

CS 344 HWK2

1. Consider the binary numbers $x = 11011010$ and $y = 10011011$. As in class, decomposing x into a , b and y into c , d , compute the product xy using the divide-and-conquer method by forming $w_1 = a + b$, $w_2 = c + d$, $u = w_1w_2$, $v = ac$, $w = bd$. Recall $xy = 2^nv + 2^{n/2}(u - v - w) + w$. Each multiplication by 2 amounts to a shift.

2. You are given an infinite array $A[i]$ in which the first n elements are integers in sorted order and the rest are filled with ∞ . You are not given n . Describe an algorithm that takes as input an integer x and finds a position in the array containing x , if such a position exists, in $O(\log n)$ time.

3. Solve each of the recurrence relations and give θ bound for each. You can use the master theorem if applicable.

(a) $T(n) = 5T(n/4) + n$.

(b) $T(n) = 7T(n/7) + n$.

(c) $T(n) = 9T(n/4) + n^2$.

(d) $T(n) = 8T(n/2) + n^3$.

(e) $T(n) = 49T(n/25) + n^{3.5} \log n$.

(f) $T(n) = T(\sqrt{n}) + 1$.

(g) $T(n) = T(\sqrt{n}) + \log n$.

4. Find the coefficients of the polynomial of degree 2, $p(x) = a_2x^2 + a_1x + a_0$ such that $p(1) = -1$, $p(2) = 1$, $p(3) = 0$.

5. (20 pts) Consider the array 25, 14, 63, 29, 63, 47, 12, 21.

Apply the Split procedure in Quicksort, as described in class, to this array using the first element as the pivot.