Indian Institute of Technology Kharagpur QUESTION-CUM-ANSWERSCRIPT		Stamp/Signature of the Invigilator
MID-SEMESTER EXAMINATION		SEMESTER (SPRING-2023)

Roll Number										Section		Name	
Subject Number	M	A	1	1	0	0	4		Subject Name			Linear Algebra, Numerical and Complex Analysis/Mathematics-II (MA10002)	
Department/Centre/School													

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones (smart watch) or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession; even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, the exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer-script and do not tear off any page. Report to the invigilator if the answer-script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the Examination Hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and consumption of any kind of beverages is strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer-script to the invigilator. **In any case, you are not allowed to take away the answer-script with you.** After the completion of the examination, do not leave your seat until invigilators collect all the answer-scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as '**unfair means**'. Don't adopt unfair means and also don't indulge in unseemly behavior.
11. **Please see overleaf for more instructions.**

Violation of any of the above instructions may lead to severe punishment.

Signature of the Student

To be Filled by the Examiner

Question Number	1	2	3	4	5			Total
Marks Obtained								

Marks Obtained (in words)	Signature of the Examiner	Signature of the Scrutinizer

Instructions and Guidelines to the Students

1. No queries will be entertained during the examination.
2. Examination is of two hours duration.
3. The question-cum-answer booklet has 24 pages and 5 questions with a total of 30 marks.
4. All the questions are compulsory.
5. **Answer each question in the space provided below to that question only. Otherwise it will not be evaluated.**
6. **Use supplementary sheet for rough work. Any answer written in the supplementary sheet will not be evaluated.**
7. **Notations:** $\mathbb{R}$ denotes the set of all real numbers.

1. Let $\mathbb{P}_2(\mathbb{R})$ be the vector space of all polynomials with real coefficients and degree at most 2, with usual polynomial addition and scalar multiplication.

   (a) Which of the following subsets of $\mathbb{P}_2(\mathbb{R})$ are subspaces? Justify your answer.

   (i) $W_1 = \{a_0 + a_1 x + a_2 x^2 : a_0 \geq 0, a_1 \geq 0, a_2 \geq 0\}$,

   (ii) $W_2 = \{a_0 + 2a_1 x + 3a_2 x^2 : a_0, a_1, a_2 \in \mathbb{R} \text{ and } a_0 + a_1 + a_2 = 0\}$.

   (b) Let $V_1 = V_2 = \mathbb{P}_2(\mathbb{R})$. Let $T : V_1 \to V_2$ be a linear transformation defined by $T(a + bx + cx^2) = b + 2cx + ax^2$. What is the matrix representation of T with respect to the ordered bases B_1, B_2 of V_1, V_2 respectively, where $B_1 = \{1 + x, 1 + x^2, 2\}$, $B_2 = \{1 + x + x^2, 1 - x + x^2, 2\}$?

[3 + 3 Marks]

2. (a) Determine whether the polynomial $p(t) = t^2 + 4t - 3$ over $\mathbb{R}$ can be written as a linear combination of the polynomials $p_1(t) = t^2 - 2t + 5$, $p_2(t) = 2t^2 - 3t$, and $p_3(t) = t + 3$.

(b) Let A be a 2×2 matrix with real entries. Suppose $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector of A corresponding to the eigenvalue 3 and that $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is an eigenvector of A corresponding to the eigenvalue (-2). Then find $A^2 \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

[3 + 3 Marks]

3. (a) Let $\{(2,3,-1),(9,8,0),(4,-5,a+b)\}$ and $\{(2,3,-1),(9,8,0),(1,-15,a+3b)\}$ be two linearly dependent subsets of $\mathbb{R}^3$. Then find the values of a and b.

(b) Using elementary row operations, find for what values of λ and μ, the following system of linear equations

$$
\begin{aligned}
2x + 3y + 5z &= 9 \\
7x + 3y - 5z &= 8 \\
2x + 3y + \lambda z &= \mu
\end{aligned}
$$

has (i) no solution, (ii) a unique solution, (iii) an infinite number of solutions.

[3 + 3 Marks]

4. (a) Let $U = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + 3z = 0\}$ and $V = \{(x, y, z) \in \mathbb{R}^3 : 2x + 3y + 4z = 0\}$ be two subspaces of $\mathbb{R}^3$. Find a basis and the dimension of the subspace $U \cap V$.

(b) Find the value(s) of α for which the following homogeneous system of equations

$$2x + 3y + z = 0; \;\; 5x + 7y + 2z = 0; \;\; 6x + 2y + \alpha z = 0$$

has infinitely many solutions. With the calculated value(s) of α, hence find the dimension of the solution space.

[3 + 3 Marks]

5. (a) Using Gaussian Elimination method find the solution of the following system of linear equations:

$$3y + z = -9$$

$$4x + 2y + z = 0$$

$$16x + 3y + z = -1$$

(b) Let $\mathbb{M}_{2\times 2}(\mathbb{R})$ be the vector space of all 2×2 real matrices with usual matrix addition and scalar multiplication and let a linear transformation $T : \mathbb{M}_{2\times 2}(\mathbb{R}) \to \mathbb{R}^2$ be given by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a + b,\ c + d).$$

Find its kernel and range. Hence verify the statement of rank-nullity theorem.

[3 + 3 Marks]