

Practical 16

Figure 1:

16. Locate the zeros and poles of $g(z) = \frac{\pi \cot(\pi z)}{z^2}$ and determine their order. Also justify that $\text{Res}(g, 0) = -\pi^2/3$.

1

```
→ load(coma);
coma v.1.84, (Wilhelm Haager, 2017-02-24)
(%o1) /usr/share/maxima/5.43.2/share/contrib/coma/coma.mac
```

```
→ g(s):=%pi*cot(%pi*s)/s^2;
(%o2) g(s):= \frac{\pi \cot(\pi s)}{s^2}
```

```
→ zeros(g(s));
(%o3) []
```

```
→ poles(g(s));
(%o4) []
```

$g(s) = \pi \cos(\pi s) / s^2 \sin(\pi s)$.
 $\pi \cos(0) = \pi$ and $s^2 \sin(\pi s)$ has a zero
of order 3 at $s = 0$.
 $\Rightarrow g(s)$ has a pole of order 3 at 0.

```
→ residue(g(s), s, 0);
(%o5) -\frac{\pi^2}{3}
```

2

Figure 2:

Find $\text{Res}[g, 0]$ if $g(z) = \frac{3}{2z + z^2 - z^3}$.

→ `f(s):=3/(2·s+s^2-s^3);`

(%o12) $f(s) := \frac{3}{2s + s^2 - s^3}$

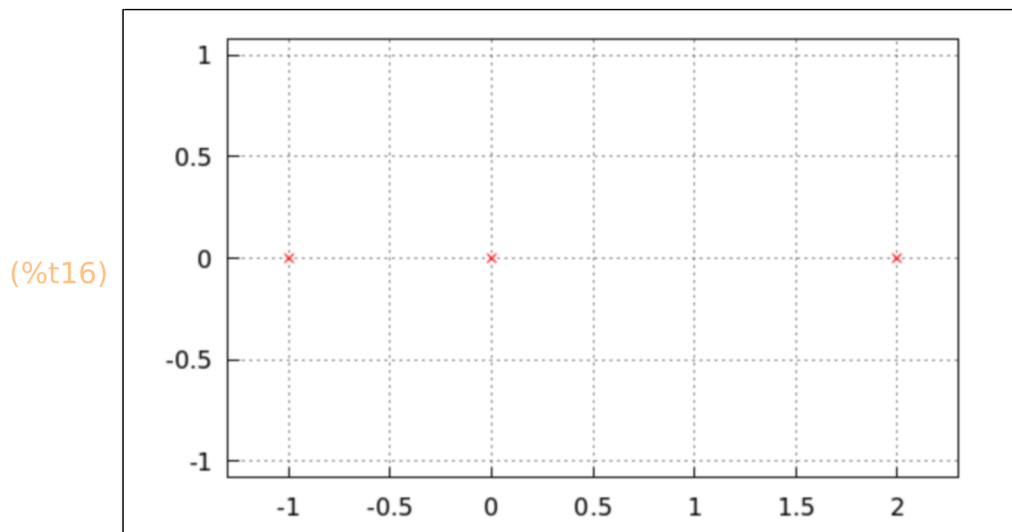
→ `zeros(f(s));`

(%o13) **`[]`**

→ `poles(f(s));`

(%o14) **`[0, -1.0, 2.0]`**

→ `poles_and_zeros(f(s));`



→ `residue(f(s), s, 0);`

(%o18) $\frac{3}{2}$

→ `residue(f(s), s, -1);`

(%o19) -1

→ `residue(f(s), s, 2);`

(%o20) $-\frac{1}{2}$

3

Exercise

Figure 3:

1. Find $\text{Res}[f, 0]$ for

(a) $f(z) = z^{-1} \exp z.$

(b) $f(z) = z^{-3} \cosh 4z.$

(c) $f(z) = \csc z.$

(d) $f(z) = \frac{z^2 + 4z + 5}{z^2 + z}.$

(e) $f(z) = \cot z.$