Practical 11

Show that
∫zdz over C1 = ∫zdz over C2 = 4 + 2i
where C1 is the line segment from –1 – i to 3 + i and
C2 is the portion of the parabola x = y^2 + 2y joining –1 – i to 3 + i.
Make plots of two contours C1 and C2 joining –1 – i to 3 + i .

1

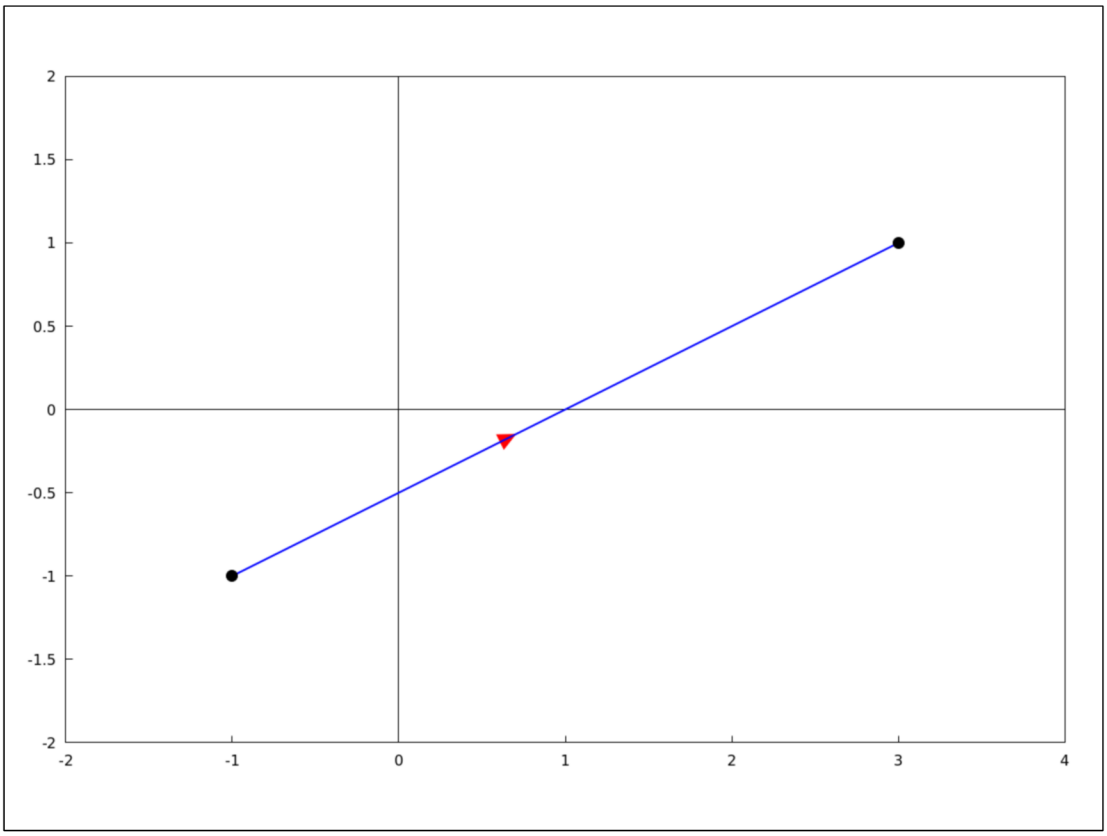
C1

```
(%i1) kill(all);
(%o0) done
(%i2) z(t):=(t)+%i·(1+((1/2)·(t−3)));
```

$$(\%o2)\quad z(t):=t+\%i\left(1+\frac{1}{2}(t-3)\right)$$

```
wxdraw2d(
  xaxis = true, xaxis_type = solid, xrange = [−2,  4],
  yaxis = true, yaxis_type = solid, yrange = [−2,  2],
  proportional_axes = xy,

  head_length = 0.3,
  head_angle = 10,
  color = red,
  vector([1/2,  −1/4],  [1/5,  1/10]),

  nticks = 500,
  color = blue,
  line_width = 2,
  parametric(realpart(z(t)),  imagpart(z(t)),  t,  −1,  3),

  color = black,
  point_type = 7,
  point_size = 2,
  points( [ [realpart(z(3)),  imagpart(z(3))],
      [realpart(z(−1)),  imagpart(z(−1))] ] )

);
```

(%t13)



(%o13)

1.1

evaluate the integral of z over C1

```
(%i) kill(all);
(%o0) done

(%i) cIntegral(p, q, a, b):=block(
       f(z):=z,
       z(t):=(p)+%i·(q),
       rectform( integrate( rectform( f(z(t))·diff(z(t), t) ), t, a, b))
       );
(%o1) cIntegral(p,q,a,b):=block(f(z):=z,z(t):=p+%i q,
      rectform(∫_a^b rectform(f(z(t))(d/(d t) z(t))) d t))

(%i) cIntegral(t, 1+((1/2)·(t−3)), −1, 3);
(%o2) 2 %i+4
```

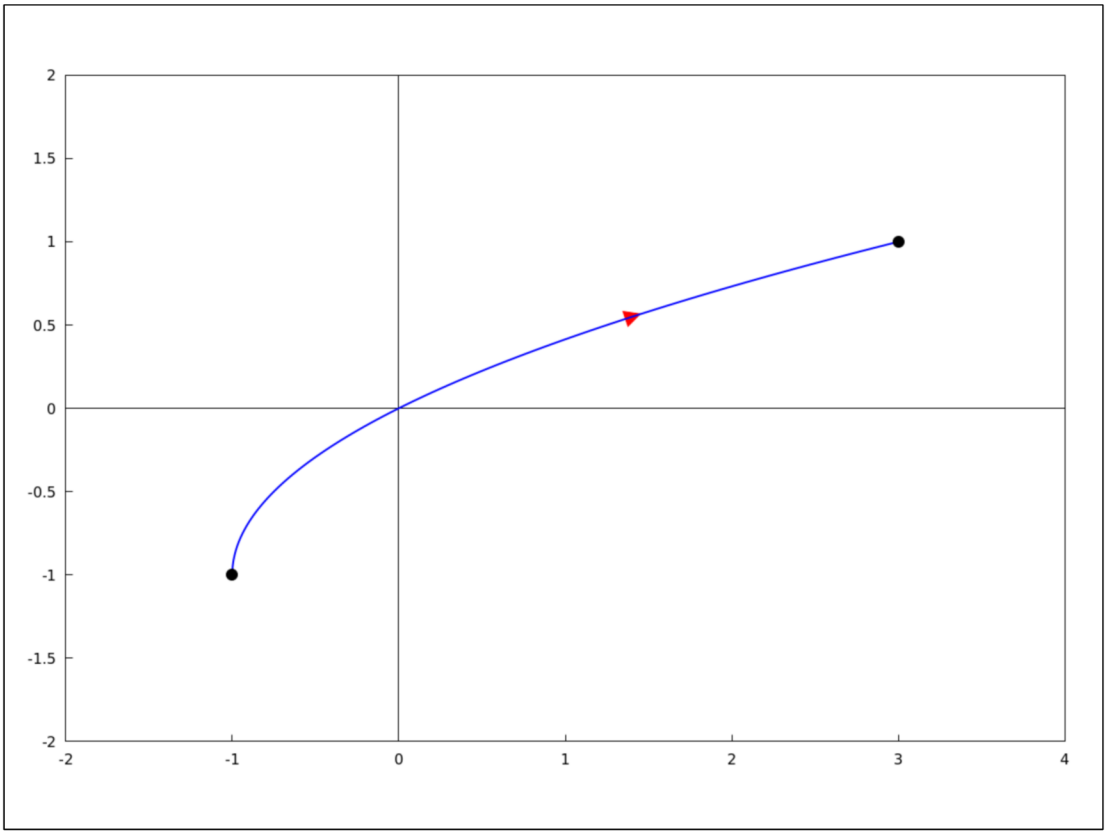
2

C2

```
(%i) kill(all);
(%o0) done

(%i) z(t):=(t^2+2·t)+%i·(t);
(%o1) z(t):=t^2+2 t+%i t
```

```
wxdraw2d(
  xaxis = true, xaxis_type = solid, xrange = [−2,  4],
  yaxis = true, yaxis_type = solid, yrange = [−2,  2],
  proportional_axes = xy,

  head_length = 0.3,
  head_angle = 10,
  color = red,
  vector([5/4,  1/2],  [1/5,  1/15]),

  nticks = 500,
  color = blue,
  line_width = 2,
  parametric(realpart(z(t)),  imagpart(z(t)),  t,  −1,  1),

  color = black,
  point_type = 7,
  point_size = 2,
  points( [ [realpart(z(1)),  imagpart(z(1))],
        [realpart(z(−1)),  imagpart(z(−1))] ] )

);
```

(%t7)



(%o7)

2.1

evaluate the integral of z over C2

```
(%i) kill(all);
(%o0) done

(%i) clntegral(p, q, a, b):=block(
       f(z):=z,
       z(t):=(p)+%i·(q),
       rectform( integrate( rectform( f(z(t))·diff(z(t), t) ), t, a, b))
       );
(%o1) clntegral(p,q,a,b):=block(f(z):=z,z(t):=p+%i q,
      rectform(∫_a^b rectform(f(z(t))(d/dt z(t)))dt))

(%i) clntegral((t^2+2·t), t, −1, 1);
(%o2) 2 %i +4
```

3

Exercise

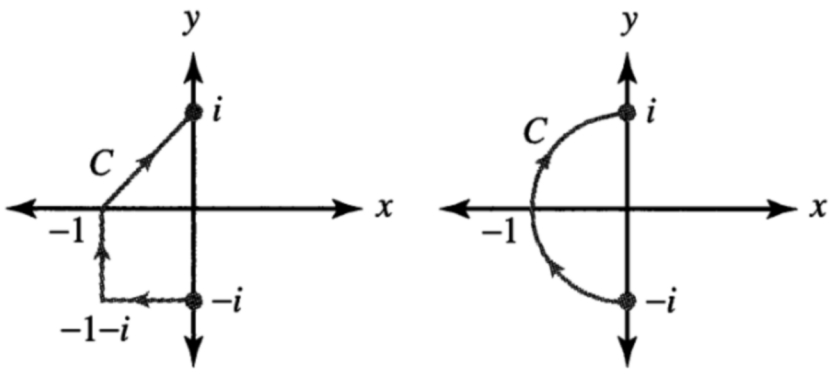
Evaluate the integral of y over C in the cases

Figure 1:

(a) The polygonal path C with vertices $-i$, $-1-i$, -1, and i.

Figure 2:

(b) The contour C that is oriented clockwise, as shown in



Also plot the contour C in each case.