

Practical 9

Plot the line segment 'L' joining the point
 $A = 0$ to $B = 2 + (\pi/4)i$ and give an exact
 value of
 $\int_L e^z dz$ over L

1

plot of the line segment L

```
(%i1) kill(all);
```

```
(%o0) done
```

```
(%i1) z(t):=(t)+%i*((%pi/8)*t);
```

```
(%o1) z(t):=t+%i\left(\frac{\pi}{8}t\right)
```

```
(%i2) wxdraw2d(
    xaxis = true, xaxis_type = solid, xrange = [-1, 3],
    yaxis = true, yaxis_type = solid, yrange = [-1, 3/2],
    proportional_axes = xy,

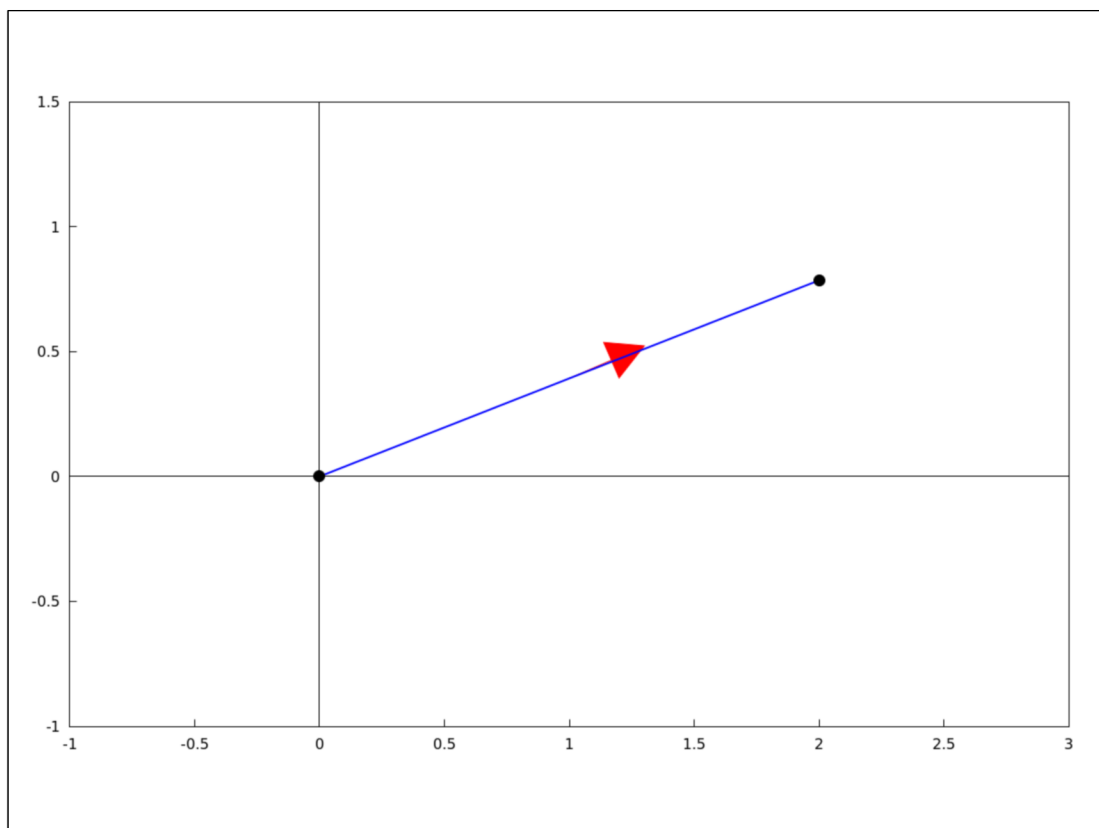
    head_length = 0.5,
    head_angle = 10,
    color = red,
    vector([1, %pi/8], [0.3, 0.13]),

    color = blue,
    line_width = 2,
    parametric(realpart(z(t)), imagpart(z(t)), t, 0, 2),

    color = black,
    point_type = 7,
    point_size = 2,
    points([[realpart(z(0)), imagpart(z(0))], [realpart(z(2)), imagpart(z(2))]])

);
```

(%t2)



(%o2)

2

evaluate the integral of e^z over L

```
(%i3) kill(all);
```

```
(%o0) done
```

```
(%i1) cIntegral(p, q, a, b):=block(
    f(z):=exp(z),
    g(t):=(p)+%i*(q),
    rectform( integrate( rectform( f(g(t))*diff(g(t), t) ), t, a, b))
);
```

```
(%o1) cIntegral(p, q, a, b):=block(f(z):=exp(z), g(t):=p+%i q
, rectform(

$$\int_a^b \text{rectform}\left(f(g(t))\left(\frac{d}{dt}g(t)\right)dt\right)$$


```

```
(%i2) cIntegral(t, (%pi/8)*t, 0, 2);
```

```
(%o2) 
$$\frac{e^2 i}{\sqrt{2}} + \frac{e^2 - \sqrt{2}}{\sqrt{2}}$$

```

3

Exercise

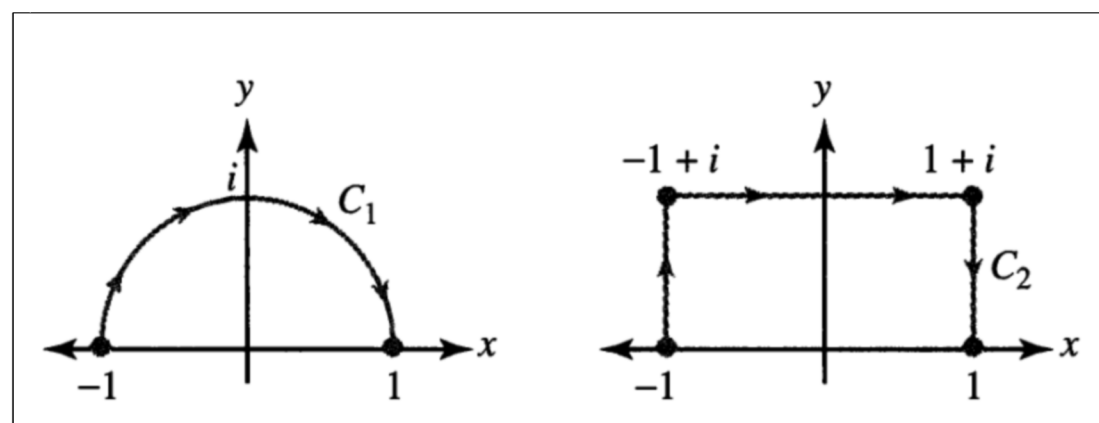
Show that

Figure 1:

$$\int_{C_1} \bar{z} dz = -\pi i \quad \text{but that} \quad \int_{C_2} \bar{z} dz = -4i,$$

where the contours C_1 and C_2 are shown below

Figure 2:



Also plot the contours