Practical 12

[redacted]

Use ML inequality to show that

| I | ≤ 1/2√5,

where I = ∫ dz/(z^2+1) (over C)
and C is the straight line segment
from 2 to 2 + i.
While solving, represent the distance
from the point z to the points i and – i,
respectively,
i.e. |z-i| amd |z+i| on the complex plane C.

Figure 1:

12. Use ML inequality to show that $\left|\int_C \frac{1}{z^2+1} dz\right| \le \frac{1}{2\sqrt{5}}$, where C is the straight line segment from 2 to $2 + i$. While solving, represent the distance from the point z to the points i and $-i$, respectively, i.e. $|z-i|$ and $|z+i|$ on the complex plane $\mathbb{C}$.

1

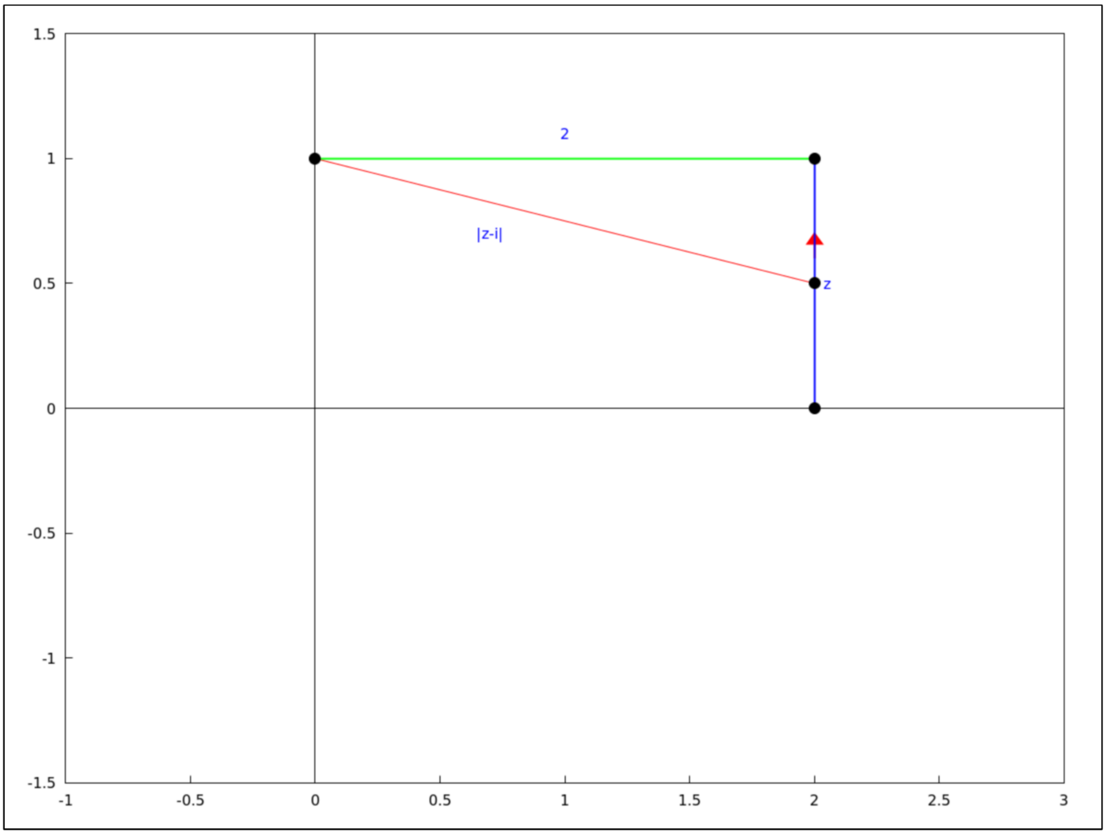
```
→ kill(all);
(%o0) done
```

Note that |z^2 +1 | = |z - i | |z + i|

A lower bound for | z - i | on C.

```
➔ wxdraw2d(
    xaxis = true, xaxis_type = solid, xrange = [−1, 3],
    yaxis = true, yaxis_type = solid, yrange = [−3/2, 3/2],
    proportional_axes = xy,

    color = red,
    parametric(t, 1+(−1/4)·(t), t, 0, 2),

    line_width = 2,
    head_length = 0.2,
    head_angle = 10,
    vector([2, 0.6], [0, 0.1]),

    color = green,
    parametric(t, 1, t, 0, 2),

    color = blue,
    parametric(2, t, t, 0, 1),
    label(["2", 1, 1.1]),
    label(["z", 2.05, 0.5]),
    label(["|z−i|", 0.7, 0.7]),

    color = black,
    point_type = 7,
    point_size = 2,
    points([[2, 0], [2, 1], [0, 1], [2, 1/2]])

);
```

(%t4)



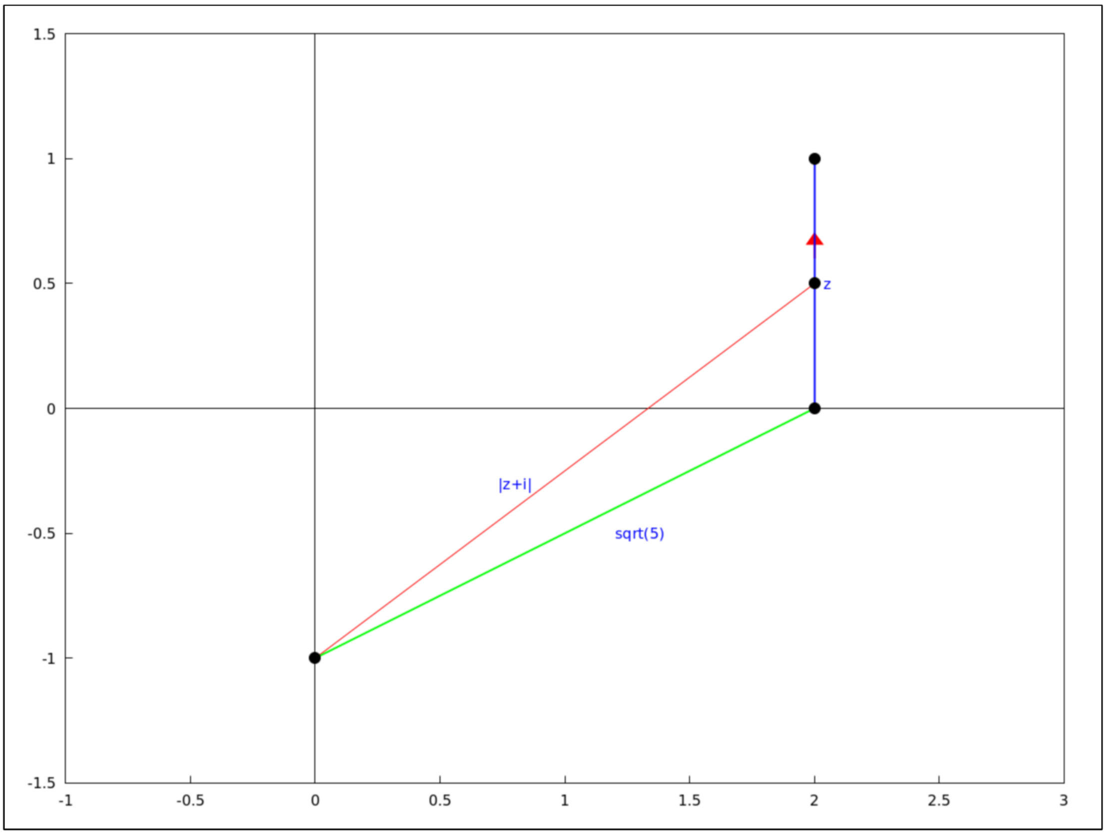
(%o4)

from the figure, $|z-i| >= 2$ when z is on C.

A lower bound for $|z + i|$ on C.

```
→ wxdraw2d(
    xaxis = true, xaxis_type = solid, xrange = [−1, 3],
    yaxis = true, yaxis_type = solid, yrange = [−3/2, 3/2],
    proportional_axes = xy,

    color = red,
    parametric(t, −1+(3/4)·(t), t, 0, 2),

    line_width = 2,
    head_length = 0.2,
    head_angle = 10,
    vector([2, 0.6], [0, 0.1]),

    color = green,
    parametric(t, −1+(1/2)·(t), t, 0, 2),

    color = blue,
    parametric(2, t, t, 0, 1),
    label(["sqrt(5)", 1.3, −0.5]),
    label(["z", 2.05, 0.5]),
    label(["|z+i|", 0.8, −0.3]),

    color = black,
    point_type = 7,
    point_size = 2,
    points([[2, 0], [2, 1], [0, −1], [2, 1/2]])


);
```

(%t5)



from the figure, |z+i| >= sqrt(5) when z is on C.

Now, |z^2 +1 | = |z - i | |z + i| >= 2 * sqrt(5) when z is on C.

Therefore, |1/(z^2 +1) | <= 1/(2 * sqrt(5)) , when z is on C.

That is, M = 1/(2 * sqrt(5)).

2

```
L:1;
M:1/(2·sqrt(5));
M·L;
```

(%o3) 1

(%o4) $\frac{1}{2\sqrt{5}}$

(%o5) $\frac{1}{2\sqrt{5}}$

By ML inequality, |I| <= 1/(2 * sqrt(5))

3

Exercise

3.1

Figure 2:

2. Let C denote the line segment from $z = i$ to $z = 1$ (Fig. 49), and show that

$$\left| \int_C \frac{dz}{z^4} \right| \le 4\sqrt{2}$$

without evaluating the integral.

Suggestion: Observe that of all the points on the line segment, the midpoint is closest to the origin, that distance being $d = \sqrt{2}/2$.

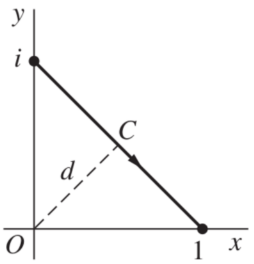


FIGURE 49

3.2

Figure 3:

3. Show that if C is the boundary of the triangle with vertices at the points 0, $3i$, and -4, oriented in the counterclockwise direction (see Fig. 50), then

$$\left| \int_C (e^z - \overline{z})\, dz \right| \leq 60.$$

Suggestion: Note that $|e^z - \overline{z}| \leq e^x + \sqrt{x^2 + y^2}$ when $z = x + iy$.

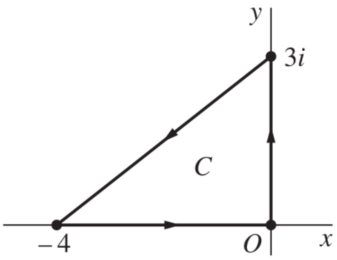


FIGURE 50