CS 323 - Spring 2020 **Exam 2**

Name:

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1. [**10 points**] Given the System of equations $A\bar{x} = \bar{b}$:

$$\begin{aligned} x_1 + 3x_2 + 3x_3 &= b_1 \\ x_1 + 4x_2 + 3x_3 &= b_2 \\ 2x_1 + 7x_2 + 7x_3 &= b_3 \end{aligned}$$

   Use Gaussian Elimination with back substitution and no pivoting to compute A^{-1}.

   IMPORTANT: you cannot use Gauss-Jordan. You may do all your calculations with 4 digits using your calculator instead of using fractions, but you can use fractions if you prefer.

2. [**15 points**] Given the matrix

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 2 & 7 & 7 \end{pmatrix}$$

   Use Gaussian Elimination with no pivoting to compute L, U such that $A = LU$ (L is a lower triangular matrix, and U is an **IMPORTANT:** You may do all your calculations with 4 digits using your calculator instead of using fractions, but you can use fractions if you prefer.

3. Given the following system of equations

$$\begin{aligned} 8x_1 + 4x_2 - 3x_3 &= 3 \\ x_1 - 10x_2 + 2x_3 &= 2 \\ 3x_1 + 3x_2 - 7x_3 &= 1 \end{aligned}$$

   and

$$\bar{x}^0 = (0, 0, 0)$$

   (a) Find an approximation to the solution of the system of equations with error bound 10^{-5} using:
      i. [**5 points**] Jacobi
      ii. [**5 points**] Gauss-Seidel

   (b) [**3 points**] Explain why these iterative methods converge or do not converge.

   (c) [**2 points**] Compare the number of iterations needed to solve the problem using Jacobi to the number of iterations neede using Gauss-Seidel and explain why one is more efficient than the other.

4. [**15 points**] Given the matrices:

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 3 & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

If $A = LU$, find the solution to $A\bar{x} = \bar{b}$ using forward/backward substitution for the following value of $\bar{b}$.

$$\bar{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

5. [**15 points**] Given the points

x	y
1	-1
2	1
3	-1

Use Lagrange's method to find a 2nd degree polynomial that goes through each one of the given points.

IMPORTANT: You must simplify your polynomial.

6. Given the following table:

x	1.0	1.3	1.6	1.9
	0.7651977	-0.4837057		
		0.6200860	-0.5489460	-0.0494433
			0.4554022	

(a) [**10 points**] Compute all the missing values of the divided differences table.
IMPORTANT: All values can be computed from the ones given.

(b) [**3 points**] Find the approximation polynomial (no need to simplify)

(c) [**2 points**] Evaluate the polyonomial in $x = 1.5$.

7. Given $x_i = 0.025, 0.4, 0.60, 0.75, 1.0$ and $y_i = \sin \pi x_i$.

(a) [**10 points**] Construct a free cubic spline that approximates the given (x_i, y_i) values. (free cubic spline means that we use $c_0 = 0$ and $c_n = 0$ as we did in class).

(b) [**3 points**] Integrate the spline over $[0, 1]$ and compare the result to $\int_0^1 \sin \pi x = 0$.

(c) [**2 points**] Use the derivatives of the spline to approximate $f'(0.5)$ and $f''(0.5)$. Compare the approximation to the actual values.