Assignment 5: CS 663

1 Q1

Q. Suppose you are standing in a well-illuminated room with a large window, and you take a picture of the scene outside. The window undesirably acts as a semi-reflecting surface, and hence the picture will contain a reflection of the scene inside the room, besides the scene outside. While solutions exist for separating the two components from a single picture, here you will look at a simpler-to-solve version of this problem where you would take two pictures. The first picture g_1 is taken by adjusting your camera lens so that the scene outside (f_1) is in focus (we will assume that the scene outside has negligible depth variation when compared to the distance from the camera, and so it makes sense to say that the entire scene outside is in focus), and the reflection off the window surface (f_2) will now be defocussed or blurred. This can be written as $g_1 = f_1 + h_2 * f_2$ where h_2 stands for the blur kernel that acted on f_2. The second picture g_2 is taken by focusing the camera onto the surface of the window, with the scene outside being defocussed. This can be written as $g_2 = h_1 * f_1 + f_2$ where h_1 is the blur kernel acting on f_1. Given g_1 and g_2, and assuming h_1 and h_2 are known, your task is to derive a formula to determine f_1 and f_2. Note that we are making the simplifying assumption that there was no relative motion between the camera and the scene outside while the two pictures were being acquired, and that there were no changes whatsoever to the scene outside or inside. Even with all these assumptions, you will notice something inherently problematic about the formula you will derive. What is it?

Solution:

- Take Fourier Transform on both sides of the 2 given equations:
  (lower case denotes signal in the time-domain while upper case denotes the frequency domain representation of the same signal e.g. $\mathcal{F}(g_1(t)) = G_1(f)$. The arguments t and f are dropped for clarity.)

$$g_1 = f_1 + h_2 * f_2 \implies G_1 = F_1 + H_2F_2 \tag{1}$$

$$g_2 = h_1 * f_1 + f_2 \implies G_2 = H_1F_1 + F_2 \tag{2}$$

  Note that * denotes convolution in the time domain.

- Multiply (1) with H_1 to get:

$$G_1H_1 = F_1H_1 + H_2F_2H_1 \tag{3}$$

- Subtract (2) from (3) to get:

$$G_1H_1 - G_2 = H_2F_2H_1 - F_2 \implies F_2 = \frac{G_1H_1 - G_2}{H_1H_2 - 1} \tag{4}$$

- Along similar lines, multiply (2) with H_2 to get

$$G_2H_2 = H_1F_1H_2 + F_2H_2 \tag{5}$$

- Subtract (1) from (5) to get:

$$G_2H_2 - G_1 = H_1F_1H_2 - F_1 \implies F_1 = \frac{G_2H_2 - G_1}{H_1H_2 - 1} \tag{6}$$

- We can apply the Inverse Fourier Transform $\mathcal{F}^{-1}$ to both sides of (4) and (6):

$$\begin{aligned} f_2 &= \mathcal{F}^{-1}(\frac{G_1H_1 - G_2}{H_1H_2 - 1}) \\ f_1 &= \mathcal{F}^{-1}(\frac{G_2H_2 - G_1}{H_1H_2 - 1}) \end{aligned} \tag{7}$$

- The problem with this approach is that it will not work when $H_1H_2 = 1$. This can be seen from the time domain representation of the signals. In the time domain, this is equivalent to $h_1 * h_2 = \delta(t)$. Convolve the original equations with h_1 and h_2 to get:

$$\begin{aligned} g_1 = f_1 + h_2 * f_2 &\implies g_1 * h_1 = f_1 * h_1 + (h_1 * h_2) * f_2 \\ g_2 = f_2 + h_1 * f_1 &\implies g_2 * h_2 = f_2 * h_2 + (h_2 * h_1) * f_1 \end{aligned} \tag{8}$$

- For any function $y(x)$, we have:

$$y(x) * \delta(x) = y(x) \tag{sifting property}$$

- Utilising this property, the equations in (8) reduce to:

$$\begin{aligned} g_1 * h_1 = f_1 * h_1 + \delta * f_2 &\implies g_1 * h_1 = f_1 * h_1 + f_2 = g_2 \\ g_2 * h_2 = f_2 * h_2 + \delta * f_1 &\implies g_2 * h_2 = f_2 * h_2 + f_1 = g_1 \end{aligned} \tag{9}$$

- The above 2 equations: $g_1 * h_1 = g_2$ and $g_2 * h_2 = g_1$ are the same because we can convolve the first one with h_2 to get:

$$g_1 * (h_1 * h_2) = g_2 * h_2 \implies g_1 * \delta = g_2 * h_2 \implies g_1 = g_2 * h_2 \tag{10}$$

  This is the same as the 2nd equation in (9).

- This implies that when $h_1 * h_2 = \delta$ (or equivalently, $H_1H_2 = 1$), we have **only one independent** equation while there are **2 unknowns:** f_1 **and** f_2. As a result, we cannot recover the original images f_1 and f_2.