D.Y. Patil College of Engineering & Technology
Kasaba Bawada, Kolhapur 416006
(An Autonomous Institute)

Academic Year: 2023-24 **Sem:** I **Class:** F. Y .B. Tech.
Course: Linear Algebra and Calculus **Course Code:** 231FYL101

Tutorial-IX

Differential Calculus-II: Euler's theorem on homogeneous functions

Course Outcomes (COs): After successful completion of this tutorial, the students will be able to:	
101.5	Apply the knowledge of partial differentiation.

Q. **Solve the following questions**

1. If $u = \frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}$
2. If $u = \sin^{-1}\left[\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right]$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$
3. If $u = \frac{xy}{x+y}$, Prove that $x^2\frac{\partial^2 u}{\partial^2 x} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial^2 y} = 0$
4. If $u = x\sin^{-1}\left(\frac{y}{x}\right) + y\tan^{-1}\left(\frac{y}{x}\right)$, Prove that $x^2\frac{\partial^2 u}{\partial^2 x} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial^2 y} = 0$
5. If $u = \sin^{-1}\left[\frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}\right]^{\frac{1}{2}}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{12}\tan u$
6. If $u = log\left[\frac{x^3+y^3}{x^2+y^2}\right]$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$
7. If $u = \sin^{-1}[x^2 + y^2]^{\frac{1}{5}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2}{5}\tan u$
8. If $u = \log[x^3 + y^3 - x^2y - xy^2]$, prove that $x^2\frac{\partial^2 u}{\partial^2 x} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial^2 y} = -3$
9. If $u = \tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$, Prove that $x^2\frac{\partial^2 u}{\partial^2 x} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial^2 y} = \sin 2u(1 - 4sin^2 u)$
10. If $u = \sin^{-1}\left[\frac{\sqrt{x^2+y^2}}{x+y}\right]$,Prove that $x^2\frac{\partial^2 u}{\partial^2 x} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial^2 y} = 0$

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Tutorial-VIII

Differential Calculus-I: Indeterminate forms and L'Hospital's rule

Course Outcomes (COs): After successful completion of this tutorial, the students will be able to:	
101.4	Apply Taylor theorem to find the expansion of functions and identify the indeterminate forms

Q. Evaluate the following limits

1. $\lim_{x\to 0} \frac{\log(1-x^2)}{\log cosx}$ **Ans.:** $\boldsymbol{L = 2}$

2. $\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$ **Ans.:** $\boldsymbol{L = \frac{3}{2}}$

3. $\lim_{x\to \frac{\pi}{2}} \frac{\log(x-\frac{\pi}{2})}{\tan x}$ **Ans.:** $\boldsymbol{L = 0}$

4. $\lim_{x\to 0} \frac{\log \tan 2x}{\log tanx}$ **Ans.:** $\boldsymbol{L = 1}$

5. $\lim_{x\to 2} \left[\frac{1}{x-2} - \frac{1}{\log(x-1)}\right]$ **Ans.:** $\boldsymbol{L = \frac{1}{2}}$

6. $\lim_{x\to 3} \left[\frac{1}{x-3} - \frac{1}{\log(x-2)}\right]$ **Ans.:** $\boldsymbol{L = \frac{-1}{2}}$

7. $\lim_{x\to 1} (1-x^2)^{\frac{1}{\log(1-x)}}$ **Ans.:** $\boldsymbol{L = e}$

8. $\lim_{x\to 0} (a^x + x)^{\frac{1}{x}}$ **Ans.:** $\boldsymbol{L = ae}$

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Tutorial-X

Differential Calculus-II: Partial derivatives, Jacobian and its properties

Course Outcomes (COs): After successful completion of this tutorial, the students will be able to:	
101.5	Apply the knowledge of partial differentiation.

Q. **Solve the following questions**

1. If $z = x^3 + xy + y^3$, then evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$
2. If $u = (1 - 2xy + y^2)^{\frac{-1}{2}}$, Prove that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$
3. If $z = y^2e^x + x^2y^3 + 1$, Prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$
4. If $u = y^2 log(x^2 + y^2)$, Find $\frac{\partial^2 u}{\partial x \partial y}$ $at\ x = 2, y = 1$
5. If $u = xsiny,\ v = ysinx$, then find $\frac{\partial(u,v)}{\partial(x,y)}$
6. If $u = x^2 - y^2,\ v = 2xy$, Calculate $\frac{\partial(u,v)}{\partial(x,y)}$
7. If $u = x^2,\ v = y^2$, Calculate $\frac{\partial(u,v)}{\partial(x,y)}$
8. If $x = u^2 - v^2,\ y = u^2 + v^2$, Find $\frac{\partial(x,y)}{\partial(u,v)}$
9. If $x = u(1 + v),\ y = v(1 + u)$, Find $\frac{\partial(x,y)}{\partial(u,v)}$
10. If $x = uv,\ y = \frac{u+v}{u-v}$, Calculate $\frac{\partial(x,y)}{\partial(u,v)}$