Practical 3 -- Solutions

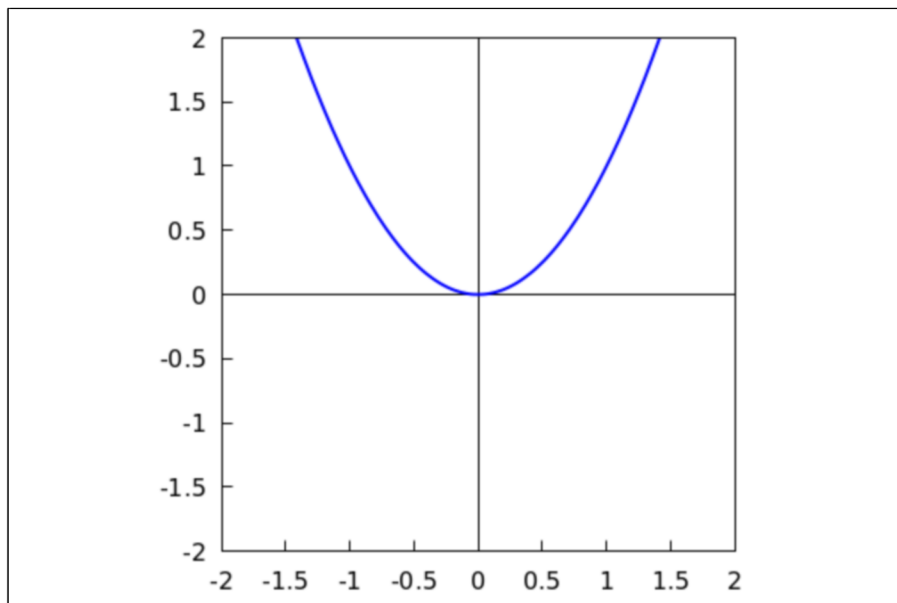## *1*

Rotate the parabola (t, t^2),
t in [-2, 2], by an angle pi/6.

➜   kill(all);

(%o0) *done*

➜   s(t):=t +%i·t^2;

(%o1) s*(t):=*$t+$%i t^{2}

➜   wxdraw2d(
        xaxis = true, xaxis_type = solid, xrange = $[-2,\ 2]$,
        yaxis = true, yaxis_type = solid, yrange = $[-2,\ 2]$,
        proportional_axes = xy,
        nticks = 200,
        line_width = 2,
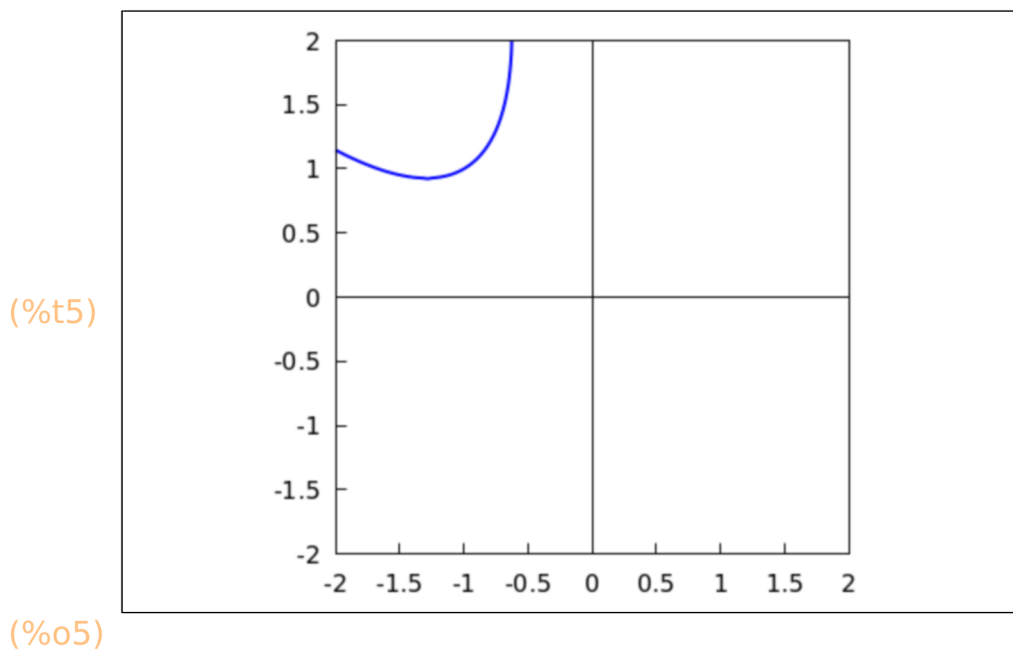   parametric(realpart(s(t)), imagpart(s(t)), t, −2, 2)
    );

(%t2)

(%o2)

rotate the parabola about the origin by
an angle of pi/6

→   theta:%pi/6;
     r(t):=s(t)·exp(%i·theta)+(−1+%i);

(theta) $\dfrac{\pi}{6}$

(%o4) r$(t):=$s(t) exp$($%i $theta)+(-1+$%i$)$

→   wxdraw2d(
      xaxis = true, xaxis_type = solid, xrange = [−2, 2],
      yaxis = true, yaxis_type = solid, yrange = [−2, 2],
      proportional_axes = xy,
      nticks = 200,
      line_width = 2,
      parametric(realpart(r(t)), imagpart(r(t)), t, −2, 2)
    );

(%t5)

(%o5)

2

Rotate the curve (t, t^3),
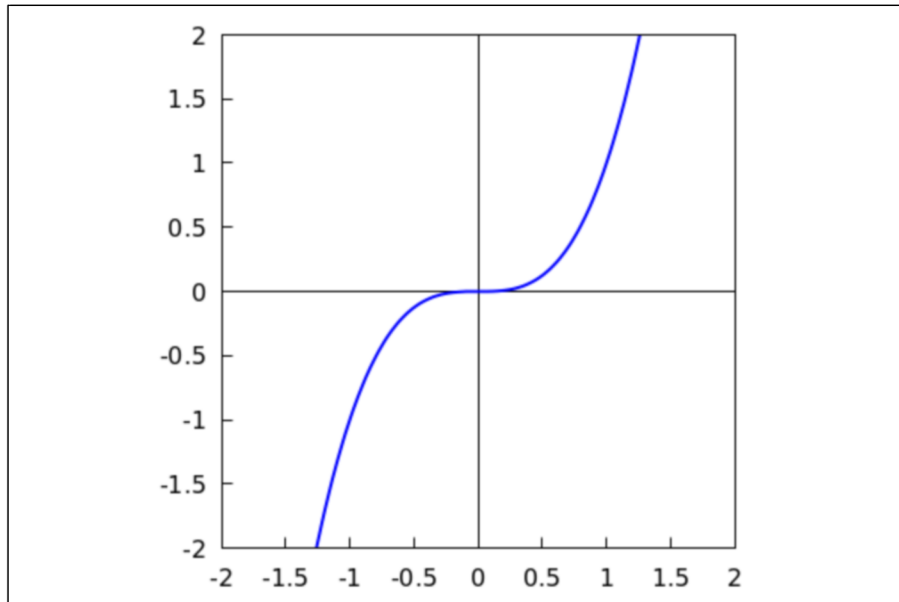t in [-2, 2], by an angle pi/6.

→   kill(all);
(%o0) *done*

→   s(t):=t +%i·t^3;

(%o1) s$(t):=t+$%i t^3

→   `wxdraw2d(`
        `xaxis = true, xaxis_type = solid, xrange = [−2, 2],`
        `yaxis = true, yaxis_type = solid, yrange = [−2, 2],`
        `proportional_axes = xy,`
        `nticks = 200,`
        `line_width = 2,`
        `parametric(realpart(s(t)), imagpart(s(t)), t, −2, 2)`
      `);`

(%t2)

(%o2)

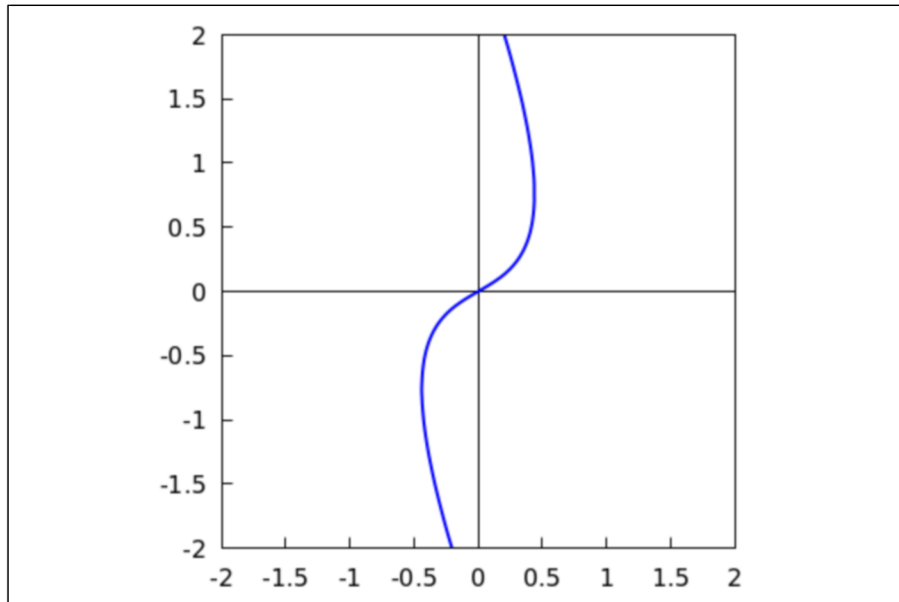rotate the curve about the origin by
an angle of pi/6

→   `theta:%pi/6;`
      `r(t):=s(t)·exp(%i·theta);`

(theta) $\dfrac{\pi}{6}$

(%o4) $r(t):=s(t)\exp(\%i\,\theta)$

➜   wxdraw2d(
      xaxis = true, xaxis_type = solid, xrange = [−2, 2],
      yaxis = true, yaxis_type = solid, yrange = [−2, 2],
      proportional_axes = xy,
      nticks = 200,
      line_width = 2,
      parametric(realpart(r(t)), imagpart(r(t)), t, −2, 2)
    );

(%t5)

(%o5)

3

Rotate the hyperbola y^2-x^2=1,
by an angle pi/6.

➜   kill(all);
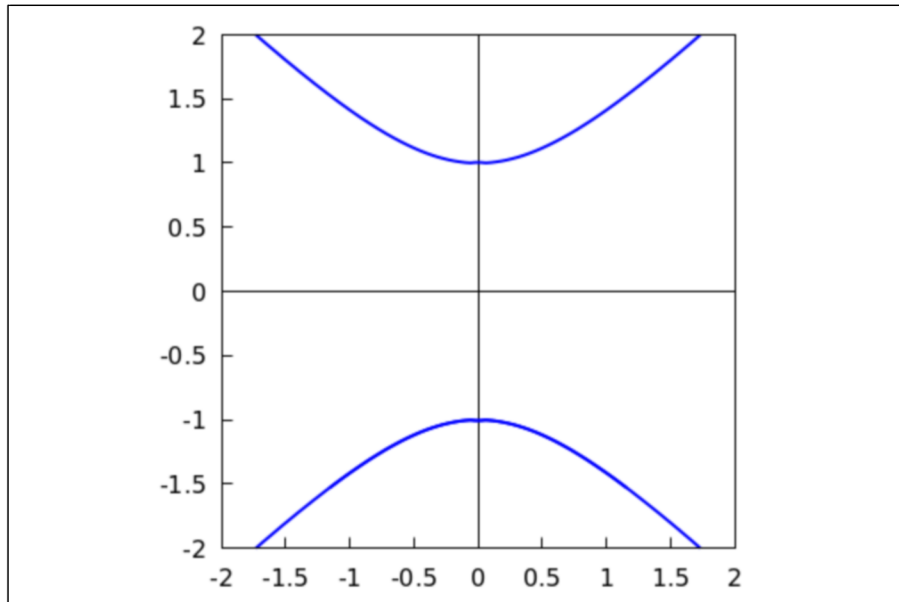(%o0) *done*

➜   s(t):=tan(t) +%i·sec(t);
(%o1) s*(t):=* tan*(t)* + %i sec*(t)*

➜   wxdraw2d(
    xaxis = true, xaxis_type = solid, xrange = [−2, 2],
    yaxis = true, yaxis_type = solid, yrange = [−2, 2],
    proportional_axes = xy,
    nticks = 200,
    line_width = 2,
    parametric(realpart(s(t)), imagpart(s(t)), t, −4, 4)
    );

(%t7)

(%o7)

rotate the hyperbola about the origin by
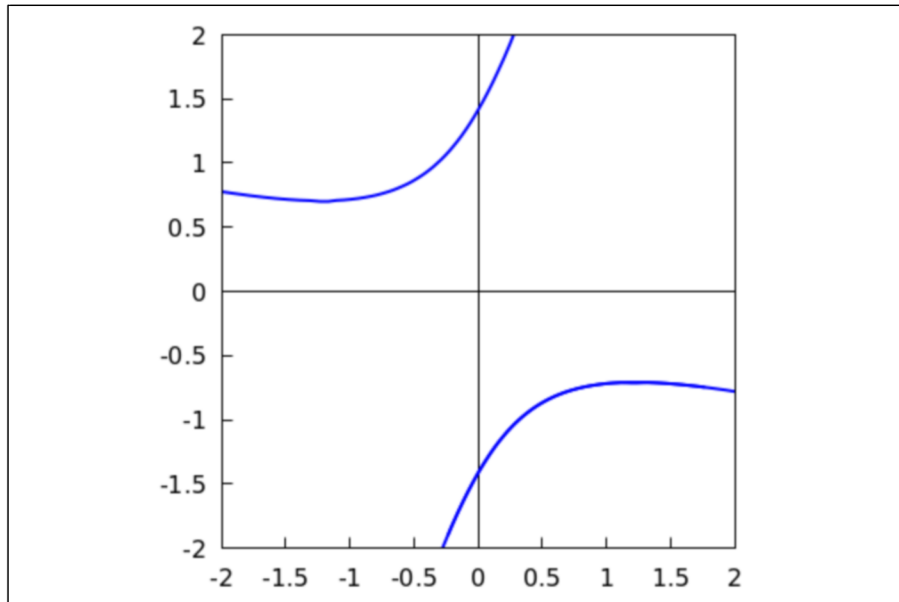an angle of pi/6

➜   theta:%pi/6;
    r(t):=s(t)·exp(%i·theta);

(theta) $\dfrac{\pi}{6}$

(%o9) $r(t):=s(t)\exp(\%i\,theta)$

➜   wxdraw2d(
        xaxis = true, xaxis_type = solid, xrange = [−2, 2],
        yaxis = true, yaxis_type = solid, yrange = [−2, 2],
        proportional_axes = xy,
        nticks = 200,
        line_width = 2,
        parametric(realpart(r(t)), imagpart(r(t)), t, −4, 4)
    );

(%t11)

(%o11)

4

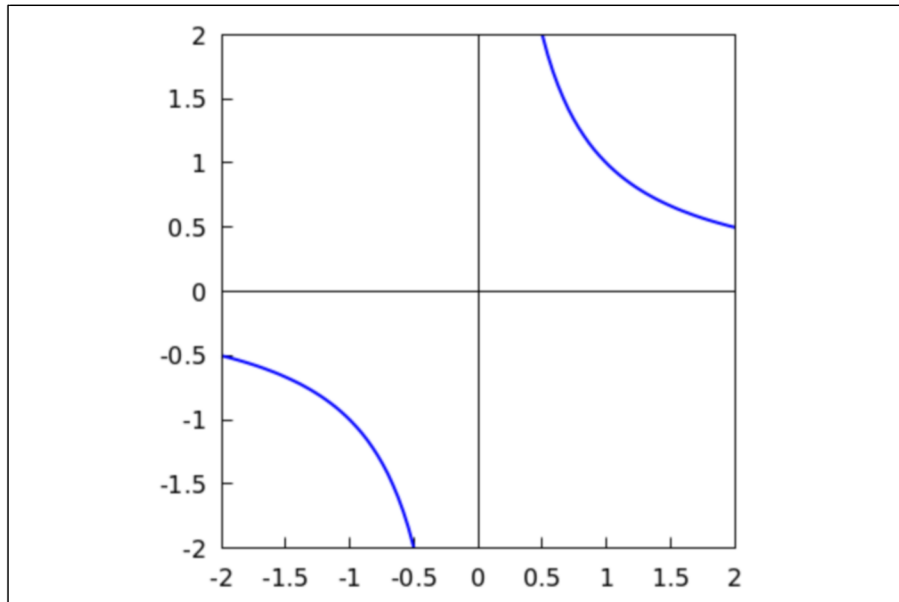Rotate the hyperbola xy=1
by an angle pi/3.

➜   kill(all);
(%o0) *done*

➜   s(t):=t +%i·(1/t);

(%o1) $s(t):=t+\%i\,\dfrac{1}{t}$

➔   wxdraw2d(
        xaxis = true, xaxis_type = solid, xrange = [−2, 2],
        yaxis = true, yaxis_type = solid, yrange = [−2, 2],
        proportional_axes = xy,
        nticks = 200,
        line_width = 2,
        parametric(realpart(s(t)), imagpart(s(t)), t, −4, 4)
    );

(%t2)

(%o2)

rotate the hyperbola about the origin by
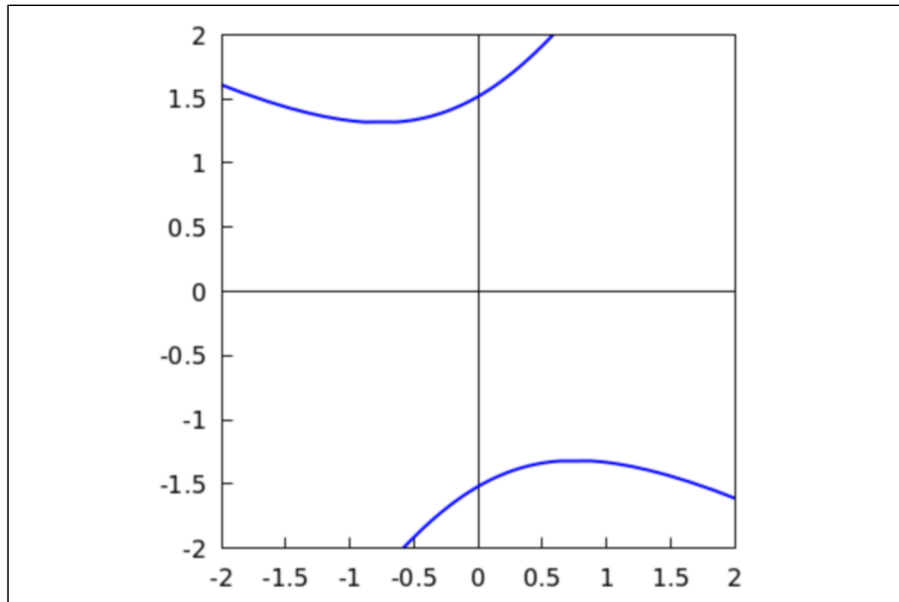an angle of pi/3

➔   theta:%pi/3;
        r(t):=s(t)·exp(%i·theta);

(theta) $\dfrac{\pi}{3}$

(%o4) $r(t):=s(t)\exp(\%i\,theta)$

→   wxdraw2d(
           xaxis = true, xaxis_type = solid, xrange = [−2, 2],
           yaxis = true, yaxis_type = solid, yrange = [−2, 2],
           proportional_axes = xy,
           nticks = 200,
           line_width = 2,
           parametric(realpart(r(t)), imagpart(r(t)), t, −4, 4)
       );

(%t5)

(%o5)

5

Rotate the triangle joining the points
0, 2, 1+i sqrt(3)
by an angle pi/12.

→   kill(all);
(%o0) *done*

→   s1(t):=t +%i·0;
       s2(t):=t +%i·(−t·sqrt(3)+2·sqrt(3));
       s3(t):=t +%i·(t·sqrt(3));

(%o1) $\text{s1}(t):=t+\%i\,0$

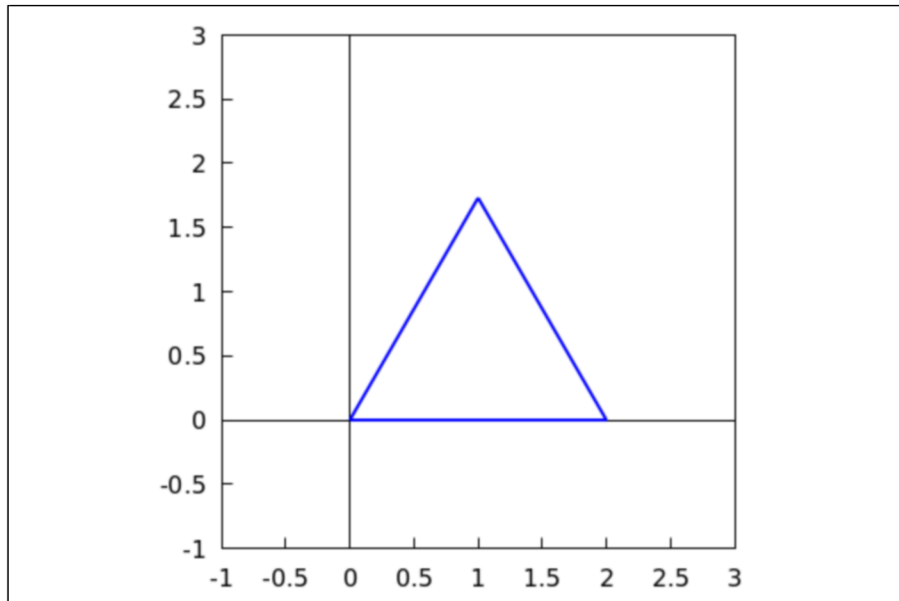(%o2) $\text{s2}(t):=t+\%i\,((-t)\sqrt{3}+2\sqrt{3})$

(%o3) $\text{s3}(t):=t+\%i\,(t\sqrt{3})$

```
wxdraw2d(
    xaxis = true, xaxis_type = solid, xrange = [−1, 3],
    yaxis = true, yaxis_type = solid, yrange = [−1, 3],
    proportional_axes = xy,
    nticks = 200,
    line_width = 2,
    parametric(realpart(s1(t)),  imagpart(s1(t)),  t,  0,  2),
    parametric(realpart(s2(t)),  imagpart(s2(t)),  t,  1,  2),
    parametric(realpart(s3(t)),  imagpart(s3(t)),  t,  0,  1)
);
```

(%t4)

(%o4)

rotate the triangle about the origin by
an angle of pi/12

```
theta:%pi/12;
r1(t):=s1(t)·exp(%i·theta);
r2(t):=s2(t)·exp(%i·theta);
r3(t):=s3(t)·exp(%i·theta);
```

(theta) $\dfrac{\pi}{12}$

(%o10) $r1(t):=s1(t)\exp(\%i\,theta)$
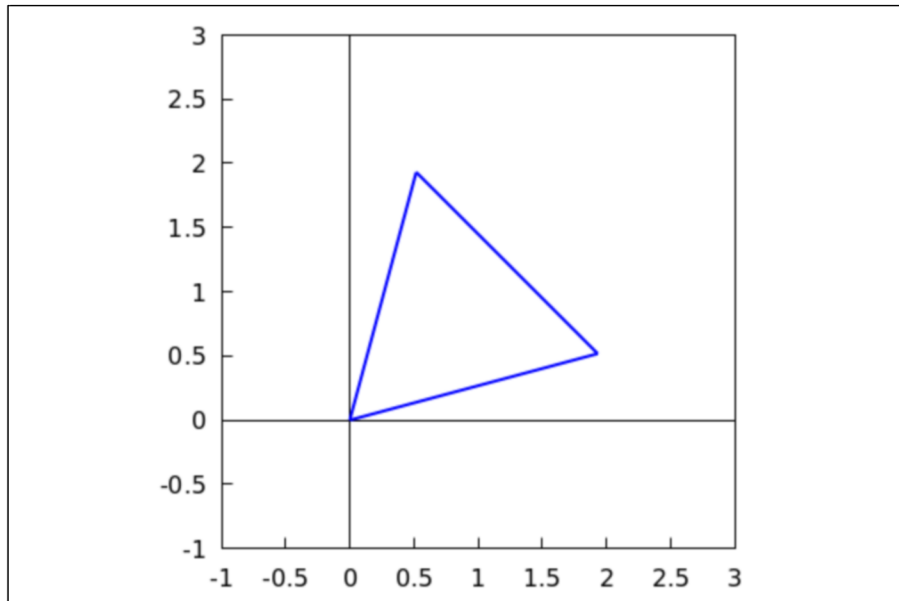
(%o11) $r2(t):=s2(t)\exp(\%i\,theta)$

(%o12) $r3(t):=s3(t)\exp(\%i\,theta)$

➜ wxdraw2d(
    xaxis = true, xaxis_type = solid, xrange = [−1, 3],
    yaxis = true, yaxis_type = solid, yrange = [−1, 3],
    proportional_axes = xy,
    nticks = 200,
    line_width = 2,
    parametric(realpart(r1(t)), imagpart(r1(t)), t, 0, 2),
    parametric(realpart(r2(t)), imagpart(r2(t)), t, 1, 2),
    parametric(realpart(r3(t)), imagpart(r3(t)), t, 0, 1)
);

(%t13)

(%o13)

6

Find the image of the rectangle
0, 1, 1+2i, 2i
under the mapping
w = (1+i)z+2

➜ kill(all);
(%o0) *done*

➜ s1(t):=t +%i·0;
    s2(t):=1 +%i·t;
    s3(t):=t +%i·2;
    s4(t):=0 +%i·t;

(%o5) $s1(t):=t+\%i\,0$
(%o6) $s2(t):=1+\%i\,t$
(%o7) $s3(t):=t+\%i\,2$
(%o8) $s4(t):=0+\%i\,t$

```
wxdraw2d(
    xaxis = true, xaxis_type = solid, xrange = [−1, 4],
    yaxis = true, yaxis_type = solid, yrange = [−1, 4],
    proportional_axes = xy,
    nticks = 200,
    line_width = 2,
    parametric(realpart(s1(t)),  imagpart(s1(t)),  t,  0,  1),
    parametric(realpart(s2(t)),  imagpart(s2(t)),  t,  0,  2),
    parametric(realpart(s3(t)),  imagpart(s3(t)),  t,  0,  1),
    parametric(realpart(s4(t)),  imagpart(s4(t)),  t,  0,  2)
);
```

(%t17)

(%o17)
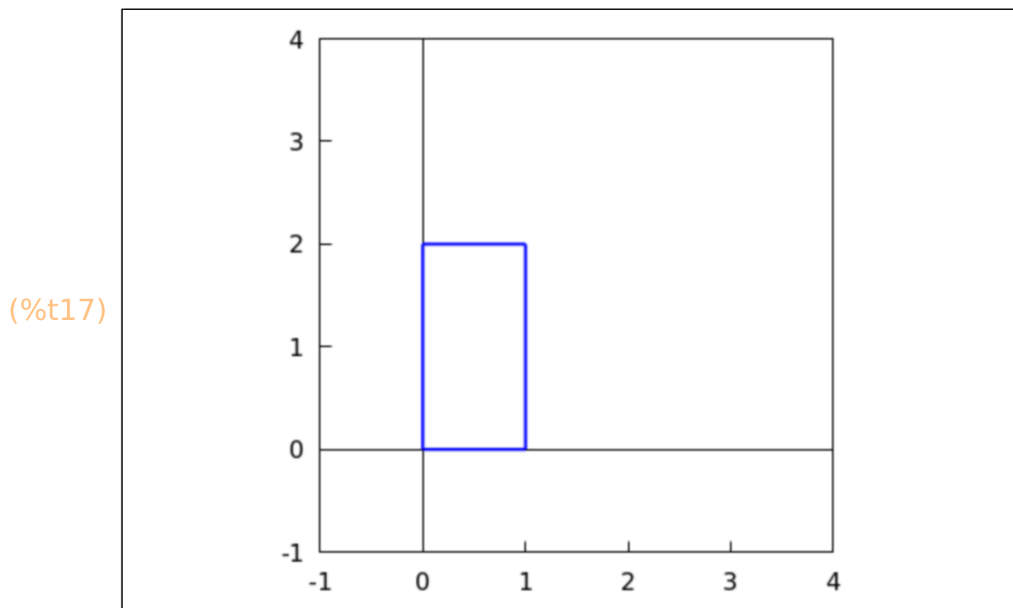
rotate the triangle about the origin by
an angle of pi/12

```
theta:%pi/12;
r1(t):=s1(t)·(1+%i)+ 2;
r2(t):=s2(t)·(1+%i)+ 2;
r3(t):=s3(t)·(1+%i)+ 2;
r4(t):=s4(t)·(1+%i)+ 2;
```

(theta) $\dfrac{\pi}{12}$

(%o11) $r1(t):=s1(t)(1+\%i)+2$

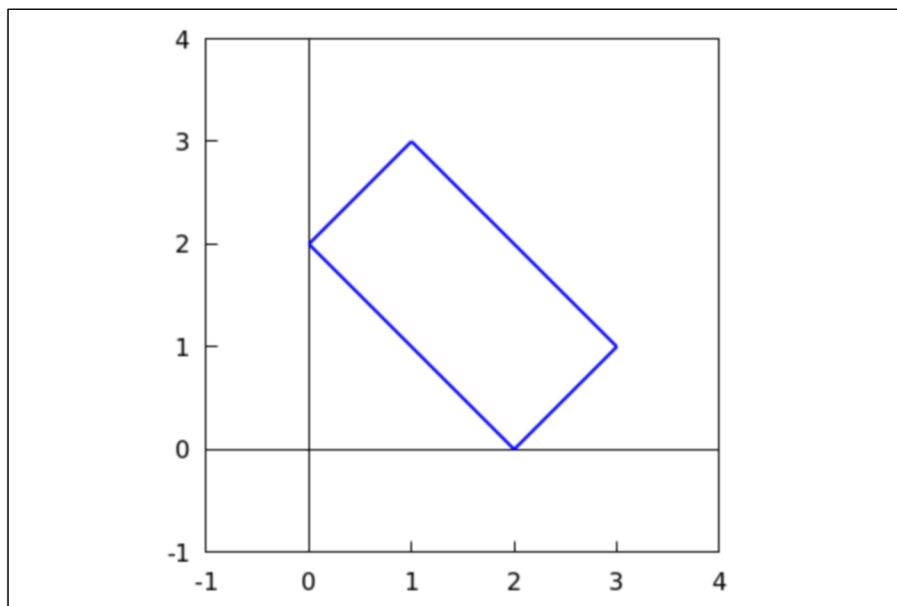(%o12) $r2(t):=s2(t)(1+\%i)+2$

(%o13) $r3(t):=s3(t)(1+\%i)+2$

(%o14) $r4(t):=s4(t)(1+\%i)+2$

➜   wxdraw2d(
        xaxis = true, xaxis_type = solid, xrange = [−1, 4],
        yaxis = true, yaxis_type = solid, yrange = [−1, 4],
        proportional_axes = xy,
        nticks = 200,
        line_width = 2,
        parametric(realpart(r1(t)), imagpart(r1(t)), t, 0, 1),
        parametric(realpart(r2(t)), imagpart(r2(t)), t, 0, 2),
        parametric(realpart(r3(t)), imagpart(r3(t)), t, 0, 1),
        parametric(realpart(r4(t)), imagpart(r4(t)), t, 0, 2)
    );

(%t16)

(%o16)