

# Module 12: Proportional Reasoning - Complete Notes

## What You'll Learn

In this module, you'll master **direct and inverse proportion** — understanding how quantities scale together or against each other.

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## Concept Explained (Like a YouTube Video)

### The Basics

#### Direct Proportion

When one increases, the other increases at the same rate.

More workers → More work done

Double input → Double output

$$y = k \times x \text{ (} k \text{ is the constant)}$$

#### Inverse Proportion

When one increases, the other decreases.

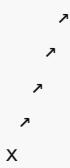
More workers → Less time needed

Double speed → Half the time

$$y = k / x \text{ (} k \text{ is the constant)}$$

### Visual

DIRECT:  $x$  increases,  $y$  increases



INVERSE:  $x$  increases,  $y$  decreases



## Programming Connection

### Code Examples

```
# Example 1: Direct Proportion

def calculate_direct(known_x, known_y, new_x):
    """If y is directly proportional to x, find new y"""
    k = known_y / known_x # constant of proportionality
    return k * new_x

# 5 items cost $20, how much for 8 items?
print(calculate_direct(5, 20, 8)) # 32.0

# 3 workers produce 150 units, how many for 7 workers?
print(calculate_direct(3, 150, 7)) # 350.0
```

```
# Example 2: Inverse Proportion

def calculate_inverse(known_x, known_y, new_x):
    """If y is inversely proportional to x, find new y"""
    k = known_x * known_y # constant
    return k / new_x

# 4 workers take 6 days, how long for 8 workers?
print(calculate_inverse(4, 6, 8)) # 3.0 days

# 3 pipes fill tank in 12 hours, how long for 6 pipes?
print(calculate_inverse(3, 12, 6)) # 6.0 hours
```

```
# Example 3: Scale Factor

def scale_dimensions(width, height, factor):
    """Scale dimensions by factor"""
    return (width * factor, height * factor)

def scale_to_width(orig_w, orig_h, new_w):
    """Scale proportionally to target width"""
    factor = new_w / orig_w
    return (new_w, orig_h * factor)

# Scale 800x600 to width 400 (maintain aspect ratio)
print(scale_to_width(800, 600, 400)) # (400, 300.0)
```

```
# Example 4: Estimate Using Proportion

def estimate_time(known_items, known_time, target_items):
    """Estimate time for different item count (direct proportion)"""
    return (known_time / known_items) * target_items

# 50 tests take 10 minutes, estimate for 200 tests
print(estimate_time(50, 10, 200)) # 40.0 minutes
```

## SDET/Testing Application

```
# SDET Scenario: Estimate Test Execution Time

def estimate_test_duration(benchmark_tests, benchmark_time, actual_tests):
    """Estimate total duration based on benchmark"""
    # Direct proportion: more tests = more time
    estimated = calculate_direct(benchmark_tests, benchmark_time, actual_tests)

    return {
        "benchmark": f"{benchmark_tests} tests in {benchmark_time} min",
        "actual_tests": actual_tests,
        "estimated_time": f"{estimated:.1f} min",
        "per_test": f"{benchmark_time/benchmark_tests:.2f} min"
    }

result = estimate_test_duration(50, 10, 175)
print(f"Estimated: {result['estimated_time']}") # 35.0 min
```

```
# SDET Scenario: Parallel Execution Time

def parallel_execution_time(tests, single_runner_time, num_runners):
    """Calculate time with parallel runners (inverse proportion)"""
    # Inverse: more runners = less time
    parallel_time = single_runner_time / num_runners

    return {
        "total_tests": tests,
        "single_runner": f"{single_runner_time} min",
        "num_runners": num_runners,
        "parallel_time": f"{parallel_time:.1f} min",
        "speedup": f"{num_runners}x faster"
    }

result = parallel_execution_time(100, 60, 4)
print(f"With 4 runners: {result['parallel_time']}") # 15.0 min
```

## Practice Problems

### Problem 1: Easy

**Challenge:** A car travels 150km in 2 hours. How far in 5 hours?

### Problem 2: Medium

**Challenge:** 6 painters finish a job in 4 days. With 8 painters, how many days?

### Problem 3: Application

**Scenario:** Your single test runner takes 120 minutes. Budget is 30 minutes.

**Challenge:** How many parallel runners do you need?

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## 🔑 Key Takeaways

- ✓ **Direct:** More input → More output (multiply)
  - ✓ **Inverse:** More input → Less of other (divide)
  - ✓ **Scale factor:** Maintain proportions while resizing
  - ✓ **Cross-multiply:**  $a/b = c/d \rightarrow a \times d = b \times c$
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 Save as: *Module\_12\_Proportional\_Reasoning.md*