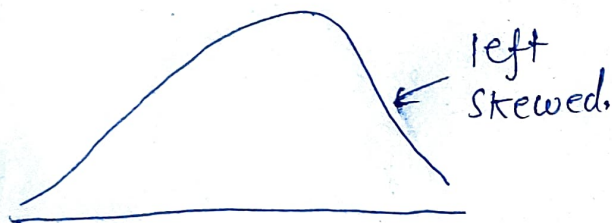
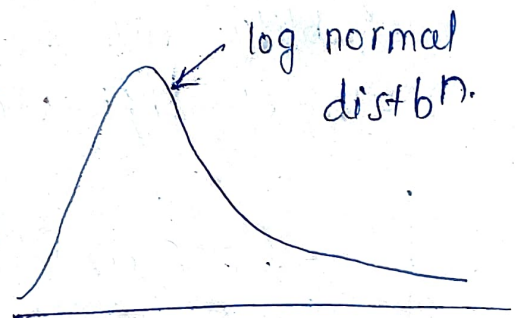
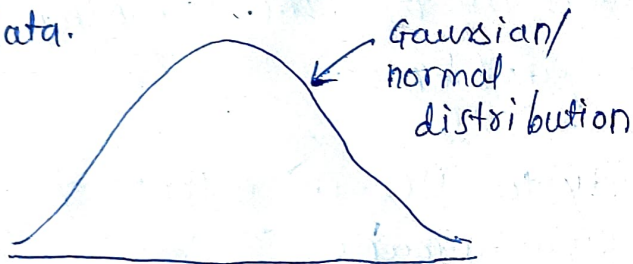


Stats Basic

- ① central limit theorem
- ② probability
- ③ permutations and combinations
- ④ covariance, pearson co-relation, Spearman co-relation.
- ⑤ Bernauli's distribution
- ⑥ Binomial distribution.
- ⑦ Power law (pareto distribution)

* Central limit theorem

population data.



In any population data (N) with any distribution where sample data (n) , consider m such samples as below:

$$S_1 = \{x_1, x_2, x_3, \dots, x_n\} \Rightarrow \bar{x}_1$$

$$S_2 = \{x_1, x_2, x_3, \dots, x_n\} \Rightarrow \bar{x}_2$$

$$S_m = \{x_1, x_2, x_3, \dots, x_n\} \Rightarrow \bar{x}_m$$

for $n \geq 30$

when the means of m samples are plotted will follow a gaussian distribution (approximately).

* sampling Technique: sampling with replacement.

* probability

It is a measure of likely hood of an event.

Eg: Tossing a fair coin.

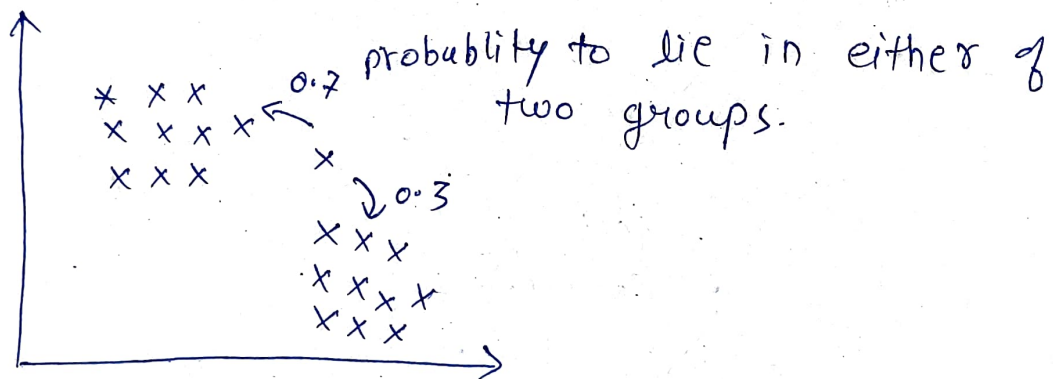
$$\text{prob}(H) = 0.5$$

$$P(T) = 0.5$$

Eg: Rolling a dice

$$P(1) = \frac{1}{6}, \quad P(2) = \frac{1}{6}, \quad P(3) = \frac{1}{6} \dots$$

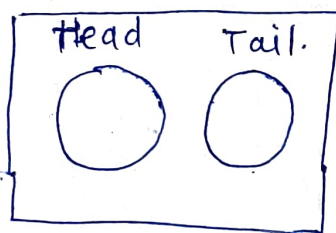
* probability will be used in machine learning.
ie in classification problems.



① Mutually Exclusive events

Two events are said to be mutually exclusive if they cannot occur at same time.

Eg: 1) Tossing a coin 2) Rolling a dice.



② Non-mutual Exclusive Event

Two events can occur at the same time.

Eg: Picking randomly a card from a deck of cards, two events,

"heart" and "king" can be selected



A) mutually exclusive event

Que: 1) what is the probability of a coin landing on heads or tails.

(Addition rule for mutually exclusive event)

$$P(A \text{ or } B) = P(A) + P(B)$$

$$= 0.5 + 0.5$$

$$= 1$$

2) what is the probability of getting 1 or 6 or 3 while rolling a dice.

$$P(1 \text{ or } 6 \text{ or } 3) = P(1) + P(6) + P(3)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

$$= 0.5$$

B) Non-mutually Exclusive Event

Que: 1 In Bag of marbles.

10 Red, 6 Green 3 (R&G)

when picking randomly from a bag of marbles
what is probability of choosing a marble that
is red or green.

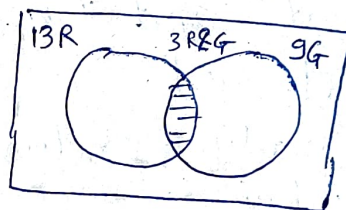
(Addition rule for non-mutual Exclusive event)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\therefore P(\text{Red}) = \cancel{10/19} \cdot 13/19$$

$$P(\text{Green}) = \cancel{6/19} \cdot 9/19$$

$$P(\text{R\&G}) = \cancel{3/19} \cdot 3/19$$



$$P(\text{Red or Green}) = \cancel{10/19} + \cancel{6/19} - \cancel{3/19}$$
$$= \cancel{13/19}$$

$$= 13/19 + 9/19 - 3/19 = 19/19 = 1$$

Que: 2 Deck of cards

probability of choosing Heart or Queen card.

$$P(\heartsuit \text{ or } Q) = P(\heartsuit) + P(Q) - P(\heartsuit \text{ and } Q)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= 4/13$$

Multiplication rule.

1) Dependent Event:

Two events are dependent, if they affect one another.

Eg: Bag of marble $\left\{ \begin{array}{l} 4 \text{ w} - \circ \circ \circ \circ \\ 3 \text{ y} - \circ \circ \circ \end{array} \right\}$.

$$P(w) = \frac{4}{7} \xrightarrow[\text{that 1 yellow}]{\text{After}} P(y) = \frac{3}{6}$$

ie $P(y)$ depends on $P(w)$.

Que: what is the probability of rolling a 5 and then a 3 with a normal 6 sided dice.

(Independent event)

$$\Rightarrow P(5) = \frac{1}{6}, \quad P(3) = \frac{1}{6}$$

multiplication rule for independent event.

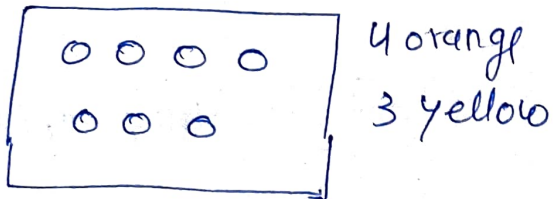
$$P(A \text{ and } B) = P(A) * P(B)$$

$$P(\text{Req}) = P(5) \times P(3)$$

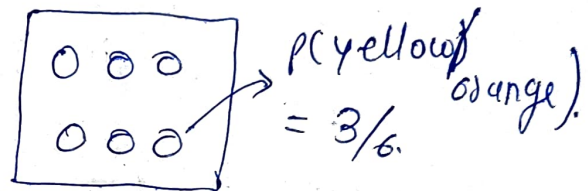
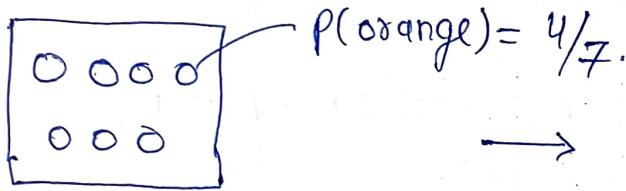
$$= \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{36}$$

Que: 2 Bag of marbles



what is probability of drawing a "orange"
and then a "yellow" marble from the bag:
(Dependent Event)



conditional probability

$$P(o \text{ and } y) = P(o) \times P(y/o)$$

$$= \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$$

ie $P(\text{yellow when orange has happened})$

$$= P(y/o) = 3/6$$

* Permutations

chocolate

{ Dairy milk, Kit Kat, milky bar, sneakers, 5 stars }

$$* \underline{5} \times \underline{4} \times \underline{3} = 60 \text{ ways to pick 3 chocolates.}$$

Note: with permutations order matters..

ie All possible Arrangements will be counted.

$$\text{permutation} = {}^n P_r$$

$n \equiv$ total No. of objects

$r \equiv$ No. of selections

$${}^n P_r = \frac{n!}{(n-r)!}$$

\therefore for above example

$$n=5$$

$$r=3$$

$$\text{permutations} = {}^5 P_3 = \frac{5!}{(5-3)!} = \frac{5 \times 4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1}}$$

$$* \text{ combinations } {}^n C_r = 60$$

(Repetitions will not occur)

ie unique arrangements are only allowed.

$${}^n C_r = \frac{n!}{r! (n-r)!}$$

\therefore for above chocolate example

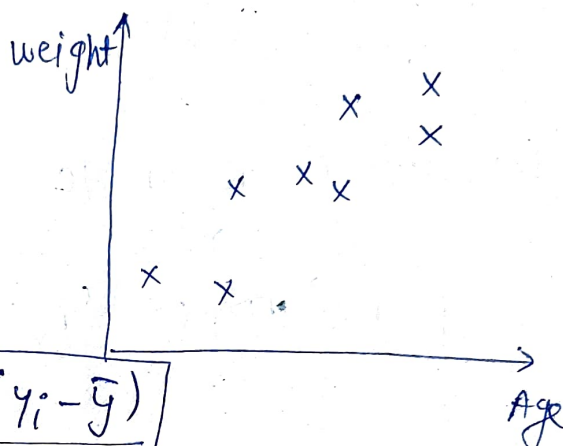
$${}^5 C_3 = \frac{5!}{3! (5-3)!} = \frac{5!}{3! \times 2!} = \frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3} \times \cancel{2} \times 1}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{2} \times 1} = 10 \text{ combinations.}$$

* Covariance (Feature selection)

Age	weight
12	40
13	45
15	48
17	60
18	62

Age ↑ weight ↑

Age ↓ weight ↓



$$\text{Cov}(X, Y) = \frac{\sum (x_i - \bar{x}) \times (y_i - \bar{y})}{n-1}$$

Note: Variance (X), $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{\sum (x_i - \bar{x}) (x_i - \bar{x})}{n-1}$

$$\sigma^2 = \text{Cov}(X, X)$$

covariance of (X, X) is variance of X.

for above example

$$\bar{x} = \frac{12+13+15+17+18}{5} = 15$$

$$\bar{y} = 51$$

$$\text{Cov}(\text{Age}, \text{weight}) = (12-15)(40-51) + (13-15)(45-51) + (15-15)(48-51) + (17-15)(60-51) + (18-15)(62-51)$$

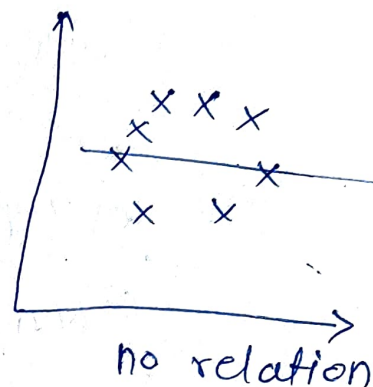
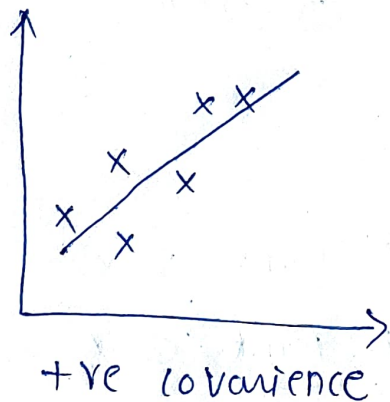
$$= \frac{(-3 \times -11) + (-2 \times -6) + 0 + 2 \times 9 + 3 \times 11}{5}$$

$$\text{Cov}(\text{Age}, \text{weight}) = 19.2$$

* +ve covariance $\Rightarrow X \uparrow Y \uparrow$ or $X \downarrow Y \downarrow$

* -ve covariance $\Rightarrow X \downarrow Y \uparrow$ or $X \uparrow Y \downarrow$

* Zero covariance \Rightarrow No relation b/w X and Y .

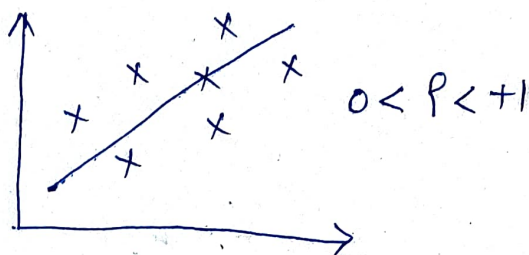
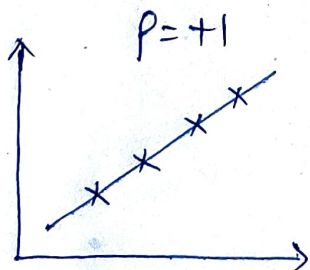
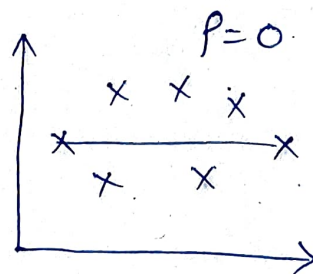
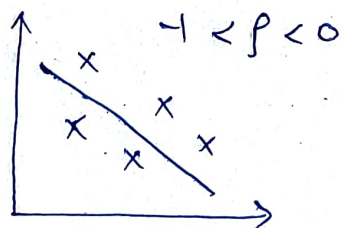
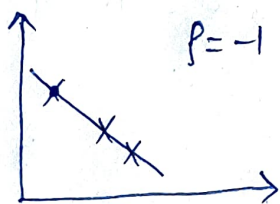


* Pearson co-relation coefficient $(-1 \text{ to } 1)$.

$$r(x, y) = \frac{\text{cov}(x, y)}{\sigma_x \times \sigma_y}$$

* covariance has no limit to its value so to limit it, we use $PCC (r)$.

* more +ve value towards +1 more +ve co-related variables X and Y .



* Spearman's Rank co-relation.

* Pearson co-relation holds good for only linear data

* For non-linear data, we have to use Spearman's Rank co-relation.

$$r_s = \frac{\text{cov}(R(x), R(y))}{\sigma_{R(x)} \times \sigma_{R(y)}}$$

$R(x) \Rightarrow$ Rank of x .

$R(y) \Rightarrow$ Rank of y .

X	Y	R(x)	R(y)
10	4	4	1
8	6	3	2
7	8	2	3
6	10	1	4

$$\text{mean}(R(y)) =$$

$$\text{mean}(R(x)) = \frac{4+3+2+1}{4} = 2.5$$

$$\sigma_{R(x)}^2 = \sigma_{R(y)}^2 =$$

$$= \frac{(4-2.5)^2 + (3-2.5)^2 + (2-2.5)^2 + (1-2.5)^2}{4-1}$$

$$\sigma^2 = 1.67$$

$$\text{cov}(R(x), R(y)) =$$

$$\frac{(4-2.5)(1-2.5) + (3-2.5)(2-2.5) + (2-2.5)(3-2.5) + (1-2.5)(4-2.5)}{4-1}$$

$$\sigma_{R(y)} = \sigma_{R(x)} = 1.29$$

$$\text{cov}(R(x), R(y)) = -1.67$$

$$r_s = \frac{\text{cov}(R(x), R(y))}{\sigma_{R(x)} \times \sigma_{R(y)}} = \frac{-1.67}{1.29 \times 1.29} = -1$$

$$\therefore r_s = -1$$

where and
* why these co-relations will be used.??
I/p



- 1) x and o/p High co-related. \Rightarrow x is important feature
- 2) y and o/p very low co-related \Rightarrow y can be dropped.
- 3) x and z are 95% co-related \Rightarrow either of x and z can be dropped.
I/p o/p

Eg:

Experience	Degree	City
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Salary

- 1) exp and salary \Rightarrow +ve co-relation
- 2) city and salary \Rightarrow +ve co-relation.
- 3) exp and degree \Rightarrow no relation
- 4) ~~exp~~ deg and salary \Rightarrow +ve co-relation.