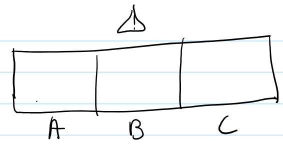
Will be on Final Exam

Friday, December 6, 2019 10:41 AM



$X_t \in \{A, B, C\}$
$E_t \in \{cold, hot\}$

Initial distribution $P(X_1) = \begin{bmatrix} \frac{1}{2}, & \frac{1}{4}, & \frac{1}{4} \end{bmatrix}$
(A, B, C)

Transition model

$P(X_{t+1} | X_t)$

X_t \ X_{t+1}	A	B	C
A	0.1	0.9	0
B	0	0.1	0.9
C	0	0	1.0

Observational model

$P(E_t | X_t)$

X_t \ E_t	cold	hot
A	0.8	0.2
B	0.2	0.8
C	0.9	0.1

$\Rightarrow$ **Filtering**

Find $P(X_2 | cold_1, hot_2) = ?$ (We will predict x_2 based all we know of past and future).

1st step Pay1

$P(x_1 | cold_1) = \alpha\, P(cold_1 | X_1) \cdot P(X_1)$

$= \alpha \left[P(cold_1 | X_1 = A), P(cold_1 | X_1 = B), P(cold_1 | X_1 = C) \right]$
$\cdot \left[P(X_1 = A), P(X_1 = B), P(X_1 = C) \right]$

$= \alpha \left[0.8 \cdot \frac{1}{2}, 0.2 \times \frac{1}{4}, 0.9 \times \frac{1}{4} \right]$

$= \alpha \left[0.4, 0.05, 0.225 \right]$

$= \left[\frac{0.4}{0.675}, \frac{0.05}{0.675}, \frac{0.225}{0.675} \right]$

$= [0.59, 0.07, 0.33]$ (Adds to 1 so good)

- $P(X_2 | cold_1, hot_2) = \alpha\, P(hot_2 | X_2) \times \sum_{X_1 \in \{A,B,C\}} P(X_1 | cold_1)\, P(X_2 | X_1)$

What is probility X_2 based on first day weather and second day weather

Probabity of being hot on day 2

Probabity of X_1 based on first day weathr (Step 1)

- $P(X_2 = A | cold_1, hot_2) = \alpha P(hot_2 | X_2 = A) \sum_{X_1 \in \{A,B,C\}} P(X_1 | cold_1) \cdot P(X_2 = A | X_1)$

$= \alpha\, P(hot_2 | X_2 = A) \cdot \left[P(X_1 = A | cold_1) \cdot P(X_2 = A | X_1 = A) + P(X_1 = B | cold_1) \cdot P(X_2 = A | X_1 = B) + P(X_1 = C | cold_1) \cdot P(X_2 = A | X_1 = C) \right]$ this is 0

$= \alpha \cdot 0.2 \cdot [0.59 \cdot 0.1 + 0.07 \cdot 0 + 0.33 \cdot 0]$

Different from step1

$= \alpha \cdot 0.012$ Repeat for B and C for α_1.

$P(X_2 = A | cold_1, hot_2) = \alpha \cdot 0.012$
$P(X_2 = B | cold_1, hot_2) =$ Replace X_2 by B $= \alpha\, 0.05$

(to find α. Add them all to 1.

$P(X_2=A \mid cold_1, hot_2) = \alpha \cdot 0.012$

$P(X_2=B \mid cold_1, hot_2) = \leftarrow$ Replace X_2 by B $= \alpha\, 0.05$

$P(X_2=C \mid cold_1, hot_2) = \leftarrow$ Replace X_2 by C. $= \alpha\, 0.01$

[to find α. Add them all to 1. we will get $\alpha[0.05 + 0.01 + 0.012] = 1$]

Solve for it all using α

and final answer. $P(X_2 \mid cold_1, hot_2) = [\quad]$

- Extra question.

$$P(X_3 \mid cold_1, hot_2, hot_3) = \alpha \cdot P(cold_3 \mid X_3) \cdot \sum_{X_2=\{A,B,C\}} P(X_2 \mid cold_1, hot_2)\, P(X_3 \mid X_2)$$

No alpha here

$\Rightarrow$ Prediction $P(cold_3 \mid X_2) = (?)$

Do it for $X_2 = \{A, B, C\}$

$$= \sum_{X_3 \in \{A,B,C\}} P(cold_3 \mid X_3) \cdot P(X_3 \mid X_2)$$

$$= P(cold_3 \mid X_3=A) \cdot P(X_3=A \mid X_2) + P(cold_3 \mid X_3=B) \cdot P(X_3=B \mid X_2) + P(cold_3 \mid X_3=C) \cdot P(X_3=C \mid X_2)$$

$$= P(cold_3 \mid X_3=A) \cdot P(X_3=A \mid X_2=A) + P(cold_3 \mid X_3=B) \cdot P(X_3=B \mid X_2=A) + P(cold_3 \mid X_3=C) \cdot P(X_3=C \mid X_2=A)$$

Repeat for $X_2=B$ and $X_2=C$

$$+ P(cold_3 \mid X_3=A) \cdot P(X_3=A \mid X_2=B) + P(cold_3 \mid X_3=B) \cdot P(X_3=B \mid X_2=B) + P(cold_3 \mid X_3=C) \cdot P(X_3=C \mid X_2=B)$$

$$+ P(cold_3 \mid X_3=A) \cdot P(X_3=A \mid X_2=C) + P(cold_3 \mid X_3=B) \cdot P(X_3=B \mid X_2=C) + P(cold_3 \mid X_3=C) \cdot P(X_3=C \mid X_2=C)$$

$\Rightarrow$ $P(X_2 \mid cold_1, hot_2, cold_3)$ Smoothing (Finding X_2 based on what we know from **past AND future**.)

So basically we improve our belief (filtering) of earlier stage)

$$= \alpha\, \underbrace{P(cold_3 \mid X_2)}_{\text{prediction}} \cdot \underbrace{P(X_2 \mid cold_1, hot_2)}_{\text{filtering}}$$

$$= \alpha \cdot [X_2=A, X_2=B, X_2=C] \cdot [X_2=A, X_2=B, X_2=C]$$

answers from filtering

$= \alpha [\quad , \quad , \quad] \leftarrow$ Normalize it.