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FOSP ASSIGNMENT 2

Q1)

Sol. $x[n] = \delta[n] + 2u[n-1] + u[n-2] + \delta[n-3] + -3u[n-4]$

$= \delta[n] = \{0, \underset{\uparrow}{1}, 0, 0 \ldots\}$

$u[n-1] = \{\ldots 0, \underset{\uparrow}{0}, 1, 1, 1 \ldots\}$

$u[n-2] = \{\ldots 0, \overset{\uparrow}{0}, 0, 1, 1 \ldots\}$

$\delta[n-3] = \{\ldots 0, \overset{\uparrow}{0}, 0, 0, 1, 0 \ldots\}$

$u[n-4] = \{\ldots 0, \underset{\uparrow}{0}, 0, 0, 0, 1, 1 \ldots\}$

$x[n] = \{\ldots 0, \underset{\uparrow}{1}, 2, 3, 4, 0 \ldots\}$

$x[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$

4pt. DFT

$x[n] \to T[x[n]] \to X[k]$

$X[k] = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$X(k) = \{10, -2+2j, -2, -2-2j\}$

b) $x[n] = 3\cos(0.5\pi n)$

$\omega \cancel{n} = 0.5\pi \cancel{n}$

$2\cancel{\pi} f = 0.5\cancel{\pi}$

$f = 0.25 = \frac{1}{4} = \frac{k}{N}$

$N = 4$

Thus $x[n]$ is periodic

$x[n] = \{\underset{\uparrow}{1}, 0, -1, 0\}$

DFT:

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-1 \\ 1+1 \\ 1-1 \\ 1+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \end{bmatrix}$$

$X[k] = \{0, 2, 0, 2\}$

c) $x[n] = \{1, 2, 3, 4, 0, 0, 0, 0\}$

$$X[k] = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w^1 & w^2 & w^3 & w^4 & w^5 & w^6 & w^7 \\ 1 & w^2 & w^4 & w^6 & w^8 & w^{10} & w^{12} & w^{14} \\ 1 & w^3 & w^6 & w^9 & w^{12} & w^{15} & w^{18} & w^{21} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & w^5 & w^{10} & w^{15} & w^{20} & w^{25} & w^{30} & w^{35} \\ 1 & w^6 & w^{12} & w^{18} & w^{24} & w^{30} & w^{36} & w^{42} \\ 1 & w^7 & w^{14} & w^{21} & w^{28} & w^{35} & w^{42} & w^{49} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$w^1 = \cos\left(\frac{-\pi}{4}\right) + \sin(-\pi/4)\, j$

$w^1 = 0.707 - 0.707j$ $\qquad$ $w^9 = 0.707 - 0.707j$

$w^2 = -j$ $\qquad$ $w^{10} = -j$

$w^3 = -0.707 - 0.707j$ $\qquad$ $w^{11} = -0.707 - 0.707j$

$w^4 = -1$ $\qquad$ $w^{12} = -1$

$w^5 = -0.707 + 0.707j$ $\qquad$ $w^{13} = -0.707 + 0.707j$

$w^6 = j$ $\qquad$ $w^{14} = j$

$w^7 = 0.707 + 0.707j$ $\qquad$ $w^{15} = 0.707 + 0.707j$

$w^8 = 1$ $\qquad$ $w^{16} = 1$

and so on

$$X[k] = \begin{bmatrix} 1+2+3+4 \\ w^1+2w^2+3w^3+4w^4 \\ w^2+2w^4+3w^6+4w^8 \\ w^3+2w^6+3w^9+4w^{12} \\ 1-2+3-4 \\ w^5+2w^{10}+3w^{15}+4w^{20} \\ w^6+2w^{12}+3w^{18}+4w^{24} \\ w^7+2w^{14}+3w^{21}+4w^{28} \end{bmatrix} = \begin{bmatrix} 10 \\ -5.414 - \\ \\ \\ -2 \\ \\ \\ \\ \end{bmatrix}$$

$$X[k] = \begin{bmatrix} 1+2+3+4 \\ 1+2w^1+3w^2+4w^3 \\ 1+2w^2+3w^4+4w^6 \\ 1+2w^3+3w^6+4w^9 \\ 1-2+3-4 \\ 1+2w^5+3w^{10}+4w^{15} \\ 1+2w^6+3w^{12}+4w^{18} \\ 1+2w^7+3w^{14}+4w^{21} \end{bmatrix} = \begin{bmatrix} 10 \\ -0.414-7.242j \\ -2+2j \\ -2.414+1.242j \\ -2 \\ 2.414+1.242j \\ -2-2j \\ -0.414+7.242j \end{bmatrix}$$

Properties:

$x[n] = \{1,2,3,4\}$

$P[k] = 8X[k]$

$X[k] \xrightarrow{IDFT} x[n]$

$x[n] \xrightarrow{DFT} X[k]$

$ax[n] \xrightarrow{DFT} aX[k]$

$\therefore$ For $P[k]$, $a = 8$

$P[k] \xrightarrow{IDFT} 8x[n]$

$= \{8, 16, 24, 32\}$

$Q[k] = 8 + X[k]$

$ax_1[n] + bx_2[n] \xrightarrow{DFT} aX_1[k] + bX_2[k]$

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$Q[k] \xrightarrow{IDFT} 8 + x[n]$

~~$q[n] = \{9, 10, 11, 12\}$~~

$Q[k] = \{9, 10, 11, 12\}$

Q3 $X[k] = \{1, 2, 3, 4\}$

$p[n] = x[n-1]$

$x[n-l] \xrightarrow{DFT} X[k] \cdot \omega^{kl}$

$x[n-1] \xrightarrow{DFT} X[k] \cdot \omega^{k}$

ω^{k} for $N=4 = \{1, -j, -1, j\}$

$$x[n-1] \xrightarrow{DFT} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix} = \begin{bmatrix} 1 \\ -2j \\ -3 \\ 4j \end{bmatrix}$$

$\therefore P[k] = \{1, -2j, -3, 4j\}$

b) $q[n] = x[n+1]$

$x[n+l] \xrightarrow{DFT} X[k] \cdot \omega^{kl}$

$\therefore x[n-1] \xrightarrow{DFT} X[k] \cdot \omega^{-k}$

$\omega^{-k} = \{1, j, -1, -j\}$

$$x[n-1] \xrightarrow{DFT} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 \\ j \\ -1 \\ -j \end{bmatrix} = \begin{bmatrix} 1 \\ 2j \\ -3 \\ -4j \end{bmatrix}$$

$Q[k] = \{1, 2j, -3, -4j\}$

$a[n] = \{1, 2, 3, 4\}$

a) $A[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$

b) $b[n] = \{3, 4, 1, 2\}$

$N = 4$

$\cancel{b[n-1] = \{}$ $a[n-1] = \{4, 1, 2, 3\}$

$a[n-2] = \{3, 4, 1, 2\} = b[n]$

$a[n-2] = a[n - \frac{N}{2}]$

$a[n - \frac{N}{2}] \xrightarrow{DFT} (-1)^{k} A[k]$

$\therefore$ $a[n-2] \xrightarrow{DFT} (-1)^k \{10, -2+2j, -2, -2-2j\}$

$B[k] = \{10, 2-2j, -2, 2+2j\}$

c) $c[n] = \{4, 6, 4, 6\}$

$a[n] = \{1, 2, 3, 4\}$

$a[n-2] = \{3, 4, 1, 2\}$

$a[n] + a[n-2] = \{4, 6, 4, 6\} = c[n]$

$a[n] + a[n-2] \xrightarrow{DFT} A[k] + (-1)^k A[k]$

$\cancel{B[k]}$ $C[k] = \{20, 0, -4, 0\}$

d) $d[n] = \{-2, -2, 2, 2\}$

$a[n] = \{1, 2, 3, 4\}$

$a[n-2] = \{3, 4, 1, 2\}$

$a[n] - a[n-2] = \{-2, -2, 2, 2\} = d[n]$

$a[n] - a[n-2] \xrightarrow{DFT} A[k] - (-1)^k A[k]$

$D[k] = \{0, -4+4j, 0, -4-4j\}$

e) $e[n] = \{5, 3, 5, 7\}$

$a[n] = \{1, 2, 3, 4\}$

$a[n-1] = \{4, 1, 2, 3\}$

$x[n] + x[n-1] = e[n]$

$x[n] + x[n-1] \xrightarrow{DFT} X[k] + X[k-1]$

$X[k-1] = X[k] \cdot \omega_N^k$

$\therefore E[k] = X[k] + X[k]\omega_N^k$

$= X[k](1+\omega_N^k)$

$$= \cancel{X[l} \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{bmatrix} 1+1 \\ +1-j \\ 0 \\ 1+j \end{bmatrix} = \begin{bmatrix} 20 \\ 4j \\ 0 \\ -4j \end{bmatrix}$$

FREQUENCY SHIFT:

Q) a) $X[k] = \{1, 2, 3, 4\}$

$p[n] = (-1)^n x[n]$

$x[n] \xrightarrow{DFT} X[k]$

$x[n]\, \omega^{-kl} \xrightarrow{DFT} X[k-l]$

$(-1)^n = \{1, -1, 1, -2\}$

$\omega^{-k(2)} = \{1, -1, 1, -1\}$

$l = 2$

$\cancel{x[n] \cdot \omega^{-kl}}$

$P[k] = X[k-2]$

$X[k-1] = \{4, 1, 2, 3\}$

$X[k-2] = \{3, 4, 1, 2\}$

$\therefore P[k] = \{3, 4, 1, 2\}$

b) $q[n] = x[n] \cos\left(\frac{n\pi}{2}\right)$

$\cos(n\pi/2)$ for $N=4$: $\{\cancel{1,0,} 1, 0, -1, 0\}$ ...(I)

$\cancel{x}\; x[n]\, \omega^{-kl} \xrightarrow{DFT} X[k-l]$

$$w^{-kl} = \{\cos\left(\frac{2\pi kl}{4}\right) + j\sin\left(\frac{2\pi kl}{4}\right), \cos\left(\frac{4\pi kl}{4}\right) + j\sin\left(\frac{4\pi kl}{4}\right),$$
$$\cos\left(\frac{6\pi kl}{4}\right) + j\sin\left(\frac{6\pi kl}{4}\right), \cos\left(\frac{8\pi kl}{4}\right) + j\sin\left(\frac{8\pi kl}{4}\right)\} \quad \ldots (\text{II})$$

For (I)

$$\cos\left(\frac{n\pi}{2}\right) = \frac{1}{2}\left(w^{-k} + w^{k}\right)$$

$$\therefore q[n] = x[n] \times \frac{1}{2}\left(w^{k} + w^{-k}\right)$$

$$\therefore Q[k] = \frac{1}{2} X[k-1] + \frac{1}{2} X[k+1]$$

$$= \frac{1}{2}\{$$

$$X[k-1] = \{4, 1, 2, 3\}$$
$$X[k+1] = \{2, 3, 4, 1\}$$
$$Q[k] = \frac{1}{2}\left(X[k-1] + X[k+1]\right)$$
$$= \frac{1}{2}\left(\{6, 4, 6, 4\}\right)$$

$$Q[k] = \{3, 2, 3, 2\}$$

Time Reversal Property:

Q) $X[k] = \{1, 2, 3, 4\}$

a) $p[n] = x[-n]$

~~x[n X[]~~ $x[n] \xrightarrow{DFT} X[k]$

$x[-n] \xrightarrow{DFT} X[-k]$

$$X[-k] = \{1, 4, 3, 2\}$$
$$P[k] = \{1, 4, 3, 2\}$$

b) $q[n] = x[-n+1]$

$q[n] = x[-(n-1)]$

$\cancel{x[-}$ $x[-(n-1)] \xrightarrow{DFT} X[-(k+1)]$

$x[n-1] \xrightarrow{DFT} X[n]\, w^{k}$

$x[-n] \xrightarrow{DFT} X[-n]$

$X[-k] = \{1, 4, 3, 2\}$

$X[-n+1] = X[-(n-1)]$

$X[n-1] = X[n]\, w^{k}$

$$\begin{bmatrix} 1 \\ 4 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix} = \begin{bmatrix} 1 \\ -4j \\ -3 \\ 2j \end{bmatrix}$$

$X[-(n-1)] = \{ \cancel{\{1, 4j, -3, -2j\}} \; \{1, -4j, -3, 2j\}$

~~Time Reversal Property:~~

~~$X[k] = \{1, 2, 3, 4\}$~~

c) $r[n] = X[-n-1]$

$X[-(n+1)]$

$x[-n] \xrightarrow{DFT} X[-k]$

$x[n+1] \xrightarrow{DFT} X[k]\, w^{-k}$

$w^{-k} = \{1, j, -1, -j\}$

$X[-k] = \{1, 4, 3, 2\}$

$$X[-(n+1)] = \begin{bmatrix} 1 \\ 4 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ j \\ -1 \\ -j \end{bmatrix} = \begin{bmatrix} 1 \\ 4j \\ -3 \\ -2j \end{bmatrix}$$

$R[k] = \{1, 4j, -3, -2j\}$

Q7 $x[n] = \{1,1,1,1,0,0,0,0\}$

$X[k] \xrightarrow{DFT} x[n]$

a) $a[n] = \{1,0,0,0,0,1,1,1\}$

$x[-n] = \{1,0,0,0,0,1,1,1\} = a[n]$

Property:

$x[-n] \xrightarrow{DFT} X[-k]$

$\Rightarrow a[n] \xrightarrow{DFT} A[k]$

$\therefore A[k] = X[-k]$

b) $b[n] = \{2,1,1,1,0,1,1,1\}$

$x[n] = \{1,1,1,1,0,0,0,0\}$

$x[-n] = \{1,0,0,0,0,1,1,1\}$

$x[n]+x[-n] = \{2,1,1,1,0,1,1,1\} = b[n]$

~~x[n] + x~~ Property

$x[n] \xrightarrow{DFT} X[k]$

$x[-n] \xrightarrow{DFT} X[-k]$

$x[n] + x[-n] \longrightarrow X[k] + X[-k]$

$b[n] \xrightarrow{DFT} B[k]$

$B[k] = X[k] + X[-k]$

Convolution property:

Q14 $x[n] = \{1,2,3,2\}$

$h[n] = \{1,0,3,4\}$

a) Circular convolution

$$= \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 \\ 2 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 18 \\ 20 \\ 14 \\ 12 \end{bmatrix}$$

$x[n] * h[n] = \{18,20,14,12\}$

b)
$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ -2+4j \\ 0 \\ -2-4j \end{bmatrix}$$

Circular convolution

$$X[k] \cdot H[k] = \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix} \begin{bmatrix} 8 \\ -2+4j \\ 0 \\ -2-4j \end{bmatrix} = \begin{bmatrix} 64 \\ 4-8j \\ 0 \\ +4+8j \end{bmatrix}$$

IDFT:

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 64 \\ 4-8j \\ 0 \\ 4+8j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 72 \\ 80 \\ 56 \\ 48 \end{bmatrix} = \begin{bmatrix} 18 \\ 20 \\ 14 \\ 12 \end{bmatrix}$$

$x[n] * h[n] = \{18, 20, 14, 12\}$

Parsevals Energy Theorem

Q16 $x[n] = \{1, 2, 3, 2\}$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

(a) Energy $= \frac{1}{N} \Sigma (X[k])^2$

$= \frac{1}{4} \times (64+4+0+4)$

$E = 18$

b) Energy $= \Sigma (x[n])^2$

$= 1+4+9+4$

$E = 18$

Q15 $x[n] = \{1,2,3,4\}$

Sol $X[k] = \{8,-2,0,-2\}$

$q[n] = x[n] \circledast x[n]$

$Q[k] = X[k] \cdot X[k]$

$$= \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix} \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 64 \\ 4 \\ 0 \\ 4 \end{bmatrix}$$

$$q[n] = \frac{1}{N} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 64 \\ 4 \\ 0 \\ 4 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 72 \\ 64 \\ 56 \\ 64 \end{bmatrix}$$

$q[n] = \{18, 16, 14, 16\}$

Complex Conjugate

Q7

$$x[n] = \begin{bmatrix} 1+j \\ 2+2j \\ 3+3j \\ 4+2j \end{bmatrix} \quad X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1+j \\ 2+2j \\ 3+3j \\ 4+2j \end{bmatrix}$$

$= \{10+8j, -2, -2, -2-4j\}$

$\cancel{X^*[k] \xrightarrow{DFT}}$ $x^*[n] \xrightarrow{DFT} X^*[-k]$

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$X[k] = p[n] + jq[n]$

$P[k] = \frac{X[k] + X^*[k]}{2}$

$Q[k] = \frac{X[k] - X^*[-k]}{2j}$

$X[k] = [10+8j, -2, -2, -2-4j]$

$X^*[+k] = [10-8j, -2, -2, -2+4j]$

$X^*[-k] = [10-8j, -2+4j, -2, -2]$

$P[k] = \frac{1}{2} X[k] + X^*[k]$

$= \{\frac{1}{2}\} [20, -4+4j, -4, -4-4j]$

$P[k] = [10, -2+2j, -2, -2-2j]$

$Q[k] = \frac{1}{2j} [16j, -4j, 0, -4j]$

$Q[k] = [8, -2, 0, -2]$

Symmetry property:

Q $P[k] = \{0, -j, 2j, -1, 2j, j\}$

Symmetry about $N/2 = 6/2 = 3$

Complex conjugate

$Q[k] = \{1, 2, \;, \;, 0, 1-j, -2, \;\}$

$N = 8$

$N/2 = 4$

Symmetry about $N/2 = 4$

$\therefore Q[k] = \{1, 2, -2, 1+j, 0, 1-j, -2, 2\}$

DFT Property of Even & Odd

Q $X[k] = \{1, 4+2j, 6+4j, 2j, 6, -2j, 6-4j, 4-2j\}$

$$p[n] = \frac{1}{2}\left[x[n] + x[-n]\right]$$

$$x[n] \xrightarrow{DFT} X[k] \qquad x[-n] \xrightarrow{DFT} X[-k]$$

$$p[n] \xrightarrow{DFT} P[k]$$

$$\xrightarrow{DFT} \frac{1}{2}\left(X[k] + X[-k]\right)$$

$$P[k] = \frac{1}{2}\Big[\{1, 4+2j, 6+4j, 2j, 6, -2j, 6-4j, 4-2j\} +$$

$$\{1, 4-2j, 6-4j, \overset{-2j,\,6}{\cancel{6-2j}}, 2j, 6+4j, 4+2j\}\Big]$$

$$= \frac{1}{2}\left(\{2, 8, 12, \cancel{6}, \overset{12}{\cancel{8}}, \overset{0}{\cancel{12}}, 8\}\right)$$

$$P[k] = \{1, 4, 6, \cancel{8}\ 0, 6, 0, 4\}$$