Tutorial - 6

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F2 Batch

1.) we know that -

$V_{avg} = \sqrt{\frac{8kT}{\pi m}}$, $V_{rms} = \sqrt{\frac{3kT}{m}}$

So, $\frac{V_{rms}}{V_{avg}} = \sqrt{\frac{3\pi}{8}} = 1.09$

$\Rightarrow V_{rms} = 1.09\, V_{avg}$

2.) Molecular Mass of $N_2 = 28u = 28 \times 1.66^{-27}$ kg
$= 4.65 \times 10^{-26}$ kg

By formula → $\boxed{V_{rms} = \sqrt{\frac{3kT}{m}}}$

Now at $T = 273$ k, $k = 1.38 \times 10^{-23}$ J/K.

So $V_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 273}{4.65 \times 10^{-26}}}$

$= 493$ m/s

3.) Comparisons of the Distributions →

Maxwell Boltzmann	Bose Einstein	Fermi Dirac
* Particles are identical & distinguishable	Particles are identical & indistinguishable	Particles are identical & indistinguishable
* No Pauli Principle	No Pauli Principle	Obeys Pauli Principle
* Particles are called classical Particles	Particles are called Bosons	Particles are called Fermions.
* Applicable for any spin.	Particles with Integral spin. 0,1,2	Particles with odd half integral spin. $\frac{1}{2}, \frac{3}{2}$..
* Example - Gas Molecules	Example - Photons, phonons	Example - electron in metals, protons.

$f_{MB}(\varepsilon) = \dfrac{A}{e^{\varepsilon/kT}}$ | $f_{MB}(\varepsilon) = \dfrac{1}{Ae^{\varepsilon/kT} - 1}$ | $f_{MB}(\varepsilon) = \dfrac{1}{Ae^{\varepsilon/kT} + 1}$

For $\boxed{\varepsilon >> KT} \rightarrow$

All Distribution functions have Same Behaviour.

4.) We know that $\rightarrow$

$n(\varepsilon_1) = g(\varepsilon_1) f(\varepsilon_1)$ ———— $n=2$

$n(\varepsilon_2) = g(\varepsilon_2) f(\varepsilon_2)$ ———— $n=1$

But $g(\varepsilon_n) = 2n^2$ $\therefore$ $g(\varepsilon_1) = 2$, $g(\varepsilon_2) = 8$

$n(\varepsilon_1) = 2Ae^{-\varepsilon_1/kT}$, $n(\varepsilon_2) = 8Ae^{-\varepsilon_2/kT}$

$$\therefore \frac{n(\varepsilon_2)}{n(\varepsilon_1)} = 4e^{-(\varepsilon_2 - \varepsilon_1)/kT} \quad \text{------ (1)}$$

$\varepsilon = \dfrac{13.6}{n^2}$, $\varepsilon_1 = -13.6\text{ eV}$, $\varepsilon_2 = \varepsilon_1/4$

$$\frac{n(\varepsilon_2)}{n(\varepsilon_1)} = \frac{1}{1000} \quad \text{—— (2)}$$

from (1) and (2) we have

$$\frac{\varepsilon_2 - \varepsilon_1}{kT} = \ln 4000 \Rightarrow T = \frac{\varepsilon_2 - \varepsilon_1}{k \ln 4000}$$

$$T = \frac{3 \times 13.6}{4 \times 8.62 \times 10^{-5}} \times \frac{1}{\ln 4000}$$

$$\boxed{T = 1.43 \times 10^4 \text{ K}}$$

5.) In Maxwell Boltzmann ways $\Rightarrow$

possible ways $= 2^2 = 4$

$\boxed{a}\ \boxed{b}$, $\boxed{b}\ \boxed{a}$, $\boxed{ab}\ \boxed{}$, $\boxed{}\ \boxed{ab}$

Bose Einstein ways $\rightarrow$

possible ways = 3

$\boxed{a}\ \boxed{a}$, $\boxed{aa}\ \boxed{}$, $\boxed{}\ \boxed{aa}$

Fermi Dirac Ways $\rightarrow$

possible ways = 1 $\quad \boxed{a}\ \boxed{a}$

6.) Electronic configuration of zinc is $[Ar]\,3d^{10}\,4s^2$

Here, we can see that in ground state, zinc has completely filled K, L and M shells and two free electrons in 4s subshell,

Thus there are two electrons per atom.

No. of atoms per unit volume is ratio of density to the mass per atom. Then we get $\rightarrow$

$$E_F = \frac{h^2}{2m}\left(3N/8\pi V\right)^{2/3} = \frac{h^2}{2m}\left(\frac{3\times 2\,\rho_{Zn}}{8\pi m_{Zn}}\right)^{2/3} = \frac{(6.626\times10^{-34})^2}{2\times 0.85\times 9.11\times10^{-31}} \times$$

$$\left[\frac{3(2)(7.13\times10^{3})}{8\pi(65.4)(1.66\times10^{-27})}\right]^{2/3} = 1.78\times10^{-18}\,J$$

$$= \frac{1.78\times10^{-18}}{1.6\times10^{-19}}\,eV = \boxed{11\,eV}$$

7.) we know that →

$$n(v)\,dv = 4\pi N\left[\frac{m}{2\pi kT}\right]^{3/2} v^2 e^{-mv^2/2kT}\,dv$$

At $v = v_p$, $\frac{d}{dv} n(v) = 0.$

we get →

$$v_p = \sqrt{\frac{2kT}{m}}$$

8.) Fermi Dirac Probability Function →

$$f_{FD}(\mathcal{E}) = \frac{1}{e^{(\mathcal{E}-\mathcal{E}_F)/kT} + 1} \quad \text{— (1)}$$

Given: $f_{FD}(\mathcal{E}) = \frac{5}{100} = 0.05$ — (2)

from (1) & (2) →

we get, $e^{(\mathcal{E}-\mathcal{E}_F)/kT} + 1 = 20$

$$\Rightarrow e^{(\mathcal{E}-\mathcal{E}_F)/kT} = 19$$

Taking log on both sides →

$$(\mathcal{E}-\mathcal{E}_F)/kT = \ln 19$$

$$\mathcal{E} - \mathcal{E}_F = kT \ln 19$$

$$\boxed{\mathcal{E} = \mathcal{E}_F + 2.94\,kT}$$