

# Time Complexity 1

## Arithmetic Progression

An arithmetic progression or arithmetic sequence is a **sequence** of **numbers** such that the difference between the consecutive terms is constant.

For instance, the sequence 5, 7, 9, 11, 13, 15, . . . is an arithmetic progression with a common difference of 2.

If the initial term of an arithmetic progression is  $a_1$  and the common difference of successive members is  $d$ , then the  $n$ -th term of the sequence  $a_n$  is given by:

$$a_n = a_1 + (n - 1)d$$

Sum of AP series is given by -

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d].$$

### Derivation of Sum Formula:

To derive the above formula, begin by expressing the arithmetic series in two different ways:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n - 2)d) + (a_1 + (n - 1)d)$$

$$S_n = (a_n - (n - 1)d) + (a_n - (n - 2)d) + \cdots + (a_n - 2d) + (a_n - d) + a_n.$$

Adding both sides of the two equations, all terms involving  $d$  cancel:

$$2S_n = n(a_1 + a_n).$$

Dividing both sides by 2 produces a common form of the equation:

$$S_n = \frac{n}{2}(a_1 + a_n).$$

An alternate form results from re-inserting the substitution:  $a_n = a_1 + (n - 1)d$

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d].$$

Reference: [https://en.wikipedia.org/wiki/Arithmetic\\_progression](https://en.wikipedia.org/wiki/Arithmetic_progression)

## Geometric Progression -

A geometric progression, also known as a geometric sequence, is a [sequence](#) of non-zero [numbers](#) where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the *common ratio*.

For example, the sequence 2, 6, 18, 54, ... is a geometric progression with common ratio 3. Similarly 10, 5, 2.5, 1.25, ... is a geometric sequence with a common ratio 1/2.

The general form of a geometric sequence is  $a, ar, ar^2, ar^3, ar^4, \dots$  where  $r \neq 0$  is the common ratio and  $a \neq 0$  is a [scale factor](#), equal to the sequence's start value.

The  $n$ -th term of a geometric sequence with initial value  $a = a_1$  and common ratio  $r$  is given by

$$a_n = ar^{n-1}.$$

Reference: [https://en.wikipedia.org/wiki/Geometric\\_progression](https://en.wikipedia.org/wiki/Geometric_progression)