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Discrete Mathematics II Final Exam

Name ______________

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Section 0

Directions* Read each exam question carefully before you answer. Show all relevant work so that partial credit may be awarded for an intelligent attempt at a solution. Please leave your answers in closed form whenever possible. Be extremely careful and good luck!

1. At a certain college, a recent survey taken reveals that the probability a student consumes alcoholic beverages is one out of every eleven students. Given that the process is binomially distributed and $n=25$ students are randomly selected, what is the probability; (a) exactly three students (b) less than three students consume alcoholic beverages? What is the expected number of students that consume alcoholic beverages?

certain surgical procedure is a Poissonly Distributed process with the probability of a patient suffering unwanted side effects is $P = .0000007$. Given that the procedure is performed $n = 8{,}000{,}000$ times, what is the probability that;
(a) exactly four people (b) exactly eight people will suffer unwanted side effects?

3. A probability distribution has a moment generating function, $M_x(t) = \frac{3}{(5-e^{2t})}$. What is the numeric value for the mean (μ) and the standard deviation of this distribution?

organization wishes to sell raffle tickets to raise money or their specific cause. A person is allowed to purchase only one lottery ticket. The payoff amounts and corresponding probabilities are given in the chart below;

X	\$20	\$25	\$40	\$50	\$75	\$80	\$100	\$125	\$150
P(X)	.25	.15	.12	.10	.08	.075	.06	.05	.04

(a) If you (the student) obtained a lottery ticket for free, what amount of money can you expect to win?

(b) If the organization charges \$50 per ticket, what is the amount of money you now can expect to win?

(c) What is a fair price to pay to play the game?

(d) With your newfound knowledge of discrete mathematics, do you feel confident playing this "game?"

die is "loaded" is such a way that a "2" appears three times as likely as any other face and a "5" appears three times as likely as a "2." You roll this die <u>three</u> times. Let X be the discrete random variable corresponding to the number of times a "2" appears in three rolls. What is (a) the probability distribution f(x) for X?

(b) the cumulative distribution F(x) for X?

If you receive $10 for any face, except a "2" where you lose $12, what is the expected value of this "game?"

Given the two-person, zero-sum game between Player A and Player B, with payoff matrix P, what is the value of the game and what should the strategy be of each player?

(b) Is this game stable?

Player A

$$
\text{Player B}\quad
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{cccc} \mathrm{I} & \mathrm{II} & \mathrm{III} & \mathrm{IV} \end{array} \\
\begin{pmatrix}
3 & -4 & 2 & -2 \\
-2 & 2 & -5 & -3 \\
-4 & -1 & -4 & -2 \\
0 & 7 & 1 & -1 \\
6 & -8 & -1 & -1
\end{pmatrix}
\end{array}
$$

7. Given the payoff matrix for the two-person, zero-sum game, $P = \begin{pmatrix} 9 & -5 \\ -8 & 6 \end{pmatrix}$, what are/is the (a) strategies for each player?

(b) value of the game?

the 2020-2021 academic year, Rutgers University will eliminate all bus service to the Livingston Campus. Instead, a monorail system will transport students exclusively from the Busch campus to the Livingston Campus. Busses will, however, continue to transport students between the Busch, College Avenue and Douglass College campuses. Busses from the Busch Campus are five times as likely to stop on College Avenue and four times as likely to stop on Douglass as they are on Busch campus. Busses from College Avenue are three times as likely to stop on Busch campus and four times as likely to stop on Douglass as they are on College Avenue. Lastly, busses from Douglass are five times as likely to stop on Busch and twice as likely to stop on College Avenue as they are on Douglass campus? What is the;

(a) regular stochastic matrix for this process?

(b) the fixed probability vector $\vec{t}$, such that $\vec{t}P=\vec{t}$, that yields the probability of a bus stops on each campus?

A frog has six lily pads with which to sit. At random intervals, the frog will jump from lily pad i to lily pad $i+1$ with probability p or jump from lily pad i to lily pad $i-1$ with probability $q=1-p$. If, however, the frog lands on lily pad 1 or lily pad 6, the frog, respectively, immediately jumps to lily pad 2 or lily pad 5 to avoid being eaten by an alligator. If $p=.55$ and $q=1-p=.45$, what is the transition matrix for the random walk? If, initially, the frog begins at lily pad 2 ($p^{(0)}=(0,1,0,0,0,0)$) after three iterations of $p^{(n)}P$, $n=1,2,3$, what are the probabilities each lily pad will be occupied by the frog?

Given the regular stochastic matrix, P, what is the fixed probability vector $\vec{t}$, such that $\vec{t}P=\vec{t}$? What is the numerical value of $Tr(P)$? What is the numerical value of <u>one</u> of the eigenvalues of P?

$$P=\begin{pmatrix} 1/8 & 1/2 & 1/8 & x \\ 1/4 & 3/8 & y & 1/8 \\ 1/8 & 1/8 & w & w \\ 0 & z & 1/8 & 3/4 \end{pmatrix}$$