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$$\frac{3x^2 + x + 1}{x^3 - 6x^2 + 11x - 6}$$

$$(x-1)(x^2 - 5x + 6)$$

$$= (x-1)(x-2)(x-3)$$

$$f(x) = 3x^2 + x + 1 \quad \underline{1, 2, 3}$$

$$= \frac{(x-1)(x-2)f(3)}{(3-1)(3-2)} + \frac{(x-1)(x-3)f(2)}{(2-1)(2-3)} + \frac{(x-2)(x-3)f(1)}{(1-2)(1-3)}$$

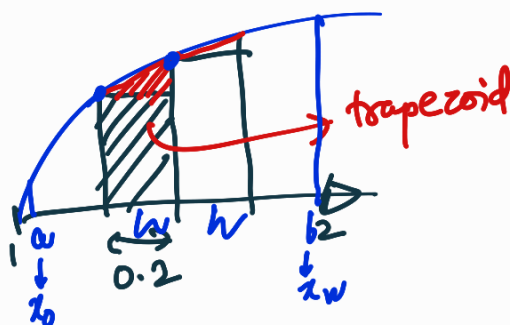
$$= \frac{3}{2}(x-1)(x-2) - 15(x-1)(x-3) + \frac{5}{2}(x-2)(x-3)$$

$$\frac{3}{8} \text{ pt. } y_0 = \frac{1}{2}(y_1 + y_2) - \frac{1}{8} \left[ \frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_1 - y_3) \right]$$

$$y_2 = \frac{1}{(-1)(-2)(-3)} y_{-3} + \frac{(x-1)(x+1)(x+3)}{2 \cdot 4 \cdot 6} y_3 + \frac{(x-1)(x-3)(x+3)}{(-2) \cdot (-4) \cdot (2)} y_1 + \frac{(x+1)(x-3)(x+2)}{2 \cdot (-2) \cdot 4} y_1$$

$$y_0 = -\frac{3}{16} y_{-3} + \dots$$

Numerical Integration

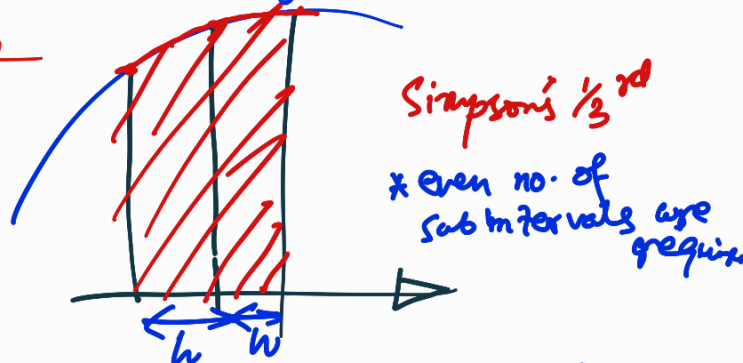


$$\int_a^b f(x) dx = \frac{h}{2} (f(a) + f(b)) + h (f(x_1) + f(x_2) + \dots + f(x_{n-1}))$$

$$|E| \leq \frac{b-a}{12} h^2 \max_{a \leq x \leq b} |f''(x)|$$

$$a = x_0 < x_1 < x_2 < \dots < x_n = b$$

$$b = a + nh$$



Simpson's  $\frac{1}{3}$  rule

\* even no. of sub intervals are required

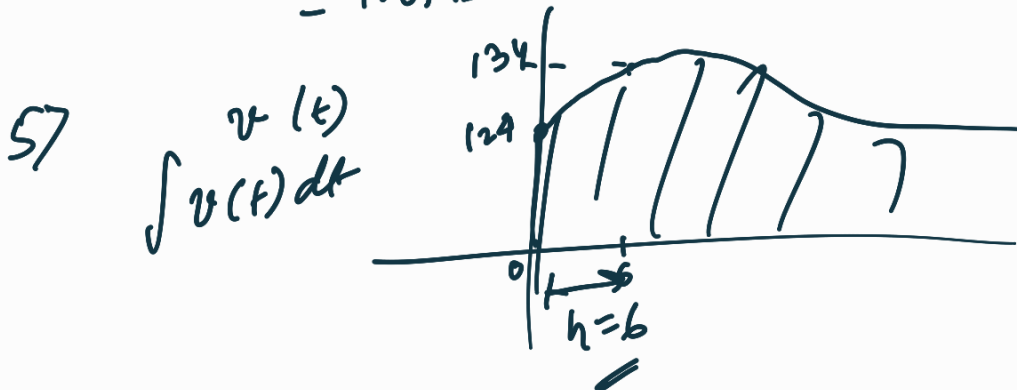
$$\int_a^b f(x) dx = \frac{h}{3} (f(a) + f(b)) + \frac{2h}{3} (f(a+2d) + f(a+4d) + \dots) + \frac{4h}{3} (f(a+d) + f(a+3d) + \dots)$$

$$|E| \leq \frac{b-a}{180} h^4 \max_{a \leq x \leq b} |f^{(4)}(x)|$$

$$4) \int_{0.2}^{1.4} (\sin x - \ln x + e^x) dx$$

$$h = \frac{b-a}{6} \Rightarrow h = \frac{1.2}{6} = 0.2$$

$$I_{\text{trap}} = 0.1 (f(0.2) + f(1.4)) + 0.2 (f(0.4) + f(0.6) + f(0.8) + f(1.0) + f(1.2)) = 4.07151845$$



$$6) |E|_{\text{trap}} \leq \frac{b-a}{12} h^2 \max_{a \leq x \leq b} |f^{(2)}(x)|$$

$$\int_0^1 e^x dx \quad f(x) = e^x, \quad f^{(2)}(x) = e^x$$

$$10^{-5} = \frac{1}{12} h^2 e \Rightarrow h \approx 0.018061$$

$$7) f(x) = ax^2 + bx + c$$

$$\int_{-h}^h f(x) dx = \frac{h}{3} (f(-h) + f(h)) + \frac{4h}{3} (f(0))$$



$$= \frac{h}{3} (2ah^2 + 2c) + \frac{4h}{3} (c)$$

$$= \frac{2a}{3} h^3 + 2hc$$

$$\int_{-h}^h (ax^2 + bx + c) dx = 2 \int_0^h (ax^2 + c) dx = 2 \left( \frac{ah^3}{3} + ch \right)$$

8)

$$\int_4^{5.2} \ln x \, dx$$

$$\frac{4, 4.2, 4.4, 4.6, 4.8, 5.0, 5.2}{\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow}$$

$$5.2 = 4 + 6h \Rightarrow h = 0.2$$

$$\frac{0.2}{3} (f(4) + f(5.2)) + \frac{0.4}{3} (f(4.4) + f(4.8)) + \frac{0.8}{3} (f(4.2) + f(4.6) + f(5.0))$$

$$= 1.82784725$$

9)

$$|E|_{\text{trap}} \leq \frac{b-a}{12} h^2 \max_{a \leq x \leq b} |f^{(2)}(x)|$$

$$|E|_{\text{trap}} \leq \frac{b-a}{180} h^4 \max_{a \leq x \leq b} |f^{(4)}(x)|$$

$$a=0, b=2, f(x) = \frac{1}{x+4}$$

$$f'(x) = -\frac{1}{(x+4)^2}, f^{(2)}(x) = \frac{2}{(x+4)^3}$$

$$f^{(3)} = -\frac{6}{(x+4)^4}, f^{(4)} = \frac{24}{(x+4)^5}$$

$$b = a + nh$$

$$10^{-4} = \frac{2}{12} h^2 \frac{2}{4^3} \Rightarrow h = \dots \Rightarrow n = \frac{b-a}{h} = 14.4337$$

$$10^{-4} = \frac{2}{180} h^4 \frac{24}{4^5} \Rightarrow h = \dots \Rightarrow n = \frac{b-a}{h} = 2.5408$$

$$\therefore n = 4$$