$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$, $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$

$\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$, $\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x = e^a$

Limits Class - 11th

- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
- $\lim_{x \to 0} \frac{e^{mx} - 1}{mx} = 1$
- $\lim_{x \to 0} \frac{a^{mx} - 1}{mx} = \log_e a$
- $\lim_{x \to 0} \frac{1}{ax} \log(1+ax) = 1$
- $\lim_{x \to 0} \frac{\sin(mx)}{mx} = 1$, $\frac{\sin^{-1}(x)}{x} = 1$
- $\lim_{x \to 0} \cos(ax) = 1$
- $\lim_{x \to 0} \frac{\tan(ax)}{(ax)} = 1$, $\frac{\tan^{-1}(x)}{x} = 1$

- **Working rule**

* Don't use limit until it →

(a) Common factorisation cancellation

Rationalisation

(b) $1 - \cos 2\theta = 2\sin^2\theta$

(c) Use C-D formula

(d) $90 \pm \theta$ formula

- $\frac{\log(1+ax)}{ax} = 1$
- $\lim_{x \to \infty} e^x = \infty$
- $\lim_{x \to -\infty} e^x = 0$
- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

LHL

$$\lim_{\substack{x\to 0-h\\ h\to 0}}\left[\frac{1-\cos 3x}{x\sin x}\right]$$

$$\lim_{\substack{x\to 0-h\\ h\to 0}}\left[\frac{2\sin^2\left(\frac{3x}{2}\right)}{x\sin x}\right]$$

$$= \left[\frac{2\sin\frac{3x}{2}\cdot\sin\frac{3x}{2}\times\frac{9}{4}}{\frac{3x}{2}\cdot\frac{3x}{2}\cdot\frac{\sin x}{x}}\right]$$

$$= 2\times 9/4$$

$$= 9/2$$

RHL

$$\lim_{\substack{x\to 0+h\\ h\to 0}}\left[\frac{1-\cos 3x}{x\sin x}\right]$$

$$= \left[\frac{2\sin^2 3x/2}{x\sin x}\right]$$

$$= 2\times\frac{9}{4}$$

$$\Rightarrow 9/2$$

Rules :

- **Constant,** $y = k \cdot f(x) \Rightarrow \frac{dy}{dx} = k \cdot \frac{d f(x)}{dx}$

- **Addition and Subtraction,**

$$y = f(x) \pm g(x) \pm h(x)$$

$$\frac{dy}{dx} = \frac{d[f(x)]}{dx} \pm \frac{d[g(x)]}{dx} \pm \frac{d[h(x)]}{dx}$$

- **Product rule,** $y = f(x) \times g(x)$

$$\frac{dy}{dx} = f(x) \times \frac{d[g(x)]}{dx} + g(x) \frac{d[f(x)]}{dx}$$

$$y = U \cdot V$$

$$y = U \cdot V' + V \cdot U'$$

- **Quotient rule,** $y = \frac{f(x)}{g(x)}$

$$\frac{dy}{dx} = \frac{g(x) \times \frac{d f(x)}{dx} - f(x) \times \frac{d[g(x)]}{dx}}{[g(x)]^2}$$

$$y = \frac{N}{D}$$

$$\frac{dy}{dx} \Rightarrow \frac{N' \cdot D - D' N}{D^2}$$

$\frac{1}{x}$

x^{-1}

$-1 x^{-1-1}$

$\frac{-1}{x^2}$

- **Chain rule,** $y = \sqrt{x^{3/5}}$, let $x^{3/5} = t$

$y = \sqrt{t}$ $\quad$ $t = x^{3/5}$

$y = t^{1/2}$ $\quad$ $t = x^{3/5}$

Standard Results

- $y = \text{Const.}(k)$ $\quad \frac{dy}{dx} \Rightarrow 0$
- $y = x^n$ $\quad$ " $\Rightarrow nx^{n-1}$
- $y = \sqrt{x}$ $\quad$ " $\Rightarrow \frac{1}{2\sqrt{x}}$
- $y = (\exp)^n$ $\quad$ " $\Rightarrow n(\exp)^{n-1} \times (\exp)'$
- $y = e^{\exp}$ $\quad$ " $\Rightarrow e^{\exp} \times (\exp)' \times \underset{(1)}{\log_e e}$
- $y = a^{\exp}$ $\quad$ " $\Rightarrow a^{\exp} \times (\exp)' \times \log a$
- $y = \log_e(\exp)$ $\quad$ " $\Rightarrow \frac{1}{\exp} \times (\exp)'$
- $y = \sqrt{\exp}$ $\quad$ " $\Rightarrow \frac{1}{2\sqrt{\exp}} \times (\exp)'$
- $y = \left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$ $\quad$ ($\log e = 1$)

Imp • $y = x^x \Rightarrow x^x(1 + \ln x)$

• $\cos 2\theta = 1 - 2\sin^2\theta$ $\quad$ • $\sin 2\theta = 2\sin\theta\cos\theta$

Imp • $y = \log_a x = \frac{1}{x \ln a}$ $\longrightarrow$ e.g. $\frac{d(\log_a x)}{dx} = \frac{1}{x \log_e a}$

- $\log(a \times b \times c) = \log a + \log b + \log c$
- $\log\left(\frac{a \times c}{b \times f}\right) = \log a - \log b + \log c - \log f$ → mostly used
- ~~$\log(a^m)$ = $\log(a)^m$~~ $\cdot \log(a^m) = \log(a)^m = m \log a$
- $\log_x (x) = \log_a a = \log_e e = \log_s s = 1$
- $e^{\log x} = x$ $\left(e^{\log 5x} = 5x\right)$
- Base change, $\log_B A = \frac{\log_e A}{\log_e B}$

* When there is nothing in base of the log, we take that blank as 'e', this log is called a natural log, we directly apply derivatives on natural log, but not on other log.

$$\log 5 = \log_e 5$$
(Natural log)

* $\log\left(\frac{a}{b}\right) = \log a - \log b$

$$\log\left(\frac{a}{b}\right) \neq \frac{\log a}{\log b}$$
(because they have diff log)

• Trigo

- $y = \sin(\text{exp}) \longrightarrow \frac{dy}{dx} = \cos(\text{exp}) \times (\text{exp})'$

- $y = \cos(\text{exp}) \longrightarrow \frac{dy}{dx} = -\sin(\text{exp}) \times (\text{exp})'$

- $y = \tan(\text{exp}) \longrightarrow \frac{dy}{dx} = \sec^2(\text{exp}) \times (\text{exp})'$

- $y = \cot(\text{exp}) \longrightarrow \frac{dy}{dx} = -\text{cosec}^2(\text{exp}) \times (\text{exp})'$

- $y = \sec(\text{exp}) \longrightarrow \frac{dy}{dx} = \sec(\text{exp}) \tan(\text{exp}) \times (\text{exp})'$

- $y = \text{cosec}(\text{exp}) \longrightarrow \frac{dy}{dx} = -\text{cosec}(\text{exp}) \cot(\text{exp}) \times (\text{exp})'$

• Inverse Trigo

- $y = \sin^{-1}(ax) \longrightarrow \frac{dy}{dx} = \frac{+1}{\sqrt{1-(ax)^2}} \times (ax)'$

- $y = \cos^{-1}(ax) \longrightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-(ax)^2}} \times (ax)'$

- $y = \tan^{-1}(ax) \longrightarrow \frac{dy}{dx} = \frac{+1}{1+(ax)^2} \times (ax)'$

- $y = \cot^{-1}(ax) \longrightarrow \frac{dy}{dx} = \frac{-1}{1+(ax)^2} \times (ax)'$

- $y = \sec^{-1}(ax) \longrightarrow \frac{dy}{dx} = \frac{+1}{|ax|\sqrt{(ax)^2-1}} \times (ax)'$

- $y = \operatorname{cosec}^{-1}(ax) \longrightarrow \frac{dy}{dx} = \frac{-1}{|ax|\sqrt{(ax)^2-1}} \times (ax)'$

• Uses of log :-

- Product of 3 or more factors.

- $y = \frac{u \times v}{w} \Rightarrow \frac{dy}{dx} = \frac{(uv)'w - w'(uv)}{(w)^2}$

If product and Quotient rules are applicable.

$$y = [f(x)]^{g(x)}$$

Very Imp

e.g → $y = x^4$
$y = e^{4x}$ [No log]
$y = a^{3x}$
$y = (x)^{3x}$ → [Must, log]

$y = \sin x^{(\tan x)}$ [Must log]

$(xy)' = x'y + y'x$

$= y + \frac{dy}{dx} \cdot x$

Property : 6

- $\sin^{-1}(-x) = -\sin^{-1}(x)$
- $\tan^{-1}(-x) = -\tan^{-1}(x)$
- $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$
- $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$
- $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$
- $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$

Q. $\sin^{-1}\left(-\sin \frac{7\pi}{3}\right)$

$$-\sin^{-1}\left(\sin\left(2\theta + \frac{\pi}{3}\right)\right)$$

$$-\sin^{-1}\left(\sin \frac{\pi}{3}\right)$$

$$-\pi/3$$

Property : 7 (Imp formulas)

- $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$
- $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$
- $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[x \cdot y - \sqrt{1-x^2}\sqrt{1-y^2}\right]$
- $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left[x \cdot y + \sqrt{1-x^2}\sqrt{1-y^2}\right]$
- $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$ condition: $xy<1$, $x,y>0$
- $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left[\frac{x-y}{1+xy}\right]$

Property-10 (Most Imp)

- Whenever — Always put

(i) $\sqrt{1-x^2}$ $\longrightarrow$ $x = \sin\theta$ (always)

$\sqrt{a^2-x^2}$ $\quad x = a\sin\theta$

eg → $\sqrt{25-x^2} = 5\sin\theta$

$\sqrt{11-x^2} = \sqrt{11}\sin\theta$

(ii) $\sqrt{1+x^2}$ $\longrightarrow$ $x = \tan\theta$

$\sqrt{a^2+x^2}$ $\quad x = a\tan\theta$

(iii) $\sqrt{x^2-1}$ $\longrightarrow$ $x = \sec\theta$

$\sqrt{x^2-a^2}$ $\quad x = a\sec\theta$

(iv) $\sqrt{\frac{1+x}{1-x}}$ $\longrightarrow$ $x = \cos 2\theta$

$\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}$ $\quad \sqrt{x} = \cos 2\theta$

$\sqrt{\frac{9-x}{9+x}}$ $\quad x = 9\cos\theta$

$\sqrt{\frac{1-x^2}{1+x^2}}$ $\quad x^2 = \cos 2\theta$

$\sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}$ $\quad \sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}} = \cot\theta$

(v) $x\sqrt{1-y^2} \pm y\sqrt{1-x^2} \longrightarrow$ $x = \sin\alpha$, $y = \sin\beta$

(vi) $\dfrac{a\cos\theta \pm b\sin\theta}{r} \longrightarrow$ $a = r\cos\alpha$, $b = r\sin\alpha$

or

$a = r\sin\alpha$, $b = r\cos\alpha$

~~(vii)~~

If $a^2 + b^2 = r^2$

Q.2) $\dfrac{3\cos x - 4\sin x}{5}$

$3 = 5\cos\alpha$

$4 = 5\sin\alpha$

• Types of problems

- evaluate
  - $\sin^{-1}\left(\frac{1}{2}\right) + 3\tan^{-1}(\sqrt{3}) - \operatorname{cosec}(-2)$
- Simplest form
- Prove that
- Solve the equation

Integra
tion

- **Rules ÷**

1) $I = \int K\,dx = K\int dx$

2) $I = \int [f(x) \pm g(x) \pm h(x)]\,dx$

$= \int f(x)\,dx \pm \int g(x)\,dx \pm \int h(x)\,dx$

3) $I = \int \underset{\mathrm{I}}{f(x)} \cdot \underset{\mathrm{II}}{g(x)}\,dx$

$\Rightarrow \mathrm{I}\int \mathrm{II} - \int (\mathrm{I}' \int \mathrm{II})$

(Product rule)

for deciding I, II we used $\boxed{\text{I - Late}}$ rule in product

(Inverse Trigo

L - Logarithm

A - Algebra

T - Trigo

e - exponential

- **Techniques ÷** → Substitution, By-parts, Partial fraction

- **Results ÷**

- $I = \int K\,dx = K\int dx = Kx + C$
- $I = \int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

$= \int (ax+b)^n\,dx = \dfrac{(ax+b)^{n+1}}{(n+1)\cdot a} + C$

- $I = \int \frac{1}{x}\,dx = \log|x| + C$, $\int \frac{1}{ax+b}\,dx = \dfrac{\ln|ax+b|}{a} + C$
- $I = \int e^{ax}\,dx = \dfrac{e^{ax}}{a} + C$
- $I = \int a^{mx}\,dx = \dfrac{a^{mx}}{(\log a)\cdot m} + C$

- $\int 1\,dx = x + C$

- $\int \cos(ax)\,dx = \frac{\sin(ax)}{a} + C$

- $\int \sin(ax)\,dx = -\frac{\cos(ax)}{a} + C$

- $\int \sec^2(ax)\,dx = \frac{\tan(ax)}{a} + C$

- $\int \operatorname{cosec}^2(ax)\,dx = -\frac{\cot(ax)}{a} + C$

- $\int \sec(ax)\tan(ax)\,dx = \frac{\sec(ax)}{a} + C$

- $\int \operatorname{cosec}(ax)\cot(ax)\,dx = -\frac{\operatorname{cosec}(ax)}{a} + C$

- $\int \tan(ax)\,dx = \frac{\log|\sec(ax)|}{a} + C$

- $\int \cot(ax)\,dx = \frac{\log_e|\sin(ax)|}{a} + C$

- $\int \sec(ax)\,dx = \frac{\ln|\sec(ax) + \tan(ax)|}{a} + C$

- $\int \operatorname{cosec}(ax)\,dx = \frac{\ln|\operatorname{cosec}(ax) - \cot(ax)|}{a} + C$

- $\int \frac{1}{\sqrt{1-(ax)^2}}\,dx = \frac{\sin^{-1}(ax)}{a} + C$

- $\int \frac{-1}{\sqrt{1-(ax)^2}}\,dx = \frac{\cos^{-1}(ax)}{a} + C$

- $\int \frac{1}{1+(ax)^2}\,dx = \frac{\tan^{-1}(ax)}{a} + C$

- $\int \frac{-1}{1+(ax)^2}\,dx = \frac{\cot^{-1}(ax)}{a} + C$

- $\int \frac{dx}{ax\sqrt{(ax)^2-1}} = \frac{\sec^{-1}(ax)}{a} + C$

- $\int \frac{-dx}{ax\sqrt{(ax)^2-1}} = \frac{\operatorname{cosec}^{-1}(ax)}{a} + C$

$\int (\sec^2 x - 1)\tan^2 x$

$\int \sec^2 \tan^2 x - \int \tan^2 x$

$\tan x = t$

$\sec^2 x\,dx = dt$

$\sin x\,dx = dt$

• always

$1 + \cos 4x = 2\cos^2 2x$

Not open in $\sin^2 x + \cos^2 x$

How to get results

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\sin(ax) + C] = a\cos(ax)$$

$$d[\sin(ax) + C] = a\cos(ax)\,dx$$

$$d\left[\frac{\sin(ax)}{a} + C_1\right] = \cos(ax)\,dx$$

$$\int d\left[\frac{\sin(ax)}{a} + C_1\right] = \int \cos(ax)\,dx$$

$$\boxed{\int \cos(ax)\,dx = \frac{\sin(ax)}{a} + C}$$

• Guru Mantra → Unknown → Known

Always Change

• deg of Num $\geq$ deg of Deno $\Rightarrow \frac{N}{D} = Q + \frac{R}{\text{Divisor}}$

• $\sin^2\theta \rightarrow \frac{1 - \cos 2\theta}{2}$

• $\cos^2\theta \rightarrow \frac{1 + \cos 2\theta}{2}$

• $\tan^2\theta \rightarrow \sec^2\theta - 1$

• $\cot^2\theta \rightarrow \operatorname{cosec}^2\theta - 1$

$2\cos x \sin y = \sin(x+y) - \sin(x-y)$

• $2\sin A \cos B \rightarrow \sin(A+B) + \sin(A-B)$

• $-2\sin A \sin B \rightarrow \cos(A+B) - \cos(A-B)$

- $\cancel{2\cot A \cot B}$ $2\cos A \cdot \cos B \Rightarrow \cos(A+B) + \cos(A-B)$

- $\cancel{\sin^3 x = \frac{3\sin x - \sin 3x}{4}}$ $\sin^3 x$ $= \frac{3\sin x - \sin 3x}{4}$

- $\cos^3 x = \frac{3\cos x + \cos 3x}{4}$

- $2\sin A \cdot \cos A = \sin 2A$
- $\cos 2x = \cos^2 x - \sin^2 x$

  $1 - 2\sin^2 x$

  $2\cos^2 x - 1$

  $\frac{1 - \tan^2 x}{1 + \tan^2 x}$

(I) Substitution

(I) (II)

unknown to known by substitution

(I),

$$I = \int \frac{f'(x)}{f(x)}\,dx$$

$$f(x) = t$$

$$f'(x)\,dx = dt$$

$$\boxed{I = \int \frac{dt}{t} = \ln|f(x)| + C}$$

(II)

$$I = \int f'(x)\,[f(x)]^n\,dx$$

$$f(x) = t$$

$$f'(x)\,dx = dt$$

$$I = \int t^n\,dt = \frac{t^{n+1}}{n+1} + C$$

$$\boxed{= \frac{[f(x)]^{n+1}}{n+1} + C}$$

Formulae (Most Imp

- $\cancel{\int \sqrt{a^2-x^2}\,dx}$

- $$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

- $$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

- $$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

- $$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

- $$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

- $$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

- $$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left|x+\sqrt{a^2+x^2}\right| + C$$

4 Types of Question

① $\int \frac{dx}{Quad}$

② $\int \frac{dx}{\sqrt{Quad}}$

Use this Method →

* Coeff. of $x^2 = 1$
* $\frac{1}{2}$ (coeff of x)
* $\pm \left[\frac{1}{2} (\text{coeff of } x)\right]^2$

* Make perfect sq form

then, apply; $\frac{dx}{x^2 \pm A^2}$

(use direct results)

③ $\int \frac{\text{linear}\, dx}{Quad}$

④ $\int \frac{\text{linear}\, dx}{\sqrt{Quad}}$

Use this →

* Put,

Linear $= A\left[\frac{d}{dx}(Quad)\right] + B$

* find A and B

* $\int \frac{A[Quad]' + B}{Quad}$

$\Rightarrow A\int \frac{(Quad)'}{Quad} + B\int \frac{dx}{Quad}$

↳ for this use substitution, $\int \frac{f'(x)}{f(x)}$

Q₂ $I = \int \frac{2x-1}{3x^2+2x-4}\, dx$

* Put

$2x - 1 = A[6x + 2] + B \quad \cdots (1)$

coeff of $x \Rightarrow 2 = 6A$, $A = 1/3$

coeff of $x^0 \Rightarrow -1 = 2A + B$, $B = -5/3$

Standard results

1. $$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

2. $$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\ln\left|x + \sqrt{a^2 + x^2}\right| + C$$

3. $$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right| + C$$

Type-I (All are distinct factors)

$$I = \int \frac{2x+1 \, dx}{(x+5)(x^2-5x+6)}$$

$$= \int \frac{(2x+1)\, dx}{(x+5)(x-2)(x-3)}$$

$$\frac{2x+1}{(x-5)(x-2)(x-3)} = \frac{A}{x-5} + \frac{B}{x-2} + \frac{C}{x-3} \quad \text{---(i)}$$

$$\frac{2x+1}{(x-5)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-5)(x-3) + C(x-5)(x-2)}{(x-5)(x-2)(x-3)}$$

$$2x+1 = A(x-2)(x-3) + B(x-5)(x-3) + C(x-5)(x-2)$$

$x = 5$, $11 = 6A$, $A = 11/6$ | $x-5=0$, $x=5$

$x = 2$, $5 = 3B$, $B = 5/3$ | $x-2=0$, $x=2$

$x = 3$, $7 = -2C$, $C = -7/2$ | $x-3=0$, $x=3$

$$\int \frac{2x+1}{(x-5)(x-2)(x-3)} = \int \frac{11/6}{(x-5)} + \int \frac{5/3}{(x-2)} + \int \frac{-7/2}{(x-3)}$$

$$I = \frac{11}{6} \ln|x-5| + \frac{5}{3} \ln|x-2| - \frac{7}{2} \ln|x-3| + C$$

(Change limits)

6 Properties

(a) Property 1 : Definite Integral is independent of change of variable. i.e.,

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$$

(b) Property 2 :

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

* If a=b

$$\int_a^a f(x)\,dx = F(a) - F(a) = \underline{0}$$

Imp. <u>Property 3:</u>

$$\int_a^b |f(x)|\, dx$$

$+ \quad - \quad +$

$a \quad c \quad d \quad b$

$$\Rightarrow \int_a^c +f(x)\,dx + \int_c^d -f(x)\,dx + \int_d^b f(x)\,dx$$

eg $\to$ 1) $I = \int |ax+b|\, dx$

$$|ax+b| = \begin{cases} +(ax+b) & \text{If } ax+b > 0 \\ 0 & ,\ ax+b = 0 \\ -(ax+b) & \text{If } ax+b < 0 \end{cases}$$

2) $\int_0^{2\pi} |\sin x|\, dx$

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

$$= \int_0^{\pi} +\sin x\,dx + \int_{\pi}^{2\pi} (-\sin x)\, dx$$

3) $\int_0^{2\pi} |\cos x|\, dx$

$$\Rightarrow \int_0^{\pi/2} +\cos x\,dx + \int_{\pi/2}^{3\pi/2} -\cos x\,dx + \int_{3\pi/2}^{2\pi} +\cos x\,dx$$

- <u>Property 4:</u>

$$\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx$$

Imp. <u>Property 5:</u>

(i) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(ii) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$\hookrightarrow$ Put, $x = a+b-x$

e.g→ $$I = \int_0^{\pi/2} \frac{\sin^{7/5}x + \cos^{7/5}x}{\sin^{7/5} - \cos^{7/5}x}\,dx \quad \text{---} (i)$$

Put $x = \pi/2 - x$

$$I = \int_0^{\pi/2} \frac{\cos^{7/5}x + \sin^{7/5}x}{\cos^{7/5}x - \sin^{7/5}x} \quad \text{----} (ii)$$

Add (i) and (ii)

$2I = 0$

$\underline{I = 0}$

e.g→ $$I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/5}x\,dx}{\sin^{3/5}x + \cos^{3/5}x}$$

Put $x = a+b-x$

$x = \pi/2 - x$

$$I = \int_{\pi/6}^{\pi/3} \frac{\cos^{3/5}x\,dx}{\cos^{3/5}x + \sin^{3/5}x}$$

add ① and ②

$$2I = \int_{\pi/6}^{\pi/3} 1\,dx$$

$$2I = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6}$$

$2I = \pi/6$, $\underline{I = \pi/12}$

- **Property: 6 :**

even f^n : $f(-x) = f(x)$

odd f^n : $f(-x) = -f(x)$

* for symmetrical limits

$$\int_{-a}^{+a} f(x)\,dx \begin{cases} 2\times\int_0^a f(x)\,dx & \text{If } f(x) \text{ is even} \\ 0 & \text{If } f(x) \text{ is odd} \end{cases}$$

eg → $$I = \int_{-\pi}^{\pi} \sin^7 x\,dx$$

$= 0$

$$I = \int_{-\pi/3}^{\pi/3} \underbrace{x \sin x}_{\text{even}}\,dx$$

$$= 2\times\int_0^{\pi/3} \underset{\mathrm{I}}{x}\,\underset{\mathrm{II}}{\sin x}\,dx$$