ASSIGNMENT-3
PHYSICS-II

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BATCH: F2

Q1: Draw a diagram that depicts the variation of interatomic force as a function of spacing in terms of its attractive and repulsive components. Derive expression for equilibrium spacing.

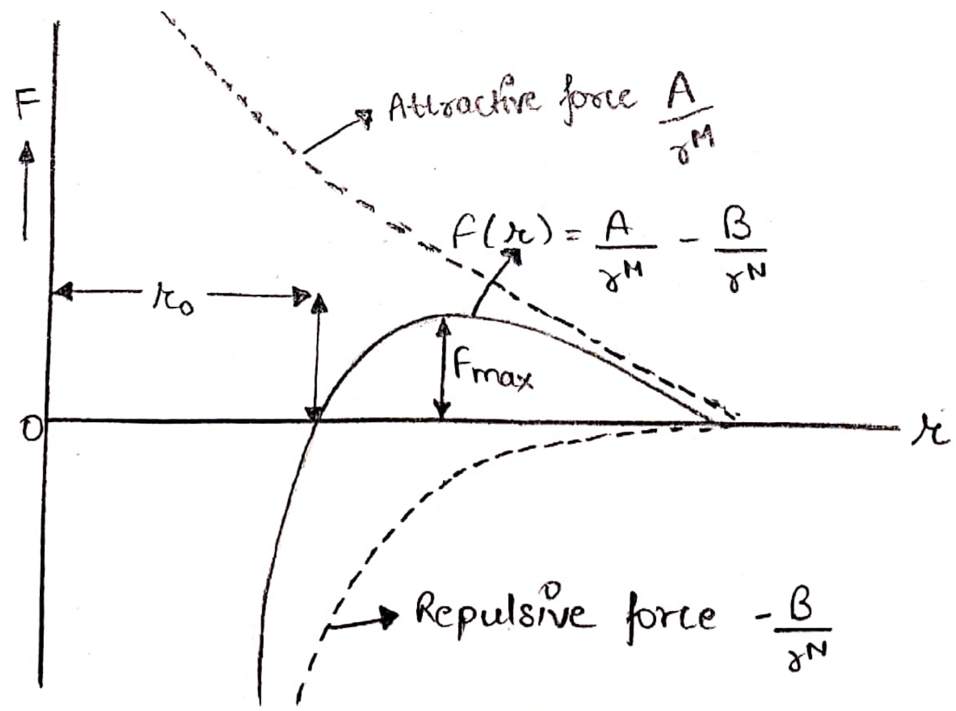
Ans- The equilibrium spacing will occur when the Bond Energy (F_n) is minimum. This is when the net force between the two atoms is zero.

$$\boxed{F_n = F_{at} + F_{r} = 0}$$

where F_{at} = attractive force
F_r = repulsive force

The force between atoms is given as-

$$\boxed{F = \frac{du}{dr}}$$ where u = Bond Energy
r = atomic separation



Suppose two atoms exert attractive and repulsive forces on each other such that bonding force is-

$$\boxed{F(r) = \frac{A}{r^M} - \frac{B}{r^N}}$$

where $N > M$
$r \rightarrow$ centre spacing between atoms

at Equilibrium when $r = r_0$

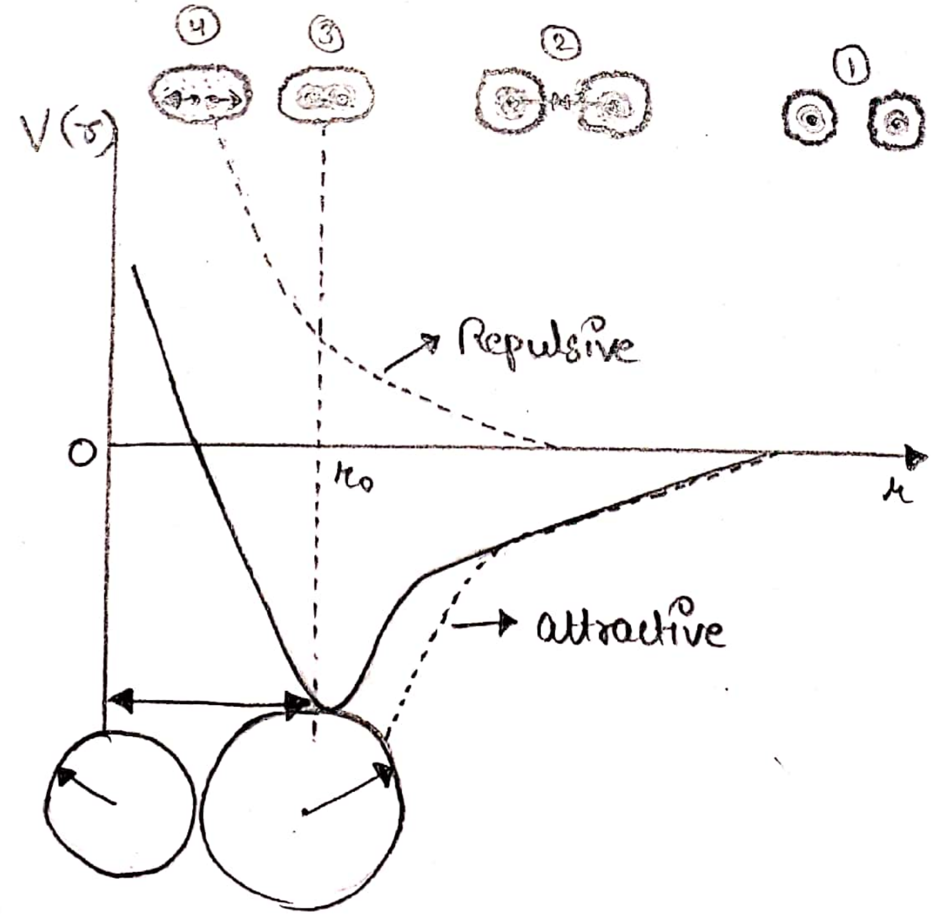
$f(r) = 0$

Thus, $\frac{A}{r^M} - \frac{B}{r^N} = 0 \Rightarrow r_0^{N-M} = \frac{B}{A}$

$$\Rightarrow \quad r_0 = [B/A]^{1/N-M}$$

Q2: Plot the variation of attractive potential energy, repulsive potential energy and resultant potential energy with interatomic distance, when two atoms are brought nearer. Derive the expression for equilibrium spacing of two atoms for which potential energy is minimum and hence obtain the dissociation energy.

Ans -



$$u(r) = -a/r^m + b/r^n$$

at Equilibrium repulsive force becomes equal to attractive force

Let r_0 be the distance between the atoms for this min.(u(r)) to occur-

$u(r=r_0)_{min} \longrightarrow -ve$, $V_0 \rightarrow$ Eq. spacing of system

$u(r)$ is minimum at $r=r_0$

$$\Rightarrow \left[\frac{du}{dr}\right]_{r=r_0} = 0 \Rightarrow \frac{ma}{r_0^{m+1}} - \frac{nb}{r_0^{n+1}} = 0$$

$$\Rightarrow \boxed{r_0 = \left[\frac{nb}{ma}\right]^{1/n-m}}$$

The Energy required to dissociate the two atom of molecule into an infinite separation is called <u>Dissociation Energy</u>.

$$u(r) = -a/r^m + b/r^n$$

$u(r)$ is min at $\underline{r=r_0}$

$$\Rightarrow u(r) = -a/r^m + b/r^n \quad \text{—— (1)}$$

$$\left[\frac{du}{dr}\right]_{r=r_0} = 0$$

$$\Rightarrow r_0^n = r_0^m\left[\frac{bn}{am}\right]$$

Putting value of r_0^n in equation (1)

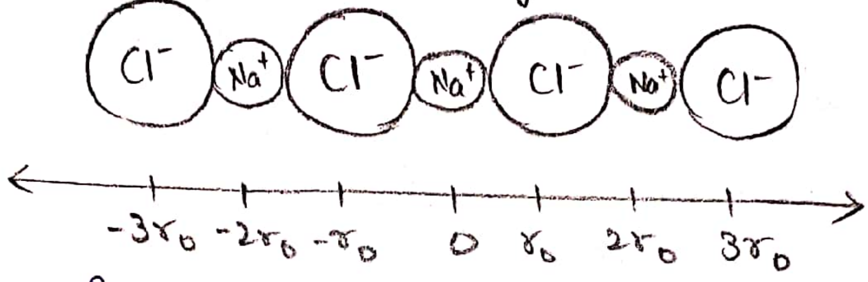
$$\Rightarrow u_{min} = -a/r_0^m + \cancel{b}\left[\frac{a}{\cancel{b}}\right]\left[\frac{m}{n}\right]\frac{1}{r_0^m}$$

$$= +\frac{a}{r_0^m}\left[\frac{m}{n} - 1\right]$$

$$\boxed{u_{min} = -\frac{a}{r_0^m}\left[1 - \frac{m}{n}\right]}$$

Q3: Derive an expression for lattice energy in ionic crystals and prove that the Madelung constant for molecule in NaCl like ionic crystal is $2\ln 2$.

Ans -



Attractive Coulomb Energy due to nearest neighbour

$$\left[\frac{-e^2}{4\pi\varepsilon_0 r_0}\right] + \left[\frac{-e^2}{4\pi\varepsilon_0 r_0}\right] = \frac{-2e^2}{4\pi\varepsilon_0 r_0}$$

Repulsive Energy due to two +ve ions at distance $2r_0$ is

$$= \frac{2e^2}{4\pi\varepsilon_0 (2r_0)}$$

Attractive Coulomb Energy due to next neighbour → at a distance $3r_0$ is → $\dfrac{-2e^2}{4\pi\varepsilon_0 (3r_0)}$

Thus, total Energy due to all ions is →

$$= \frac{-2e^2}{4\pi\varepsilon_0 r_0} + \frac{2e^2}{4\pi\varepsilon_0 (2r_0)} - \frac{2e^2}{4\pi\varepsilon_0 (3r_0)} \cdots\cdots$$

$$= \frac{-2e^2}{4\pi\varepsilon_0 r_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right]$$

$$= -\frac{2e^2}{4\pi\varepsilon_0 r_0}\log(1+1)$$

$$= \frac{-e^2}{4\pi\varepsilon_0 r_0}(2\log 2)$$

Now, $2\log 2$ is Madelung constant per molecule of ionic solid.

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