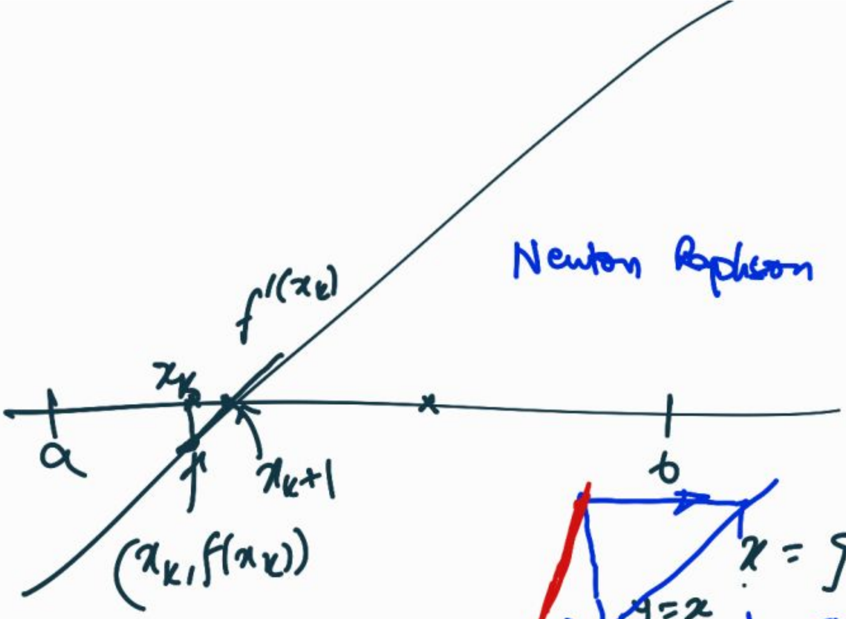
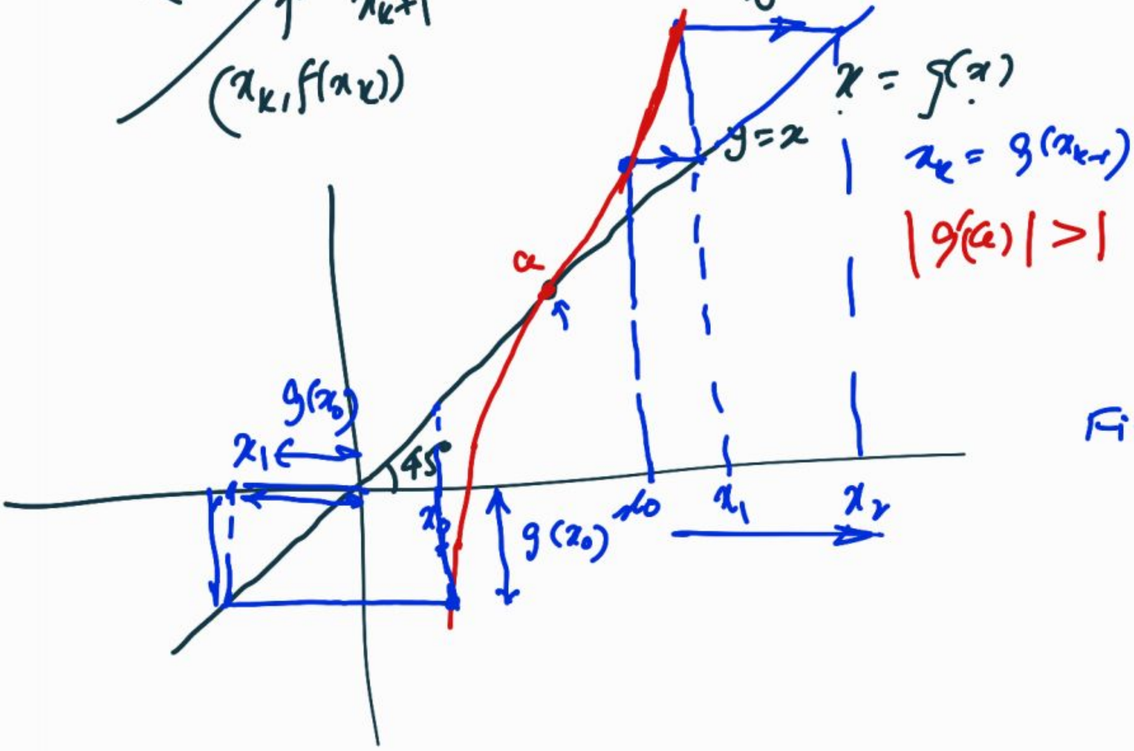
$$f'(x_k) = \frac{f(x_k) - 0}{x_k - x_{k+1}}$$

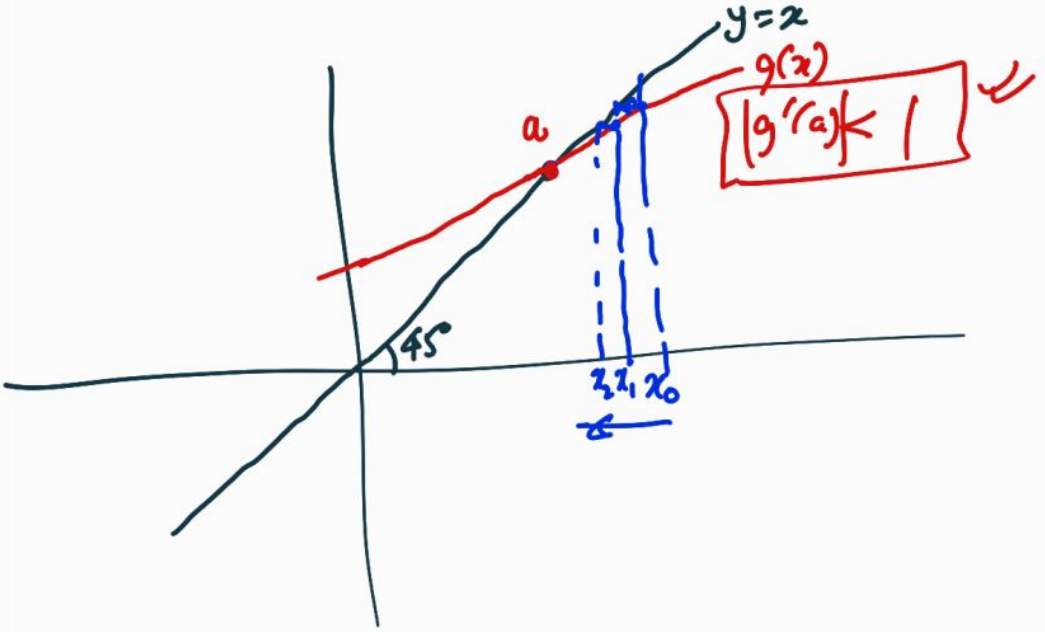
$$\Rightarrow x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

Newton Raphson





Fixed pt. it.



1) $f(x) = -x^4 + x^2 + A$, $f'(x) = -4x^3 + 2x$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - \left(\frac{-x_k^4 + x_k^2 + A}{-4x_k^3 + 2x_k} \right)$$

$$= \frac{-4x_k^4 + 2x_k^2 + x_k^4 - x_k^2 - A}{-4x_k^3 + 2x_k}$$

$$x_{k+1} = \frac{-3x_k^4 + x_k^2 - A}{-4x_k^3 + 2x_k}$$

$$k=0, \quad x_1 = -x_0 = \frac{-3x_0^4 + x_0^2 - A}{-4x_0^3 + 2x_0}$$

$$\Rightarrow 4x_0^4 - 2x_0^2 = -3x_0^4 + x_0^2 - A$$

$$\Rightarrow A = -7x_0^4 + 3x_0^2$$

$$= -7\left(\tfrac{1}{3}\right)^4 + 3\left(\tfrac{1}{3}\right)^2 = \frac{20}{81}$$

27 $\quad f(x) = x^5 - x^3 + 2x^2 - 1 = 0$

$x_0 = 1, \quad f'(x) = 5x^4 - 3x^2 + 4x$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

n	0	1	2
x	1	$\frac{5}{6}$	

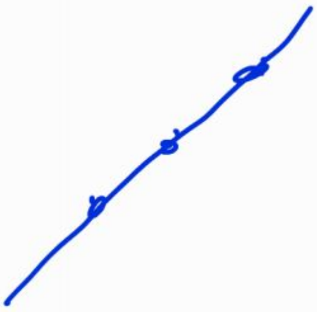
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1}{6}$$

$f(x) = x^2 - N, \quad f(\sqrt{N}) = 0$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^2 - N}{2x_k}$$

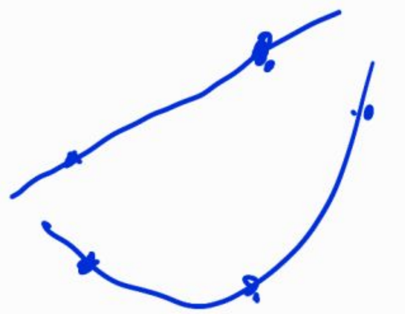
$$x_{k+1} = \frac{x_k^2 + N}{2x_k} = \frac{x_k + \frac{N}{x_k}}{2}$$

$N = 2, \quad x_0 = 1.5, \quad x_1 = \dfrac{1.5 + \frac{2}{1.5}}{2} = 1.4167$



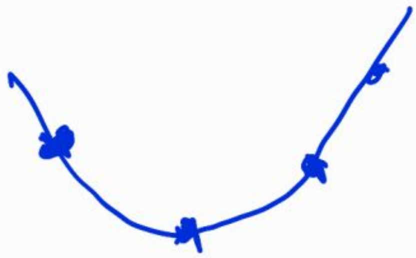
Interpolation $\longrightarrow p(x)$

- Lagrange
- Newton forward
- Newton backward



$x_0 \quad x_1 \quad x_2$

$x_0 ; x_1, x_2, x_3$



$$p(x) = a(x-x_0)(x-x_1)(x-x_2) + b(x-x_0)(x-x_1)(x-x_3) + c(x-x_0)(x-x_2)(x-x_3) + d(x-x_1)(x-x_2)(x-x_3)$$

$$p(x_0) = d(x_0-x_1)(x_0-x_2)(x_0-x_3) = f(x_0)$$

$$\Rightarrow d = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$c = \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$b = \cdots, \quad a = \cdots$

x	-4	-2	0	2	4	6
$f(x)$	-139	-21	1	23	141	451

$$p(x) = \frac{(x+4)(x+2)(x)(x-2)(x-4)\,f(6)}{(6+4)(6+2)(6)(6-2)(6-4)} + \frac{(x+4)(x+2)(x)(x-2)(x-6)\,f(4)}{(4+4)(4+2)(4)(4-2)(4-6)} + \cdots$$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
-4 x_0	$\boxed{-139}$	$\boxed{118}$	$\boxed{-96}$	$\boxed{96}$	$\boxed{0}$	$\boxed{0}$
-2 x_1	-21	22	0	96	0	x
0 x_2	1	22	96	96	x	x
2 x_3	23	118	192	x	x	x
4 x_4	141	310	x	x	x	0
6 x_5	451	x	x	x		

$x_0 \to$; d ; d ; d

$$f'(x) = \frac{f(x+h)-f(x)}{h} = \frac{f(x_1)-f(x_0)}{(x_1-x_0)}$$

$$\Delta f(x_i) = f(x_{i+1}) - f(x_i)$$

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \dots$$

$$f(x_0+h) = f(x_0) + (x-x_0)\frac{\Delta f(x_0)}{d}$$

$x_0 + h \to x$

$x = x_0 + h$

$$+ (x-x_0)(x-x_1)\frac{\Delta^2 f(x_0)}{2!\, d^2}$$

$$+ (x-x_0)(x-x_1)(x-x_2)\frac{\Delta^3 f(x_0)}{3!\, d^3}$$

$$+ \dots + (x-x_0)(x-x_1)(x-x_2)\dots(x-x_4)\frac{\Delta^5 f(x_0)}{5!\, d^5}$$

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	∇^3	∇^4	∇^5
-4 x_0	-139	x	x	x		
-2 x_1	-21	118	x	x		
0 x_2	1	22	-96			
2 x_3	23	22	0	96		
4 x_4	141	118	96	96	0	
6 x_5	$\boxed{451}$	$\boxed{310}$	$\boxed{192}$	$\boxed{96}$	$\boxed{0}$	$\boxed{0}$

$$\nabla f(x_i) = f(x_i) - f(x_{i-1})$$

$$f(x) = f(x_5) + (x - x_5)\frac{\nabla f(x_5)}{1!\,d}$$
$$+ (x - x_5)(x - x_4)\frac{\nabla^2 f(x_5)}{2!\,d^2}$$
$$+ \cdots + (x - x_5)(x - x_4)(x - x_3)(x - x_2)(x - x_1)\frac{\nabla^5 f(x_5)}{5!\,d^5}$$

---

5) $$\Delta \ln f(x) = \ln\left(1 + \frac{\Delta f(x)}{f(x)}\right)$$

$$\Delta \ln f(x_i) = \ln f(x_{i+1}) - \ln f(x_i)$$
$$= \ln\left(\frac{f(x_{i+1})}{f(x_i)}\right)$$
$$= \ln\left(\frac{f(x_i) + \Delta f(x_i)}{f(x_i)}\right)$$

$$\Delta \ln f(x) = \ln\left(1 + \frac{\Delta f(x)}{f(x)}\right)$$

$$f(x) = \underbrace{f(x_0) + (x - x_0)\frac{\Delta f(x_0)}{1!\,d}}_{p(x)} + (x - x_0)(x - x_1)\frac{\Delta^2 f(c)}{2!\,d^2}$$

$$\therefore\ E = (x - x_0)(x - x_1)\frac{\Delta^2 f(c)}{2!\,d^2}$$

$$\cancel{(x - x_0)(x}$$

$$\left(\frac{x_0 + x_1}{2} - x_0\right)\left(\frac{x_0 + x_1}{2} - x_1\right)$$

$$= \left(\frac{x_1 - x_0}{2}\right)\left(-\frac{x_1 - x_0}{2}\right) = \left(\frac{d}{2}\right)\left(-\frac{d}{2}\right) = -\frac{d^2}{4}$$

$$|E| \leq \frac{d^2}{4} \times \frac{1}{2}\frac{\max|\Delta^2 f(x)|}{d^2}$$

$$= \frac{1}{8}\max|\Delta^2 f(x)|$$

8)

Marks	No. of Students
30-40	31
40-50	42
50-60	51
60-70	35
70-80	31

marks (x_i)	cf (No. of students getting marks $\le x_i$)
x_0 30	0
x_1 40	31
x_2 50	73
x_3 60	124
x_4 70	159
x_5 80	190

$f(45)$ ✓

x	$f(x)$	Δ	Δ^2	Δ^3	Δ^4	Δ^5
30	0	31	11	-2	-23	60
40	31	42	9	-25	37	
50	73	51	-16	12		
60	124	35	-4			
70	159	31				
80	190					

$$f(x) = f(30) + (x-30)\frac{\Delta f(30)}{1!\ 10} + (x-30)(x-40)\frac{\Delta^2 f(30)}{2!\ 10^2} + \dots$$

$$f(45) = 15\times\frac{31}{10} + \frac{15\times 5\times 11}{2\times 10^2} + \frac{15\times 5\times(-5)\times(-2)}{3!\ 10^3}$$

$$+ 15\times 5\times(-5)\times(-15)\times\frac{(-23)}{4!\times 10^4}$$

$$+ 15\times 5\times(-5)\times(-15)\times(-25)\times\frac{60}{5!\times 10^5}$$

$$\simeq \underline{\underline{50.07}}$$