

Homework 5

CS 323 - Numerical Analysis

1. Given the following linear system:

$$A = \begin{pmatrix} 3.02 & -1.05 & 2.53 \\ 4.33 & 0.56 & -1.78 \\ -0.83 & -0.54 & 1.47 \end{pmatrix}, \quad \text{and} \quad \bar{b} = \begin{pmatrix} -1.61 \\ 7.23 \\ -3.38 \end{pmatrix}$$

- (a) Compute the condition number of the matrix
 - (b) Find the solution to the system using Gaussian Elimination with pivoting and using six-digit arithmetic.
 - (c) Use Gaussian Elimination with pivoting and using three digit arithmetic.
 - (d) Comment about the difference between both solutions
2. As we did in class, some curve fitting problems can be converted to linear least square approximation problems. One example is the fitting function $y = ax^b$. For this exercise suppose that you are given a set of points (x_i, y_i) $1 \leq i \leq n$ and you want to find the values of a and b for the following fitting function

$$y = \frac{1}{(ax + b)^3}$$

Use the least squares method to find the formulas to compute a and b .

3. Given the following data, do a quadratic least squares polynomial fit using $P_2(x) = a_0 + a_1x + a_2x^2$.

x_i	y_i	x_i	y_i	x_i	y_i
-1.0	7.904	-0.3	0.335	0.4	-0.711
-0.9	7.452	-0.2	-0.271	0.5	0.224
-0.8	5.827	-0.1	-0.963	0.6	0.689
-0.7	4.400	0	-0.847	0.7	0.861
-0.6	2.908	0.1	-1.278	0.8	1.358
-0.5	2.144	0.2	-1.335	0.9	2.613
-0.4	0.581	0.3	-0.656	1.0	4.599

4. For $n, m \geq 0$ and $n \neq m$, show

$$\int_{-1}^1 \frac{T_n(x)T_m(x)}{\sqrt{1-x^2}} dx = 0$$

This is called the *orthogonality relation* for Chebyshev polynomials.

Hint: Use $T_n(x) = \cos(n \cos^{-1}(x))$ and $x = \cos \theta$.