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Homework #4

(1) given

$x_1 = 1$ & $f(x_1) = .7$ $x_4 = 4$ & $f(x_4) = .75$

$x_2 = 2$ & $f(x_2) = .73$ $x_5 = 5$ & $f(x_5) = .6$

$x_3 = 3$ & $f(x_3) = .8$

$$L_1(x) = \frac{.7(x-2)(x-3)(x-4)(x-5)}{(-1)(-2)(-3)(-4)}$$

$$L_1(x) = \left(\frac{(7)(x)}{10} - \frac{7}{5}\right)\left(\frac{(x-3)(x-4)(x-5)}{24}\right)$$

$$L_2(x) = \frac{(.73)(x-1)(x-3)(x-4)(x-5)}{(1)(-1)(-2)(-3)}$$

$$L_2(x) = -\left(\left(\frac{(73)(x)}{100} - \frac{73}{100}\right)\left(\frac{(x-3)(x-4)(x-5)}{6}\right)\right)$$

$$L_3(x) = \frac{(.8)(x-1)(x-2)(x-4)(x-5)}{(2)(1)(-1)(-2)}$$

$$L_3(x) = \left(\left(\frac{(4)(x)}{5} - \frac{4}{5}\right)\left(\frac{(x-2)(x-4)(x-5)}{4}\right)\right)$$

$$L_4(x) = \frac{(.75)(x-1)(x-2)(x-3)(x-5)}{(3)(2)(1)(-1)}$$

$$L_4(x) = -\left(\left(\frac{(3)(x)}{4} - \frac{3}{4}\right)\left(\frac{(x-2)(x-3)(x-5)}{6}\right)\right)$$

$$L_5(x) = \frac{(.6)(x-1)(x-2)(x-3)(x-4)}{(4)(3)(2)(1)}$$

so, net polynomial is $L(x) = L_1 + L_2 + L_3 + L_4 + L_5$

$$L_5(x) = \left(\left(\frac{3x}{5} - \frac{3}{5}\right)\left(\frac{(x-2)(x-3)(x-4)}{24}\right)\right)$$ 8%

So,

$$L(x) = \left(\frac{(3)(x^{4})}{400}\right) - \left(\frac{(61)(x^{3})}{600}\right) + \left(\frac{177x^2}{400}\right) - \left(\frac{419x}{600}\right) + \left(\frac{21}{20}\right)$$

$$= 0.0075x^4 - 0.101667x^3 + 0.4425x^2 - 0.69833x + 1.05$$

2) Given data:

x	1.0	1.3	1.6	1.9	2.2
y	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

At $x=1.5$ using Nevelle's method:→

x_i	f_i	f_i^0	f_i^1	f_i^2	f_i^3	f_i^4
x_1	y_1	f_1^0	f_1^1	f_1^2	f_1^3	f_1^4
x_2	y_2	f_2^0	f_2^1	f_2^2	f_2^3	
x_3	y_3	f_3^0	f_3^1	f_3^2		
x_4	y_4	f_4^0	f_4^1			
x_5	y_5	f_5^0				

$$f_1^2 = \frac{(x-x_i)^2 \; f_i^{\circ}+1 - (x-x_{i+n}) \; f_i(n-1)}{x_{i+n} - x_i}$$

for

$$f_i^1 = \frac{(x-x_i) \; f_i^0+1 - (x-x_{i+1}) \; f_i^{\circ}}{x_i+1 - x_2}$$

$\therefore f_1^1 = 0.52334067$

$$f_1^1 = \frac{(1.5-1) \times 0.6220860 - (1.5-1.3) \times 0.7656}{1.3-1.0}$$

$$f_2^1 = \frac{(1.5-1.6) \times 0.2818166 - (1.5-1.9)(.4554022)}{1.9-1.6}$$

$\therefore f_2^1 \Rightarrow 0.51326340$

$$f_2^3 = \frac{(1.5-1.3)(0.51373613 - (1.5-2.2) \times 0.5137613}{2.2-1.3}$$

$\therefore f_2^3 \Rightarrow 0.51183022$

$$f_1^4 = \frac{(1.5-1.0)(.51103021) - (1.5-2.2)(.51181269402)}{2.2-1.0}$$

$\therefore f_1^4 = 0.51081999$

x	f_i	f_i^1	f_i^2	f_i^3	f_i^4
1.0	0.7651977				
1.3	0.6200860	0.5233449			
1.6	0.4554022	0.5102968	0.5124715		
1.9	0.2818186	0.51326234	0.5112857	0.5118127	
2.2	0.1103623	0.5104270	0.5137361	0.5118302	0.5118200

(3)

	x	y
0	0.628318530	0.5877852523358846
1	1.2566370616	0.5948561851889
2	1.884955925	0.9510565162190 97
3	2.5132741232	0.58778525 2026982

(a)

	0	1	2	3
0	0.5877852523358846	0.011262758	0.442191368	0.248850926
1		0.5948618551889	0.566910434	-0.911264313
2			0.95105651621909	-0.578163701
3				0.5877852520269 82

$f_{01} = \dfrac{0.594861851889 - 0.5877852823358846}{1.2566370616 - 0.6283185308} = 0.011262758$ (0.628318531)

$\dfrac{0.9510565162190 97 - 0.5948561851889}{0.628318531} = 0.566910434$

$\dfrac{0.5877852520269 82 - 0.95105651621909 7}{0.62831853 1} = -0.578163701$

$f_{02} = \dfrac{0.566910434 - 0.011262758}{1.884955924 - 0.6283185308} = 0.442191368$ (1.256577343)

$\dfrac{-0.578163701 - 0.566910434}{1.256577393} = -0.911264313$

$f_{03} = \dfrac{-0.911264313 - 0.442191368}{2.51327412 32 - 0.6283185308} = -0.248850926$ (1.884917592)

b) $P_3(x) = 0.5877852358846 + x + 0.442191368(x)(x - 0.62831-)$ 85305
$+ 0.442191368(x - 0.6283185308)(x - 1.2566370616) +$
$0.248850926 (x - 0.6283185308)(x - 1.2366370616)$
$(x - 1.884955924)$

c) $P(1.5) = 2.528333381$

4. Let $f(x) = x^4 + \sqrt{2}x^3 + \pi x$

verify whether, $f[1,2,3,4] = f[0,1,\pi,e,-1]$

Now, $f(1) = 1 + \sqrt{2} + \pi$

$f(2) = 2^4 + \sqrt{2} \cdot 2^3 + 2\pi = 16 + 8\sqrt{2} + 2\pi$

$f(3) = (3)^4 + \sqrt{2} \cdot 3^3 + 3\pi = 81 + 27\sqrt{2} + 3\pi$

$f(4) = (4)^4 + 4^3\sqrt{3} + 4\pi = 256 + 64\sqrt{3} + 4\pi$

f,

$f(0) = 0 \neq f[1,2,3,4]$

i.e in Range of $f[0,1,\pi,e,-1]$ 0 is contain

but in range of $[1,2,3,4]$ 0 does not contain

i.e. $0 \in f(0,1,\pi,e,-1]$

& $0 \notin f[1,2,3,4]$

$\Rightarrow f[1,2,3,4] \neq f[0,1,\pi,e,-1]$

$\Rightarrow$ No, they are not equal

(5) Quadratic spline $S_i(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2$
where $i = 0, 1, 2, \ldots, n-1$
here we need to find $3n$ unknowns: a_i, b_i & c_i
$a_i = a_0, a_1, a_2, \ldots, a_{n-1}$
$b_i = b_0, b_1, b_2, \ldots, b_{n-1}$
$c_i = c_0, c_1, c_2, \ldots, c_{n-1}$
so that $S_i(x)$ satisfy the following conditions:
(1) $S_i(x_i) = y_i$, $i = 0, 1, 2, \ldots n$ condition for interpolation, a total of $n+1$ equations
(2) $S_i(x_i+1) = S_{i+1}(x_i+1)$, $i = 0, 1, 2, \ldots, n-2$ i condition for continuity @ interior points a total of $n-1$ equations
(3) $S_i'(x_i+1) = S_{i+1}'(x_i+1)$, $i = 0, 1, 2, \ldots, n-2$; condition for continuous slope @ interior points; $n-1$ equations.
(4) one additional condition from the following 3 conditions:
4a) $S'(x_0) = \alpha$ 4.c) $S'(x_0) = S'(x_n)$
4.b) $S'(x_n) = \beta$
Since we have $3n$ equations for $3n$ unknowns, we can again solve a_i, b_i & c_i uniquely from the conditions (1), $S_i(x_i) = y_i$ $i = 0, 1, 2, \ldots n$ we can obtain a_i for $i = 0, 1, 2, \ldots n-1$
$S_i(x_i) = a_i = y_i$, $i = 0, 1, 2, \ldots n-1$
& an equation in b_{n-1} & c_{n-1}
$S_{n-1}(x_{n-1}) = y_{n-1} + b_{n-1}(x_n - x_{n-1}) + c_{n-1}(x_n - x_{n-1})^2$
$= y_n \rightarrow 1.1$
from condition (2) we $S_i(x_i+1) = S_{i+1}(x_i+1)$, $i = 0, 1, \ldots n-2$, we can obtain $n-1$ equations in b_i, c_i for $i = 0, 1, \ldots n-2$
$S_i(x_i+1) = y_i + b_i(x_{i+1} - x_i) + c_i(x_{i+1} - x_i)^2$
$= S_{i+1}(x_i+1) = y_{i+1}$, $i = 0, 1, \ldots n-2 \Rightarrow$ 2-1
Similarly, from condition (3)
$S_i'(x_i+1) = S_{i+1}'(x_i+1)$; $i = 0, 1, \ldots n-2$ we can obtain $n-2$ equations in b_i & c_i for $i = 0, 1, \ldots n-2$

$S_i'(x_{i+1}) = b_i + 2c_i(x_{i+1} - x_i)$

$= S_{i+1}'(x_{i+1}) = b_{i+1}, \quad i = 0, 1, \ldots, n-2 \Rightarrow$ 3.1

w/ 1 more equation from condition given in 4,

4.a) $S'(x_0) = S_0'(x_0) = b_0 + 2c_0(x_1 - x_0) = \alpha$

4.b) $S'(x_n) = S_{n-1}'(x_0) = b_{n-1} + 2c_{n-1}(x_n - x_{n-1}) = \beta$

4.c) $S'(x_0) = b_0 + 2c_0(x_1 - x_0) = S'(x_n) = b_{n-1} + 2c_{n-1}(x_n - x_{n-1})$

We have 2n equations to solve b_i's & c_i's.

Example :->

Let $f(x) = \sqrt{x+1}$ Given $(0,1), (3,2), (8,3)$

(a) Construct a quadratic spline $Q(x)$ suppose that we also know $f'(0) = 1/2$

(b) approx $f(2)$ by $S(2)$ & $f(5)$ by $S(5)$;

(c) approx $f'(3)$ by $S'(3)$ & $f'(5)$ by $S'(5)$; &

(d) approx $\int_0^3 f(x)\,dx$ by $\int_0^3 S(x)\,dx$

(a) $S(x) = \begin{cases} S_0(x) = a_0 + b_0 x + c_0 x^2 & \text{if } 0 \le x \le 3 \\ S_1(x) = a_1 + b_1(x-3) + c_1(x-3)^2 & \text{if } 3 \le x \le 8 \end{cases}$

$= \begin{cases} S_0(x) = 1 + b_0 x + c_0 x^2 & \text{if } 0 \le x \le 3 \\ S_1(x) = 2 + b_1(x-3) + c_1(x-3)^2 & \text{if } 3 \le x \le 8 \end{cases}$

we solve b_0, b_1, c_0 & c_1 as follows from condition 1.1

$S_1(8) = 2 + b_1(5) + \cancel{c_1(5)^2}\, c_1 5^2$

$= 2 + 5b_1 + 25c_1 = y_2 = 3 \Rightarrow$ 1.2

from condition 2.1

$S_0(3) = 1 + b_0(3) + c_0(3)^2 = 1 + 3b_0 + 9c_0$

$= S_1(3) = y_1 = 2 \Rightarrow$ 2.2

from condition 3.1,

$S_0'(3) = b_0 + 2c_0(3) = b_0 + 6c_0 = S_0'(3) = b_1 \Rightarrow$ 3.2

from condition 4.1

$S_0'(0) = b_0 = 1/2$

Solve c_0 in (2.2) $\Rightarrow c_0 = 1/9\ (2-1-3b_0)$
$= 1/9\ (1-3/2) = -1/18$

Solve b_1 in (3.2) $\Rightarrow b_1 = 1/2 + 6(-1/18) = 1/6$

Solve c_1 in (1.2) $\Rightarrow c_1 = 1/25\ (3-2-5/6) = 1/150$

hence,

$$s(x) = \begin{cases} s_0(x) = 1 + 1/2\,x + 1/18\,x^2 & \text{if } 0 \le x \le 3 \\ s_1(x) = 2 + 1/6(x-3) + 1/150\,(x-3)^2 & \text{if } 3 \le x \le 5 \end{cases}$$

(b) $f(2) \approx s(2) = s_0(2) = 1 + 1/2(2) - 1/18\ (2)^2$
$= 16/9 = 1.7777778$

$f(5) \approx s(5) = s_1(5) = 2 + 1/6(2) + 1/150(2)^2 = 59/25 = 2.36$

$|\sqrt{2}+1 - 16/9| = 0.04573$

$|\sqrt{5+1} - 59/25| = 0.0894897$

(c) $f'(3) \approx s'(3) = s_0'(3) = s_1'(3) = 1/6$

$f'(5) \approx s'(5) = s_1'(5) = 1/6 + 2/150\ (5-3)$
$= 29/150 = 0.19333$

(d) $\int_0^3 f(x)dx \approx \int_0^3 s(x)dx = \int_0^3 s_0(x)dx$

$= \int_0^3 (1 + 1/2\,x - 1/18\,x^2)dx$

$= 4.75$