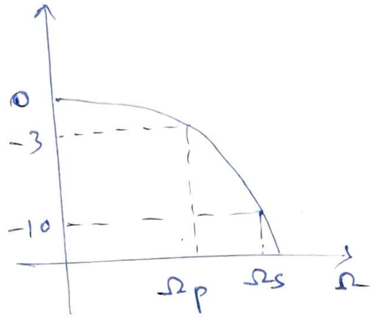
WEEK-1 / QUESTION-3

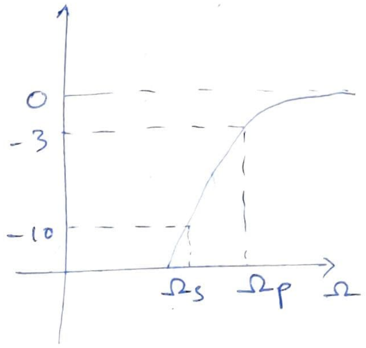
$\alpha_p = 3\,dB$; $\omega_c = \omega_p = 2 \times \pi \times 1000 = 2000\pi$ rad/s

$\alpha_s = 10\,dB$; $\omega_s = 2 \times \pi \times 350 = 700\pi$ rad/s

$T = \frac{1}{f} = \frac{1}{5000} = 2 \times 10^{-4}$ sec.



Low pass



High pass

To design a butterworth filter:-

prewarping the digital frequencies;

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p T}{2} = \frac{2}{2 \times 10^{-4}} \tan \frac{(2000\pi \times 2 \times 10^{-4})}{2}$$

$$= 10^4 \tan(0.2\pi)$$

$$= 7265 \text{ rad/s.}$$

$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s T}{2} = \frac{2}{2 \times 10^{-4}} \tan \frac{(700\pi \times 2 \times 10^{-4})}{2}$$

$$= 10^4 \tan(0.07\pi)$$

$$= 2235 \text{ rad/s.}$$

The order of the filter:-

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1 a_s} - 1}{10^{0.1 a_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

$$= \frac{\log \sqrt{\dfrac{10 - 1}{10^{0.3} - 1}}}{\log \dfrac{7265}{2235}}$$

$$= \frac{\log(3)}{\log(3.25)} = \frac{0.4771}{0.5118} = 0.932$$

$\Rightarrow \quad N = 1.$

tr. func. for High pass filter.

$$H(s) = \frac{1}{s+1}\Big|_{s = \frac{7265}{s}}$$

$$= \frac{s}{s + 7265}$$

using Bilinear transformation.

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265}\Big|_{s = \frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{10000\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right) + 7265} = \frac{0.5792\,(1-z^{-1})}{1 - 0.15842\,z^{-1}}$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{0.5792\,(1 - z^{-1})}{1 - 0.1584\, z^{-1}}$$

$$\Rightarrow Y(z) - 0.1584\, z^{-1}\, Y(z) = 0.5792\, X(z) - z^{-1} X(z)$$

$$0.5792$$

Applying IZT :-

$$\Rightarrow y(n) - 0.1584\, y(n-1) = 0.5792\, x(n) - x(n-1)$$

$$0.5792.$$