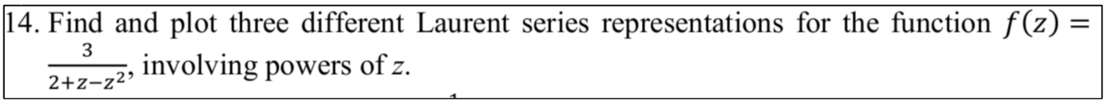
Practical 14

[redacted]

Figure 1:



14. Find and plot three different Laurent series representations for the function $f(z) = \frac{3}{2+z-z^2}$, involving powers of z.

1

1.1

```
kill(all);
```

(%o0) *done*

```
f(z):=3/(2+z−z^2);
g(z):=1/(1+z);
h(z):=(1/2)·(1/(1−(z/2)));
```

(%o1) $f(z) := \frac{3}{2+z-z^2}$

(%o2) $g(z) := \frac{1}{1+z}$

(%o3) $h(z) := \frac{1}{2}\frac{1}{1-\frac{z}{2}}$

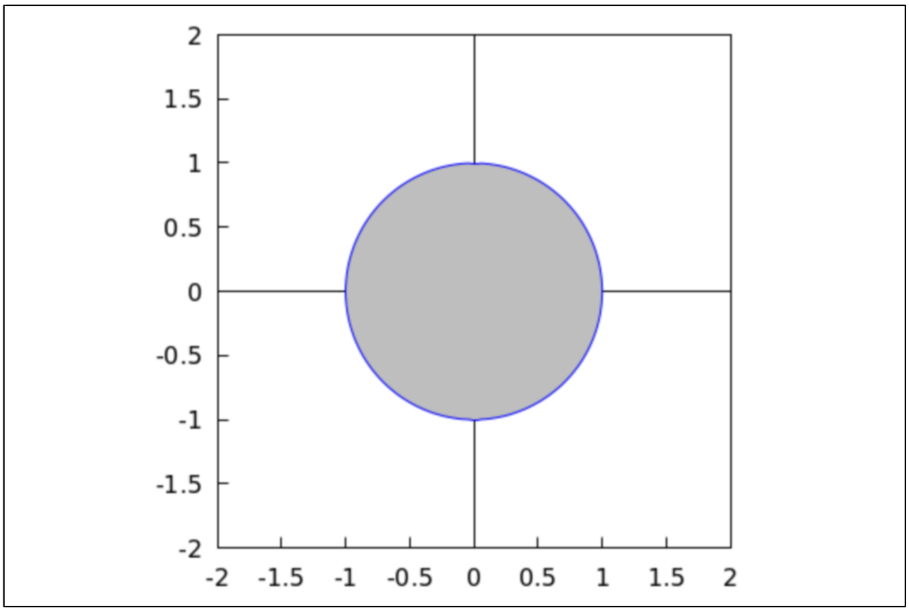
1.2

|z| < 1

```
wxdraw2d(
    xaxis = true, xaxis_type = solid,  xrange = [−2,  2],
    yaxis = true, yaxis_type = solid,  yrange = [−2,  2],
    proportional_axes = xy,

    nticks = 200,
    fill_color = gray,
    ellipse(0,   0,   1,   1,   0,   360)

);
```

(%t1) 

(%o1)

```
f(z):=3/(2+z−z^2);
g(z):=1/(1+z);
h(z):=(1/2)·(1/(1−(z/2)));
```

(%o7) $f(z):=\frac{3}{2+z-z^2}$

(%o8) $g(z):=\frac{1}{1+z}$

(%o9) $h(z):=\frac{1}{2}\frac{1}{1-\frac{z}{2}}$

```
taylor(g(z),   z,   0,   4);
```

(%o13)/T/ $1-z+z^2-z^3+z^4+...$

```
taylor(h(z),   z,   0,   4);
```

(%o14)/T/ $\frac{1}{2}+\frac{z}{4}+\frac{z^2}{8}+\frac{z^3}{16}+\frac{z^4}{32}+...$

```
→ taylor(g(z), z, 0, 4)+taylor(h(z), z, 0, 4);
```

(%o15)/T/ $\frac{3}{2}-\frac{3\,z}{4}+\frac{9\,z^2}{8}-\frac{15\,z^3}{16}+\frac{33\,z^4}{32}+...$

1.3

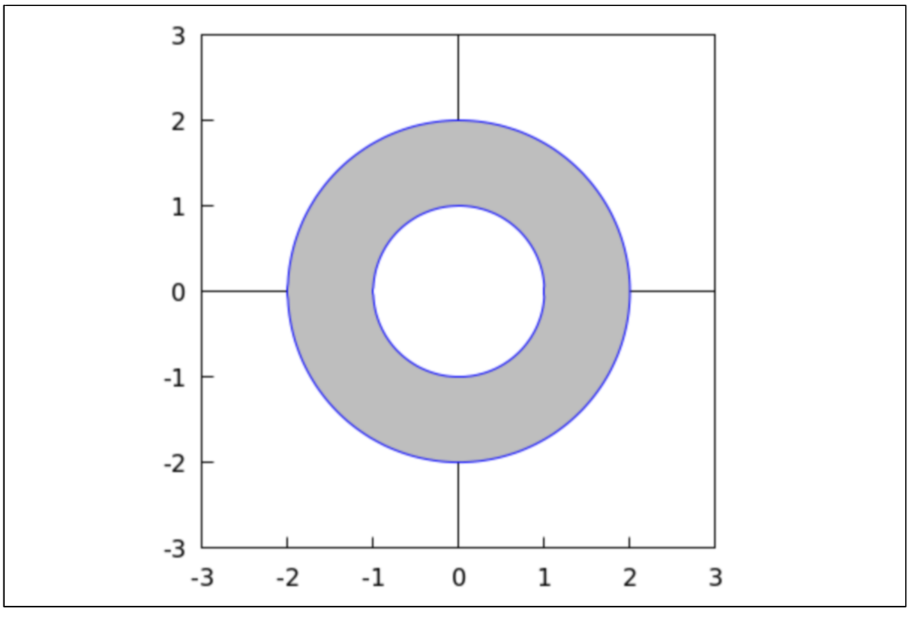
$1 < |z| < 2$

```
→ wxdraw2d(
    xaxis = true, xaxis_type = solid, xrange = [−3, 3],
    yaxis = true, yaxis_type = solid, yrange = [−3, 3],
    proportional_axes = xy,

    nticks = 200,
    fill_color = gray,
    ellipse(0, 0, 2, 2, 0, 360),
    fill_color = white,
    ellipse(0, 0, 1, 1, 0, 360)

);
```

(%t3)



(%o3)

```
f(z):=3/(2+z−z^2);
g(z):=1/(1+z);
h(z):=(1/2)·(1/(1−(z/2)));
```

(%o16) $f(z):=\frac{3}{2+z-z^2}$

(%o17) $g(z):=\frac{1}{1+z}$

(%o18) $h(z):=\frac{1}{2}\frac{1}{1-\frac{z}{2}}$

```
taylor(g(z), [z, 0, 4, 'asymp])+taylor(h(z), z, 0, 4);
```

(%o19)/T/ $\frac{z^4}{32}+\frac{z^3}{16}+\frac{z^2}{8}+\frac{z}{4}+\frac{1}{2}+\frac{1}{z}-\frac{1}{z^2}+\frac{1}{z^3}-\frac{1}{z^4}+...$

1.4

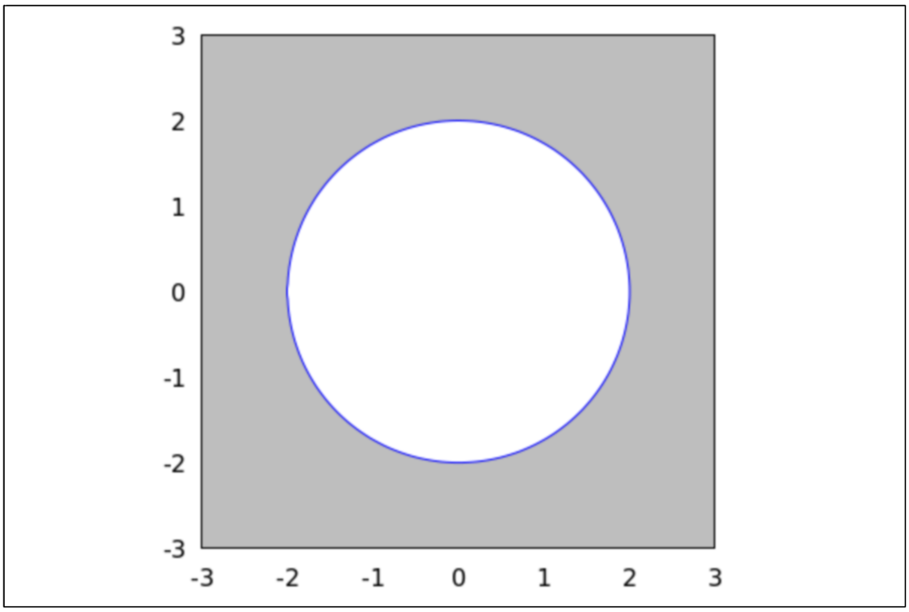
|z| > 2

```
wxdraw2d(
    xaxis = true, xaxis_type = solid,  xrange = [−3,  3],
    yaxis = true, yaxis_type = solid,  yrange = [−3,  3],
    proportional_axes = xy,

    nticks = 200,
    fill_color = gray,
    ellipse(0,   0,   6,   6,   0,   360),
    fill_color = white,
    ellipse(0,   0,   2,   2,   0,   360)

);
```

(%t6)



(%o6)

```
f(z):=3/(2+z−z^2);
g(z):=1/(1+z);
h(z):=(1/2)·(1/(1−(z/2)));
```

(%o20) $f(z):=\frac{3}{2+z-z^2}$

(%o21) $g(z):=\frac{1}{1+z}$

(%o22) $h(z):=\frac{1}{2}\frac{1}{1-\frac{z}{2}}$

```
taylor(g(z),   [z,   0,   4,  'asymp])+taylor(h(z),  [z,   0,   4,  'asymp]);
```

(%o23)/T/ $-\frac{3}{z^2}-\frac{3}{z^3}-\frac{9}{z^4}+...$

2

Exercise

Figure 2:

1. Find two Laurent series expansions for $f(z) = \frac{1}{z^3 - z^4}$ that involve powers of z.

Figure 3:

9. Find two Laurent series for $z^{-1}(4-z)^{-2}$ involving powers of z and state where they are valid.

10. Find three Laurent series for $\left(z^2 - 5z + 6\right)^{-1}$ centered at $\alpha = 0$.