Practical 3: Complex Parametric Forms of Conic Sections

Objective

To represent conic sections (circle, ellipse, hyperbola, and parabola) using complex parametric functions of the form $s(t) = x(t) + iy(t)$, where $x(t)$ and $y(t)$ are real-valued functions.

General Notation

- $s(t)$: Complex-valued function representing the curve.
- a: Shape parameter — semi-major axis (ellipse), scale (parabola, hyperbola).
- b: Semi-minor axis (ellipse and vertical hyperbola).
- r: Radius (for circle).
- t: Real parameter.

Complex Parametric Forms of Conic Sections

Conic Type	Real Equation	Complex Parametric Form	Comment
Circle	$x^2 + y^2 = r^2$	$s(t) = r\cos t + ir\sin t$	Circle of radius r, centered at origin.
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$s(t) = a\cos t + ib\sin t$	Stretched circle. a: semi-major, b: semi-minor axis.
Hyperbola (vertical)	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$	$s(t) = a\tan t + ib\sec t$	Derived from identity $\sec^2 t - \tan^2 t = 1$. Opens up/down.
Hyperbola (rectangular)	$xy = 1$	$s(t) = t + i\frac{1}{t}$	Exponential form where $x = e^t, y = e^{-t}$. Symmetric hyperbola.
Parabola (horizontal)	$y^2 = 4ax$	$s(t) = at^2 + i \cdot 2at$	Opens rightward. Vertex at origin.

Parabola (vertical)	$x^2 = 4ay$	$s(t) = 2at + i \cdot at^2$	Opens upward. Vertex at origin.

Conclusion

Using complex-valued parametric functions, conic sections can be uniformly expressed and visualized as $s(t) = x(t) + iy(t)$. This is especially useful in software like Maxima, where plotting complex functions is efficient and elegant.