Data Structure and Algorithm Laboratory

Group D

By Ms.Monika Sharad Deshmukh

Problem Statement

Given sequence $k = k1 < \ldots < kn$ of n sorted keys, with a search probability pi for each key ki . Build the Binary search tree that has the least search cost given the access probability for each key

Optimal Binary Search Tree

An optimal binary search tree, sometimes called a weight-balanced binary tree, is a binary search tree which provides the smallest possible search time that is minimum cost for a given sequence of accesses (or access probabilities)

Example

```
Input:  keys[] = {10, 12, 20}, freq[] = {34, 8, 50}
There can be following possible BSTs
    10                12                   20         10              20
      \             /    \                /             \            /
      12          10     20             12               20        10
        \                              /                 /           \
         20                          10                12             12
     I               II                III             IV             V
Among all possible BSTs, cost of the fifth BST is minimum.
Cost of the fifth BST is 1*50 + 2*34 + 3*8 = 142
```

Solved Example

	0	1	2	3	4
Key		10	20	30	40
Pi		3	3	1	1
qi	2	3	1	1	1

- $w(i, j) = w(i, j-1) + pj + qj$
- $C(i, j) = \min_{i<k<=j} \{c(i,k-1)+c(k,j)\} + w(i,j)$

	0	1	2	3	4
J-i=0	W_{00}= C_{00}= r_{00}=	W_{11}= C_{11}= r_{11}=	W_{22}= C_{22}= r_{22}=	W_{33}= C_{33}= r_{33}=	W_{44}= C_{44}= r_{44}=
J-i=1	W_{01}= C_{01}= r_{01}=	W_{12}= C_{12}= r_{12}=	W_{23}= C_{23}= r_{23}=	W_{34}= C_{34}= r_{34}=	
J-i=2	W_{02}= C_{02}= r_{02}=	W_{13}= C_{13}= r_{13}=	W_{24}= C_{24}= r_{24}=		
J-i=3	W_{03}= C_{03}= r_{03}=	W_{14}= C_{14}= r_{14}=			
J-i=4	W_{04}= C_{04}= r_{04}=				

- Initial step:

  $w(0,0)=q0= 2$ $w(1,1)= q1=3$
  $w(2,2)=q2= 1$ $w(3,3)= q3=1$
  $w(4,4) =q4=1$

And
$c(0,0)=c(1,1)=c(2,2)= c(3,3)= c(4,4)=0$
$r(0,0)=r(1,1)=r(2,2)= r(3,3)= r(4,4)=0$

	0	1	2	3	4
J-i=0	W_{00}= 2 C_{00}= 0 r_{00}= 0	W_{11}= 3 C_{11}= 0 r_{11}= 0	W_{22}= 1 C_{22}= 0 r_{22}= 0	W_{33}= 1 C_{33}= 0 r_{33}= 0	W_{44}= 1 C_{44}=0 r_{44}= 0
J-i=1	W_{01}= C_{01}= r_{01}=	W_{12}= C_{12}= r_{12}=	W_{23}= C_{23}= r_{23}=	W_{34}= C_{34}= r_{34}=	
J-i=2	W_{02}= C_{02}= r_{02}=	W_{13}= C_{13}= r_{13}=	W_{24}= C_{24}= r_{24}=		
J-i=3	W_{03}= C_{03}= r_{03}=	W_{14}= C_{14}= r_{14}=			
J-i=4	W_{04}= C_{04}= r_{04}=				

- For second step

w(i , j) = w(i,j-1)+ pj + qj

So,w(0,1) = w(0,0) + p1 +q1

=2+3+3=8

Similarly, w(1,2) = 7 w(2,3) =3 w(3,4)=3

w(0,2)= 12 w(1,3)=9 w(2,4)=5

w(0,3)=14 w(1,4)=11

w(0,4)=16

	0	**1**	**2**	**3**	**4**
Key		10	20	30	40
Pi		3	3	1	1
qi	2	3	1	1	1

	0	1	2	3	4
J-i=0	W_{00}= 2 C_{00}= 0 r_{00}= 0	W_{11}= 3 C_{11}= 0 r_{11}= 0	W_{22}= 1 C_{22}= 0 r_{22}= 0	W_{33}= 1 C_{33}= 0 r_{33}= 0	W_{44}= 1 C_{44}=0 r_{44}= 0
J-i=1	W_{01}=8 C_{01}= r_{01}=	W_{12}=7 C_{12}= r_{12}=	W_{23}=3 C_{23}= r_{23}=	W_{34}=3 C_{34}= r_{34}=	
J-i=2	W_{02}=12 C_{02}= r_{02}=	W_{13}=9 C_{13}= r_{13}=	W_{24}=5 C_{24}= r_{24}=		
J-i=3	W_{03}=14 C_{03}= r_{03}=	W_{14}=11 C_{14}= r_{14}=			
J-i=4	W_{04}=16 C_{04}= r_{04}=				

- $C(i\ ,\ j) = \min_{i<k<=j} \{c(i,k-1)+c(k,j)\} + w(i,j)$
- $C(0,1)=\min_{0<k<=1} \{c(0,1-1)+c(1,1)\} + w(0,1)$

Here K can be 1 only

Therefore

- $C(0,1)=\{c(0,0)+c(1,1)\} + w(0,1)$

  $=0+0+8$

  $=8$

Similarly

$C(1,2)=7 \quad c(2,3)=3 \quad c(3,4)=3$ and

As k is having only 1 value equal to j

$r(0,1)= 1 \quad r(1,2)=2 \quad r(2,3)=3 \quad r(3,4)=4$

	0	1	2	3	4
J-i=0	W_{00}= 2 C_{00}= 0 r_{00}= 0	W_{11}= 3 C_{11}= 0 r_{11}= 0	W_{22}= 1 C_{22}= 0 r_{22}= 0	W_{33}= 1 C_{33}= 0 r_{33}= 0	W_{44}= 1 C_{44}=0 r_{44}= 0
J-i=1	W_{01}=8 C_{01}=8 r_{01}=1	W_{12}=7 C_{12}=7 r_{12}=2	W_{23}=3 C_{23}=3 r_{23}=3	W_{34}=3 C_{34}=3 r_{34}=4	
J-i=2	W_{02}=12 C_{02}= r_{02}=	W_{13}=9 C_{13}= r_{13}=	W_{24}=5 C_{24}= r_{24}=		
J-i=3	W_{03}=14 C_{03}= r_{03}=	W_{14}=11 C_{14}= r_{14}=			
J-i=4	W_{04}=16 C_{04}= r_{04}=				

- $C(i\ ,\ j) = \min\limits_{i<k<=j} \{c(i,k-1)+c(k,j)\} + w(i,j)$
- $C(0\ ,\ 2) = \min\limits_{0<k<=2} \{c(0,k-1)+c(k,2)\} + w(0,2)$

Here K can have values 1 or 2 so,

C(0 , 2)

$=\min \{c(0,1-1)+c(1,2),\ c(0,2-1)+c(2,2)\} + w(0,2)$

$=\min\{c(0,0)+c(1,2),c(0,1)+c(2,2)\}+ w(0,2)$

$=\min\{\underline{0+7},8+0\}+12$

$=\min\{\underline{7},8\}+12$

=7+12 (min value is by k=1 so, r(0,2)=1)

=19

so c(0,2) = 19 and r(0,2)=1

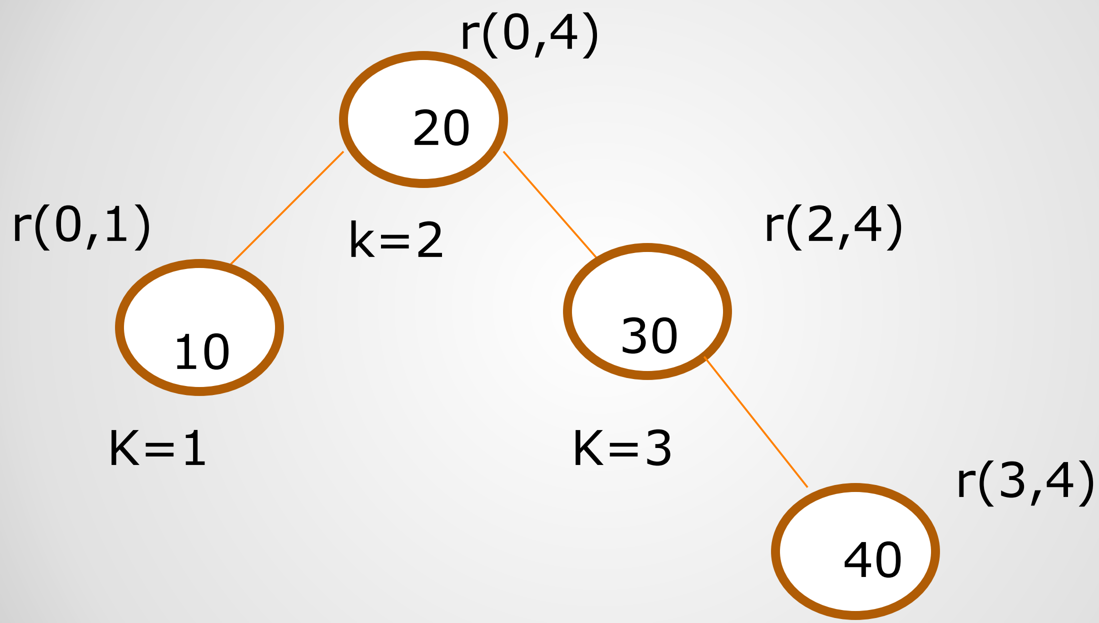
- Similarly using same formula we can find

$C(1,3)=12$	$r(1,3)=2$
$C(2,4)= 8$	$r(2,4)=3$
$C(0,3)= 25$	$r(0,3)=2$
$C(1,4)=19$	$r(1,4)=2$
$C(0,4)= 3$	**$r(0,4)=2$**

r(i,j) value is the k value which gives minimum cost

	0	1	2	3	4
J-i=0	W_{00}= 2 C_{00}= 0 r_{00}= 0	W_{11}= 3 C_{11}= 0 r_{11}= 0	W_{22}= 1 C_{22}= 0 r_{22}= 0	W_{33}= 1 C_{33}= 0 r_{33}= 0	W_{44}= 1 C_{44}=0 r_{44}= 0
J-i=1	W_{01}=8 C_{01}=8 r_{01}=1	W_{12}=7 C_{12}=7 r_{12}=2	W_{23}=3 C_{23}=3 r_{23}=3	W_{34}=3 C_{34}=3 r_{34}=4	
J-i=2	W_{02}=12 C_{02}=19 r_{02}=1	W_{13}=9 C_{13}=12 r_{13}=2	W_{24}=5 C_{24}=8 r_{24}=3		
J-i=3	W_{03}=14 C_{03}=25 r_{03}=2	W_{14}=11 C_{14}=19 r_{14}=2			
J-i=4	W_{04}=16 C_{04}=32 r_{04}=2				

- Now we can easily build an OBST using table



Thank you