

## Homework 2

1)  $p(x) = x^4 + 5x^3 - 9x^2 - 85x - 130$   $\epsilon = 0.00001$

(a)  $p'(x) = 4x^3 + 15x^2 - 18x - 85$

iteration 1

-4	1	5	-9	-85	-130	$x_1 = x_0 - \frac{p(x_0)}{p'(x_0)}$
		-4	-4	52	132	
	1	1	-18	33	-4	$= -4 - \frac{-4}{-29} = -4.13793$

error =  $|x_0 - x_1| = 0.13793$

iteration 2

-4.13793	1	5	-9	-85	-136	$x_1 = -4.13793 - \frac{0.8429}{3.70861} = -4.12329$
		-4.13793	-3.56752	0.002	136.57	
	1	0.862	-12.567	-32.997	0.5429	
		-4.13793	13.555	-4.0882		error = 0.01463
		-3.2758	0.988	-3.70866		

iteration 3

-4.12329	1	5	-9	-85	-136	$x_1 = -4.12329 - \frac{9.007}{-30.1009} = -4.123105$
		-4.12329	3.6197	52.035	136.0067	
	1	0.8767	-12.6149	-32.989	0.0087	
		-4.12329	13.3866	-3.1817		error = 0.00086
		-3.2416	0.77169	-36.1669		

iteration 4

-4.123105	1	5	-9	-85	-136	$x_1 = -4.123105 - \frac{0.00001099}{-36.155} = -4.123105826$
		-4.123105	-3.6155	52.0152	136.0000011	
	1	0.87689	-12.6155	-32.9848	0.000001099	
		-4.123105	13.389	-3.1704		error = 0.000003
		-3.246211	0.7689	-36.1415		

b)  $x_1 = -4.123105826$

-4.123105826	1	5	-9	-85	-136	
		-4.123105826	-3.6155	52.015	136	
	1	0.87689	-12.6155	-32.9840	0	

$p_1(x) = x^3 + 0.87689x^2 - 12.61556x - 32.9845$



$$P_1(x) = x^3 + 0.87689x^2 - 12.6155x - 32.9848$$

iteration 1

(c)	4	1	0.87689	-12.6155	-32.9848	$x_1 = 4 - 8.41606$
			4	19.50756	27.50824	$42 - 39862$
	1		4.87689	6.89206	-5.41650	$= 4.1277$
			4	35.007		error = 0.18738
	1		8.87689	42.37962		

Iteration 2

4.1277	1	0.87689	-12.6155	-32.9848	$x_1 = 4.1277 - 0.2122$
		4.1277	20.6579	33.197	$4.5.7386$
	1	5.0046	8.0424	0.2122	
					error = 0.00264 = 4.1231

iteration 3

4.123	1	0.87689	-12.6155	-32.9848	$x_1 = -4.1231 - 0.000657$
		4.1231	20.6155	32.1850	$4.5.6258$
	1	4.999	8.00004	0.000255	$= 4.12303727$
		4.1231	37.6105		
	1	-9.1231	45.6156		error = -0.0000657

$r_1 = 4.123103727$

d) 4.123103

1	5	-9	-85	-136
	4.1231	37.6155	117.983	135.997
1	9.1231	28.6155	32.984	-0.0007
	4.123103	54.615	343.12	1550.9245
1	13.246	83.2307	376.156	1558.924

$r_1 = 4.123104$  error = 0.00000046

$\epsilon$  is  $10^{-5}$ , so 4.123103727 is a root of  $f(x)$



2)  $f(x) = e^{6x} - 3(\ln 2)^2 e^{2x} e^{4x} \ln 8 - (\ln 2)^3$   
 $f'(x) = 6e^{6x} + 6(\ln 2)^2 e^{2x} - 4 \ln 8 e^{6x}$

i	$x_0$	$x_1$	$ x_1 - x_0 $
1	-0.5	-0.352665	0.147335
2	-0.352665	-0.2854863	0.06717
3	-0.2854863	-0.2437798	0.037708
4	-0.2437798	-0.224823	0.02895
5	-0.224823	-0.210373	0.014457
6	-0.210373	-0.201086	0.0092891
7	-0.201086	-0.195036	0.0060504
8	-0.195036	-0.191063	0.00397313
9	-0.191063	-0.18844	0.0026226
10	-0.18844	-0.186703	0.00173
11	-0.186703	-0.1855	0.001203
12	-0.1855	-0.184784	0.000716
13	-0.184784	-0.18427	0.000514
14	-0.18427	-0.183934	0.000336
15	-0.183934	-0.183708	0.000226
16	-0.183708	-0.183557	0.000151
17	-0.183557	-0.183457	0.0001002
18	-0.183457	-0.18339	0.000068
19	-0.18339	-0.18339	0.00004452
20	-0.183345	-0.183315	0.000029517
21	-0.183313	-0.183295	0.000018
22	-0.183295	-0.183282	0.00001286
23	-0.183283	-0.183273	0.00001086

$f_1 = -0.183262$



3)  $f(x) = e^{6x} + 3(\ln 2)^2 e^{2x} - \ln 8 (e^{4x}) - (x^2)^3$

$x_6$	$x_1$	$f(x_0)$	$f(x_1)$	$x_2$
0	-1	0.02897285	-0.1756572	-0.14246089
-1	-0.142710114	-0.17356542	0.00026672	-0.1432004
-0.142710114	-0.1437004	0.000200777	0.00015516	-0.1504102
-0.1432004	-0.15604162	0.0001858616	0.0000728	-0.166524
-0.15604162	-0.1616424	0.0000582957	0.00002047	-0.1670627

error

0.085728985

0.0064903

0.01234602

0.00564138

0.00541382

4. 
$$\begin{bmatrix} 8 & 3 & 0 & 20 \\ 0 & 12 & 6 & 30 \\ 1 & 0 & 10 & 10 \end{bmatrix} \xrightarrow{r_3 \leftrightarrow r_1} \begin{bmatrix} 1 & 0 & 10 & 10 \\ 0 & 12 & 6 & 30 \\ 8 & 3 & 0 & 20 \end{bmatrix} \xrightarrow{r_3 - 8r_1} \begin{bmatrix} 1 & 0 & 10 & 10 \\ 0 & 12 & 6 & 30 \\ 0 & 3 & -80 & -60 \end{bmatrix}$$

$\frac{1}{8} r_1 \rightarrow r_1$   $\begin{bmatrix} 1 & 0 & 10 & 10 \\ 0 & 12 & 6 & 30 \\ 0 & 3 & -80 & -60 \end{bmatrix} \xrightarrow{\frac{1}{12} r_2 \rightarrow r_2} \begin{bmatrix} 1 & 0 & 10 & 10 \\ 0 & 1 & 0.5 & 2.5 \\ 0 & 3 & -80 & -60 \end{bmatrix}$

$\frac{1}{12} r_2 \rightarrow r_2$

a)  $x_3 = 135/163$   $x_2 + \frac{1}{2} (135/163) = 5/2$   $x_1 + 3/8 (340/163) = 37/12$

$x_2 = \frac{340}{163}$   $x_1 = 280/1693$

b) 
$$\begin{bmatrix} 8 & 3 & 0 \\ 0 & 12 & 6 \\ 1 & 0 & 10 \end{bmatrix} \rightarrow 8 \begin{bmatrix} 12 & 6 \\ 0 & 10 \end{bmatrix} - 3 \begin{bmatrix} 0 & 6 \\ 1 & 10 \end{bmatrix} + 0 \begin{bmatrix} 0 & 10 \\ 1 & 10 \end{bmatrix}$$

$8[(12)(10)] - 3[(-6)(1)]$   $(960 + 18 = 978)$

$$\begin{bmatrix} 20 & 3 & 0 \\ 30 & 12 & 6 \\ 10 & 0 & 10 \end{bmatrix} \rightarrow 20 \begin{bmatrix} 12 & 6 \\ 0 & 10 \end{bmatrix} - 3 \begin{bmatrix} 30 & 6 \\ 10 & 10 \end{bmatrix} + 0 \begin{bmatrix} 30 & 12 \\ 10 & 0 \end{bmatrix}$$

$20[(120)(10)] - 3[(30)(10) - (10)(10)] = 3200$

$$\begin{bmatrix} 20 & 3 & 0 \\ 0 & 30 & 6 \\ 1 & 10 & 10 \end{bmatrix} \rightarrow 8 \begin{bmatrix} 30 & 6 \\ 10 & 10 \end{bmatrix} - 20 \begin{bmatrix} 0 & 6 \\ 1 & 10 \end{bmatrix} + 6 \begin{bmatrix} 0 & 30 \\ 1 & 10 \end{bmatrix}$$

$8[(30)(10) - (6)(18)] - 20[(-6)] + 6[340/163]$

$$\begin{bmatrix} 8 & 3 & 20 \\ 0 & 12 & 30 \\ 1 & 10 & 10 \end{bmatrix} \rightarrow 8 \begin{bmatrix} 12 & 30 \\ 0 & 10 \end{bmatrix} - 3 \begin{bmatrix} 0 & 30 \\ 1 & 10 \end{bmatrix} + 20 \begin{bmatrix} 0 & 12 \\ 1 & 10 \end{bmatrix}$$

$8[(120) - 3(-30) + 20(-12)] = 135/163$



II

$$2x_1 + x_2 + 5x_3 = 1$$

$$2x_1 + 2x_2 + 2x_3 = 1$$

$$4x_1 + x_2 = 2$$

$$a) \begin{bmatrix} 2 & 1 & 5 & | & 1 \\ 2 & 2 & 2 & | & 1 \\ 4 & 1 & 0 & | & 2 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{bmatrix} 2 & 2 & 2 & | & 1 \\ 2 & 1 & 5 & | & 1 \\ 4 & 1 & 0 & | & 2 \end{bmatrix} \xrightarrow{1/2 r_1 \rightarrow r_1}$$

$$\begin{bmatrix} 1 & 1/2 & 0 & | & 1/2 \\ 2 & 2 & 2 & | & 1 \\ 4 & 1 & 0 & | & 2 \end{bmatrix} \xrightarrow{\begin{matrix} -r_2 + r_1 \rightarrow r_2 \\ -r_3 + r_1 \rightarrow r_3 \end{matrix}} \begin{bmatrix} 1 & 1/2 & 0 & | & 1/2 \\ 0 & 1 & 2 & | & 1/2 \\ 0 & -1 & 0 & | & 3/2 \end{bmatrix} \xrightarrow{r_2 + r_3 \rightarrow r_3}$$

$$\begin{bmatrix} 1 & 1/2 & 0 & | & 1/2 \\ 0 & 1 & 2 & | & 1/2 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \begin{cases} x_3 = 0 \\ x_2 = 0 \end{cases} \begin{matrix} x_2 - 3(0) = 0 \\ x_1 + 1/2(0) + 0 = 1/2 \end{matrix} \begin{cases} x_2 = 0 \\ x_1 = 1/2 \end{cases}$$

$$b) \begin{bmatrix} 2 & 1 & 5 \\ 2 & 2 & 2 \\ 4 & 1 & 0 \end{bmatrix} \Rightarrow \begin{matrix} 2 \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 2 \\ 4 & 0 \end{vmatrix} + 5 \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} \\ 2[(2)(1) - (2)(4)] - 1[(2)(0) - 2(4)] + 5[2(1) - 4] \\ = -26 \end{matrix}$$

$$\begin{bmatrix} 2 & 1 & 5 \\ 2 & 2 & 2 \\ 4 & 1 & 0 \end{bmatrix} \Rightarrow \begin{matrix} 1 \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 2 \\ 2 & 0 \end{vmatrix} + 5 \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} \\ 1[-2] = -1(4) + 5[1-4] \\ = -13 \mid -26 \rightarrow 1/2 \end{matrix}$$

$$\begin{bmatrix} 2 & 1 & 5 \\ 2 & 2 & 2 \\ 4 & 1 & 0 \end{bmatrix} \Rightarrow \begin{matrix} 2 \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 2 \\ 4 & 0 \end{vmatrix} + 5 \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} \\ 2[-2] - 1[-8] + 5[4-4] \\ = 0 \mid -26 = 0 \end{matrix}$$

$$\begin{bmatrix} 2 & 1 & 5 \\ 2 & 2 & 2 \\ 4 & 1 & 0 \end{bmatrix} \Rightarrow \begin{matrix} 2 \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix} + 1 \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} \\ 2[4-4] - 1[4-4] + 1[2-8] \\ = 0 \mid -26 = 0 \end{matrix}$$



III

$$x_1 + x_2 - x_3 = -3$$

$$6x_1 + 2x_2 + 2x_3 = 2$$

$$a) \left[ \begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 6 & 2 & 2 & 2 \\ -3 & 4 & 1 & 1 \end{array} \right] \xrightarrow{r_2 \leftrightarrow r_1} \left[ \begin{array}{ccc|c} 6 & 2 & 2 & 2 \\ 1 & 1 & -1 & -3 \\ -3 & 4 & 1 & 1 \end{array} \right] \xrightarrow{1/6 r_1 \rightarrow r_1}$$

$$\left[ \begin{array}{ccc|c} 1 & 1/3 & 1/3 & 1/3 \\ 1 & 1 & -1 & -3 \\ -3 & 4 & 1 & 1 \end{array} \right] \xrightarrow{-r_1 + r_2 \rightarrow r_2} \left[ \begin{array}{ccc|c} 1 & 1/3 & 1/3 & 1/3 \\ 0 & 2/3 & -4/3 & -10/3 \\ -3 & 4 & 1 & 1 \end{array} \right] \xrightarrow{3r_1 + r_3 \rightarrow r_3}$$

$$\left[ \begin{array}{ccc|c} 1 & 1/3 & 1/3 & 1/3 \\ 0 & 2/3 & -4/3 & -10/3 \\ 0 & 0 & -3 & -21/3 \end{array} \right] \xrightarrow{-1/3 r_3 \rightarrow r_3} \left[ \begin{array}{ccc|c} 1 & 1/3 & 1/3 & 1/3 \\ 0 & 2/3 & -4/3 & -10/3 \\ 0 & 0 & 1 & 7/2 \end{array} \right]$$

$x_3 = 7/2$      $x_2 + 7/2 = 5/2$      $x_1 + 1/3(-1) + 1/3(7/2) = 1/3$

$x_2 = -1$      $x_1 = -1/2$

$$b) \left[ \begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 6 & 2 & 2 & 2 \\ -3 & 4 & 1 & 1 \end{array} \right] \rightarrow \begin{array}{l} 1 [2-8] \\ -1 [6+6] \\ -1 [24+6] \end{array} = -48$$

$$\left[ \begin{array}{ccc|c} -3 & 1 & -1 & -1 \\ 2 & 2 & 2 & 2 \\ 1 & 4 & 1 & 1 \end{array} \right] \rightarrow \begin{array}{l} -3 [2-8] \\ -1 [2-2] \\ -1 [8-2] \end{array} = 12 | -48 = -4$$

$$\left[ \begin{array}{ccc|c} 1 & -3 & -1 & -1 \\ 6 & 2 & 2 & 2 \\ -3 & 4 & 1 & 1 \end{array} \right] \rightarrow \begin{array}{l} 1 [2-5] \\ +3 [6-3] \\ -1 [3-3] \end{array} = 24 \rightarrow 24 | -48 = -2$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & -3 & -3 \\ 6 & 2 & 2 & 2 \\ -3 & 4 & 1 & 1 \end{array} \right] \rightarrow \begin{array}{l} 1 [2-8] \\ -1 [6+6] \\ -3 [24+6] \end{array} = -108 \rightarrow -108 | -48 = -1/4$$

$-108 \rightarrow -108 | -48 = -1/4$