ASSIGNMENT-1

PHYSICS-II

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BATCH: R2

Q.1: Determine E_{Avg}, E_{rms} and E_{mp} by using molecular energy distribution with energies between E and $(E+dE)$ in a sample of an ideal gas that contains N molecules and whose absolute temperature is T.

Solution: we know that –

$$n(\varepsilon)d\varepsilon = g(\varepsilon)d\varepsilon\, f(\varepsilon)$$
$$= g(\varepsilon)\, A e^{-\varepsilon/kT} d\varepsilon$$

$$\therefore \boxed{n(\varepsilon)\, d\varepsilon = \frac{2\pi N}{(\pi kT)^{3/2}} \sqrt{\varepsilon}\, e^{-\varepsilon/kT} d\varepsilon}$$

Average Energy →

$$(\bar{\varepsilon}) = \frac{\text{Total Energy}}{\text{Total no. of Particles}}$$

$$= \frac{\int_0^\infty \varepsilon\, n(\varepsilon)\, d\varepsilon}{N}$$

$$= \frac{\int_0^\infty \frac{2\pi N \cdot \varepsilon\sqrt{\varepsilon}\, e^{-\varepsilon/kT}}{(\pi kT)^{3/2}}}{N}$$

$$= \frac{2\pi}{(\pi kT)^{3/2}} \int_0^\infty \varepsilon^{3/2} e^{-\varepsilon/kT} d\varepsilon$$

By using Gamma's function →

$$\boxed{\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}}$$

$$\therefore \int_0^{\infty} x^{3/2} e^{-\alpha x}\, dx = \frac{3}{4\alpha^2}\sqrt{\pi/\alpha}$$

$$\Rightarrow \quad \bar{\varepsilon} = \frac{2\pi}{(\pi kT)^{3/2}} \times \frac{3}{4\left(\frac{1}{kT}\right)^2}\sqrt{\pi kT}$$

$$* \quad \boxed{(\bar{\varepsilon}) = \frac{3}{2} kT}$$

<u>Rms</u> <u>value</u> <u>of</u> <u>Energy</u> -

$$\sqrt{(\bar{\varepsilon})^2} = \sqrt{\frac{\int_0^{\infty} \varepsilon^2 n(\varepsilon)\, d\varepsilon}{N}}$$

$$= \sqrt{\frac{1}{\cancel{N}}\int_0^{\infty} \frac{2\pi \cancel{N} \varepsilon^2 \sqrt{\varepsilon} \cdot e^{-\varepsilon/kT}}{(\pi kT)^{3/2}}\, d\varepsilon}$$

$$= \sqrt{\int_0^{\infty} \frac{2\pi \varepsilon^{3/2} e^{-\varepsilon/kT}}{(\pi kT)^{3/2}}\, d\varepsilon}$$

Now using Gamma's formula.

$$\int_0^{\infty} x^{n} e^{-\alpha x}\, dx = \frac{n!}{\alpha^{n+1}}$$

$$\sqrt{(\bar{\varepsilon})^2} = \sqrt{\frac{15}{4}(kT)^2}$$

$$* \quad \boxed{\sqrt{(\bar{\varepsilon})^2} = \sqrt{15}\,\frac{kT}{2}.}$$

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Most Probable Energy →

for obtaining the expression for most probable Energy

put $\frac{dn(\mathcal{E})}{d\mathcal{E}} = 0$

Hence, we get →

$$\frac{2\pi N}{(\pi kT)^{3/2}}\left[\frac{1}{2}\mathcal{E}^{-1/2}e^{-\mathcal{E}/kT} + \mathcal{E}^{1/2}\left(\frac{-1}{kT}\right)e^{-\mathcal{E}/kT}\right] = 0.$$

$$\Rightarrow \mathcal{E}^{1/2}e^{-\mathcal{E}/kT}\left[\frac{1}{2\mathcal{E}} - \frac{1}{kT}\right] = 0.$$

$$\therefore \frac{\mathcal{E}}{kT} = \frac{1}{2}$$

$$\Rightarrow * \boxed{\mathcal{E} = \frac{1}{2}kT}$$

Q.2: Write down the number of particles with velocities v and $v+dv$ from molecular energy distribution and calculate ratio between $E_{avg} : E_{rms} : E_{mp}$.

Solution- We know that $n(v)d(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv$

Since $E_{avg} = \frac{3}{2}kT$, $E_{rms} = \frac{\sqrt{15}}{2}kT$; $E_{mp} = \frac{1}{2}kT$

$$\therefore E_{avg} : E_{rms} : E_{mp} = \frac{3}{\cancel{2}}\cancel{kT} : \frac{\sqrt{15}}{\cancel{2}}\cancel{kT} : \frac{1}{\cancel{2}}\cancel{kT}$$

$$= \boxed{3 : \sqrt{15} : 1} *$$

Also, $n(\mathcal{E})d\mathcal{E} = \frac{2\pi N}{(\pi kT)^{3/2}}\sqrt{\mathcal{E}}\, e^{-\mathcal{E}/kT} d\mathcal{E}$

Now $\mathcal{E} = \frac{1}{2}mv^2$

$$\therefore d\varepsilon = mv\,dv$$

$$\therefore n(v)\,dv = \frac{2\pi N}{(\pi KT)^{3/2}} \sqrt{\frac{1}{2}mv^2}\, e^{-\frac{1}{2}\times\frac{mv^2}{KT}}\, mv\,dv$$

$$* \boxed{n(v)\,dv = \frac{4\pi N}{(2\pi KT)^{3/2}} m^{3/2} v^2 e^{-\frac{1}{2}\cdot\frac{mv^2}{KT}} dv}$$

Q.3: Prove average energy of a free electron gas at T=0 is 3/5 of Fermi energy.

Solution- we know, in Fermi Dirac Energy Distribution →

$$n(\varepsilon)\,d\varepsilon = g(\varepsilon) f_{FD}(\varepsilon)\,d\varepsilon$$

Also, $g(\varepsilon) = \frac{3N}{2} \varepsilon_F^{-3/2} \varepsilon^{1/2}$

$$n(\varepsilon)\,d\varepsilon = \frac{3N}{2} \frac{\varepsilon_F^{-3/2}\varepsilon^{1/2}}{e^{(\varepsilon-\varepsilon_F)/KT}+1} d\varepsilon$$

Average Energy, $\bar{\varepsilon} = \frac{\int_0^\infty \varepsilon n(\varepsilon)\,d\varepsilon}{}$

$$\bar{\varepsilon}_0 = 3\varepsilon_F^{-3/2} \int_0^{\varepsilon_F} \frac{\varepsilon^{3/2}}{e^{(\varepsilon-\varepsilon_F)/KT}+1} d\varepsilon \quad N$$

at T=0 : $e^{(\varepsilon-\varepsilon_F)/KT} = e^{-\infty} = 0$

Hence $* \boxed{\bar{\varepsilon}_0 = \frac{3}{5}\varepsilon_F}$

Thus, average Energy of a free electron gas at T=0 is 3/5 of Fermi Energy.

NOTE: * marked are the results.