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IT/Computer Engineering Department

Subject : Foundation of Signal Processing Class : TE Computer / IT SEM-VI

Assignment-2 Topic : Discrete Fourier Transform Date : 25-2-2023

NOTE : [1] All questions are Compulsory.
[2] Do not write answers for FAQs in your assignment. But you should be in a position to answer FAQs at the time of correction.
[3] Deadline for submission of corrected assignment is 12th March 2023.
[4] After deadline you will have to attempt all questions.

Module : 02

References :

[1] Proakis and Manolakis, "Digital Signal Processing", 4th Edition, Pearson Education.

[2] Ashok Ambardar, 'Digital Signal Processing ' , Cengage Learning, 2007, ISBN : 978-81-315-0179-5.

[3]. Emmanuel C. Ifeachor, Barrie W. Jervis, "Digital Signal Processing: A Practical Approach", Pearson Education ISBN 0-201-59619- 9

[4] S. Salivahanan, A. Vallavaraj, C. Gnanapriya, 'Digital Signal Processing ' TataMcgraw Hill Publication First edition (2010). ISBN 978-0-07-066924-6.

• DFT- IDFT EQUATION

Q(1) (a) Let $x[n] = \delta[n] + 2\,u[n-1] + u[n-2] + \delta[n-3] - 3\,u[n-4]$. Find X[k].

(b) Let $x[n] = 3\cos(0.5\pi n)$. Find X[k].

Hint : x[n] is periodic with period N=4. Find first four values of x[n].

(c) Let $x[n] = \{\underset{\uparrow}{1}\ 2\ 3\ 4\ 0\ 0\ 0\ 0\}$ Find 8 point DFT of x[n].

- **Properties of DFT**

[1] Scaling and Linearity property

Q(2) Let x[n] = { 1, 2, 3, 4 }
Find inverse DFT of the following without using DFT/iDFT equations.
(a) P[k] = 8 X[K] (b) Q[k] = 8 + X[k]

[2] Time Shift Property

Q(3) Let x[n] be 4 point sequence with X[k] = {1, 2, 3, 4}.
Find the DFT of the following sequences using X[k] and not otherwise.
(a) p[n]= x[n-1] (b) q[n] = x[n+1] **(c)**

Q(4) Given a[n] = { 1, 2, 3, 4 }.Find the DFT of the following signals using A[k].

(a) Find A[k].
(b) Let b[n] = { 3, 4, 1, 2 }
(c) Let c[n] = { 4, 6, 4, 6 }
(d) Let d[n] = { –2, –2, 2, 2 }
(e) Let e[n] = { 5, 3, 5, 7 }

ANS :

(b)	$B[k] = (-1)^k A[k]$	(d)	$D[k] = [1+(-1)^k] A[k]$
(c)	$C[k] = [1+(-1)^k] A[k]$	(e)	$E[k] = A[k] + W_N^k A[k]$

Q(5) Given x(n) = { 1, 1 , 1, 1, 0, 0, 0, 0}. Let X[k] be DFT of x[n].

(a) Let a [n] = { 1, 1, 1, 1, 1, 1, 1, 1 } Find A[k] using X[k]
(b) Let b[n] = { 1, 1, 1, 1, –1, –1, –1, –1} Find B[k] using X[k]
(c) Let c [n] = { 1, 0, 0, 0, –1, 0, 0, 0} Find C[k] using X[k]
(d) Let d[n] = { 2, 0, 0, 0, 0, 2, 2, 2 } Find D[k] using X[k]

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[3] Frequency Shift Property

Q(6) Let x[n] be four point sequence with X[k] = { 1, 2, 3, 4 }.
Find the DFT of the following sequences using X[k].
(a) $p[n] = (-1)^n x[n]$ (b) $q[n] = x[n] \cos(\frac{n\pi}{2})$

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[4] Time Reversal Property

Q(7) Given $x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$. Let X[k] be DFT of x[n].

(a) Let $a[n] = \{1, 0, 0, 0, 0, 1, 1, 1\}$ Find A[k] using X[k]

(b) Let $b[n] = \{2, 1, 1, 1, 0, 1, 1, 1\}$ Find B[k] using X[k]

Q(8) Let x[n] be four point sequence with $X[k] = \{1, 2, 3, 4\}$.
Find the DFT of the following sequences using X[k].
(a) p[n]= x[-n] (b) q[n] = x[-n+1] (c) r[n] = x[-n-1]

ANS : (a) $P[k] = \{1, 4, 3, 2\}$ (b) $Q[k] = \{1, -4j, -3, 2j\}$ (c) $R[k] = \{1, 4j, -3, -2j\}$

Q(9) Given $x[n] = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & 4 \le n \le 7 \end{cases}$

Compute the DFT of the following sequence using X[k] only.

(a) $p[n] = \begin{cases} 1 & n = 0 \\ 0 & 1 \le n \le 4 \\ 1 & 5 \le n \le 7 \end{cases}$ (b) $q[n] = \begin{cases} 0 & 0 \le n \le 1 \\ 1 & 2 \le n \le 5 \\ 0 & 6 \le n \le 7 \end{cases}$

Hint : (a) $p[n] = x[-n]$ $P[k] = X[-k]$ (b) $q[n] = x[n-2]$ $Q[k] = W_N^{2k} X[k]$

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[5] Symmetry Property

Q(10) For the DFT of each real sequence compute boxed quantities
(a) P[k] = { 0, □, 2+j, -1, □, j }
(b) Q[k] = { 1, 2, □, □, 0, 1-j, -2, □ }

ANS : (a) P[1] = -j P[4]= 2-j (b) Q[2] = -2 Q[3] =1+j Q[7] = 2

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[6] DFT Property of Even Signal and Odd Signal

Q(12) Let x[n] be the finite duration sequence of length 8. Its corresponding DFT X[k] is, X[k] = {(1), (4 + j2), (6 + j4), (2j), (6), (-2j), (6 – j4), (4 – j2)} A new sequence p[n] of length 8 is defined as p[n] = ½ {x[n]+x[-n]} Find P[k] i.e. DFT of p[n] without performing DFT/ iDFT operations.

Hint : $\mathbf{p[n] = x_e[n]}$ So $P[k] = X_e[k]$.

ANS $P[k] = \{1, 4, 6, 0, 6, 0, 6, 4\}$

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[7] Complex Conjugate Property

Q(13) Given x[n] = { (1 + j), (2 + j2), (3 + j3), (4 + j2) }

(a) Find X[k].

(b) Find DFT of $x^*[n]$using X[k] and not otherwise.

(c) Let p[n] = { 1, 2, 3, 4 } and q[n] = { 1, 2, 3, 2 } Find P[k] and Q[k] using X[k].

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[8] Convolution Property

Q(14) Let x[n] = (1, 2, 3, 2 } and h[n] = { 1, 0, 3, 4 }

(a) Find Circular Convolution using Time domain method.

(b) Find Circular Circular using DFT.

Q(15) Given x[n] = { 1, 2, 3, 4 }. And X[k]= { 8, -2, 0, -2 }.

(a) Find inverse DFT of P[k] = $X^2[k]$ without using DFT/iDFT equations.

(b) Find the DFT of q[n]=x[n]⊗ x[n] using X[k] and not otherwise.

ANS : (a) p[n] = { 26, 28, 26, 20 } (b) [k] = { 1, 4, 9, 16 }

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[9] Circular Correlation Property of DFT

Q(15) Let x[n] = (1, 2, 3, 2 } and h[n] = { 1, 2, 3, 4 }

(a) Find Circular Cross Correlation using Time domain method.

(b) Find Circular Cross Correlation using DFT.

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[10] Parseval's Energy Theorem

Q(16) Let x[n] = (1, 2, 3, 2 } Find X[k]

(a) Find Energy of the signal using X[k].

(b) Find Energy of the signal using x[n].

Q(17) Let x[n]= {1, –2, 3, –4, 5, –6} without evaluating its DFT /iDFT compute the following

(a) X[0] (b) X[3] (c) $\sum_{n=0}^{5} |X[k]|^2$

➢ Linear FIR Filtering

Q(24) Given h[n]= { 1, 0, 2 } Find the response of a Digital FIR filter to the input x[n] = { 1, 2, 3, 4, 0, 0, 1, 2, 3, 4 } using Overlap **Add** Method.

Q(25) Given h[n]= { 1, 0, 2 } Find the response of a Digital FIR filter to the input x[n] = { 1, 2, 3, 4, 0, 0, 1, 2, 3, 4 } using Overlap **Save** Method.

➢ Home Work Practice Problems

Q(1) Let X[k] be DFT of 4 point sequence x[n]..
Find the DFT of the following sequences interms of X[k].

(A) x[n-1]	(D) x[-n+1]	(G) $2\delta[n] + x[n]$	(J) $e^{j(n-2)\pi} x[n-2]$
(B) x[n+1]	(E) x[-n-1]	(H) 2 + x[n]	(K) $e^{jn\pi} x[n-2]$
(C) x[-n]	(F) x[n] * x[n]	(I) $e^{jn\pi} x[n]$	(L) x*[n]

Q(2) Let X[k] be DFT of 4 point sequence x[n]..
Find inverse DFT of the following in terms of x[n].

(1) X[k-2]	(4) X[-k+2]	(7) 2 X[K]	(10) $e^{j1.5\pi k} X[-k]$
(2) X[k+2]	(5) X[-k-2]	(8) $e^{j\pi k} X[k]$	(11) X*[-k]
(3) X[-k]	(6) $X^2[k]$	(9) $e^{j\pi k} X[k-2]$	(12) 8 + X[K]

Q(4) A sequence x[n] = { x[0], x[1], x[2], x[3] } Let DFT { x[n] } = X[k] = { 1, 2 , 3, 2 }

(a) Identify the signal type.
(b) Let p[n] = { x [0] , x[1], x [2], x [3], 0, 0, 0, 0 }. Find P[k] only for even values of k.

Q(6) Given x(n) = { 1, 1 , 1, 1, 0, 0, 0, 0}.

Find DFT of the following sequences in terms of X[k].

A) a [n] = { 0, 0, 0, 0, 1, 1, 1, 1 }	E) e [n] = { 1, 1, 1, 1, 1, 1, 1, 1}
B) b [n] = { 1, 0, 0, 0, 0, 1, 1, 1}	F) f [n] = { 0, 0, 1, 1, 1, 1, 0, 0}
C) c [n] = { 1, 0, 0, 0, –1, 0, 0, 0}	G) g [n] = { 2, 1, 1, 1, 0, 0, 0, 0}
D) d [n] = { 1, 1, 1, 1, –1, –1, –1, –1}	H) p [n] = { 1, 0.5, 0.5, 0.5, 0, 0.5, 0.5, 0.5}

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ANS :

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A) $A[k] = (-1)^k X[k]$	E) $E[k] = X[k] + A[k]$
B) $B[k] = X[-k]$	F) $F[k] = W_N^{2k} X[k]$
C) $C[k] = B[k] - A[k]$	G) $G[k] = X[k] + 1$
D) $D[k] = X[k] - A[k]$	H) $P[k] = X_e[k] = \text{Real}\{X[k]\}$

Drill Problems :

Q(1) For the given sequence x[n]={ 2, 0, 0, 1 }. Perform the following operations:-

(a) Find out 4 point DFT of x[n].

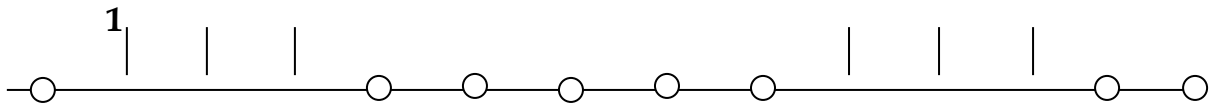
(b) Plot x[n], its periodic extension $x_p[n]$ and $x_p[n-3]$ and x[((n-3))]

(c) Add phase angle in (a) with factor $\left[\frac{-2\pi rk}{N}\right]$ where N=4, r=3, k=0,1,2,3.
Then find x′[n].

(d) Comment on results you had in point (b) and (c).

Q(2) Consider a finite duration sequence x(n) = {0, 1, 2, 3, 4}.
Find the sequence y(n) with five point DFT Y(k) = Re | X(k) |.

Q(3) A periodic sequence with a period 8 is shown below :



What time origin should be selected so that the corresponding DFT be real. Justify your answer. How many such sequences are possible? Give such sequences.

Q(4) A certain sequence $x_1[n]$ has DFT $X_1[k]$, $0 \le n \le 7$, $0 \le k \le 7$. $X_1[k] = X_R[k] + X_I[k]$ where R and I denote real and imaginary parts. If $x[n] = x_1[n] + x_2[n]$ and $X[k] = 2\,(X_R[k])$ for $0 \le k \le 7$. What is the relation between $x_1[n]$ and $x_2[n]$? Justify your answer.

Q(5) Compute the DFT of each of the following.

(a) $x[n] = \cos(\frac{4\pi n}{N})$ (b) $x[n] = \sin(\frac{4\pi n}{N})$

Q(6) Compute the N point DFT's of the sequence

a) $x[n] = e^{j(\frac{2\pi\, n\, k_o}{N})}$ c) $x[n] = \sin(\frac{2\pi\, n\, k_o}{N})$

b) $x[n] = \cos(\frac{2\pi\, n\, k_o}{N})$ d).$x[n] = \begin{matrix} 1 & n \text{ even} \\ 1 & n \text{ odd} \end{matrix}$

Q(7) If x[k] is DFT of the sequence x[n] Determine N point DFTs of the sequence $p[n] = x[n]\cos(\frac{2\pi k_0 n}{N})$ $q[n] = x[n]\sin(\frac{2\pi k_0 n}{N})$

Q(8) For the sequence $x_1(n) = \mathrm{Cos}\left(\frac{2\pi}{N}\right)n$ $x_2(n) = \mathrm{Sin}\left(\frac{2\pi}{N}\right)n$

Determine N point, circular convolution $x_1(n) \otimes x_2(n)$.

Q(9) Compute the energy of N pt sequence $x[n] = \cos(\frac{2\pi kn}{N})$ $0 \le n \le N-1$

Q(10) Given $x[n] = a^n u[n]$

(a) Determine DTFT X(w) of x[n] b) Determine DFT X[k]

(b) How X[k] is related to X[w].

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Frequently Asked Questions on DFT

(1) Define Discrete Fourier Transform of x[n].

(2) How many complex multiplications and additions are required to find DFT ?

(3) How many real multiplications and additions are required to find DFT.

(4) What is the DFT of δ[n] ?

(5) What is the DFT of N pt signal u[n] ?

(6) What is the DFT of 4 pt x[n] where x[n] = δ[n] + u[n] ?

(7) Why DFT results are periodic ?

(8) DFT gives discrete spectrum or continuous spectrum ? Justify ?

(9) What do you mean by spectrum is Discrete or continuous.

(10) Find DFT of x[n] where x[n] = u[n] + 2 u[n-2] – 3 u[n-4]

(11) Find DFT of 10 pt x[n] where x[n] = δ[n] + δ[n-5] ?

(12) What is DFT property of EVEN signal ?

(13) What is the DFT of real and even signal.?

(14) What is the DFT of Imaginary and Even signal ?

(15) What is DFT property of ODD signal ?

(16) What is the DFT of real and Odd signal ?

(17) What is the DFT of Imaginary and Odd signal ?

(18) If DT signal is **expanded** in time domain what will be the effect in frequency domain?

(19) If DT signal is **compressed** in time domain what will be the effect in frequency domain?

(20) If DT signal is **appended** by zeros in time domain what will be the effect in frequency domain?

(21) How to find energy of signal from its DFT ?

(22) How to find CC using DFT ?

(23) How to find LC using CC ?

(24) How to find LC using DFT ?

(25) How to find output of the filter using DFT ?

(26) What is the length of linearly convolved signals ?

(27) What do you mean by aliasing in circular convolution ?

(28) What is DTFT ?

(29) If DTFT is Fourier Transform of DT signal then What is DFT ?

(30) Describe the relation between DFT and DTFT.

(31) Derive DFT equation .

(32) Why DFT ? What is need of Sampling DTFT ?

(33) How to find DFT of infinite length sequence ?

(34) What is Power Density Spectrum of Periodic DT Signals ?

(35) What is Energy Density Spectrum of DT Aperiodic Signals

(36) Find DTFT and Energy Density Spectrum of x[n] = u[n].

(37) What is the necessary condition to find DTFT of any signal. ?

(38) DTFT gives continuous spectra or discrete spectra?.

(39) What is the relation between DFT and DTFT ?