Algorithm 1 Inverse Kinematics with Redundancy Handling and Joint Limits

Require: $\mathbf{x}_{target}$: Target position and quaternion orientation
$\mathbf{q}_{seed}$: Initial joint configuration (seed)
$\mathbf{q}_{min}$: Vector of lower joint limits
$\mathbf{q}_{max}$: Vector of upper joint limits
ϵ: Tolerance for convergence
λ: Damping factor
α: Secondary objective gain
maxIterations: maximum number of iterations

Initialize $\mathbf{q} \leftarrow \mathbf{q}_{seed}$
$\lambda \leftarrow 1 \times 10^{-6}$
$\alpha \leftarrow 0.01$
for $i = 1$ to maxIterations **do**
  Compute forward kinematics $\mathbf{T} \leftarrow \mathbf{T}(\mathbf{q})$
  Compute Jacobian $\mathbf{J} \leftarrow \mathbf{J}(\mathbf{q})$
  Compute error $\mathbf{e} \leftarrow \mathbf{x}_{target} - \mathbf{x}(\mathbf{T})$
  if $\|\mathbf{e}\| < \epsilon$ **then**
    return $\mathbf{q}$
  end if
  Compute damped pseudo-inverse $\mathbf{J}_{\text{pinv}} \leftarrow \mathbf{J}^T(\mathbf{J}\mathbf{J}^T + \lambda\mathbf{I})^{-1}$
  Compute null-space projection matrix $\mathbf{N} \leftarrow \mathbf{I} - \mathbf{J}_{\text{pinv}}\mathbf{J}$
  Compute secondary objective:
  $\mathbf{q}_{mid} \leftarrow \frac{\mathbf{q}_{max}+\mathbf{q}_{min}}{2}$
  $\mathbf{q}_{diff} \leftarrow \mathbf{q} - \mathbf{q}_{mid}$
  $\mathbf{q}_{sec} \leftarrow -\alpha\mathbf{q}_{diff}$
  Compute joint update $\Delta\mathbf{q} \leftarrow \mathbf{J}_{\text{pinv}}\mathbf{e} + \mathbf{N}\mathbf{q}_{sec}$
  Update joint angles $\mathbf{q} \leftarrow \mathbf{q} + \Delta\mathbf{q}$
  Enforce joint limits:
  for $j = 1$ to size($\mathbf{q}$) **do**
    if $\mathbf{q}[j] < \mathbf{q}_{min}[j]$ **then**
      $\mathbf{q}[j] \leftarrow \mathbf{q}_{min}[j]$
    end if
    if $\mathbf{q}[j] > \mathbf{q}_{max}[j]$ **then**
      $\mathbf{q}[j] \leftarrow \mathbf{q}_{max}[j]$
    end if
  end for
end for
Print "Max number of iterations reached. Solution found is proximal."
return $\mathbf{q}$