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HW – 1

(1) We observe that sign of (a) is negative, F(b) is positive, therefore $f(a) \times f(b)$ is negative, There must be a real root between a and b

Iteration	a	b	c	|b − a|	F(a) F(b)
0	0.5−	2	1.25+	1.5	negative
1	0.5−	1.25	0.875+	0.75	negative
2	0.5−	0.875	0.6875−	0.375	positive
3	0.6875−	0.875	0.78125−	0.1875	positive

If we calculate the true value of x for this equation, it about 0.83666, we find the at iteration#3, it shows there are some roots exists in the interval [0.78125, 0.825) which there are very close to the true value of x

(2) Method 1: Newton's Method formula:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad f'(x) = 6 - 8x + 1.5x^2$$

If we try $x_0 = 0$, then,

i	x_0	x_1	Error
0	1	2	1
1	2	1.5	0.5
2	1.5	1.38095	0.119
3	1.38095	1.36927	0.0117
4	1.36927	1.36910	0.000167

The real roots exists in the interval [6.15637, 6.15633] as well, which it has 0.00045 tolerance error < 0.01

~~Method 2~~

$\cancel{f(x) = 0.5x^3 - 4x^2 + 8x - 2}$

3. $x + y = 20$

$(x + \sqrt{x})(y + \sqrt{y}) = 155.55$

substitute x w/ y $(x + \sqrt{x})(20 - x + \sqrt{20 - x})$

$= 155.55$

$\hookrightarrow f(x) = 20x - x^2 + \sqrt{20 - x} \cdot x + 20\sqrt{x} - x\sqrt{x}$

$+ \sqrt{20 - x}\sqrt{x} = 155.55$

estimate x between $[6.5, 6.52]$

$F(6.5) = -0.1316 < 0$, $f(6.52) = 0.073 > 0$

i	a	b	c	|b - a|	f(a) f(c)
1	6.5	6.52	6.51	0.02	positive
2	6.51	6.52	6.515	0.01	negative
3	6.5125	6.515	6.5125	0.005	positive
4	6.51375	6.515	6.51375	0.0125	negative
5	6.514375	6.515	6.514375	0.006625	positive
6	6.514687	6.515	6.5148437	0.0003125	positive
7	6.5148437	6.515	6.514922	0.00013625	positive
8	6.514922	6.515	6.514961	0.000078	positive

One number is in the interval $(6.514961 - 0.000005) < \text{solution} < (6.514961 + 0.000005)$ which is $6.514911 < x < 6.515011$

according to $x + y = 20$, get the interval of y

4. $A = \frac{P}{i}\left(1-(1+i)^{-n}\right)$ $\quad A = 800{,}000$ $\quad \frac{1-(1+i)^{-360}}{i} \geq \frac{800}{7}$

$P \leq 7000$

$P = \frac{A \times i}{1-(1+i)^{-360}} \leq 7000$ $\quad \frac{1}{i} - \frac{1}{i(1+i)^{360}} \geq \frac{800}{7}$

$P = \frac{800000\, i}{1-(1+i)^{-360}} \leq 7000$ $\quad f(i) = \frac{1}{i} - \frac{1}{i(1+i)^{360}} - \frac{8000}{7}$

Newton's method :

i	x_0	x_1	error
0	0.008	0.008259	0.000259
1	0.008259	0.008299	0.0000399
2	0.008299	0.0083037	0.0000047

highest monthly interest is between $[0.008299, 0.0083037]$.

5. enumerate all the elements in $f, (2, 2, -1, 1)$

$\pm 0.10 \times 10^{-1}$ $\quad \pm 0.00 \times 10^{0}$ $\quad \pm 0.10 \times 10^{1}$

$\pm 0.11 \times 10^{-1}$ $\quad \pm 0.10 \times 10^{0}$ $\quad \pm 0.11 \times 10^{1}$

$\pm 0.20 \times 10^{-1}$ $\quad \pm 0.11 \times 10^{0}$ $\quad \pm 0.20 \times 10^{1}$

$\pm 0.21 \times 10^{-1}$ $\quad \pm 0.20 \times 10^{0}$ $\quad \pm 0.21 \times 10^{1}$

$\pm 0.21 \times 10^{0}$

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6. $x^3 - 7x^2 + 14x - 6 = 0$ in $[1, 3.2]$; $\epsilon = 10^{-2}$

$$c = \frac{a+b}{2}$$

a	b	c	error
1	3.2	2.1	2.2
2.1	3.2	2.65	1.8
2.65	3.2	2.925	0.55
2.925	3.2	3.0625	0.275
2.925	3.0625	2.99375	0.1375
2.99375	3.0625	3.028125	0.6875
2.99375	3.021825	3.0109	0.0344
2.99325	3.0109	3.002325	0.0172
2.99325	3.002325	2.998	0.0086
2.998	3.002325	3.0001625	0.04325

The real values are in the interval $[2.99375, 3.002325]$.

7) $P(x) = 10x^3 - 8.3x^2 + 2.75x - 0.2141 = 0$

a) $a = 0.25$, $b = 0.3$, $\epsilon = 10^{-3}$

a	b	c	error
0.25	0.3	0.275	0.05
0.275	0.3	0.2875	0.025
0.2875	0.3	0.29375	0.0175
0.2875	0.29375	0.290625	0.00625
0.2875	0.290625	0.290638	0.003125
0.289063	0.290625	0.2898375	0.001562
0.28984375	0.290625	0.2902343	0.000718

Hence, the root lies in the interval $[0.289063, 0.290625]$

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b) $x_0 = 0.28$

$f(x) = 10x^3 - 8.3x^2 + 2.295x - 0.2141$

$f'(x) = 30x^2 - 16.6x + 2.295$

x_0	x_1	error
0.28	0.27	0.01
0.27	undefined	

There exists a solution around $x = 0.27$

8) $f(x) = \sqrt{1+\frac{1}{x}} - 1$ $\quad \lim_{x\to\infty} x f(x)$

$f(x) = \sqrt{1+\frac{1}{x}} = \sqrt{1} - 1 = 0$

$\lim_{x\to\infty} x\left(\sqrt{1+\frac{1}{x}} - 1\right) = \lim_{x\to\infty} x \cdot \frac{(\sqrt{1+1/x}-1) \times (\sqrt{1+1/x}+1)}{(\sqrt{1+1/x}+1)}$

$= \lim_{x\to\infty} \frac{x\left((1+1/x)-(1)\right)}{\sqrt{1+1/x}+1}$

$= \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1}$

$= \frac{1}{\sqrt{1+0}+1}$