CS 344, Final Exam, Spring 2020
Maximum Points: 280 points

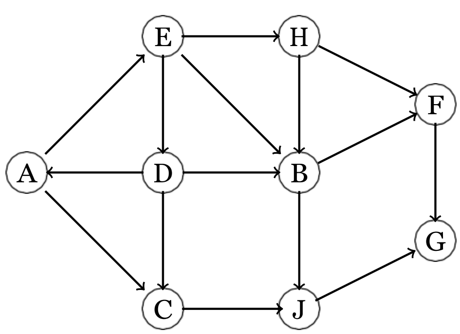
Show all work here and justify your answers.

Problem 1. (20 pts) For the recurrence relation $T(n) = 8T(\frac{n}{b}) + O(n^d)$, $T(1) = 1$ do
(a) (10 pts) Assign smallest positive integer values to b and d so that $T(n) = O(n^3)$.

(b) (10 pts) Assign values to b and d so that $T(n) = O(n^2 \log n)$.

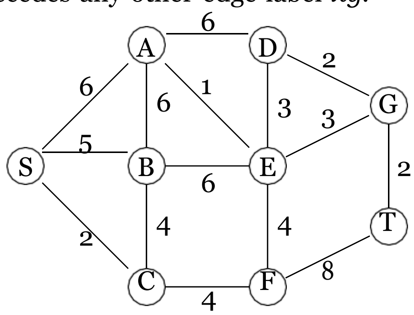
Problem 2. (20 pts) Given an unsorted array A of size n, give a linear time algorithm to output all elements between 80-th and 90-th percentile.

Problem 3. (20 pts) Do a DFS starting at A and then assuming vertices are considered in alphabetical order. Show discovery and finish times and classify edges as tree edges, back edges, etc.

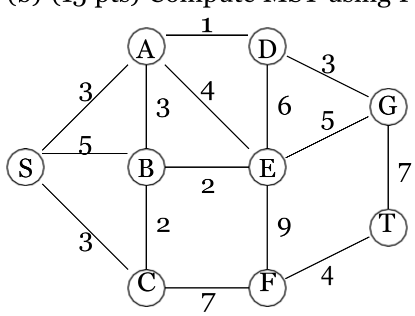


Problem 4. (60 pts) Show all your work on graphs.

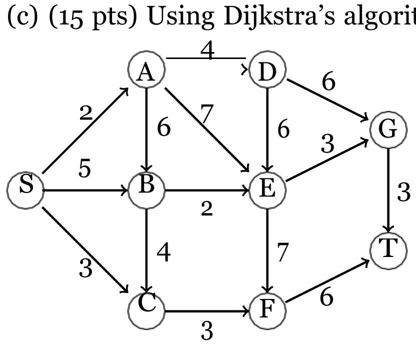
(a) (15 pts) Compute MST using Kruskal's algorithm. Show the order of the selected edges. If there is a tie, select edges in dictionary ordering, i.e. if you choose edge (u, v), then uv lexicographically (alphabetically) precedes any other edge label xy.



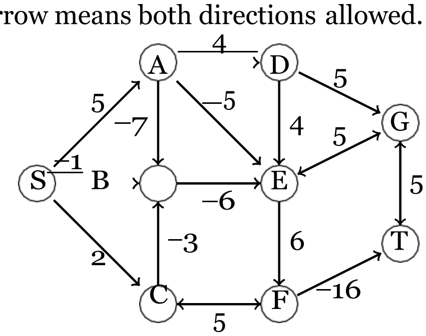
(b) (15 pts) Compute MST using Prim's algorithm. Start at S and give the order of the selected edges.



(c) (15 pts) Using Dijkstra's algorithm find the shortest path from S to all other vertices.



(d) (15 pts) By inspection find the shortest distance from S to all other vertices, finite or infinite. Double arrow means both directions allowed.



Problem 5. (20 pts) For each pattern $AAAAAB$, $ABABAC$ write $fail(k)$ for each $k = 1, 2, 3, 4, 5, 6$.

Problem 6. (20 pts)

(a) (10 pts) Given a CNF formula on n Boolean variables $x_1, \ldots, x_n$ and their negations with 5 clauses, describe a polynomial-time algorithm that would test if it is satisfiable.

(b) (10 pts) Convert the CNF to an equivalent 3-CNF formula: $(x_1 \vee x_2 \vee x_3 \vee \overline{x_4}) \wedge (x_1 \vee x_2 \vee x_3 \vee x_4 \vee \overline{x_5})$.

Problem 7. (20 pts)

(a) (10 pts) Given an undirected graph $G = (V, E)$, a subset of vertices V^t is an *independent set* if each edge in E is incident to at most one vertex in V^t. The independent set problems asks if G has an independent set of size k. Considering the graph of Problem 3 as an undirected graph, find an independent set of the largest size.

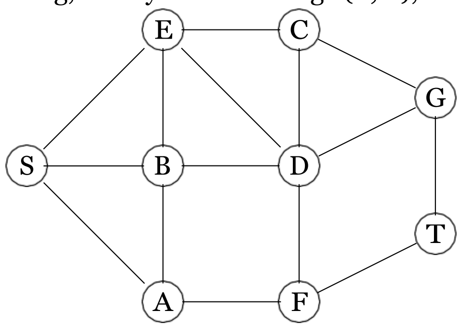
(b) (10 pts) For a given graph G, consider $\overline{G}$, the complement graph of G (same vertices, complementary edges) and the clique problem. What relationship can you state between an independent set in G and clique in $\overline{G}$? What can you conclude about the complexity class of independent set problem?

Problem 8. (30 pts) For the graph shown below do:

(a) (10 pts) Give a maximum independent set.

(b) (10 pts) Give a minimum vertex cover.

(c) (10 pts) Apply the vertex cover approximation algorithm described in lecture, using edges in dictionary ordering, i.e. if you choose edge (u, v), then uv lexicographically precedes any other edge label xy.

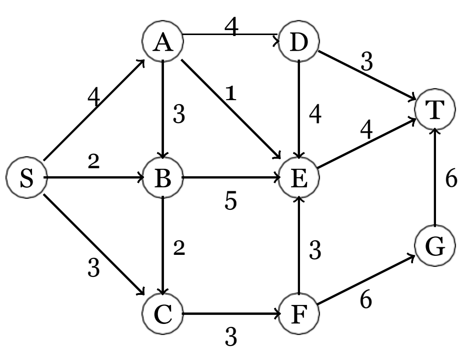


Problem 9. (30 pts)

(a) (10 pts) For the given directed graph first find the maximum flow from S to T by inspection.

(b) (10 pts) Give the corresponding residual graph.

(c) (10 pts) Give the minimum (S,T) cut from this residual graph.



Problem 10. (20 pts) Using the shortest path algorithm for graphs, but via inspection, determine if the following system of inequalities has a feasible solution. If it has a feasible solution, find one. Otherwise, say why it is not feasible.

$$x_1 - x_3 \leq 4$$
$$x_5 - x_1 \leq -2$$
$$x_1 - x_2 \leq 3$$
$$x_4 - x_3 \leq -2$$
$$x_2 - x_4 \leq -3$$
$$x_3 - x_2 \leq 8$$
$$x_4 - x_5 \leq 2.$$

Problem 11. (20 pts) True-False answers only.

(a) To decide if a graph is 2-colorable is in P.

(b) In any complete graph with nonnegative weights on edges, the weight of any minimum spanning tree is no larger than the weight of any Hamiltonian cycle.

(c) Computing the minimum vertex cover of a graph G that is a tree is solvable in polynomial time.

(d) To test if an integer n is not prime is in co-NP.