Practical 7

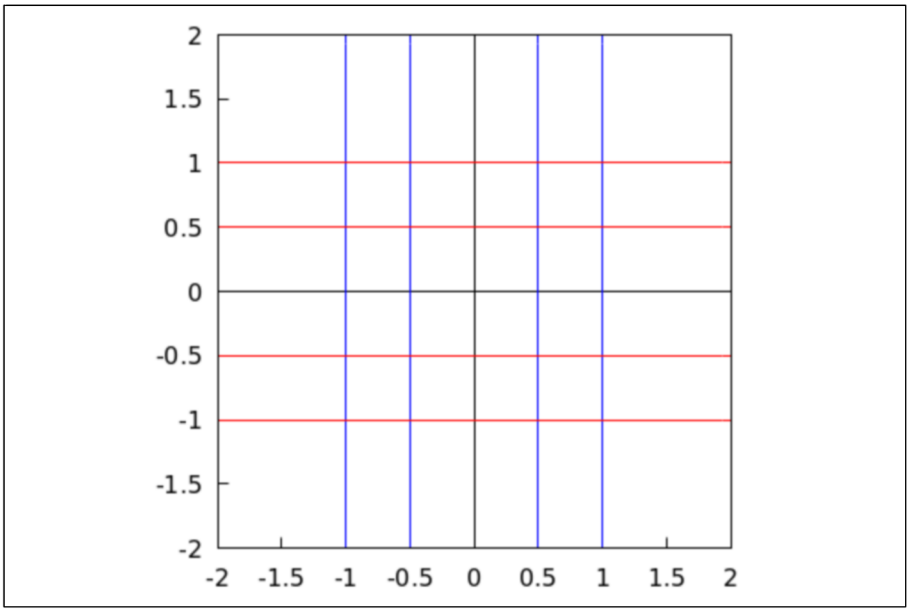
Make a plot of the vertical lines x = a,
for a = −1, − 1/2 , 1/2 , 1
and the horizontal lines
y = b,
for b = −1, −1/2 , 1/2 , 1.
Find the plot of this grid under the mapping
w = f(z) = 1/z.

1

```
(%i) kill(all);
(%o0) done

(%i) f(z):=block(
       [x, y],
       x:realpart(z),
       y:imagpart(z),
       w:rectform(1/(x+y·%i) )
     );
(%o1) f(z):=block([x,y],x:realpart(z),y:imagpart(z),w:rectform(1/(x+y %i)))

(%i) r(t, s):=(t+%i·s);
(%o2) r(t,s):=t+%i s

(%i) h:makelist(parametric(realpart(r(t, s)), imagpart(r(t, s)), t, −3, 3), s, [−1, -
     v:makelist(parametric(realpart(r(t, s)), imagpart(r(t, s)), s, −3, 3), t, [−1, -
(h) [parametric(t,−1,t,−3,3),parametric(t,−1/2,t,−3,3),
    parametric(t,1/2,t,−3,3),parametric(t,1,t,−3,3)]
(v) [parametric(−1,s,s,−3,3),parametric(−1/2,s,s,−3,3),
    parametric(1/2,s,s,−3,3),parametric(1,s,s,−3,3)]
```

```
wxdraw2d(
  xaxis = true,  xaxis_type = solid,  xrange = [−2,  2],
  yaxis = true,  yaxis_type = solid,  yrange = [−2,  2],
  proportional_axes = xy,
  v,
  color = red,
  h
);
```

(%t10)



(%o10)

```
w(t,  s):=f(r(t,  s));
```

(%o11) $$w(t,s):=f(r(t,s))$$

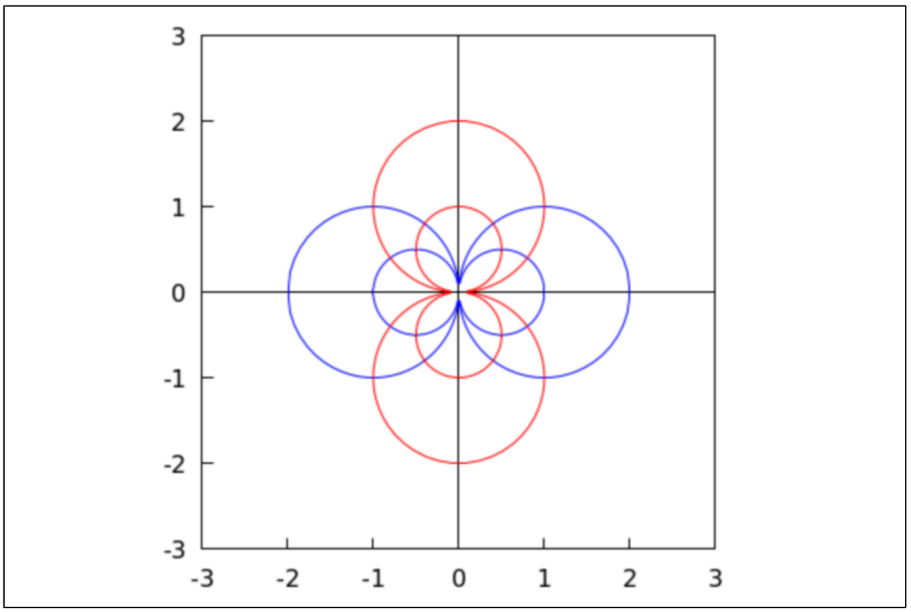
```
hImage:makelist(parametric(realpart(w(t, s)), imagpart(w(t, s)), t, -10, 10),
vImage:makelist(parametric(realpart(w(t, s)), imagpart(w(t, s)), s, -10, 10),
```

(hImage)
$$\left[\text{parametric}\left(\frac{t}{t^2+1},\frac{1}{t^2+1},t,-10,10\right),\ \text{parametric}\left(\frac{t}{t^2+\frac{1}{4}},\frac{1}{2\left(t^2+\frac{1}{4}\right)},t,-10,10\right),\ \text{parametric}\left(\frac{t}{t^2+\frac{1}{4}},-\frac{1}{2\left(t^2+\frac{1}{4}\right)},t,-10,10\right),\ \text{parametric}\left(\frac{t}{t^2+1},-\frac{1}{t^2+1},t,-10,10\right)\right]$$

(vImage)
$$\left[\text{parametric}\left(-\frac{1}{s^2+1},-\frac{s}{s^2+1},s,-10,10\right),\ \text{parametric}\left(-\frac{1}{2\left(s^2+\frac{1}{4}\right)},-\frac{s}{s^2+\frac{1}{4}},s,-10,10\right),\ \text{parametric}\left(\frac{1}{2\left(s^2+\frac{1}{4}\right)},-\frac{s}{s^2+\frac{1}{4}},s,-10,10\right),\ \text{parametric}\left(\frac{1}{s^2+1},-\frac{s}{s^2+1},s,-10,10\right)\right]$$

```
wxdraw2d(
  xaxis = true,  xaxis_type = solid,  xrange = [−3,   3],
  yaxis = true,  yaxis_type = solid,  yrange = [−3,   3],
  proportional_axes = xy,
  nticks = 500,
  vImage,
  color = red,
  hImage
);
```

(%t17)



(%o17)

2

Exercise

Find the image of the given circle or line under the reciprocal transformation w = 1/z.

Figure 1:

1. The horizontal line $\mathrm{Im}\,(z) = \frac{1}{5}$.
2. The circle $C_{\frac{1}{2}}\left(-\frac{i}{2}\right) = \left\{z : \left|z + \frac{i}{2}\right| = \frac{1}{2}\right\}$.
3. The vertical line $\mathrm{Re}\ z = -3$.
4. The circle $C_1\,(-2) = \{z : |z + 2| = 1\}$.