Rightarrow \frac{13 v_x}{246} = \frac{400}{41} + \frac{360}{41} = \frac{760}{41}$$

$$\Rightarrow v_x = 350.77\,V$$

(b)

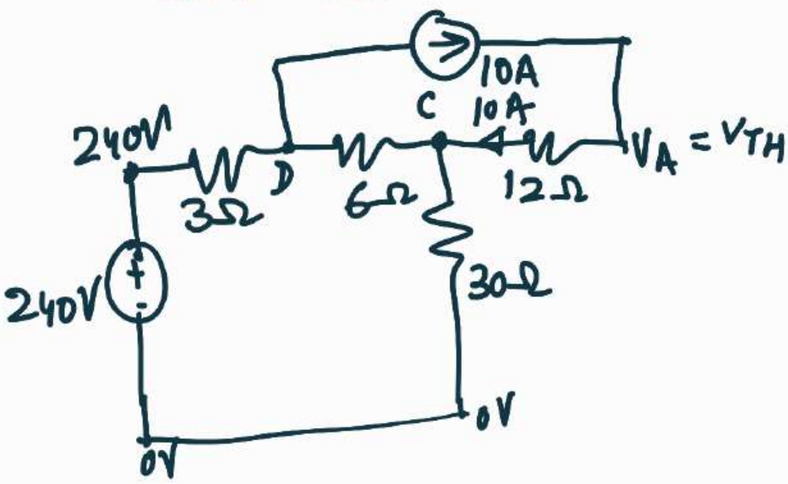


Let us first calculate thevenin resistance across A and B.



$$R_{TH} = 12 + \frac{\overset{10}{\cancel{30}} \times 9}{\underset{13}{\cancel{39}}} = \frac{246}{13}\,\Omega$$

Now let us calculate $V_A - V_B$. Take $V_B = 0V$



$$V_C = V_{TH} - 120$$

$$\frac{240 - V_D}{3} = 10 + \frac{V_D - V_C}{6}$$

$$\Rightarrow 80 - \frac{V_D}{3} = 10 + \frac{V_D}{6} - \frac{1}{6}(V_{TH} - 120)$$

$$\Rightarrow 70 = \frac{V_D}{2} - \frac{1}{6} V_{TH} + 20$$

$$\Rightarrow V_D/2 = \frac{1}{6} V_{TH} + 50$$

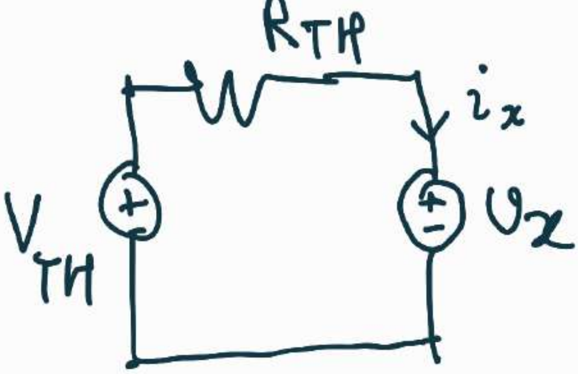
$$\Rightarrow V_D = \frac{V_{TH}}{3} + 100$$

$$\frac{V_D - V_C}{6} + 10 + \frac{0 - V_C}{30} = 0$$

$$\Rightarrow \frac{\frac{V_{TH}}{3} + 100 - V_{TH} + 120}{6} + 10 - \frac{V_{TH} - 120}{30} = 0$$

$$\Rightarrow -\frac{V_{TH}}{9} + \frac{220}{6} + 10 - \frac{V_{TH}}{30} + 4 = 0$$

$$\Rightarrow \frac{13V_{TH}}{90} = 14 + \frac{110}{3} \Rightarrow V_{TH} = \frac{4560}{13}\,V$$



$$i_x = \frac{V_{TH} - v_x}{R_{TH}}$$

Power delivered to this source $P = i_x v_x$

$$= \left(\frac{V_{TH} - v_x}{R_{TH}}\right) v_x$$

$$\frac{(V_{TH} - v_x) + (v_x)}{2} \geq \sqrt{(V_{TH} - v_x)(v_x)}$$

$$\Rightarrow (V_{TH} - v_x)(v_x) \leq \frac{V_{TH}^2}{4}$$

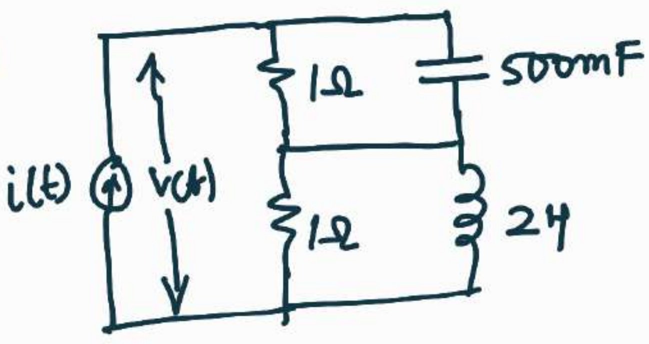
$$\Rightarrow P \leq \frac{V_{TH}^2}{4R_{TH}}, \; P_{max} \text{ when } v_x = V_{TH} - v_x$$

$$\Rightarrow V_x = \frac{V_{TH}}{2}$$

$$\therefore v_x = 175.38\,V$$

$$P_{max} = 1625.52\,W$$

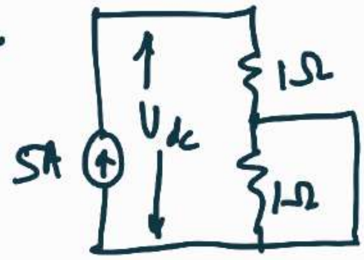
2) a)



$$i(t) = 10\sin(10t) + 5$$

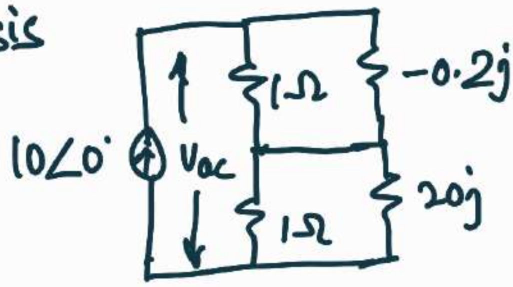
$$= i_{ac} + i_{dc}$$

dc analysis →



$v_{dc} = 5 \times 1 = 5V$

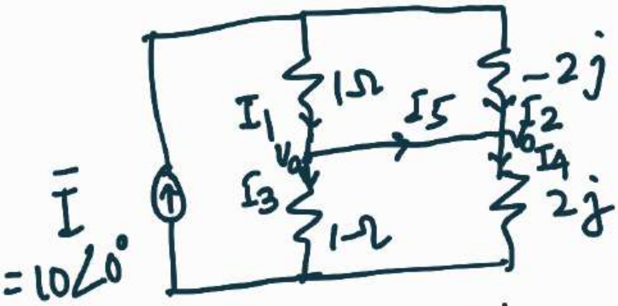
ac analysis



$$V_{ac} = i_{ac} \cdot Z$$
$$= 10\left[\frac{1\times(-0.2j)}{1-0.2j} + \frac{1\times 20j}{1+20j}\right]$$
$$= 10.457\angle -7.83^\circ$$

$\therefore$ $v(t) = 5 + 10.457 \sin(10t - 0.1367)$ V

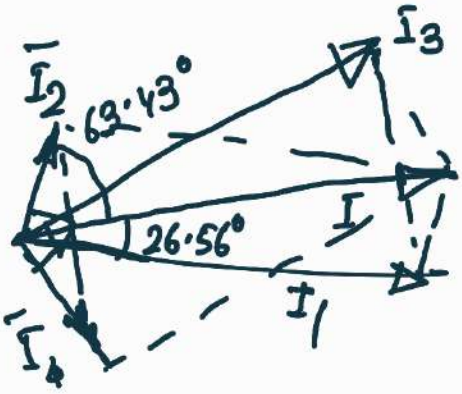
(b)

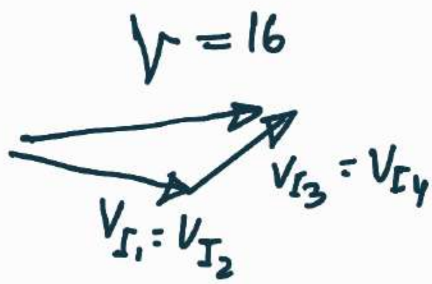


$$I_1 \times 1 = I_2 \times (-2j)$$
$$\Rightarrow \frac{I_1}{I_2} = \frac{-2j}{1}$$
$$\Rightarrow \bar{I}_1 = \frac{-2j}{1-2j}\bar{I}$$
$$= 8-4j = 8.94\angle -26.56^\circ$$
$$\bar{I}_2 = 2+4j = 4.47\angle 63.43^\circ$$

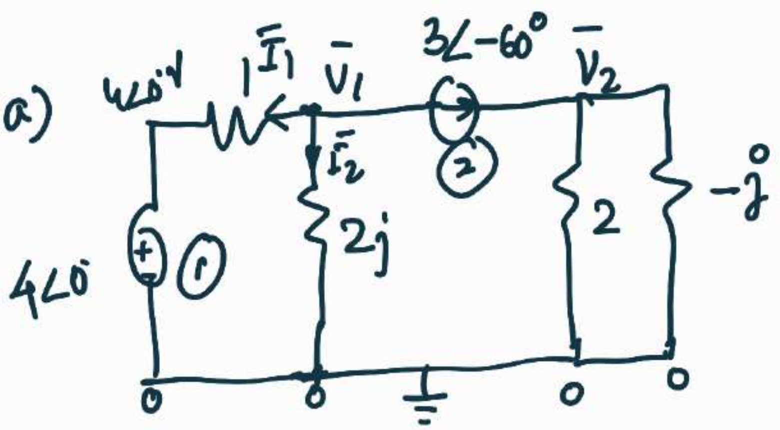
$$I_3 \times 1 = I_4 \times 2j$$
$$\Rightarrow \frac{I_3}{I_4} = \frac{2j}{1} \Rightarrow \bar{I}_3 = \frac{2j}{1+2j}\times 10$$
$$= 8+4j = 8.94\angle 26.56^\circ$$
$$\bar{I}_4 = 2-4j = 4.47\angle -63.43^\circ$$

$$\bar{I}_1 = \bar{I}_3 + \bar{I}_5 \Rightarrow \cancel{8}-4j = \cancel{8}+4j + \bar{I}_5 \Rightarrow \bar{I}_5 = -8j$$
$$= 8\angle -90^\circ$$





3) a)



$$\bar{I}_1 + \bar{I}_2 + 3\angle -60^\circ = \bar{0}$$

$$\Rightarrow \frac{\bar{V}_1 - 4}{1} + \frac{\bar{V}_1 - 0}{2j} = -3\angle -60^\circ$$

$$\Rightarrow \bar{V}_1(1 - 0.5j) = 4 - 3\angle -60^\circ \Rightarrow \bar{V}_1 = \frac{4 - 3\angle -60^\circ}{1 - 0.5j}$$

$$\bar{V}_1 = 3.225\angle 72.67^\circ$$

$$3\angle -60^\circ = \frac{\bar{V}_2 - 0}{2} + \frac{\bar{V}_2 - 0}{-j} \Rightarrow (0.5 + j)\bar{V}_2 = 3\angle -60^\circ$$

$$\Rightarrow \bar{V}_2 = 2.68\angle -123.43^\circ$$

Complex power delivered by source 1,

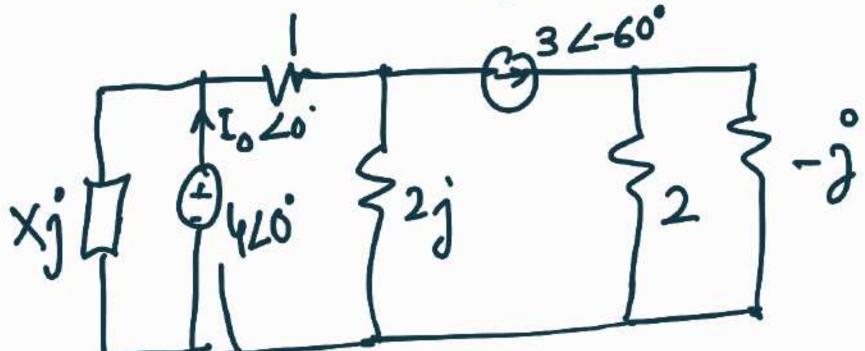
$$\bar{S}_1 = (4\angle 0^\circ) \times \left(\frac{4\angle 0^\circ - 3.225\angle 72.67^\circ}{1}\right)^*$$

$$= 17.30\angle 45.37^\circ \text{ W}$$

Complex power delivered by source 2,

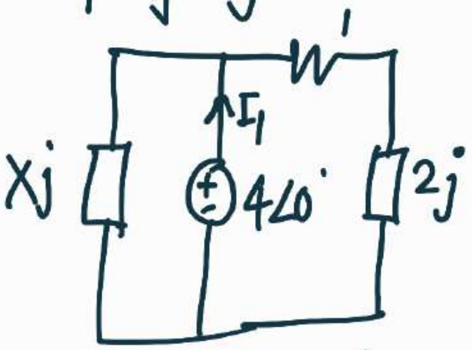
$$\bar{S}_2 = (2.68\angle -123.43^\circ - 3.225\angle 72.67^\circ) \times (3\angle -60^\circ)^*$$

$$= 17.542\angle -54.63^\circ \text{ W}$$

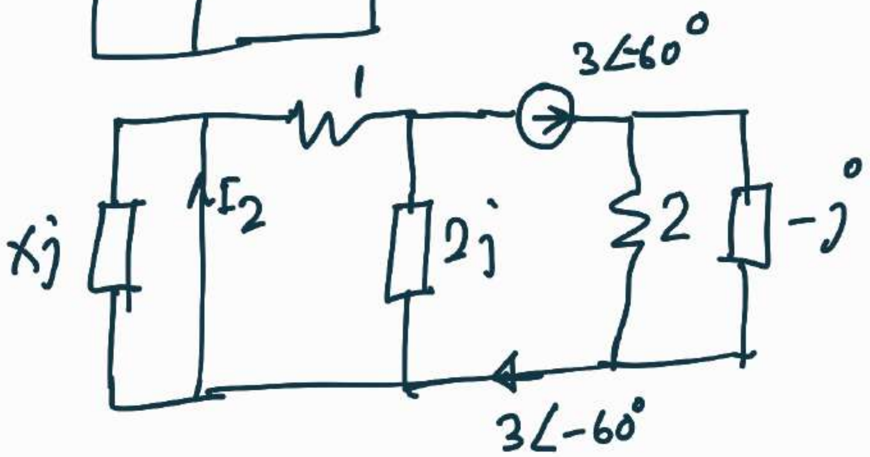


$$Q = \text{Im}(\bar{S}) = 0 \Rightarrow \theta_v = \theta_i$$

Applying method of superposition



$$I_1 = \frac{4\angle 0^\circ \,(1+(X+2)j)}{(1+2j)\,Xj}$$



$$I_2 = \frac{2j}{1+2j}\, 3\angle -60^\circ$$

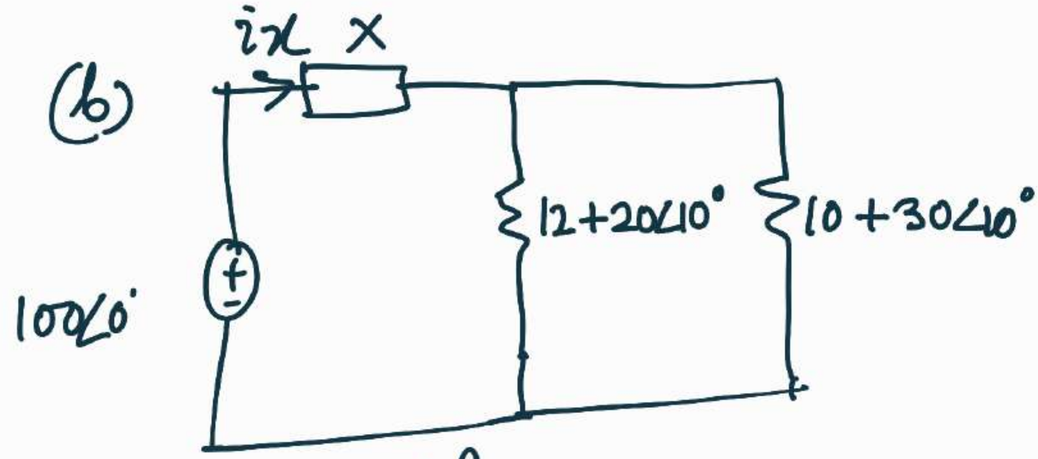
$$\operatorname{Im}\left[\frac{4\,(1+(X+2)j)}{X(-2+j)} + \frac{2j}{1+2j}\, 3\angle -60^\circ\right] = 0$$

$$\Rightarrow \operatorname{Im}\left[\frac{(-1.6-0.8j)(1+(X+2)j)}{X}\right] = 1.478461$$

$$\Rightarrow \frac{-1.6(X+2) - 0.8}{X} = 1.478461$$

$$\Rightarrow \boxed{X = -1.3\ \Omega}$$

(b)

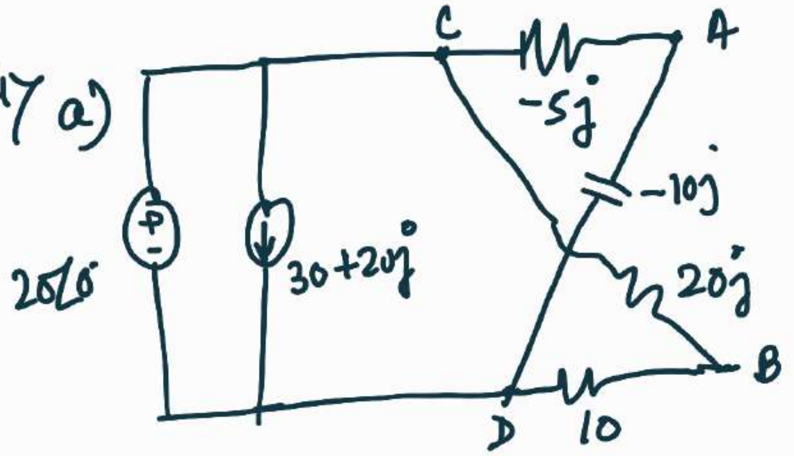


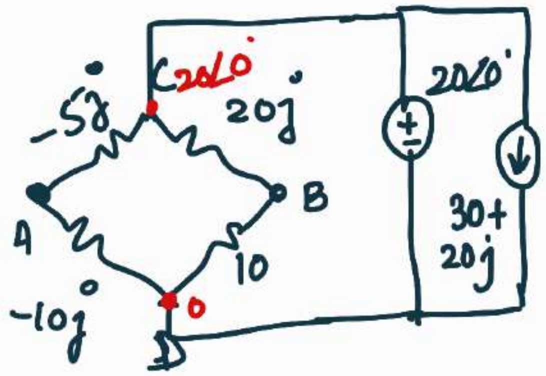
$$i_x = \frac{V}{X + \frac{(12+20\angle 10^\circ)(10+30\angle 10^\circ)}{22+50\angle 10^\circ}}$$

$$\Rightarrow i_x = \frac{V}{X + 17.596 + 2.101j}$$

$$\therefore \boxed{X = -2.101j}$$

4) a)

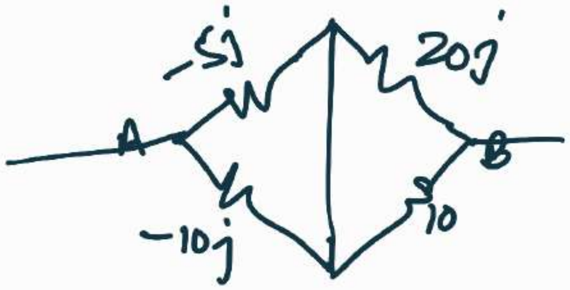




$$V_A = \frac{(-10j)\times 20}{-15j} = \frac{40}{3}\angle 0°$$

$$V_B = \frac{10\times 20}{10+20j} = 4-8j$$

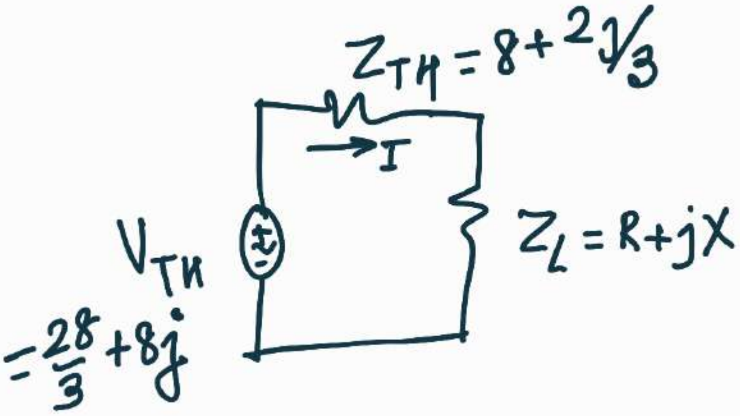
$$\therefore \bar{V}_{TH} = \frac{40}{3} - 4 + 8j = \boxed{\frac{28}{3} + 8j \ \ V}$$



$$Z_{TH} = \frac{(-5j)(-10j)}{(-15j)} + \frac{(20j)(10)}{10+20j}$$

$$= \frac{20j}{1+2j} - \frac{10j}{3}$$

$$= \boxed{8 + \frac{2j}{3}}$$



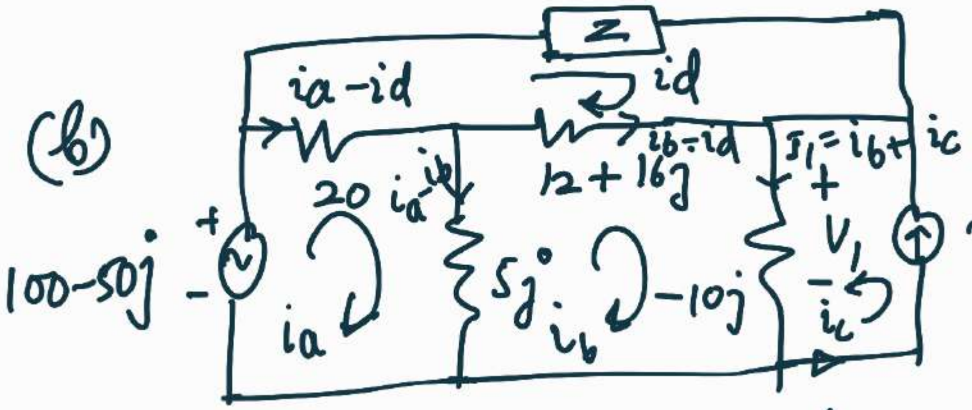
For $(P_L)_{max}$, $\boxed{Z_L = Z_{TH}^*}$

$$\Rightarrow R = 8\ \Omega,\ X = -\frac{2}{3}\ \Omega$$

$$P_{max} = Re\left[\frac{Z_L}{Z_{TH}+Z_L} V_{TH} \cdot \left(\frac{V_{TH}}{Z_{TH}+Z_L}\right)^*\right] = \frac{|V_{TH}|^2}{|Z_{TH}+Z_L|^2} Re(Z_L)$$

$$= \frac{\left(\frac{28}{3}\right)^2 + 8^2}{16^2} \times 8 = 4.72\,W$$

(b)



$$i_c = 30+20j$$

$$I_1 = \frac{140+30j}{-10j} = -3+14j = i_b + i_c$$

$$\Rightarrow i_b = -3+14j - 30 - 20j$$
$$= -33-6j$$

KVL in mesh (b)

$$+5j(i_a+33+6j) - (12+16j)(-33-6j-i_d) - 140-30j = 0$$

$$\Rightarrow 5j\,i_a + (12+16j)\,i_d = 140+30j - (12+16j)(33+6j) - 5j(33+6j)$$

$$5j\,i_a + (12+16j)\,i_d = -130-735j \quad \text{—(i)}$$

KVL in mesh (a),

$$100-50j - 20(i_a - i_d) - 5j(i_a+33+6j) = 0$$

$$\Rightarrow (-20-5j)\,i_a + 20\,i_d = 5j(33+6j) - 100 + 50j$$

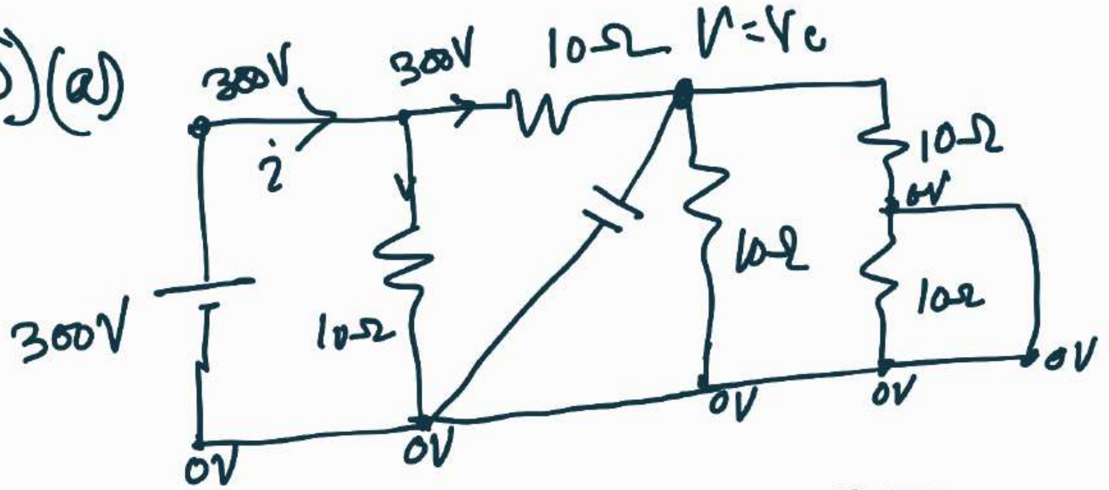
$$\Rightarrow (-20-5j)\,i_a + 20\,i_d = -130+215j \quad \text{—(ii)}$$

$$i_d = \frac{\begin{vmatrix} 5j & -130-735j \\ -20-5j & -130+215j \end{vmatrix}}{\begin{vmatrix} 5j & 12+16j \\ -20-5j & 20 \end{vmatrix}} = -30-10j$$

$$(100-50j)-(140+30j) = (i_d)\times Z$$

$$\Rightarrow Z = \frac{-40-80j}{-30-10j} = 2+2j$$

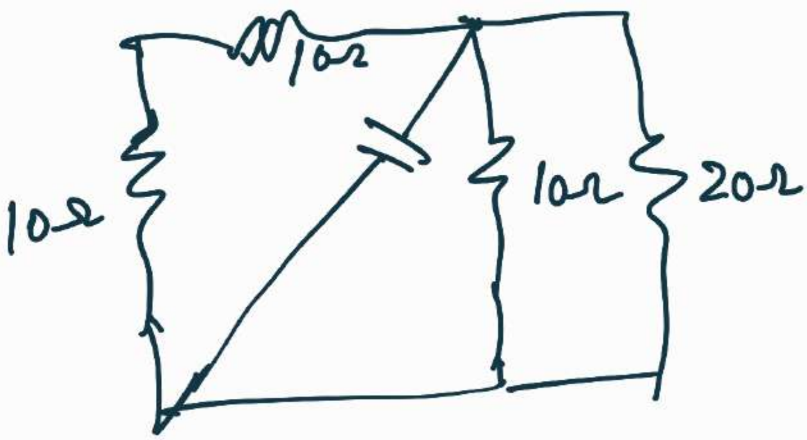
5)(a)



$$i = \frac{300}{10\,\|\,(10+5)} = \frac{300}{10\,\|\,15} = \frac{300\times 25}{10\times 15} = 50\text{ A}$$

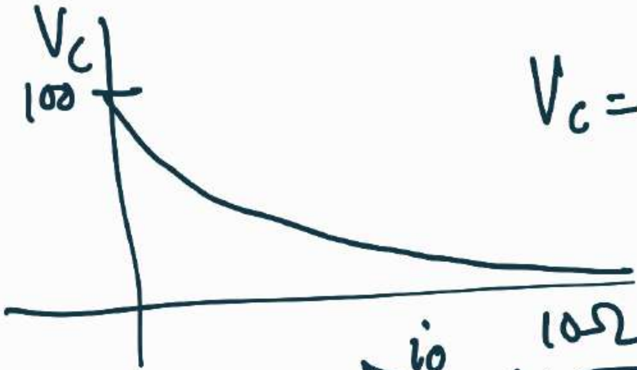
$$50 = \frac{300-0}{10} + \frac{300-v}{10}$$

$$\Rightarrow 50 = 30 + 30 - \frac{v}{10} \Rightarrow \frac{v}{10} = 10 \Rightarrow (v_c)_{t=0} = 100\text{ V}$$



$$R_{eq} = 20\,\|\,20\,\|\,10 = 5\,\Omega$$

$$\tau = RC = 5\text{ s}$$

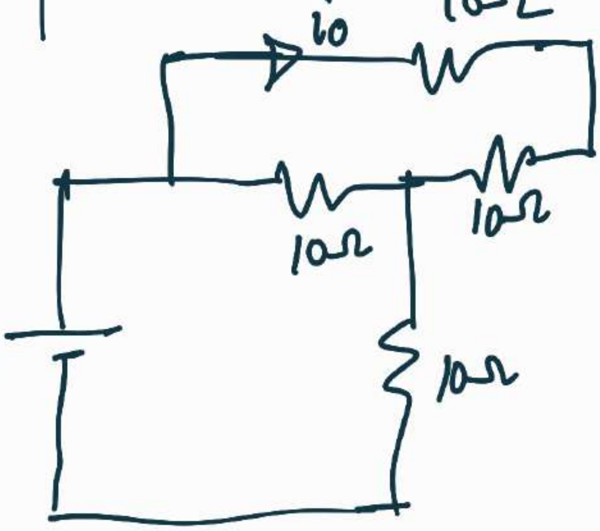


$$V_c = 100\,e^{-t/5}$$

$$V_c(t=1s) = 100\,e^{-1/5}$$

$$= \boxed{81.87\,V}$$

(b)

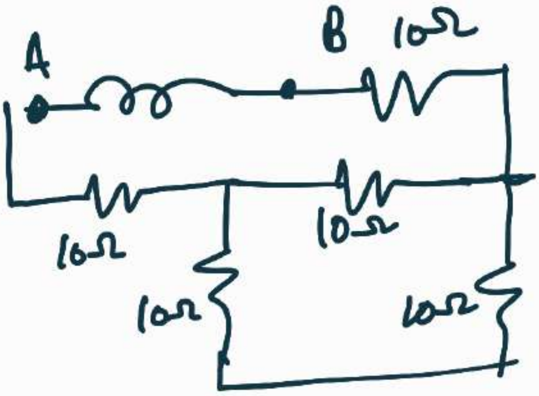


$$i_0 = \frac{10}{10+20} \times \frac{100}{10 + \frac{10\times 20}{30}}$$

$$= \frac{1\cancel{0}}{3\cancel{0}} + \frac{100}{10+\frac{20}{3}}$$

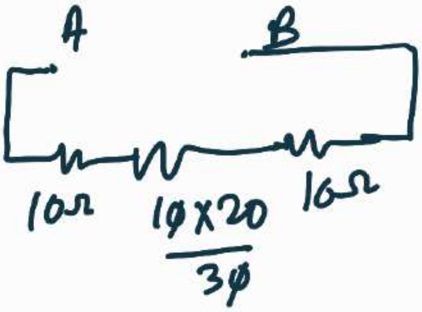
$$= \frac{100}{30+20} = \frac{100}{50}$$

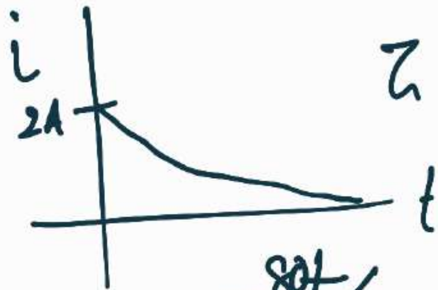
$$= 2A$$



$$R_{TH} = 20 + \frac{20}{3} = \frac{80}{3}\,\Omega$$

$$\tau = \frac{L}{R} = \frac{1\times 3}{80}$$





$$i = 2\,e^{-80t/3}\ A$$

$$i(t=0.01) = 2\,e^{-\frac{80\times 0.01}{3}}\ A$$

$$= \boxed{1.532\,A}$$