Practical 13

1

Figure 1: Show that

$$\int_C \frac{dz}{2z^{\frac{1}{2}}} = 1 + i,$$

where $z^{\frac{1}{2}}$ is the principal branch of the square root function and C is the line segment joining 4 to $8 + 6i$.

Let I denote the required integral.

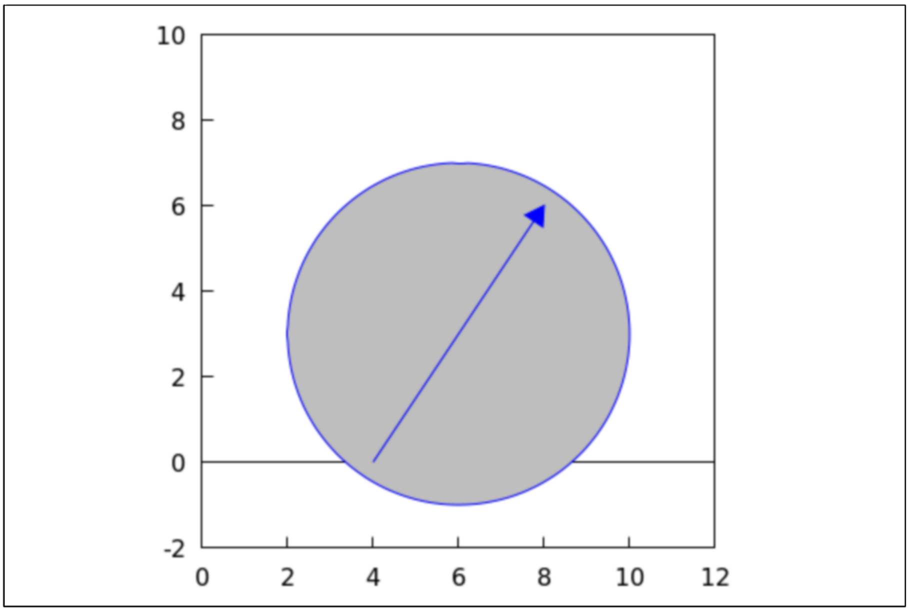
```
wxdraw2d(
    xaxis = true, xaxis_type = solid,  xrange = [0,  12],
    yaxis = true, yaxis_type = solid,  yrange = [−2,  10],
    proportional_axes = xy,

    nticks = 200,
    fill_color = gray,
    ellipse(6,   3,   4,   4,   0,   360),

    head_length = 0.5,
    head_angle = 30,
    vector([4,  0],  [4,   6])

);
```

(%t16)



(%o16)

Note that the line segment C is contained in the simply connected domain(SCD) D4(6+3i) (open disc centered at 6+3i with radius 4).

2

2.1

```
kill(all);
```

(%o0) *done*

```
f(z):=1/(2·sqrt(z));
define(F(z), integrate(f(z), z));
```

(%o1) $\mathrm{f}(z):=\frac{1}{2\sqrt{z}}$

(%o2) $\mathrm{F}(z):=\sqrt{z}$

Here the principal branch of the square root function is used in both the formulas for f and F.

By FTC applied to f(z) in D4(6+3i)(a SCD)

```
I:F(8+6·%i)−F(4);
```

(%o3) $\sqrt{6\,\%i+8}-2$

```
rectform(I);
```

(%o4) $\%i+1$

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Exercise

Find the value of the definite integral using Theorem 6.9, and explain why you are justified in using it.

Figure 2:

1. $\int_C z^2\,dz$, where C is the line segment from $1+i$ to $2+i$.

Figure 3:

2. $\int_C \cos z \, dz$, where C is the line segment from $-i$ to $1+i$.

3. $\int_C \exp z \, dz$, where C is the line segment from 2 to $i\frac{\pi}{2}$.

4. $\int_C z \exp z \, dz$, where C is the line segment from $-1 - i\frac{\pi}{2}$ to $2 + i\pi$.

5. $\int_C \frac{1+z}{z} \, dz$, where C is the line segment from 1 to i.

6. $\int_C \sin \frac{z}{2} \, dz$, where C is the line segment from 0 to $\pi - 2i$.