

# Assignment 5: CS 663

## 1 Q1

**Q.** Suppose you are standing in a well-illuminated room with a large window, and you take a picture of the scene outside. The window undesirably acts as a semi-reflecting surface, and hence the picture will contain a reflection of the scene inside the room, besides the scene outside. While solutions exist for separating the two components from a single picture, here you will look at a simpler-to-solve version of this problem where you would take two pictures. The first picture  $g_1$  is taken by adjusting your camera lens so that the scene outside ( $f_1$ ) is in focus (we will assume that the scene outside has negligible depth variation when compared to the distance from the camera, and so it makes sense to say that the entire scene outside is in focus), and the reflection off the window surface ( $f_2$ ) will now be defocussed or blurred. This can be written as  $g_1 = f_1 + h_2 * f_2$  where  $h_2$  stands for the blur kernel that acted on  $f_2$ . The second picture  $g_2$  is taken by focusing the camera onto the surface of the window, with the scene outside being defocussed. This can be written as  $g_2 = h_1 * f_1 + f_2$  where  $h_1$  is the blur kernel acting on  $f_1$ . Given  $g_1$  and  $g_2$ , and assuming  $h_1$  and  $h_2$  are known, your task is to derive a formula to determine  $f_1$  and  $f_2$ . Note that we are making the simplifying assumption that there was no relative motion between the camera and the scene outside while the two pictures were being acquired, and that there were no changes whatsoever to the scene outside or inside. Even with all these assumptions, you will notice something inherently problematic about the formula you will derive. What is it?

### Solution:

- Take Fourier Transform on both sides of the 2 given equations:  
(lower case denotes signal in the time-domain while upper case denotes the frequency domain representation of the same signal e.g.  $\mathcal{F}(g_1(t)) = G_1(f)$ . The arguments t and f are dropped for clarity.)

$$g_1 = f_1 + h_2 * f_2 \implies G_1 = F_1 + H_2 F_2 \quad (1)$$

$$g_2 = h_1 * f_1 + f_2 \implies G_2 = H_1 F_1 + F_2 \quad (2)$$

Note that  $*$  denotes convolution in the time domain.

- Multiply (1) with  $H_1$  to get:

$$G_1 H_1 = F_1 H_1 + H_2 F_2 H_1 \quad (3)$$

- Subtract (2) from (3) to get:

$$G_1 H_1 - G_2 = H_2 F_2 H_1 - F_2 \implies F_2 = \frac{G_1 H_1 - G_2}{H_1 H_2 - 1} \quad (4)$$

- Along similar lines, multiply (2) with  $H_2$  to get

$$G_2 H_2 = H_1 F_1 H_2 + F_2 H_2 \quad (5)$$

- Subtract (1) from (5) to get:

$$G_2 H_2 - G_1 = H_1 F_1 H_2 - F_1 \implies F_1 = \frac{G_2 H_2 - G_1}{H_1 H_2 - 1} \quad (6)$$

- We can apply the Inverse Fourier Transform  $\mathcal{F}^{-1}$  to both sides of (4) and (6):

$$\begin{aligned} f_2 &= \mathcal{F}^{-1} \left( \frac{G_1 H_1 - G_2}{H_1 H_2 - 1} \right) \\ f_1 &= \mathcal{F}^{-1} \left( \frac{G_2 H_2 - G_1}{H_1 H_2 - 1} \right) \end{aligned} \tag{7}$$

- The problem with this approach is that it will not work when  $H_1 H_2 = 1$  (the denominator in (7) becomes zero). This can be seen from the time domain representation of the signals. In the time domain, this is equivalent to  $h_1 * h_2 = \delta(t)$ . Convolve the original equations with  $h_1$  and  $h_2$  to get:

$$\begin{aligned} g_1 &= f_1 + h_2 * f_2 \implies g_1 * h_1 = f_1 * h_1 + (h_1 * h_2) * f_2 \\ g_2 &= f_2 + h_1 * f_1 \implies g_2 * h_2 = f_2 * h_2 + (h_2 * h_1) * f_1 \end{aligned} \tag{8}$$

- For any function  $y(x)$ , we have:

$$y(x) * \delta(x) = y(x) \quad (\text{sifting property})$$

- Utilising this property, the equations in (8) reduce to:

$$\begin{aligned} g_1 * h_1 &= f_1 * h_1 + \delta * f_2 \implies g_1 * h_1 = f_1 * h_1 + f_2 = g_2 \\ g_2 * h_2 &= f_2 * h_2 + \delta * f_1 \implies g_2 * h_2 = f_2 * h_2 + f_1 = g_1 \end{aligned} \tag{9}$$

- The above 2 equations:  $g_1 * h_1 = g_2$  and  $g_2 * h_2 = g_1$  are the same because we can convolve the first one with  $h_2$  to get:

$$g_1 * (h_1 * h_2) = g_2 * h_2 \implies g_1 * \delta = g_2 * h_2 \implies g_1 = g_2 * h_2 \tag{10}$$

This is the same as the 2nd equation in (9).

- This implies that when  $h_1 * h_2 = \delta$  (or equivalently,  $H_1 H_2 = 1$ ), we have **only one independent** equation while there are **2 unknowns**:  $f_1$  and  $f_2$ . As a result, we cannot recover the original images  $f_1$  and  $f_2$ .