Practical 17

█████████████

Figure 1:

> 17. Evaluate $\int_{C_1^+(0)} \exp\left(\frac{2}{z}\right) dz$, where $C_1^+(0)$ denotes the circle $\{z : |z| = 1\}$ with positive orientation. Similarly evaluate $\int_{C_1^+(0)} \frac{1}{z^4 + z^3 - 2z^2} dz$.

1

→ kill(all);

(%o0) *done*

→ g(s):=exp(2/s);

(%o1) $g(s):=\exp\left(\frac{2}{s}\right)$

→ residue(g(s), s, 0);

(%o2) 0

residue() is not working here.
So we find the Laurent series of g at s = 0.

→ taylor(g(s), [s, 0, 3, 'asymp]);

(%o6)/T/ $1 + \dfrac{2}{s} + \dfrac{2}{s^2} + \dfrac{4}{3\,s^3} + \ldots$

res(g, 0) = 2.

→ I:2·%pi·%i·2;

(%o7) 4 %i π

2

Figure 2:

> Find $\displaystyle\int_{C_3^+(0)} \frac{1}{z^4 + z^3 - 2z^2} dz$

→ kill(all);

(%o0) *done*

➔   g(s):=1/(s^4+s^3−2·s^2);

(%o1) $g(s):=\dfrac{1}{s^4+s^3+(-2)s^2}$

➔   load(coma);

     coma v.2.1 (Wilhelm Haager, 2019−05−21)

(%o2)

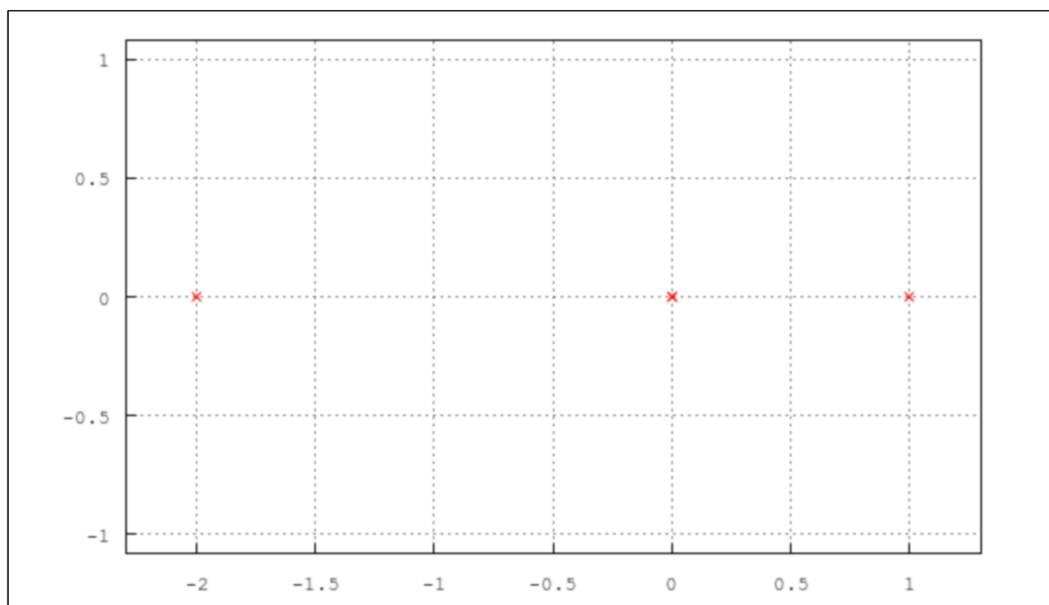   */usr/share/maxima/5.45.1/share/contrib/coma/coma.mac*

➔   zeros(g(s));

(%o3) *[ ]*

➔   poles(g(s));

(%o4) *[0,0,1.0,−2.0]*

➔   poles_and_zeros(g(s));

(%t5)

(%o5)

➔   r1:residue(g(s),   s,   0);

(%o13) $-\dfrac{1}{4}$

➔   r2:residue(g(s),   s,   1);

(%o14) $\dfrac{1}{3}$

➔   r3:residue(g(s),   s,   −2);

(%o15) $-\dfrac{1}{12}$

→   I:2·%pi·%i·(r1+r2+r3);

(%o16) 0

3

Exercise

Figure 3:

3. Evaluate

(a) $\displaystyle\int_{C_1^+(-1+i)} \frac{dz}{z^4+4}.$

(b) $\displaystyle\int_{C_2^+(i)} \frac{dz}{z\left(z^2-2z+2\right)}.$

(c) $\displaystyle\int_{C_2^+(0)} \frac{\exp z \; dz}{z^3+z}.$

(d) $\displaystyle\int_{C_2^+(0)} \frac{\sin z \; dz}{4z^2-\pi^2}.$

(e) $\displaystyle\int_{C_2^+(0)} \frac{\sin z \; dz}{z^2+1}.$