Assignment 1.

1. Which of the following represents simple harmonic motion?

a) $y = Ae^{i\omega t}$

b) $y = A \ln \omega t$

→

a] let suppose 'x' be the displacement in place of 'y'

$\therefore$ eqn will be

$x = Ae^{i\omega t}$

diffn w.r.t 't'

$\frac{dx}{dt} = Ai\omega . e^{i\omega t}$

taking double derivative.

diff w.r.t 't'

$\frac{d^2x}{dt^2} = A . i\omega . i\omega e^{i\omega t}$

$= A . i^2 . \omega^2 . e^{i\omega t}$

$= -A . \omega^2 . e^{i\omega t}$

$\frac{d^2x}{dt^2} = -Ae^{i\omega t} . \omega^2$

$a = -x . \omega^2$

$\boxed{\therefore a = -\omega^2 . x}$

The following eqn represents the eqn of S.H.M

b] $x = A \ln \omega t$

diffn w.r.t 't'

$\frac{dx}{dt} = \frac{A \times 1 \times \omega}{\omega t} = \frac{A}{t}$

taking double derivative.
diffⁿ w.r.t 't'

$$\frac{d^2x}{dt^2} = \frac{-A}{t^2}$$

$\therefore x = A \sin \omega t$ this eqⁿ not represent the eqⁿ of S.H.M

2. Calculate the amplitude, angular frequency, frequency, time period and initial phase for the simple harmonic oscillation given below.

a] $y = 2\cos \pi t$

b] $y = 3\sin(2\pi t - 1.5)$

c] $y = 3\sin(20\pi - 1.1) + 3\cos(20\pi - 1.1)$

→ a] $y = 2\cos \pi t$

Amplitude is $A = 2$ unit.

Angular frequency $\omega = \pi$ rad/s

Frequency $f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2} = 0.5$ Hz

Time period $T = \frac{1}{f} = \frac{1}{0.5} = 2$ sec

Initial phase is $\phi = 0$ rad.

b] $y = 3\sin(2\pi t - 1.5)$

Amplitude is $A = 3$ units.

Angular frequency $\omega = 2\pi$ rad/sec

Frequency $f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1$ Hz

Time period $T = \frac{1}{} = \frac{1}{} = 1$ S

Initial phase is $\phi = 1.5$ rad.

c] $y = 3\sin(20\pi - 1.1) + 3\cos(20\pi - 1.1)$

Standard form $y = A\sin(\omega t + \phi)$

$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$

$y = 3\sin(20\pi - 1.1) + 3\cos(20\pi - 1.1)$

$= 3[\sin(20\pi)\cos(1.1) - \cos(20\pi)\sin(1.1)] + 3\cos(20\pi - 1.1)$

Now combine like term

$y = 3\sin(20\pi)\cos(1.1) + 3\cos(20\pi)[1 - \sin(1.1)]$

Amplitude (A) = 3 units.

Angular Frequency $\omega = 20\pi$ rad/s

Frequency $f = \frac{\omega}{2\pi} = \frac{20\pi}{2\pi} = 10$ Hz

Time period (T) $= \frac{1}{f} = \frac{1}{10} = 0.1$ sec

initial phase $\phi = -1.1$ rad

3. A nurse measured the heart beat of the patient in terms of number of beats measured per min in 75. What will be the average heart beats of a patient in terms of time period?

→ Heart $= 75/\text{min} = \frac{1}{75}$

$\therefore f = \frac{1}{75}$

$T = \frac{1}{f} = \frac{1}{1/75}$

$T = \frac{1}{f} = \frac{1}{1/75}$

1 min is 60 sec

$T = \frac{75}{60} = 1.25 \quad s^{-1} = 0.85$

∴ The average time period per heart beat is 0.8 sec.